Illustrating the Standards for Mathematical Practice: Similarity, Slope & Graphs of Linear Functions
FACILITATOR'S AGENDA

Session 2: Similarity, Slope & Graphs of Linear Functions 2 hours

(NOTE: It would be best to complete Session 1: Congruence and Similarity through Transformations prior to engaging in Session 2: Similarity, Slope, and Graphs of Linear Functions.)

Session Goals:

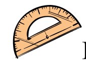
- To explore the Standards for Content and Practice through video of classroom practice
- To unpack the connection between similarity, slope & graphs of linear functions
- To examine the use of viable arguments, precise language, and geometric structure
- To consider how the Standards are likely to impact your mathematics program and to plan next steps

What is the point of this session?
For participants to gain in their understanding of the connection between similarity and graphs of linear functions as explicated in the CCSS 8th grade content standards, as well as to understand what three of the mathematical practices (viable arguments, precise language, and geometric structure) mean in practice

**The Notes section of each session agenda includes guidance and notes to support you in facilitating the activities. Icons are used to denote particular kinds of support. Indicates mathematics content information and support.*

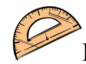
Time	Materials	Activity	Notes
10 min	PPT slides 1-6 Standards for Mathematical Practice Handout	Introduction & Goals ○ Common Core State Standards – include standards for content & standards for practice (slides 1 & 2) ○ Goals for the Session (slide 3) ○ Standards for Mathematical Practice (slide 4) ○ Standards for Mathematical Content (slide 5) ○ Intro to the body of the session: Similarity, Slope & Linearity (slide 6)	Welcome the group and introduce the session, noting that it is situated in both the content and practice standards included in the CCSS (slide 2). Share the goals of the session (slide 3) and explain that the session will explore both types of standards through video of classroom practice. If the participants do not already know one another, also take this opportunity for everyone to introduce himself or herself. Distribute Standards for Mathematical Practice Handout. Review the on the Standards for Mathematical Practice (slide 4), and ask if the participants have questions. This session will highlight standards 3, 5

			& 6. This session highlights some of the 8th grade Standards for Mathematical Content (slide 5). ○ Understand that a two-dimensional figure is **similar** to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. ○ Understand the connections between proportional relationships, lines, and linear equations. ○ Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane. Intro to the body of the session: Similarity, Slope & Linearity (slide 6). This module is based in the premise that understanding similarity, particularly in a dynamic way, is a useful foundation for understanding slope and graphs of linear functions.
20 min	PPT Slides 7-8 Graphing Similar Rectangles Problem Graph paper, rulers Colored pencils	<u>**Graphing Similar Rectangles Problem**</u> Task: (slide 7) ➢ Create four similar rectangles. List the dimensions for each rectangle. ➢ Plot the four rectangles on a coordinate graph by aligning the bottom left vertex of each rectangle with the origin (0,0), and orienting the rectangles in the same direction, thus nesting the rectangles. ➢ Write the equation for the line that passes through the origin and the upper right vertices of each rectangle. Draw the line. Discussion Prompts: (slide 8) ➢ Share multiple methods for creating similar rectangles, along with the definitions of similarity that guided each of these methods. ➢ Describe how your set of similar rectangles is a representation of dilation. - What is your definition of dilation? - What is the center of dilation? On your graph, what point acts as the center of dilation? - What is a dilation line? On your graph, where are some lines of dilation?	<u>Working on the Task</u> Distribute the Graphing Similar Rectangles Problem (slide 7). Have teachers work on the task individually or in small groups prior to discussing it as a whole group using the focus questions on slide 8. Have graph paper, rulers and colored pencils available. Try not to allot more than 5-10 minutes for teachers to work on the task, so that there are at least 10 minutes for discussion. Note that teachers should align one set of corresponding sides of the rectangles with the X-axis and another set of corresponding sides of the rectangles with the Y-axis. For example, for a non-square rectangle, they might align the rectangles so that the pair of “long sides” are always on or parallel to the X-axes and the “short sides” are on or parallel to the Y-axes, or vice versa. What’s important is that the orientation is consistent for all of their rectangles. <u>Sharing multiple methods</u> Ask teachers to talk about what criterion they used to determine whether the rectangles were similar. Try to elicit a variety of ways they define similarity, such as looking at ratios of side lengths or a scale factor. Encourage teachers to challenge themselves here by considering the use of different definitions.

		➢ What are some properties of the line you drew through the upper right vertices in relation to your set of similar rectangles?	 <u>Definitions</u> Some possible working definitions of **dilation** that teachers might generate are: - A way to produce an image that is the same shape as the original, but not the same size. - A method of stretching or shrinking a figure so that the shape is preserved. - Every point and its corresponding point are on a line that passes through the center of dilation with the corresponding points X times farther away from the center of dilation than the original point. Some possible working definitions of **center of dilation** that teachers might generate are: - The point where scaling begins. - The origin of scaling. - The point where you start dilating from. - A point that is x units from every point on an original figure and x times a scale factor units from every point on a dilated version of that figure. Some possible working definitions of **lines of dilation** that teachers might generate are: - Lines used to create dilations - Lines connecting corresponding points of similar figures if the similar figures are oriented in particular ways - A line through the center of dilation connecting points in a figure A with corresponding points in the image figure A. Some **properties of the line** connecting the lower left and upper right vertices of all the rectangles that teachers might notice are: - The line passes through the diagonals of all the rectangles - The line is a line of dilation – one of 3 shown originating from the origin & going through the corresponding vertices. -The slope of the line segment connecting the lower left vertex and the upper right vertex is the same for all rectangles that are similar to the ones pictured (and oriented and nested in the same way). -- The line cuts each rectangle into two right triangles - The slope of the line is equal to the ratio of the vertical leg to the horizontal leg of any of the right triangles formed by the line

			through the rectangles
35 min	PPT Slide 9-12 Lesson graph Transcript Video clip Tracing Paper	**Preparing for Video Clips** ○ Intro to Watching Video & Norms (slide 9) ○ Intro to Lesson Graphs & Transcripts (slide 10) ○ Intro to the Video Clip (slide 11) **Watch & Discuss the Video Clip (slide 12)** ○ Unpacking Brian & Macy's rectangles -How did Brian's group generate their set of rectangles? -Are Brian's group's rectangles dilations of each other? How do you know? -Are Macy's group's rectangles dilations of each other? How do you know? -Why must the line that connects the upper right hand vertices pass through the origin?	Norms for Watching Video (slide 9): It is important to briefly discuss the purpose of watching and analyzing video cases and to set norms for watching video. Throughout the discussions that follow, facilitators might need to remind participants of these norms, if the comments become judgmental. Intro to Lesson Graphs (slide 10): Distribute the Lesson Graph and explain that a lesson graphs is a 1-page overview of a lesson. The portion of the page devoted to each lesson activity is (roughly) proportional to the percentage of time that lesson activity took within the lesson. The grey segments represent independent/small group work by students. The white segments represent whole class work. The yellow highlighted section indicates what is happening in the clip that will be viewed during this session. Give teachers time to look at the lesson graph *prior* to watching each clip, to get a sense of the entire lesson and what came before and after the selected segment. After viewing the clip, the lesson graph can serve as a reference and/or evidence during teachers' analysis of what happened in the clip. Intro to the Video Clip (slide 11) Note that this clip comes from an 8th grade lesson, filmed in the fall. The students are working on the same problem we just worked on. They do have some prior experience with dilation and similar figures. Distribute the transcript. Explain that a transcript is provided because it can be quite useful during the discussion of the clip. Encourage teachers to turn to the transcript for evidence of what really did (and did not) happen. Watch & Discuss the Video Clip (slide 12). Teachers should watch the clip and then discuss the questions on slide 12. There are many possible structures to use for discussing the video clip. For example,

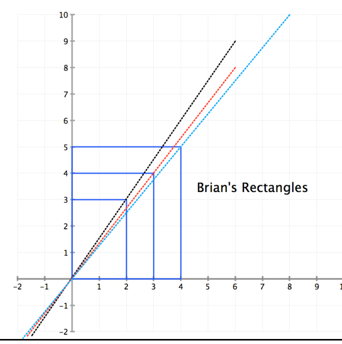
teachers could think/write about the clip individually, share with partner/small group, then discuss as whole group (think, pair, share). You might ask them to consider/discuss one guiding question at a time, or all of the questions, or divide the questions among groups of teachers. These decisions will depend on time & other factors. The session is paced assuming that there will only be time to watch the clip once.

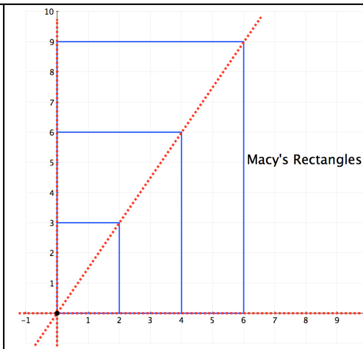


<u>Brian's Group</u>

Brian's group generated their set of rectangles by drawing rectangles on the graph, "going up by one each time", creating rectangles: 2x3, 3x4, 4x5, 5x6.

The rectangles do not share the same diagonals, connecting the top right and bottom left vertices, when plotted as nested rectangles with bottom left vertices situated at the origin (0,0). In contrast, Macy's rectangles do share this diagonal.





Brian’s rectangles are not dilation images of each other because there is not a fixed scale factor, so one of Brian’s rectangles is not “stretched” the same amount (multiplied by the same scale factor) in each direction to get the other rectangles Brian’s group drew.

<u>Macy’s group</u>
Macy’s rectangles can be understood as dilation images of each other. As can be seen in the diagram above of Macy’s rectangles, teachers might notice that the rectangles “grow proportionally”— if you pull them along the diagonal, the length and width grow at the same rate.

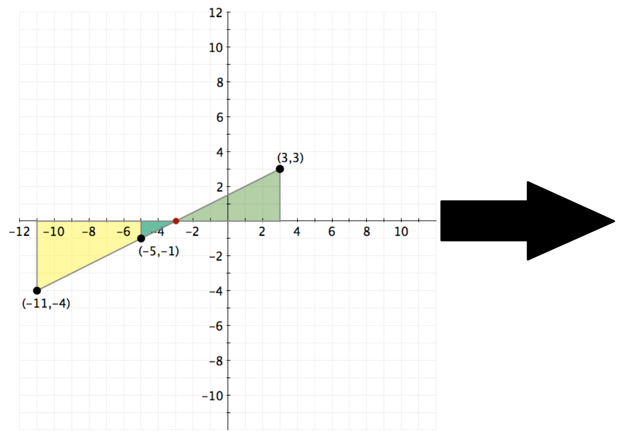
<u>Why the line must pass through the origin</u>
For nested figures that share a common vertex and are oriented the same way, the common vertex becomes a center of dilation----**if** the figures are similar. Both Brian & Macy’s groups have chosen to locate their sets of nested rectangles with the bottom left vertices on the origin. Therefore, the lines of dilation must all originate from the origin.

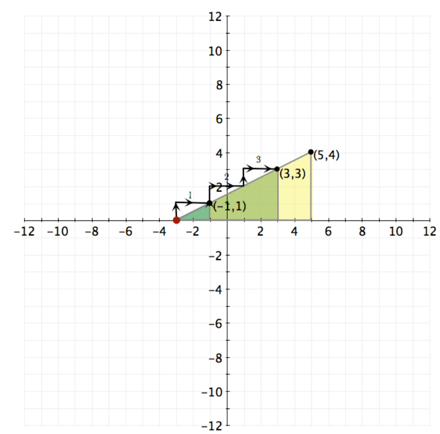
It is also possible to position the rectangles and center of dilation elsewhere on the coordinate axes. One could nest the rectangles so they share a common vertex that is not the origin. Or, one could place the rectangles in different arrangement where all of their corresponding vertices align, in which case the center of dilation may not be on any of the figures.

40 min	PPT Slides 13-14 Similar Triangles Problem Handout Graph paper, tracing paper, rulers	**<u>Similar Triangles Problem</u>** Task (slide 13): • Graph the linear function $f(x)=\frac{1}{2}x+1.5$ • Plot the following points on a coordinate graph: (-11, -4), (-5, -1), (3,3), and (-3,0) • Draw a line segment from each of the plotted points to the *x*-axis that is perpendicular with the x-axis. You should have created three right triangles. • Make two different viable arguments that the three right triangles are similar: -One viable argument using ratios -One viable argument using rotation and dilation Discussion questions: (slide 14) ➢ Thinking about your ratio arguments: - Are you looking at ratios within or between figures? - What connections can you make between the ratios and slopes of the hypotenuses of the right triangles? ➢ Thinking about your rotation & dilation arguments: - Where are the lines of dilation? - What point acts as the center of dilation in the diagram? - What connections can you make between similarity and slope?	Distribute the Similar Rectangles Problem, along with graph paper, tracing paper & rulers. Have teachers work on and discuss the problem in small groups and/or as a whole group. Have teachers compare their methods and discuss different ways to argue that the triangles are similar. Try not to allot more than 20 minutes for teachers to work on the task, so that there are at least 20 minutes for discussion. **Discussing Ratio Arguments:** - <u>"Between" Ratio Argument:</u> Using ratios of lengths *between* triangles, we can consider the ratios of corresponding sides, also called the scale factor. The line connecting the three points crosses the X axis at the point (-3, 0). So, the side lengths of the triangles can be determined by considering the distance from each vertex to (-3,0). The legs of the smallest triangle are 1x2, the legs of the second smallest triangle are 3x6, and the legs of the largest triangle are 4x8. In comparison to the 1x2 triangle, the 3x6 is scaled by a factor of 3 and the 4x8 is scaled by a factor of 4. Each triangle is scaled proportionally from the 1x2 so that any length in one triangle is related to the corresponding length in another triangle by the same scale factor. - <u>"Within" Ratio Argument:</u> Using ratios of lengths *within* triangles. The ratio of the legs on either side of the right angle in the three triangles are 1: 2, 4:8, and 3:6, which are all equivalent. - <u>Connections between ratios & slope:</u> If the triangles are nested as pictured below, the hypotenuses of the 3 triangles lie along the same line (thus with the same slope). The slope of these shared hypotenuses reflects the fact that for every unit increase in the y dimension, there is a 2 unit increase in the x dimension, and this is true *for all triangles similar to the ones pictured,* which would all have the equivalent ratios of their vertical and horizontal legs. Put another way, applying the "within ratio" relationship, the legs of each of the three right triangles are in a 1:2 ratio (i.e. 1:2, 3:6, 4:8). Therefore, the slope of the hypotenuse = the within ratio.

Discussing Rotation and Dilation Arguments

- <u>Transformations Argument:</u> A series of rotations can be applied to the triangles around the center of rotation (-3,0).to stack the triangles so that corresponding sides are oriented the same way with respect to the coordinate grid. Now we can see the triangles as dilation images of each other. For example, the smaller 1x2 triangle could be considered the original figure and the two larger figures show dilated images of that triangle (also see diagram below).





			- <u>Connections between similarity and slope:</u> Dilating a right triangle by various scale factors creates an infinite set of similar right triangles, and for that set of similar right triangles, the ratio of one leg to the other is constant. The ratio of the legs (vertical leg: horizontal leg) represents the slope of the hypotenuse of all those triangles. - In addition, slope can be a useful tool for locating corresponding points of dilated images. Lines of dilation connect the center of dilation with sets of corresponding points on similar figures. Each line of dilation has a unique slope that specifies a rate (i.e., a change in the vertical direction related to a change in the horizontal direction) at which the line moves across the coordinate axes. Corresponding points of dilated images can be located along this line.
5 min	PPT Slide 15	Session Summary (slide 15) Animation of the Similar Triangles Problem *What do you notice about the geometric structure of the set of similar right triangles?*	The animation of the Similar Triangles Problem on slide 15 is intended to provide a dynamic, visual image of dilation for teachers. In particular, the animation shows that there are an infinite number of similar triangles that are dilation images. These similar triangles share a common center of dilation. They also share a line of dilation that extends from the center and follows along their hypotenuses. Because they are right triangles, the slope of this line of dilation (1/2) is equivalent to the ratio of side lengths of the legs within each triangle. So, for the infinite set of similar right triangles that can be created by dilating the original triangle by different scale factors, the ratio of those side lengths will remain the same. The original sides would each be multiplied by that scale factor (*s*), producing new sides of 1s and 2s, and the ratio of these new sides of 1s:2s is equivalent to the original ratio of 1:2. Possible facilitator probes around the animation: • Why is the scale factor changing?

			• What do the ratios mean? Why are they all equivalent to ½?
10 min	PPT Slides 16-19 Field Guide	End of Day Reflections & Acknowledgements o End of Day Reflections (slide 16) o Acknowledgements - Noyce Foundation - WestEd - PD Series contributors	End of Day Reflections Ask teachers to reflect on the 2 questions listed on slide 16. In particular, teachers should think about the implications of this session on their instructional practice and on supporting their students in learning this content. Acknowledgements (slides 17-19) Please briefly acknowledge the Noyce Foundation. This online PD series was made possible by a generous grant from the Noyce Foundation to NCSM. In addition, video for this module comes from the project Learning and Teaching Geometry, funded by the National Science Foundation and soon to be published by WestEd. WestEd has graciously granted NCSM permission to utilize the video clip and supporting resources for this module. Contributors to the online PD series (of which this session is a part) are listed on slide 19.

COMMON CORE STATE STANDARDS FOR MATHEMATICS
Standards for Mathematical Practice

1 **Make sense of problems and persevere in solving them.**

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems.

2 **Reason abstractly and quantitatively.**

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
  - ✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
  - ✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed
  - ✓ elementary students construct arguments using objects, drawings, diagrams, and actions..
  - ✓ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions

4 **Model with mathematics.**

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 **Use appropriate tools strategically.**

Mathematically proficient students:

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 **Attend to precision.**
Mathematically proficient students:
- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ In high school, students have learned to examine claims and make explicit use of definitions.

7 **Look for and make use of structure.**
Mathematically proficient students:
- look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more.
  - ✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
  - ✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 **Look for and express regularity in repeated reasoning.**
Mathematically proficient students:
- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

STANDARDS FOR MATHEMATICAL CONTENT

6th grade content standards:
- Understand ratio concepts and use ratio reasoning to solve problems

7th grade content standards:
- Analyze proportional relationships and use them to solve real-world and mathematical problems.
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve problems involving scale drawings of geometric figures

8th grade geometry content standards:
Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Graphing Similar Rectangles Problem

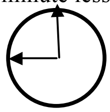
- Create four similar rectangles. List the dimensions for each rectangle.

- Plot the dimensions on a coordinate graph by aligning the bottom left vertex of each rectangle with the origin (0,0), and orienting the rectangles in the same direction, thus nesting the rectangles.

- Write the equation for the line that passes through the origin and the upper right vertices of each rectangle. Draw the line.

[53 minute lesson]

Session 8 Lesson Graph: Similarity Problems, Hannah Slovin, Class 3 (8th Grade)

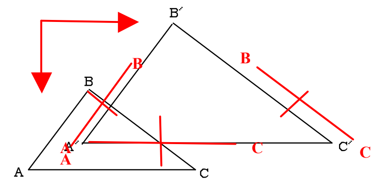


22 minutes

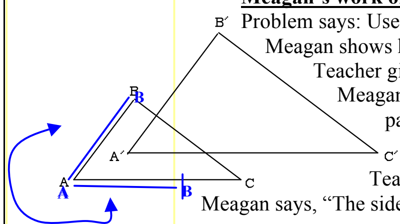
Whole Class

Brian's work on Similarity II, #3b: Examining Between-Figure Relationships

Problem says: Use the length of each original side to measure its corresponding side.
Brian traced the lengths of the "original sides" and compared them to the side lengths of the bigger triangle. In each case, the side lengths were dilated 1 ½.
Teacher: If every corresponding side on the dilated image is 1 ½ times the original, what can we say about the dilation percentage?
Brian: 150%.
Teacher: What's the ratio of the original side to the corresponding dilated side?
Andriy: 3:2. Brian, 1.5:1. One and a half of the original sides go into the dilated sides.



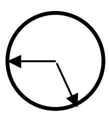
Discussion about whether this is the correct way to talk about the ratio. Andriy now says it's 2:3. Teacher probes, "What if Jackie had $2 and Kera had $3. What's the ratio for the amount of money they have?" Students say 2:3. Teacher says "It works the same way in this case. It's a comparison. 1 to 1.5. Let's talk about the ratio that way."



Meagan's work on Similarity II, #3c & d: Examining Within-Figure Ratios

Problem says: Use side AB to measure sides BC and AC. Use side A'B' to measure sides B'C' and A'C'.
Meagan shows how she traced and folded AB to find that AC is about 1 1/3 AB & CB is about 1 ¾ AB.
Teacher gives the other students tracing paper so they can follow along with Meagan.
Meagan starts to use the same approach for A'B' but says, "I don't know if I'm folding the paper right." Jensen comes up to help and says, "I think it's the same figure, just dilated."
Teacher asks, "What does he see are the same?"
Meagan says, "The sides all grew the same amount. So they should be the same fractions of each other."

Teacher: First we talked about the relationship between each original side & it's corresponding side. They're all 2:3 or 1: 1 ½. Now we're taking one side & measuring the other sides of the triangle.
Meagan adds, "Because it has all the same angles."
Teacher concludes by labeling the "between-figure relationships" (they're all the same) and the "within ratios" (they're all the same also).
Meagan adds, "Because we dilated them". Teacher agrees, "It was a dilation. They are similar triangles."



11 minutes

Small Groups.

Teacher distributes lab sheets & graph paper for **Similarity III, Lab 2, Part A #1 & 2.**

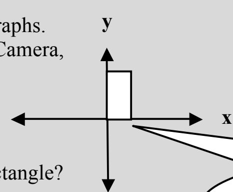
Similarity III: Lab 2

Complete all work on a separate sheet of folder paper. Work with your group. Explain all methods. Each group member is expected to turn in Lab work.

Part A
1. Create four similar rectangles. List the dimensions for each rectangle.

2. Plot the dimensions on the coordinate graph.

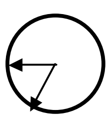
Teacher reviews coordinate graphs. Draws one on the Document Camera, labels the x & y axes.



T: How would I plot a 2x7 rectangle?

S: 2 to the right (x axis) & up 7 (y axis).
T: Make it a rectangle by drawing a line perpendicular to the y axis and a line perpendicular to the x axis.

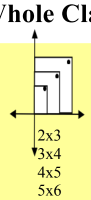
Small groups work



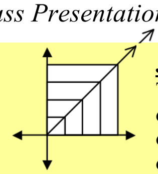
20 minutes

Clip

Whole Class *Whole Class Presentations*



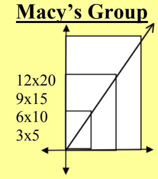
Brian's Group



Jim & Meagan's Group
They used squares. Jim draws a line through the origin. Brian's line doesn't start at the origin.

Teacher asks if anyone created rectangles with a line starting at 0,0. Macy shows hers.

Macy's Group



Discussion about the origin and center of dilation

Teacher shows Jackie's rectangles, which also have a line through the origin.
Teacher: What's the difference then (between Brian's and these)?
Meagan responds that Brian's rectangles "only go up by 1," but Macy "multiplied them by 2."
Teacher asks, "Are they similar rectangles then, if you just add on?" Students reply, "No."
Macy says that using Randy's method, the line has to go through all the vertices and the origin.
Teacher: What is the origin acting like there? Macy: The center of dilation.
Teacher: If yours isn't acting as the center of dilation, it's not going to work.
Discussion about what the line would look like if the rectangles were drawn with the shorter dimension on the y-axis. It would go up at "a lower rate."

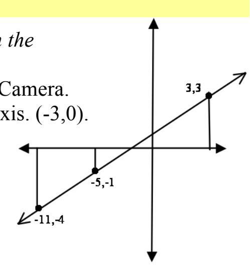
Similarity III, Lab 2, Part B #3,4 & 5

Part B
3. Plot the following points on the coordinate graph:
   (-11, -4), (-5, -1), and (3, 3)
   Draw the line that passes through all three points and mark the point where the line intersects the *x*-axis.

4. Draw a perpendicular line segment from each of the plotted points to the *x*-axis. You should have created three triangles.

5. Find two ways to prove that the three triangles are similar and describe your methods.
   Have students describe and demonstrate their methods.
   What are the lengths of the base and height of each triangle?

Teacher gives class a few minutes to work on the problem at their seats.
Noah shows his 3 triangles at the Document Camera.
Teacher asks where his line intersects the x axis. (-3,0).
Teacher: Find 2 ways to prove that the triangles are similar.
Andriy: We used the line that goes through them to create 1 side of each triangle.
Brian: Find the dilation point.
Teacher: We'll finish this tomorrow.



Brian and Macy Transcript

1 Teacher: This group did not start with numbers. They did not create them. They didn't decide on dimensions first. Ok. What did you do first, then, Brian? What did your group do?
2 Brian: This. This went up by one thing each time. And, yeah.
3 Teacher: So you just created the rectangles?
4 Brian: Yeah.
5 Teacher: On the graph. So what did your dimensions end up being then?
6 Brian: Um. 2,3. 3,4. 4,5. And 5-6.
7 Teacher: Huh. Ok. So what do you guys think about that one? 2 by 3. 3 by 4. 4 by 5. And 5 by 6.
8 Students: Whoa. What happened?
9 Teacher: What do you mean what happened? What do you mean whoa?
10 Students: No. Just the screen. The screen.
11 Teacher: Oh ok.
12 Student: It went out of focus.
13 Teacher: So what do you think about those dimensions? 2 by 3. 3 by 4. 4 by 5. 5 by 6. Yeah. What are you thinking? Jim?
14 Jim: Well, our group did squares. And if we look at it compared to his, actually the line goes up at the same rate.
15 Teacher: What do you mean the line goes up at the same rate? Go show us what you mean. What do you mean the line goes up at the same rate? You might have to zoom out.
16 Jim: How do we do that?
17 Teacher: Wide. The button that says wide. Ok, so tell us about that. Go ahead.
18 Jim: You can't really see it.
19 Teacher: Yeah, you can see it. Go ahead.
20 Jim: But then, like…
21 Teacher: Wait, excuse me. We don't have everybody paying attention. And we need to. Ok. Go ahead.
22 Jim: The line goes up at the same rate because it's increasing the lengths of the side by the same rate.
23 Teacher: Ok. So where are you looking when you say that? Just kind of point to us. What are you looking at when you say the line goes up at the same rate?
24 Jim: This line.
25 Teacher: That line. Ok. So it's straight. And it keeps going up at the same rate. And so, what… how did that help you look at the other one? Let's look at Brian's now. Or not just Brian's, his group's. So let's look at that one. How does that help you look at that?
26 Jim: Um, if you draw a line through it, it would go, oh, it would go up at the same rate.
27 Student: Yeah.
28 Teacher: It would go up at the same rate?
29 Jim: But, then, it starts at a different point.
30 Teacher: What do you mean it starts at a different point?
31 Jim: This one, it would start at one. And this one would start at zero. Because this is a rectangle. Rectangles, that they were doing. And this one is a square.
32 Teacher: So you're thinking that because it's a rectangle, it's different.
33 Jim: Yeah.
34 Teacher: Let's see about some other people who had rectangles, then. Did anyone else have a rectangle? Who? Yeah, go ahead.

35 Brian: You have to go up. The way you do Jim's one, with the rectangle, is you go up from here to there. Instead of from down to there.
36 Teacher: Right. But yours doesn't go from the place you started counting.
37 Brian: No, it doesn't.
38 Teacher: Well, I'm wondering, is that not possible with rectangles?
39 Brian: It's not.
40 Teacher: But you have one that does. Come show us yours. Let's look. So Macy, yeah, zoom out. And then tell us about yours. Go ahead. This was the one, remember, it was 3 and 5. And then 6 and 10. And then, what was your next one?
41 Student: 9 and 15
42 Teacher: 9 and… Ok. So tell us about it. So, they're rectangles. And that one goes up and goes through the 0,0. It goes through the origin. Anyone else have a set of rectangles where it goes through the origin?
43 Student: Yeah.
44 Teacher: Yours does too?
45 Jackie: Yeah.
46 Teacher: Ok. So yours goes through the… oh, Jackie drew it. May I borrow this? Ok, so here's Jackie's. They have a different rectangle than Macy's. But theirs goes through the origin, too. So I'm wondering what's the difference, then? Megan, what do you think?
47 Megan: Macy's and Jackie's goes up by two. And Brian's group only went up by one.
48 Teacher: So, what do we think about that?
49 Megan: Like they multiplied it by two. But Brian's group just went up and out a square.
50 Teacher: Ah, so Brian added one to each.
51 Megan: Yeah.
52 Teacher: And the difference was, that in group two's and group five's, what did they do instead of adding?
53 Megan: They multiplied.
54 Teacher: They multiplied. So I'm wondering if those are… do you think those are similar rectangles then? If you just add on.
55 Megan: No. Maybe.
56 Teacher: Hm. Maybe not. All right.
57 Macy: Cause, for the check, they didn't have to go from the…
58 Teacher: Say again? Talk to them.
59 Macy: Randy's check thing. They didn't have to start over here and go through all the corners to be similar.
60 Teacher: Oh, if we used Randy's method. That's right. Are you hearing this? Tell it again, Macy, how that helped you decide whether group three's were similar or not. Go ahead.
61 Macy: Because for Randy's check thing, you have to start over here. And for it to be similar, you have to go through all the corners.
62 Teacher: So it has to be able to go through that place you start counting. What is the origin acting like there?
63 Macy: The center of dilation.
64 Teacher: Yeah. It's kind of acting like the center of dilation. So, group three, you need to check yours. If yours doesn't act like a center of dilation, there, Brian, that's not going to help. Ok. If it's not acting like a center of dilation, we may need to check again. Because, yeah, Randy's method of checking for similarity that way.

<u>Similar Triangles Problem</u>

Task:
Plot the following points on a coordinate graph: (-11, -4), (-5, -1), and (3,3)

- Draw the line that passes through all three points and mark the point where the line intersects the x-axis.
- Draw a perpendicular line segment from each of the plotted points to the x-axis. You should have created three triangles.
- Find at least two ways to justify that the three triangles are similar.