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PRIME Leadership Standards

Changing Teacher Beliefs

PLCs in Guatemala

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS

The *NCSM Journal of Mathematics Education Leadership* is published at least twice yearly, in the spring and fall.

The editors of the *NCSM Journal of Mathematics Education Leadership* are interested in manuscripts that address concerns of leadership in mathematics rather than those of content or delivery. Editors are interested in publishing articles from a broad spectrum of formal and informal leaders who practice at local, regional, national, and international levels. Categories for submittal include:

- Key Topics in Leadership
- Case Studies
- Research Report and Interpretation
- Commentary on Critical Issues in Mathematics Education
- Professional Development Strategies

Note: The last two categories are intended for short pieces of 2 to 3 pages in length.

Submission/Review Procedures

Submittal of items should be done electronically to the *Journal* editor. Do not put any author identification in the body of the item being submitted, but do include author information as you would like to see it in the *Journal*. Items submitted for publication will be reviewed by two members of the NCSM Review Panel and one editor with comments and suggested revisions sent back to the author at least six weeks before publication. Final copy must be agreed to at least three weeks before publication.

Cover image: A spiral vortex generated with fractal algorithms (istockphoto.com)

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Purpose Statement

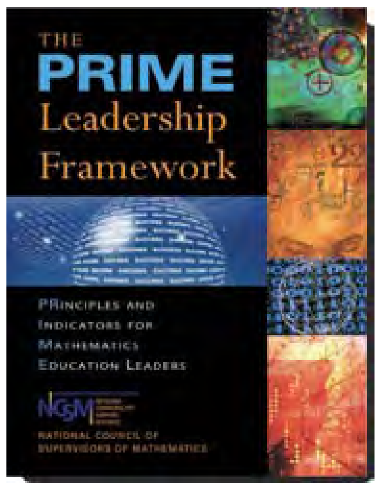
The purpose of the National Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership

Comments From the Editor

Gwendolyn Zimmermann
Adlai E. Stevenson High School
Lincolnshire, Illinois

Earlier this year, NCSM released a new mission and vision statement. Our mission speaks to our commitment to "support and sustain improved student achievement through the development of leadership skills and relationships among current and future mathematics leaders." Our vision statement challenges us as the leaders in mathematics education to collaborate with all stakeholders and develop leadership skills that will lead to improved student achievement. To that end, *The PRIME Leadership Framework* was created as both a tool and lens to focus and guide our development as leaders in mathematics education. The framework is founded on research and best practice in leadership combined with the elements essential to helping every student achieve in mathematics.



The *NCSM Journal* is the vehicle by which NCSM shares research about issues that affect mathematics education leadership. One of the assumptions of *The PRIME Leadership Framework* is that leadership practices aimed at improving student achievement must be informed by research. So as to begin to critically examine how the framework is used together with the impact it might have on equity, instruction, curriculum, assessment, and most importantly — student learning of mathematics, a working knowledge of the framework is necessary. What follows is a summary of some of the key elements of the framework.

The *PRIME* (Principles and Indicators for Mathematics Education Leaders) framework consists of four principles of leadership essential for improving mathematics education.

Equity Leadership
Principle 1: Ensure high expectations and access to meaningful mathematics learning for every student.

Teaching and Learning Leadership
Principle 2: Ensure high expectations and access to meaningful mathematics instruction every day.

Curriculum Leadership
Principle 3: Ensure relevant and meaningful mathematics in every lesson.

Assessment Leadership
Principle 4: Ensure timely, accurate monitoring of student learning and adjustment of teacher instruction for improved student learning.

On one hand these four principles are distinct. On the other hand, these principles are intertwined. A mathematics education leader who may choose to focus on equity is unlikely to do so without impacting teaching and learning, curriculum, and assessment. For example, consider a leader who chooses to focus on equity by ensuring access to meaningful mathematics. Questions begin to emerge about the content and rigor of the curriculum being provided to each student and how these students are being assessed on this content.

Within each principle are three indicators that describe the action steps necessary to positively impact student achievement resulting in 12 indicators for the *PRIME* framework. These indicators are framed around what we know are research-affirmed best practices in leadership and mathematics education.

The PRIME Leadership Framework

Principle	Indicator 1	Indicator 2	Indicator 3
Equity Leadership	Every teacher addresses gaps in mathematics achievement expectations for all student populations.	Every teacher provides each student access to relevant and meaningful mathematics experiences.	Every teacher works interdependently in a collaborative learning community to erase inequities in student learning.
Teaching and Learning Leadership	Every teacher pursues the successful learning of mathematics for every student.	Every teacher implements research-informed best practices and uses effective instructional planning and teaching strategies.	Every teacher participates in continguous and meaningful mathematics professional development and learning in order to improve his or her practice.
Curriculum Leadership	Every teacher implements the local curriculum and uses instructional resources that are coherent and reflect state standards and national curriculum recommendations.	Every teacher implements a curriculum that is focused on relevant and meaningful mathematics.	Every teacher implements the intended curriculum with needed intervention and makes certain it is attained by every student.
Assessment Leadership	Every teacher uses student assessments that are congruent and aligned by grade level or course content.	Every teacher uses formative assessment processes to inform teacher practice and student learning.	Every teacher uses summative assessment data to evaluate mathematics grade-level, course, and program effectiveness.

Built into the framework is the acknowledgement that leadership is about continual growth. Thus, within each indicator are three stages distinguished by the knowledge and practices of the leader.

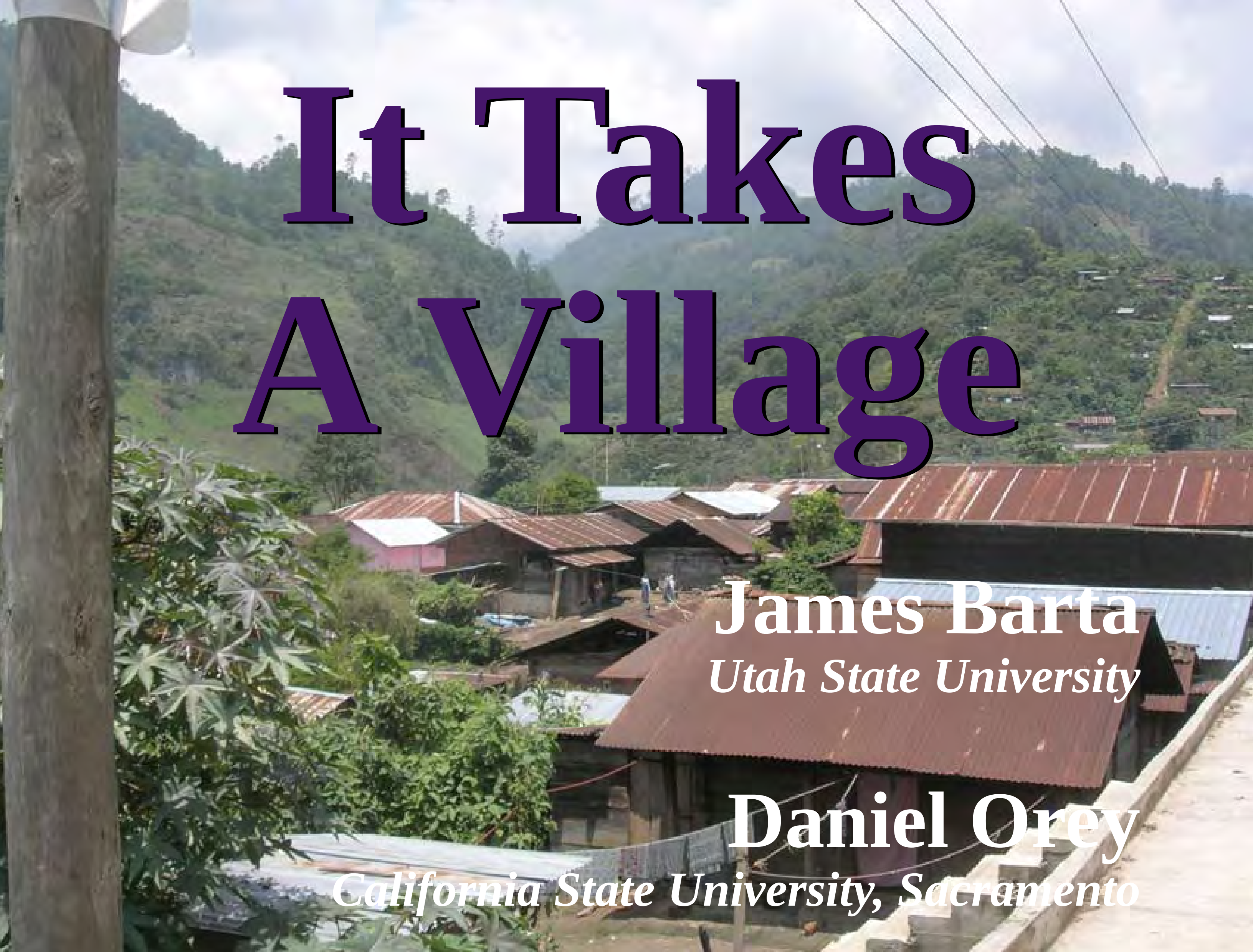
Stage 1: Leadership of Self – Leadership of self-knowledge, awareness, development, and modeling of the 12 leadership indicators; the leader is respected for his or her own teaching and learning skills. This is the “know and model” stage of leadership growth and development.

Stage 2: Leadership of Others – Leadership of all students and teachers within the mathematics program; leadership and development of other teachers, teams of teachers, and administrators toward full knowledge and development of each of the 12 leadership indicators; the leader is respected for his or her interpersonal skills and commitment for leading change among teams of teachers and colleagues. This is the “collaborate and implement” stage of leadership development.

Stage 3: Leadership in the Extended Community – Leadership of district, state, province, or beyond reform efforts through sustained deepened systemic implementation of each of the 12 leadership indicators. The leader is respected for his or her influence and engagement with an expanded community of educational stakeholders. This is the “advocate and systematize” stage of leadership and development.

The stages are viewed independently across the indicators. That is to say that a mathematics education leader may be increasing their knowledge (Stage 1) about assessment methods while at the same time they may play a pivotal role in implementing equity initiatives within their district (Stage 3). Moreover the development of mathematics education leadership is a cyclic process. As the field continues to develop in the areas of student learning and best practice of teaching, or as positions and settings change, a leader may find that they are now at Stage 1 for an indicator where they were once at a Stage 2. What is critical is that leaders are able to continuously reflect in order to identify areas for growth and action.

NCSM, which is celebrating its 40th anniversary, is a unique organization. No other national organization is dedicated to the leadership that is so critical when it comes to increasing the mathematics achievement of every student. How apropos that in celebrating 40 years of leadership in mathematics education, NCSM shares a framework of principles that require all mathematics education leaders to aspire to high standards and expectations to benefit all students.



It Takes A Village

James Barta
Utah State University

Daniel Orey
California State University, Sacramento

Culturally Responsive Professional Development and Creating Professional Learning Communities in Guatemala

"Our teachers are being transformed — they are becoming educators through these ongoing professional development opportunities. They are seeing the power of ideas, the excitement of a well-crafted lesson, and the creativity of learning in a pliable, responsive model, which incorporates and respects their local language and culture. As a result, the teachers are working long after the school day ends, coming back early, and wanting scholarships to take more course work in education. This process has caught their imagination and fueled their commitment to serve their people. There is nothing in their past experience or in the performance of their peers in other schools that would cause this redefinition of what teaching really is and can be."

Lois O'Neil
Educational Coordinator for HELPS International

First grade children in a rural highland village school in Guatemala participate excitedly as their teacher implements a collaboratively planned mathematics lesson in their home language, Ixil, created the previous day. The teacher's colleagues watch intently and note his actions and the children's responses as they make observations to be discussed at the post-instructional evaluation debriefing to follow. The instruction is in stark contrast to the typical Spanish language textbook guided education as students sit in rows of desks listening quietly to their teacher's presentation before completing a worksheet on which they practice multiple calculations using the procedure taught to them.

Teachers at the William Bonan School in Santa Avelina, Guatemala, participate regularly in a series of professional development activities designed to enhance their understanding of the mathematical content they teach and improve their teaching effectiveness. These activities are infused with local Mayan cultural connections and customary ways of knowing. Through this article readers will learn of the efforts of a team of American educators working to establish local culturally responsive professional learning communities in mathematics education focused on equity, teaching and learning, curriculum, and assessment. The model and methods described may serve as a suggested framework for other mathematics leaders involved in international efforts to improve mathematics education in the communities they serve.

It Takes A Village

The William M. Bonan School serves children from preschool to fifth grade in Santa Avelina; a small rural Mayan community nestled in the highlands of Guatemala. Its primary focus is on providing quality education to its students with a particular emphasis on reestablishing first language learning (Ixil – one of several Mayan dialects spoken in the Guatemalan highlands). For the past several years, through the efforts of HELPS International, a small team of US consultants provides educational professional development and support to the principal and her teachers. HELPS, a non-profit US based organization works in partnership with individuals, local and national governments to improve drinking water quality, medical care, education, housing, agricultural and economic development to the people of Guatemala. Educational efforts are focused to establish professional learning communities targeting optimizing mathematics instruction guided by research-based best practice while maintaining a culturally responsive pedagogy inclusive of local Mayan values and traditions.

All of the Santa Avelina teachers speak their native language, Ixil, and Spanish. They each hold a high school diploma and have participated in ongoing professional development to bolster their understanding and application of effective instruction. Their principal, Rosa Cordova Perez, holds a bachelors degree and actively supports the professional learning efforts and works to ensure that their implementation continues after the inservices are completed. The faculty's dedication to the profession and their desire to continually develop their instructional capacities provides them the motivation for continued learning.

The students are children in village families whose parents farm corn and other fruits and vegetables, raise chickens, and typically weave the beautiful and complex colorful fabrics from which clothing, scarves, bags, belts, and wraps are sewn. Most of what a family produces is for personal consumption and any excess is sold to generate much needed money for the basic necessities of living. Although a public school exists in the community, these parents send their children to this charter school to acquire an education from teachers who are constantly benefiting from inservice to improve their instruction with a special focus on home language learning and the infusion of culture in the curriculum.

A Focus on Equity

To fully understand equity issues in Santa Avelina it is important that one is knowledgeable of the history that shaped Guatemala. It is believed that numerous complex cultures have made this land their home since 10,000

BC. These prehistoric societies included some of the most advanced cultures in the world at that time and included the Olmec, Teotihuacan, Aztec, and the Maya. These cultures displayed amazing technological, scientific, and mathematical advancements reflected in networks of sociopolitical communities that were organized around highly religious customs, traditions, and ceremonies. For centuries these cultures thrived.

The Spanish conquest of Mesoamerica, which today comprises the vast area from southern Mexico and throughout Central America, took place in the 15th and 16th centuries. The invasions destroyed entire cultures and with these were lost vast amounts of intellectual, social, and political knowledge and practices. That imposition of Spanish culture and European customs on the indigenous cultures, customs including those of language, religion, education, and governance, still play a significant and influential role on the sociopolitical interactions in modern Guatemalan society.

While the majority of Guatemalans live rurally and are of ancient Mayan descent they are dominated politically by the “Ladinos,” a term used to define the social and economic elite of mixed European and indigenous ancestry. Governmental representation for the indigenous culture remains limited at best. In early 1960 tensions over equity issues related to religious, political, economic, and social power erupted and ignited 36 years of civil war. The war ravaged the villagers who were often caught between the warring factions of the government forces and the freedom fighters. Remarkably only 14 years ago in 1994, a peace treaty was signed and the conflict ended.

The teachers with whom we work were all affected either personally by the atrocities of war or had friends and relatives of all ages who suffered. Burial grounds for those who died in the war may now be the village football field, and churches walls are adorned with small black crosses dedicated to those who were assassinated or mysteriously disappeared. The war affected people of all ages and social classes. Today, the survivors continue to rebuild their communities and few indications remain to remind one of those turbulent and tragic times. The country progresses yet the injustices of the past remain albeit much more subtly and much less violently.

Poverty remains high and rural villagers such as those in Santa Avelina work hard to sustain life for themselves and their children. The educational system in Guatemala is European based. Spanish typically is the language of instruction even though the Mayan majority speaks multiple indigenous dialects. Schools follow the official Guatemalan curriculum directed by the Ministry of Education, which typically omits Mayan language, culture, and history. Public school instruction for those who can attend (some children must work in the fields) is taught in Spanish and is typically teacher-centered and focuses on rote learning of skills and concepts. In mathematics for instance, instruction seldom includes connections to local culture and language, the use of research-based instructional techniques, discourse in the classroom, or discovery and problem solving orientation. Rather intentionally, it appears that the indigenous cultures and language are greatly devalued. In the name of “education,” efforts are made to supplant them with Spanish, the “official” language, and European centered curriculum. D’Ambrosio (2001) asserts it becomes nearly impossible for children to meaningfully participate in their mathematics learning when the curriculum is so distant from what they actually experience in their lives. Rather than enriching their understanding of the role of mathematics in society, it becomes an isolated subject to study with little connection to their culture and community.

Those indigenous children who do attend school tend to stay though grade six and a few continue onto higher grades. These children usually speak their home language but often are unable to read and write with it as the children are taught Spanish in schools. Those villagers who venture to the cities in search of work are often discriminated against for their lack of Spanish language skills, cultural background, and basic educational levels. They tend to work for the more educated and affluent Ladino population in low paying service jobs.

This short history has been shared so that the reader gains a sense of the inequities present in Guatemala in general and specifically in Santa Avelina. Our educational efforts are in response to these obstacles so that the children are provided access to high quality instruction inclusive of their culture and offered in the home language called “Ixil.” This focus provides the purpose for the professional development to be described.

A Focus on Teaching and Learning

Teachers at the school have been involved in professional development efforts for more than five years and Principal Rosa Cordova Perez provides instructional leadership. Class size is typically around twenty students. Classrooms in the two-story cinderblock constructed school are airy and have the usual classroom furnishings of desks and tables.

Each student has a chair. Students sit in tabled rows and the teacher's desk and blackboard is at the front of the room.

Our professional development activities incorporate several key components. We work to help the teachers understand conceptual foundations of mathematical concepts to supplement their procedural understanding. We engage them in collaborative development of grade specific mathematics lessons that are then implemented, documented, evaluated, and revised in a lesson study approach. Lastly, we assist the teachers to better comprehend the role of culture in the process of teaching and learning. The authors have been involved in similar efforts for over a decade and a number of the insights gained are implemented with these teachers (see Barta & Shockey, 2004, and Orey & Milton, 2007). This cultural focus affects the curricular design and the context in which mathematics concepts are presented, the community connections made, and the pedagogy implemented.

Prior to our consultant work, teachers were not aware of the National Council of Teachers of Mathematics *Standards*-based instruction. Our professional development efforts center on helping the teachers learn and incorporate the NCTM (2000) *Principles and Standards for School Mathematics* into their instruction. The content standards are shared to expand the major focus on numbers and operations. The process standards are shared so teachers can envision and define their own instruction.

During one inservice session, teachers were presented with the following problem:

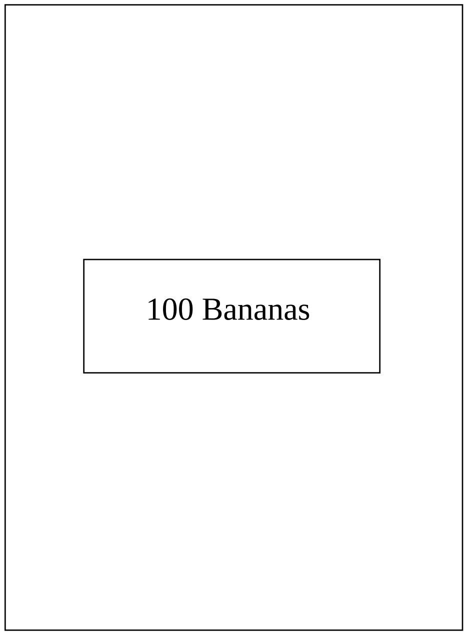
> A sack holds one hundred bananas that need to be shared with six families in the community. How many bananas will each family receive if each receive equal amounts?

At first glance this appears to be a simple division problem that could be quickly solved individually using the traditional procedure. Instead, teachers were asked to work cooperatively and interact with the drawing on this page (Figure 1) as they discussed possible solutions and answers.

Several teachers sketched different ways of subdividing the shape and ultimately divided the rectangular bag into six equal sized rectangular units. Then they used partial quotients to begin to "fill" the amount of each unit. They kept track of their amounts by writing them in the unit spaces (10 + 5 + 1=16). After this equal division process, four bananas remained. Teachers discussed giving the four to the largest family! This certainly was an appropriate real-life solution to the problem but we pressed them to use the model to find the exact amount. It was an amazing sight to hear the engaging discourse of the teachers as they presented and argued their answers. Teachers excitedly competed for the chalk with which to show others their solutions. The consultants who had formerly been seen as "the" teachers were now demonstrating their role as silent witnesses and facilitators of learning.

Figure 1

How can you share this bag of 100 bananas equally with six families? Use the figure to draw and calculate your answer. Be prepared to share your thinking.



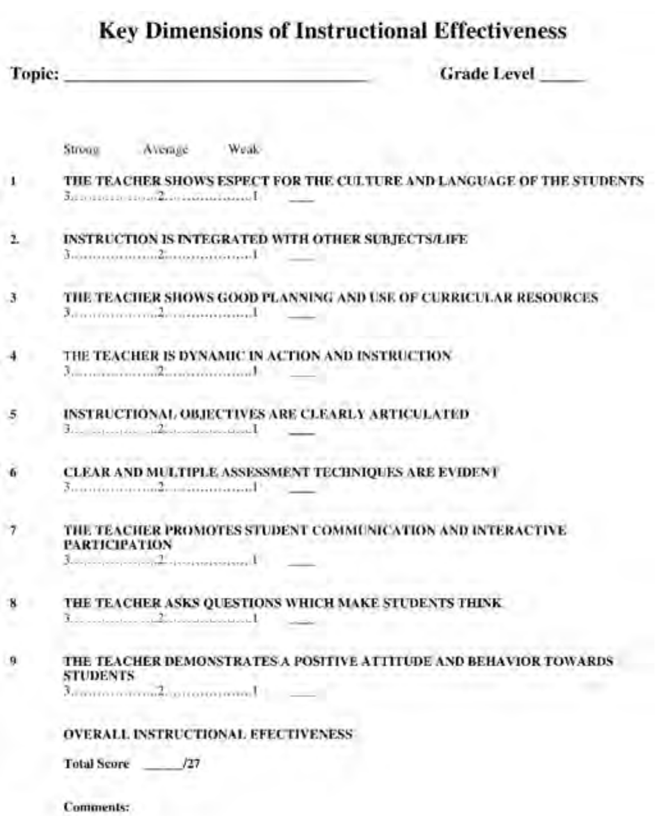
The pictorial model of the four bananas was again divided into six equal pieces and those amounts again partitioned out. Teachers were amazed to "see" that each family would get four-sixths of a banana. When we debriefed the problem and the instructional techniques the consultants displayed, several new insights emerged. Teachers stated they were beginning to finally understand what division meant and that there are alternative instructional approaches they can use with their students to deepen their own conceptual understandings. The use of discourse with learners to facilitate rather than direct teaching was a revelation. Teachers exclaimed they enjoyed being able to think for themselves and loved seeing different ways their colleagues thought about their solution. Several stated they could use this technique in their classroom to supplement the more typical individual teacher-centered instruction.

A Focus on Curriculum

As previously mentioned the formal curriculum provided by the Guatemalan Ministry of Education is Spanish language and textbook driven. The scope and sequence is determined not by what students most need to learn but rather by the topic of the day presented in the book. Teachers seldom felt they possessed enough confidence or knowledge to create their own lessons. They simply had never had experiences doing this or knew the necessary components. To help them gain this knowledge we first asked them to define what they felt were vital components of a well-designed math lesson. We could have simply handed them a form that other experts had created or located a related research-based mathematics article. It wasn't that we were dismissive of such work, rather in our professional development efforts we believe that change must occur first occur from within. We wanted to validate the knowledge these teachers already possessed so they become more confident in their own abilities to grow and learn.

The list of key instructional dimensions included a number of important aspects and after much discussion we pared the list down to nine. The dimensions follow research-based recommendations for providing effective instruction, and we were gratified our list grew from the discussion of the teachers. The list included maintaining respect for the culture and language of the students, ensuring learning objectives were clearly stated, coherence between components of the lesson, and demonstration of teacher enthusiasm. A rubric was created of the dimensions. This was then used as the basis for the development of future lessons and was later used to evaluate the quality of the implemented lessons. This will be discussed further in a following section of this article.

Figure 2



Key Dimensions of Instructional Effectiveness

Topic: ______________________ **Grade Level** _____

Strong Average Weak

1 THE TEACHER SHOWS ESPECT FOR THE CULTURE AND LANGUAGE OF THE STUDENTS
3..................2..................1 ____

2. INSTRUCTION IS INTEGRATED WITH OTHER SUBJECTS/LIFE
3..................2..................1 ____

3 THE TEACHER SHOWS GOOD PLANNING AND USE OF CURRICULAR RESOURCES
3..................2..................1 ____

4 THE TEACHER IS DYNAMIC IN ACTION AND INSTRUCTION
3..................2..................1 ____

5 INSTRUCTIONAL OBJECTIVES ARE CLEARLY ARTICULATED
3..................2..................1 ____

6 CLEAR AND MULTIPLE ASSESSMENT TECHNIQUES ARE EVIDENT
3..................2..................1 ____

7 THE TEACHER PROMOTES STUDENT COMMUNICATION AND INTERACTIVE PARTICIPATION
3..................2..................1 ____

8 THE TEACHER ASKS QUESTIONS WHICH MAKE STUDENTS THINK
3..................2..................1 ____

9 THE TEACHER DEMONSTRATES A POSITIVE ATTITUDE AND BEHAVIOR TOWARDS STUDENTS
3..................2..................1 ____

OVERALL INSTRUCTIONAL EFECTIVENESS

Total Score _____/27

Comments:

The teachers grouped into grade specific learning communities after receiving instruction in the use of Lesson Study. Guided by the dimensions they had previously discussed, teachers collectively planned and documented several optimal lessons each at a different grade level to be taught. Teachers then discussed the lessons the following day. Excitement and some anxiety were apparent as instruction of the collaboratively constructed lessons began. Never before had these teachers planned instruction together much less observed each other in their teaching. The video camera, so necessary for later use to review and deconstruct the learning event, added another level to their nervousness. Eventually, teachers not only became accustomed to being videoed but enjoyed seeing themselves and their students. Several teachers remarked they appreciated being able to use this tool to evaluate their instruction. Seldom had such reflection taken place.

This professional development effort of Lesson Study was showcased so that after having constructed supportive professional development communities the inservice teachers could more frequently use this to improve their instruction. The lessons were a wonderful sight to behold. Teachers were implementing lessons in which children were engaged in learning mathematics concepts, progressing from a concrete to pictorial to symbolic stage. Discourse was much more evident as teachers moved from a role of "telling" to one of "asking." Once students got the idea it was not only acceptable but valued for them to speak up and share their ideas, the engagement heightened.

Manipulatives, which previously were a rarity in instruction, became customary. After some playful exploration, students began to use them to mirror and to guide their growing understanding. The instruction was provided in the home language of Ixil and key vocabulary documented in writing on the board.

Cultural connections are a key element of our efforts. The following story must be shared for the reader to understand why we felt this so necessary. As students attended school to learn the traditional subjects we were told of a chasm that was developing in how some children were misperceiving their parents. Some children seemed to be begin to despise their parents for their lack of education. It was shared that when some children came home and needed help with reading, writing, or school arithmetic, parents were not able to help them even at the most elementary levels. Some children, believing their parents were not smart began acting rudely to them. In the quest for education, tension in some families grew.

Using a "Funds of Knowledge" approach (Gonzales, Moll, & Amanti, 2005), we saw it necessary to connect what we were teaching in the school with their rich culture and language experience. We wanted to sincerely validate the intelligence of the mother who weaved mathematically complex garments or the father who grew crops successfully in their field. As part of our ethnomathematical approach of connecting mathematics and culture our teachers conducted mathematical cultural interviews during our professional development inservice. Neighbors and family members were interviewed relative to daily universal activities (counting, measuring, designing, locating, explaining, and playing) incorporating mathematics (Bishop, 1991). The house builder for instance was asked how he made his calculations, designed the rooms, located spaces, explained costs or determined the necessary slope of the stairs. The mother was asked to explain her daily uses of mathematical activities from measuring the corn meal needed for the tortillas to be made to how to get the best bargain for necessary staples at the market.

With this interview information teachers created and illustrated books in their new technology center. Books had such titles as *Maria, My Mother is a Mathematician* or *Mathematics in Our Village* and depicted people the children knew displaying mathematical knowledge and application. One mother later stated that she was very glad to know she is now a "mathematician!" She explained that she never described to her children how she went about calculating the best price for items when at the market with them. She declared that from now on she will!

These books provided a contextual foundation for the instruction that followed. Our professional development efforts provided a model for looking "for the mathematics that lives in any activity." We shared that children love to know they are important and valued and their parents are smart! Our teachers began to see how easy and beneficial it was for them to adapt their standard instruction. For example, word problems could be edited to use names of class members and items and activities they felt valuable. A teacher using the "Mathematics of the House Builder" book could have students explore how to create, mark, and measure the corners for a room. Concepts such as area, perimeter, and angles can be illustrated as real life activities as the mathematics in the activity is experienced and learned.

A Focus on Assessment

Our efforts to improve assessment capabilities continue. Teachers, having learned to clearly define their instructional objectives, are challenged to write assessment procedures for the objectives prior to the design of instruction. We insist this alignment of teaching and assessment is paramount if teachers are to explicitly know what they are teaching and whether students are learning.

We stress the importance of the teaching of mathematical vocabulary and ensuring that this too is assessed. Educators at Santa Avelina are learning alternative assessment techniques to supplement the standard paper and pencil tests. Students are now being asked to draw models to represent their thinking and to write explanations of their thought processes. Teachers are learning to purposely observe multiple predetermined aspects of student involvement and learning.

Most surprising to many teachers was the idea that assessment was not complete after the student self reported or drew a model. Teachers became aware that this merely provided them data that required further analysis and documentation. Teachers began to learn to look for gaps in understanding displayed in students' errors. They began to see how assessment guides instruction and how what was or was not learned today influences what needs to be taught tomorrow. Documenting this data over time allowed teachers to look for clusters of students needing extra help or seeing trends in learning.

The assessment component of our professional development also consisted of teacher self-evaluation. As teachers debriefed the films of the lesson study they were asked three questions. Each teacher who provided the collaborative instruction was first asked to reflect upon and describe what he or she felt were effective aspects of their lesson. They were then asked what they thought was less than satisfactory. Lastly they were asked to suggest what they would do differently next time. The collaborative group was then asked to provide their input as we guided the process to ensure that all were reminded we were evaluating the cooperative lesson rather than the individual teachers. A supportive environment was maintained. Plans for lesson modifications were discussed and with the guidance of the principal, the cycle of planning, teaching, and revising as teachers strive to improve their knowledge and practice of mathematics teaching will continue.

It Takes A Village

It has indeed been a humbling and gratifying experience to be a member of the consultative team working with Guatemalan colleagues in the village of Santa Avelina. After each inservice it feels that we come away with more insight and knowledge than we shared. A community of likeminded teachers separated by thousands of miles has evolved because of this international collaborative relationship. Such is the power of mathematical leadership to transform and be transformed regardless of the distance. Mathematics as a universal language has brought us closer. Our professional development continues to be shaped and guided by our pledge to be culturally responsive and culturally respectful. Obviously we share an educational target but the humanistic interactions warm our hearts and souls. Teaching and learning is not an individual act. It requires dedicated involvement from parents, administrators, teachers, and students. For rich mathematics learning to occur it indeed "takes a village."

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A Framework for Analyzing Differences Across Mathematics Curricula

Mary Ann Huntley
University of Delaware

Textbooks have a tremendous influence on *what* and *how* mathematics is taught. In a national study teachers reported that textbooks designated for a class influence their selection of content in nearly 5 out of 10 mathematics lessons, and that textbooks influence teachers' instructional strategies in roughly 7 out of 10 lessons (Weiss, Pasley, Smith, Banilower, & Heck, 2003). Given the importance of textbooks in mathematics classrooms, it stands to reason that choosing a mathematics textbook is an important task. But at the same time, this task can be both overwhelming and time consuming (Reys & Reys, 2006). Marketing materials provided by textbook publishers can be more confusing than helpful. Indeed, it seems that all textbook publishers claim their products are research-based and will produce student success.

Teacher leaders, and others who have responsibility for choosing textbooks, often resort to making decisions by ticking off topics in tables of contents that align with their state/district standards. Another popular method for selecting textbooks is the "flip test," which involves quick browsing of several textbooks for ease of readability, appealing design and color illustrations, and ready-made teaching aids and test questions, seizing on these attributes as proxies for quality.

Another impediment to selecting textbooks is that despite the plethora of rhetoric about mathematics textbooks generated by the Math Wars, mathematics programs tend to get lumped into one of two categories — reform or traditional. These categories are too broad and do not take into account the variation that exists across textbooks. There are major differences across traditional textbooks, just as there are major differences across reform textbooks. It is important to understand these differences, as they may differentially impact instructional practice and ultimately student learning.

In this article I offer a framework for looking beyond lists of topics and surface features of mathematics textbooks. For those with responsibility for choosing textbooks, I offer the framework as a tool for better understanding and appreciating the sometimes nuanced differences across mathematics curricular programs.

Method Used to Develop the Framework

In this section I outline the process by which I developed the framework. I explain my choice of textbooks that I use to illustrate the framework, my sources of data, and the specific aspects of textbooks that constitute the backbone of the framework.

Choice of Textbooks

I present my framework in the context of two comprehensive middle-grades mathematics curricula funded by the National Science Foundation (NSF): *Connected Mathematics* (Lappan, Fey, Friel, Fitzgerald, & Phillips, 2002) and *Math Thematics* (Billstein & Williamson, 1999-2005). I could have chosen any two curricular programs — two traditional programs, two reform programs, or one of each — but I decided to compare two NSF-funded curricula for several reasons. First, despite the growing literature base about how reform mathematics materials differ from more traditional materials, how to implement reform materials, and the effects of reform

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materials on students and teachers (e.g., Goldsmith, Mark, & Kantrov, 1998; Lloyd, 2002; Senk & Thompson, 2003; Smith & Star, 2007; Trafton, Reys, & Wasman, 2001), there has been surprisingly little discourse about how one set of NSF-funded materials differs from another. NSF funded the development of five comprehensive middle-grades programs, so surely there are important differences among them! The second reason I decided to anchor this discussion around these two particular curricula, *Connected Mathematics* and *Math Thematics*, is because these materials are likely to continue being used in schools in the future, as they are the two NSF-funded comprehensive middle-grades mathematics programs with greatest market penetration. And third, while being developed with common goals, these two curricular programs represent very different approaches to middle-grades mathematics.

Sources of Data

I reviewed written materials related to *Connected Mathematics* and *Math Thematics*, including student and teacher books. I read ancillary materials related to each program, including documents the authors provide for professional development providers (Denny & Williamson, 1999; Lappan, Fey, Fitzgerald, Friel, & Phllips, 2002). I talked extensively with the lead author of *Connected Mathematics* (Glenda Lappan) and the lead author of *Math Thematics* (Rick Billstein), as well as with several middle-grades teachers who have worked closely with the authors during field testing of the materials.

Textbook Features Examined

Authors of curricula are faced with many choices that affect how students experience a given set of instructional materials. For instance, authors must wrestle with issues such as the role of problem context, the amount of basic skills practice, and emphasis of cooperative groups versus whole-group discussion. Curriculum writers make decisions about these and other issues, which are often referred to as "curriculum variables." These decisions reflect the authors' explicit and implicit beliefs about mathematics, as well as their beliefs about the teaching and learning of mathematics.

The authors of *Connected Mathematics* and *Math Thematics* made decisions about their respective curricula within specified parameters that were outlined in the NSF program solicitation from which they received initial funding (NSF, 1989). According to this solicitation the curricula had to be aligned with the National Council of Teachers of Mathematics [NCTM] *Standards* (1989), and in comparison with most existing curricula they had to place greater emphasis on mathematical investigation, mathematics presented in real-world contexts, connections among content areas of mathematics and connections and between mathematics and other disciplines, and integration of technology with mathematics. These parameters set the general bounds for curriculum writing, but left considerable room for interpretation.

I developed the framework by examining differences across *Connected Mathematics* and *Math Thematics* in terms of two sets of curriculum variables. The first set relates to content, including structural organization, depth/breadth of content, presentation of content, worked-out examples, and definitions/rules. The second set of variables relates to instruction, including instructional model, use of class time, teacher's role, students' role, use of small group work, use of tools, assessment, and homework.

Framework

The framework that I developed to compare two mathematics curricula consists of three pieces. The first piece contains descriptive information about the curricula being compared. For each curriculum this includes the title, target grade range, authors, publisher and date of publication, list of ancillary materials provided by the publisher, and context (e.g., the funding source for the materials and extent to which the materials are aligned with the NCTM *Standards*). The second and third pieces of the framework contain comparative information regarding content variables and instructional variables, respectively. To illustrate use of the framework I discuss my analyses of *Connected Mathematics* and *Math Thematics*. In Figure 1a I provide descriptive information about each curriculum. My analyses around the two sets of curriculum variables (content and instruction) are shown in Figures 1b and 1c. In each figure the curriculum variables are in the center column, together with a description of the aspects of the variable that are common across the two curricula. The corresponding left- and right-hand columns indicate differences (if any) between *Connected Mathematics* and *Math Thematics*, respectively.

Content Variables (See Figure 1b)

In both *Connected Mathematics* and *Math Thematics*, at each grade level the mathematical content is partitioned into eight pieces. In *Connected Mathematics* these pieces are called "units" and in *Math Thematics* these pieces are called "modules." The "look and feel" of *Connected Mathematics* and *Math Thematics* books are considerably

Figure 1a. Comparison of *Connected Mathematics* and *Math Thematics* — curriculum information

CURRICULUM INFORMATION		
Connected Mathematics	**Title**	*Math Thematics*
6-8	**Grades**	6-8
Glenda Lappan, Jim Fey, Susan Friel, William Fitzgerald, & Betty Phillips	**Authors**	Rick Billstein & Jim Williamson
Prentice Hall (2002)	**Publisher (Year)**	McDougal Littell (1999-2005)
• Assessment Resources • Blackline Masters & Additional Practice • Transparencies • Lesson Planner • Computer Test Bank for Assessment & Practice (CD-ROM) • Spanish Resources • Implementation Guide • Teacher's Resource Kit (includes manipulatives, dot paper, & other resources) • Student Materials Kit	**Ancillary Materials**	• Professional Development Handbook • Teacher's Resource Books (module-by-module teaching strategies, classroom management tips, & blackline masters) • Spanish Resources • Notetaking Masters • Workbook (selected pages from Teacher's Edition) • Workbook (for additional skills development) • Transparencies • Tutor Place™ (laminated cards to help students with skills) • Multi-Language Visual Glossary • Test and Practice Generator (CD-ROM) • Personal Student Tutor (interactive, student tutorial software pack) • Student Manipulative Kit • Overhead Manipulatives Kit
Developed, with funding from the National Science Foundation, to align with the NCTM *Standards* (1989)	**Context**	Developed, with funding from the National Science Foundation, to align with the NCTM *Standards* (1989)

different. The *Connected Mathematics* materials include eight soft-cover books (units) for each grade 6-8, with each unit being organized around a big idea — a cluster of related concepts, skills, procedures, and ways of thinking. There is a focus on one content strand within each *Connected Mathematics* unit, with students studying one mathematical topic deeply before moving to another. In contrast, the *Math Thematics* materials contain one hardbound book for each grade 6-8. Each book contains eight modules, and each module has a theme that connects the mathematical content to the physical or social world. Each *Math Thematics* module includes a focus on multiple content strands, with content being presented in a spiral fashion, where students continually review previously-learned material.

With more traditional textbooks students are generally given rules and worked-out examples of how to apply the rules, and then they practice those rules. Traditional books contain collections of facts and skills to be memorized or mastered by students. By contrast, *Connected Mathematics* and *Math Thematics* are both problem-based curricula — mathematical content is presented as a sequence of problems or tasks. In both *Connected Mathematics* and *Math Thematics* the majority of the problems are set in real-world contexts, and the materials first present mathematical content in concrete examples before providing abstraction and formalization of the mathematical content. Each curriculum places emphasis on developing meaning of mathematical ideas before practice and skill using those ideas. A striking difference between the curricula is that the *Connected Mathematics* student books contain only a few worked-out examples that demonstrate solution methods, and contain only a few formal definitions/rules outside of the glossary.[1] This treatment is in sharp contrast with the *Math Thematics* materials, in which each module contains a reference section that includes a summary of key concepts and worked-out examples.[2]

Instructional Variables (See Figure 1c)

The sequence of activities in traditional mathematics classrooms has been characterized by Fey (1979) as follows:

Figure 1b. Comparison of *Connected Mathematics* and *Math Thematics* — content variables

CONTENT VARIABLES

Characteristics Unique to *Connected Mathematics*	Common Characteristics	Characteristics Unique to *Math Thematics*
Eight units per grade, each in a soft-cover book	**Structural Organization** At each grade level the mathematical content is partitioned into eight pieces	Eight modules per grade in one hardcover book
Focus on one content strand within each unit; one mathematical topic is studied deeply before moving to another	**Depth/Breadth of Content** Emphasis on depth versus breadth of content coverage	Focus on multiple content strands within each module; content is presented in spiral fashion with continual review of previously-learned material
	Presentation of Content Mathematical content is presented as a sequence of problems/tasks motivated by real-world situations; Concrete examples are given before abstract formulation of ideas (students first learn "why" and then "how," i.e., understanding/meaning of concepts comes before practicing associated skills)	
Books contain few worked-out examples that demonstrate solution methods	**Worked-Out Examples**	Books contain numerous worked-out examples that demonstrate solution methods
Books contain few formal definitions/rules outside of the glossary	**Definitions/Rules**	Books contain many formal definitions/rules

Answers are given for the previous day's assignment, with the more difficult problems being worked by the teacher or students at the board. After a brief teacher-led presentation of new content and a few example problems being solved as a whole class, the remainder of the class time is devoted to students working on the homework while the teacher moves about the room answering questions. This sequence of activities, often referred to as the "transmission model of instruction," is based on the premise that students learn best by receiving information and practicing specific skills.

In contrast, reform mathematics curricula rest on the premise that students actively make sense of mathematical content. During class students are expected to investigate, discover, and make conjectures about mathematical ideas, reflecting the dynamic nature of what it means to "do mathematics." The teacher's role is that of a guide, or a facilitator, rather than a transmitter of knowledge. Students using reform mathematics materials are expected to engage in mathematical argumentation and produce mathematical evidence by talking or writing in ways that expose their reasoning to one another and to their teacher.

These characteristics of reform classrooms are consistent with the vision of mathematics teaching/learning as embodied in the *Connected Mathematics* and *Math Thematics* materials. Additionally, both sets of materials promote the "motivate[3] -explore-summary model of instruction." This model is characterized by the teacher first providing a "hook" to grab students' attention and relate the prior experiences of the students to the objectives of the lesson. In the explore phase students solve the problems presented in the curriculum materials, often working with other students in small groups. The summary phase provides closure by helping students bring mathematical ideas together in their own minds and make sense of what has just been explored. While the *Connected Mathematics* authors

[1] *Authors of Connected Mathematics encourage teachers to have students develop their own lists of definitions and examples because of their belief that students need to have descriptions of mathematical words that carry meaning at their level of verbal sophistication, which they can add to and refine as they gain new insight and encounter new examples.*

[2] *As with Connected Mathematics, each Math Thematics student book contains a glossary.*

Figure 1c. Comparison of *Connected Mathematics* and *Math Thematics* — instructional variables

INSTRUCTIONAL VARIABLES		
Characteristics Unique to *Connected Mathematics*	**Common Characteristics**	**Characteristics Unique to *Math Thematics***
The three phases occur each day; rare use of direct instruction	**Instructional Model** Three phrase instructional model: Motivate-Explore-Summary	The three phases occur over the course of several days; some use of direct instruction
Skill-based computational practice is reserved for homework	**Use of Class Time** Students work on problems from the curriculum	Some skill-based computational practice during classroom instructional time
Inquiry approach, with teachers providing only a little scaffolding	**Teacher's Role** Teachers are facilitators/guides	"Guided discovery" approach with teachers providing considerable scaffolding
	Students' Role Students actively make sense of the mathematical content by investigating, discovering, and making conjectures about mathematical ideas	
Students work in small groups 40-50% of the time	**Use of Small Group Work** Extensive use of small groups during class time	Students work in small groups 30-40% of the time
	Use of Tools Frequent use of manipulatives/technology	
	Assessment Multiple forms, both formal and informal, including student self and peer assessment	
	Homework Homework is assigned every night so students can practice content learned in class	

recommend that teachers incorporate these three phases into daily instruction, the *Math Thematics* authors recommend that teachers incorporate these three phases over the course of several days, with motivation provided one day, followed by several explorations over the course of the next several days, and then a summary.

The authors articulate other differences for how instruction with their respective curricula should play out. *Connected Mathematics* authors recommend only rare use of direct instruction, whereas *Math Thematics* authors recommend that teachers use some direct instruction of concepts and skills. *Connected Mathematics* authors believe that computational practice should be reserved for homework, whereas *Math Thematics* authors believe that some skill-based practice should occur during classroom instructional time. *Connected Mathematics* authors intend instruction to be less teacher-directed than *Math Thematics* authors, with *Math Thematics* authors using the phrase "guided discovery" to characterize instruction. *Connected Mathematics* authors recommend students work in small groups 40-50% of instructional time, and *Math Thematics* authors recommend small group work 30-40% of instructional time.

Consistent with the view of reform mathematics instruction outlined by NCTM (1989, 2000), *Connected Mathematics* and *Math Thematics* authors encourage teachers and students to make use of manipulatives and technology (as appropriate), and to use multiple forms of assessment (formal and informal, including student self and peer assessment). My talking with the lead authors of *Connected Mathematics* and *Math Thematics* revealed that they intend students to have homework every night in order to practice the content that was learned in class. Thus, for these curricular variables — tools, assessment, and homework — I found much in common, with little difference in philosophy across *Connected Mathematics* and *Math Thematics*.

[3] *In the Connected Mathematics materials, the motivate phrase is referred to as the "launch."*

Discussion

There is considerable public discourse and debate about different mathematics curricular approaches. What Reys stated several years ago bears repeating — it is time to move beyond the rhetoric and continuing controversy about various mathematics curricula and to "work together to improve children's mathematics education for the future" (Reys, 2001, p. 255). I believe one step in this direction is to discontinue the practice of lumping curricula into categories such as reform versus traditional, which disregards important differences between them. A second step (which is beyond the scope of this report) is to focus our energies on understanding how these differences differentially affect instructional practice and student learning. For example, what is the impact on students' learning when they are afforded concentrated time on one content strand before moving on to another (*Connected Mathematics*) versus a spiral approach with continual review of previously-learned material (*Math Thematics*)? The framework that I have developed and then have used to illustrate similarities and differences between *Connected Mathematics* and *Math Thematics* can be used by teacher leaders, and others who are responsible for choosing textbooks, to discern differences between any two curricula, reform or traditional. Figures 2a, 2b, and 2c contain a "stripped-down" version of the framework without reference to any specific curricula. Below I offer two specific uses of this framework.

- Textbook decision makers can complete the chart for textbooks being considered for adoption. Completing the chart, especially if done in a group setting, can result in productive discourse around curricular issues – discourse that moves beyond surface features of textbooks.

- Instantiations of the curriculum variables for the textbooks being considered can be examined to determine compatibility of the textbooks with teachers' beliefs about mathematics, and beliefs about teaching and learning mathematics. For instance, if teachers believe that students learn best by engaging in open-ended problems with minimal teacher guidance, then the following curriculum variables should be examined closely when considering a new textbook: presentation of content, worked-out examples, use of class time, teacher's role, and students' role.

Figure 2a. Framework to compare two mathematics curricula — curriculum information

CURRICULUM INFORMATION
Title *What is the title of the curriculum?*
Grades *What is the target grade range of the curriculum?*
Authors *Who are the authors of the curriculum?*
Publisher (Year) *What is the name of the publisher of the curriculum and in what year was it published?*
Ancillary Materials *What ancillary materials are provided by the publisher?*
Context *What was the funding source for the materials? To what extent do the materials align with the NCTM Standards?*

Figure 2b. Framework to compare two mathematics curricula — content variables

CONTENT VARIABLES		
Characteristics Unique to *Curriculum 1*	**Common Characteristics**	**Characteristics Unique to *Curriculum 2***
	Structural Organization *What are the physical features of the curriculum (e.g., number of units/modules per grade, softcover/hardcover)*	
	Depth/Breadth of Content *Is depth or breadth of mathematical content emphasized and how does this play out (e.g., "layer-cake"/spiral/integrated approach)?*	
	Presentation of Content *How is content presented (e.g., to what extent do students practice problems similar to worked-out examples vs. engage in a sequence of exploratory tasks; to what extent are problems set in real-world contexts)?*	
	Worked-Out Examples *What is the extent of worked-out examples?*	
	Definitions/Rules *What is the extent of definitions/rules? Where are they located (e.g., embedded in the text, glossary)?*	

Figure 2c. Framework to compare two mathematics curricula — instructional variables

INSTRUCTIONAL VARIABLES		
Characteristics Unique to *Curriculum 1*	**Common Characteristics**	**Characteristics Unique to *Curriculum 2***
	Instructional Model *What, if any, instructional model is explicitly articulated by the curriculum authors? What is the role of direct instruction?*	
	Use of Class Time *What is a typical lesson like (e.g., to what extent do students explore content, watch the teacher demonstrate procedures, work on computational practice during class time)?*	
	Teacher's Role *What is the role of the teacher during classroom instruction (e.g., what extent of scaffolding does the teacher provide)?*	
	Students' Role *What is the role of students during classroom instruction?*	
	Use of Small Group Work *To what extent do students work in groups?*	
	Use of Tools *To what extent are students expected to use manipulatives and technology?*	
	Assessment *What are major features of assessment (e.g., forms of assessment, formal/informal, self/peer)?*	
	Homework *What is the frequency and role of homework (e.g., to practice newly-learned material, to review previously-learned material)?*	

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Interactions With Curriculum: A Study of Beginning Secondary School Mathematics Teachers

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The design and dissemination of curriculum materials has been a major means of attempting to change classroom instruction, both historically and in recent years (Ball & Cohen, 1996; Davis & Krajcik, 2005; Remillard, 2005). The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics [Standards]* (1989) spurred the development of curriculum materials that were intended to help change both the content of school mathematics and the way that mathematics is taught in grades K-12. There is some evidence to suggest that these efforts have been successful (e.g., Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Senk & Thompson, 2003).

Research has also shown, however, that teachers' use of curriculum materials is shaped by, among other factors, their understanding of *Standards*-based practices, their ideas about a teacher's role in the classroom, and their ideas about students and student learning (Ball & Cohen, 1996; Wilson & Lloyd, 2000). Although practicing teachers often find it difficult to change established patterns of practice, beginning teachers have the opportunity to establish *Standards*-based teaching practices from the start. To support this potential opportunity, many *Standards*-based teacher education programs are following Remillard and Bryans' (2004) suggestion that they provide opportunities for future teachers to examine curriculum materials, to consider the mathematical and pedagogical assumptions implicit in their design, and to consider how these materials might be used in the classroom. Furthermore, based on the knowledge that the intern teaching experience is a powerful influence on teachers' future teaching (Brown & Borko, 1992; Evertson, Hawley, & Zlotnik, 1985; Parmalee, 1992; Tabachnick & Zeichner, 1984) and the increasing availability of classrooms that are using *Standards*-based materials, more programs have been able to place intern teachers in classrooms with teachers who are using these materials and are striving to teach in ways that are consistent with the vision in the *Standards* (National Council of Teachers of Mathematics, 1989, 1991, 2000).

It is well established that effective *Standards*-based teaching is difficult and requires ongoing professional development (Weiss, Arnold, Banilower, & Soar, 2001). However, it seems reasonable to expect that optimal conditions, such as those described above, would better prepare beginning teachers to implement *Standards*-oriented practices from the start, and thus, change the nature of support they would need from their school districts. In the interest of determining how such teachers can best be supported in their early years of teaching, our study investigates the teaching practices of *Standards*-prepared beginning teachers who expressed a desire to implement *Standards*-based practices. We first assess the extent to which they were able to act on their stated goals of implementing *Standards*-based teaching practices in their classrooms, and then turn our attention to ways in which the curriculum they used in their classroom supported them in doing so. We conclude by discussing implications of our findings for those charged with supporting the development of beginning mathematics teachers.

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Design of the Study

Participants

The participants in this study were seven second-year (novice) mathematics teachers (Beth[1], David, Elliot, Holly, Ingrid, Nicole, and Sarah) who had graduated from a mathematics teacher education program at a large Midwestern university, which was designed with the goal of preparing *Standards*-focused teachers who would serve as change agents in their future schools. During three 15-week mathematics education methods courses, these teachers were introduced to many of the concerns and methodologies of *Standards*-based mathematics teaching and worked with problems similar to, or actually from, *Standards*-based curriculum materials, such as the Core-Plus Mathematics Project (CPMP) or Connected Mathematics Project (CMP). In addition to focusing on *Standards*-based content, the courses themselves were taught with a *Standards*-based pedagogy focused on analysis and providing evidence to support conclusions.

All of the participants had been "good students" in their methods courses, as evidenced by their course grades and the assessment of the instructor of the third methods course. To minimize the possibility of the intern teaching experience negating what had been learned in the methods courses (Ball, 1990; Guyton & McIntyre, 1990), the participants were placed with reform-minded classroom teachers for their semester-long teaching internship. Some of the participants (Sarah, Nicole, Dave, and Holly) interned in classrooms that strictly used CPMP or CMP curriculum materials, while others (Ingrid, Beth, and Elliot) were placed with mentor teachers who used multiple textbook series (see Table 1 for specifics). Prior to the intern teaching placements, the mentor teachers had all participated in at least some professional development connected to either CPMP or CMP curriculum materials through a National Science Foundation-funded Local Systemic Change (LSC) project.

The participants who referred to their intern teaching curriculum in interviews prior to the internship all expressed excitement about using *Standards*-based curriculum materials. In particular, they talked about how the materials would allow them to be a facilitator, rather than a traditional teacher lecturing from the board. Dave and Elliot both said that the materials would fit their teaching style, while Ingrid said that they would be a really good tool for her. Both she and Sarah said the materials would allow them to be better teachers. In addition, they contrasted these materials with other materials that they felt would involve much less thinking on the part of the students and much more preparation work on the part of the teacher to design *Standards*-based instruction. Nicole reflected the general feelings of the group when she said that using *Standards*-based materials in her internship was "a big positive."

[1] *All names are pseudonyms.*

Table 1. Textbook Series Used in Internship

	Intern teaching textbook series	Beginning teaching textbook series
Dave	CPMP	CPMP
Holly	CPMP	CPMP
Ingrid	CPMP UCSMP	CPMP
Nicole	CMP	CPMP
Elliot	CPMP UCSMP	CMP
Beth	CPMP UCSMP	Merrill
Sarah	CPMP	UCSMP (8th grade)

CMP: Connected Mathematics Project (http://www.math.msu.edu/cmp);
CPMP: Core-Plus Mathematics Project (http://www.wmich.edu/cpmp/);
Merrill: Glencoe/McGraw-Hill (http://www.glencoe.com/);
UCSMP: University of Chicago School Mathematics Project
(http://social-sciences.uchicago.edu/ucsmp/Secondary.html)

Data Collection and Analysis

The data used in this study was collected as part of a four-year longitudinal project. The intent of the larger study was to examine the effects of pre-intern and intern teaching in a *Standards*-based environment on mathematics teachers' future teaching, belief structures associated with the teaching of mathematics, and job preferences and selection. Although the longitudinal study included extensive data from the participants' last two years of university coursework and their first two years in the classroom, the study reported here focuses on only interviews and classroom observations from their novice (second) year of teaching — after the teachers had completed their first "survival" year of teaching and had begun to establish patterns of instructional practice.

Each participant was observed for three consecutive teaching days, and was interviewed by the observer before and after each observation; the observer documented and videotaped each class session. The pre-observation interview questions focused on the teacher's objectives for

Figure 1. Capsule Descriptions of the Overall Quality of the Lesson (Horizon Research, Inc., 2000)

<table>
<tr><th colspan="2">Level 1:
Ineffective
Instruction</th><th>Level 2:
Elements of
Effective Instruction</th><th>Level 3:
Beginning Stages of
Effective Instruction</th><th>Level 4:
Accomplished,
Effective Instruction</th><th>Level 5:
Exemplary
Instruction</th></tr>
<tr><td colspan="2">There is little or no evidence of student thinking or engagement with important ideas of mathematics. Instruction is highly unlikely to enhance students' understanding of the discipline or to develop their capacity to successfully "do" mathematics.</td><td rowspan="2">Instruction contains some elements of effective practice, but there are serious problems in the design, implementation, content, and/or appropriateness for many students in the class. Overall, the lesson is very limited in its likelihood to enhance students' understanding of the discipline or to develop their capacity to successfully "do" mathematics.</td><td rowspan="2">Instruction is purposeful and characterized by quite a few elements of effective practice. Students are, at times, engaged in meaningful work, but there are weaknesses, ranging from substantial to fairly minor, in the design, implementation, or content of instruction. Overall, the lesson is somewhat limited in its likelihood to enhance students' understanding of the discipline or to develop their capacity to successfully "do" mathematics.</td><td rowspan="2">Instruction is purposeful and engaging for most students. Students actively participate in meaningful work. The lesson is well-designed and the teacher implements it well, but adaptation of content or pedagogy in response to student needs and interests is limited. Instruction is quite likely to enhance most students' understanding of the discipline and to develop their capacity to successfully "do" mathematics.</td><td rowspan="2">Instruction is purposeful and all students are highly engaged most or all of the time in meaningful work. The lesson is well-designed and artfully implemented, with flexibility and responsiveness to students' needs and interests. Instruction is highly likely to enhance most students' understanding of the discipline and to develop their capacity to successfully "do" mathematics.</td></tr>
<tr><td>Passive "Learning": Instruction is pedantic and uninspiring. Students are passive recipients of information from the teacher or textbook; material is presented in a way that is inaccessible to many of the students.</td><td>Activity for Activity's Sake: Students are involved in hands-on activities or other individual or group work, but it appears to be activity for activity's sake. Lesson lacks a clear sense of purpose and/or a clear link to conceptual development.</td></tr>
</table>

the class that was to be observed, as well as the teaching strategies he or she planned to use to meet these objectives. The post-observation interviews asked the teacher to reflect on the teaching session and to explain the thinking behind some of the instructional decisions he or she was observed to make. In the final post-observation interview, each participant was also asked more general questions about his or her experiences as a beginning teacher.

The LSC Observation Instrument (Horizon Research, Inc., 2000) was used by a Horizon-certified independent evaluator to rate the quality of the participants' videotaped teaching sessions. Each teaching session was rated on factors that have been found to enhance students' understanding of and success in doing mathematics, including student engagement with content, classroom culture, and lesson design and implementation (Weiss & Pasley, 2004). In addition, each lesson was given a summary rating from 1-5, descriptions of which are given in Figure 1. The pre- and post-observation interviews were audiotaped, transcribed, and coded to identify dialogue related to instructional planning, classroom activity, student thinking and understanding, and the participants' interactions with their classroom curriculum.

Success in Implementing Standards-Based Teaching Practices

The table on the next page shows the median rating that each participant received for their overall instruction. No ratings on individual observations deviated by more than one from the median value.

Based on the Horizon ratings, Dave, Holly, and Ingrid were described as being in the beginning stages of effective instruction. Their lessons involved less teacher telling than those of the other participants, provided more opportunities for students to engage in investigative tasks, and involved more collaboration between the teacher and his or her students. The general instructional pattern in these classrooms was a whole-group launch, an extended time for student investigation in small groups, and finally a whole-group discussion and summary.

Table 2. Median Instructional Ratings

	Overall Quality of Lesson
Dave	3
Holly	3
Ingrid	3
Nicole	2
Elliot	1
Beth	1
Sarah	1

Nicole was described as exhibiting at least some elements of effective instruction. Although her classroom followed a similar instructional pattern, a major difference between her instruction and the highest rated students was that she was not observed encouraging students to challenge each others' ideas or provide justifications for solutions. Another distinctive difference was her use of questioning. Nicole often asked her students questions, but she was observed to answer most of them herself.

Beth, Elliot and Sarah's Level 1: *Ineffective Instruction* ratings indicated that their practices were highly unlikely to enhance students' understanding of mathematics (Horizon Research, Inc., 2000). The Horizon instrument differentiates the reasons for a Level 1 rating as either "passive learning" or "activity for activity's sake" (see Figure 1). Sarah's instruction was described as passive learning on two of her three observations, while Beth and Elliot's instruction was characterized as activity for activity's sake. The lessons of these participants were teacher-directed and provided little opportunity for students to engage with mathematical ideas. In general, students worked on a number of short tasks during the class period and then checked their answers, as opposed to working for longer periods of time on challenging tasks that required group discussion and the sharing of ideas. This pattern is strikingly different than the pattern observed in Dave, Holly and Ingrid's classrooms.

In their interviews, the four highest-rated teachers expressed their concern for student thinking. Dave and Nicole spoke about a need to let students struggle a bit with new material and were comfortable letting students do so. Holly spoke of the importance of getting students involved in activities where they had to do the thinking. Ingrid talked about her desire to make multiple ideas public, saying, "Everybody thinks of things differently and so to hear more viewpoints rather than just from the same people who think in the same way, might open a door or put a light on for another student."

In contrast, Beth, Elliot and Sarah had a common focus on getting correct answers, often using a single teacher-prescribed method. For example, Sarah often had students present their solutions at the board; however, the focus of these presentations was on the procedures the students used to compute their answers, rather than on the thinking behind them. Sarah expressed the desire to have her students learn the "right" way to do things. At one point in a lesson, a student began presenting a method that the class had not yet learned, and Sarah said, "No, no, no!" to stop his presentation. When asked about this action, she told the interviewer that hearing about a different way to solve the problem would confuse her students.

Even though the seven teachers in this study successfully completed the same *Standards*-based teacher preparation program, intern taught using *Standards*-based curricula, and verbalized visions of teaching aligned with the *Standards*, their beginning instruction varied from ineffective to beginning stages of effective teaching—as measured by the LSC instrument's *Standards*-based criteria. This raises the question of what contributed to these differences. During our analysis, the curriculum materials they used in their beginning teaching classrooms and the relationship between those materials and the beginning teachers' visions of mathematics teaching emerged as critical factors.

Interactions with Curriculum Materials

Three distinct groups of teachers emerged from the data — those for whom their curriculum and vision of teaching were in clear alignment (Dave, Holly, Ingrid, Nicole), those for whom the alignment was ambiguous (Elliot), and those for whom there was an obvious mismatch between curriculum and vision (Beth, Sarah). Furthermore, there appeared to be a relationship between these groupings and the instructional ratings. In the following, we highlight the different ways that the novice teachers *participated with the curriculum* (Remillard, 2005) used in their classroom in pursuit of their vision of teaching.

Clear Alignment

The four most effective instructors all used the CPMP materials. These materials center instruction around investigations that promote student thinking and allow for multiple solution strategies, and thus represent an alignment with the teachers' stated vision of teaching. They also include extensive teacher guides that provide the teacher with more information and ideas to assist them in using the curriculum than do the teacher guides available with most traditional mathematics textbooks (Lloyd, 2002a). As all reported using the teacher guides to at least some extent, this may have been one factor contributing to these teachers' more effective instruction. This is not to say, however, that all of these instructors used the CPMP curriculum in an identical manner. Instead, each instructor engaged with the curriculum and adapted it in ways that they felt would best support their students' learning; this finding is consistent with other research (Remillard & Bryans, 2004).

The major changes to the curriculum identified in Dave, Holly and Ingrid's classrooms related to the problems that they assigned students both in and out of class. Dave said that he tended to not assign the 'extending problems' in the textbook very often, while Holly spoke of assigning extra homework problems — pulled out of a more traditional textbook — on topics with which her students were struggling. Ingrid, on the other hand, rearranged the lesson slightly so that the checkpoint questions were incorporated into the investigation rather than used as a distinct opportunity at the end of the investigation to reflect on the learning that had occurred. She also talked about occasionally writing her own review worksheets for the end of a unit, and periodically assigning additional challenge problems for students who wanted to earn extra credit. Although these three instructors all altered the curriculum in some way, none of them made significant changes to the student investigations that form the core of each lesson in this curriculum. In other words, these teachers adapted the materials for use with the students in their classroom in ways that didn't undermine the stated instructional goals of the materials.

Nicole, on the other hand, altered the curriculum in a quite different way. In one of the observed lessons, Nicole rephrased the questions in the investigation, reducing it to a step-by-step worksheet. She justified these changes by explaining that her previous class had struggled with the investigation. Nicole hoped these changes would give her students the more concrete guidance she thought they needed. As has also been found to be the case in other studies (e.g. Ball & Cohen, 1996; Manouchehri & Goodman, 1998), the changes Nicole made to the curriculum were in response to her ideas about what her students brought to the classroom; these included her beliefs about her students' mathematical background and their ability to persevere in solving a problem. Although these changes were well-intended, the effect of such alterations was a reduction in the challenge and investigative nature of the task. This has been shown to be detrimental to student learning, as it provides less opportunity for student thinking and for students to develop a conceptual understanding of the mathematics (Smith, 2000; Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998). By changing the curriculum in ways that were not consistent with the stated goals of the curriculum, Nicole actually created a substantially different learning experience for her students.

The four teachers who used the CPMP curriculum materials all expressed a high degree of satisfaction with them. In fact, based on their positive experiences with them during their intern-teaching experience, three of the four had intentionally sought out teaching positions where they would be using such materials. The participants were also aware of how the curriculum materials influenced their practice. Nicole said that she couldn't imagine what her practice would look like were she not using the CPMP curriculum materials, since using them made it easy for her to teach in the way she wanted. Dave shared this opinion, saying that he loved CPMP and hoped that someday something even better would come along. Holly considered herself somewhat of an ambassador for the CPMP program, both in her own school and with teachers in other schools; she talked to other teachers, parents, and even school board members about the materials' positive effect on student learning. These teachers' positive attitudes and strong belief in the benefits of the CPMP program contributed to their ability to implement the materials with some degree of success. This is consistent with other research findings regarding the influence of teachers' beliefs about curriculum materials on how they use the materials in their practice (Davis, 2004; Lloyd, 2002a, 2002b; Remillard, 2005).

Talking the Talk

Although Elliot used CMP, a middle-school curriculum that is very similar to CPMP, his practice was quite different than that of Dave, Holly, Ingrid and even Nicole. In his final interview, Elliot said:

> Connected Math (CMP) is an awesome curriculum to be a teacher of, because it's all there for you. It shows you how it relates to the Standards. It shows you all that; it's all there, all ready for you. It's awesome for a first-year teacher to teach. It's an incredible amount of stuff that I was able to learn through this.

In other interviews, Elliot echoed this enthusiasm for the CMP curriculum materials. He said that CMP "does what no other curricul[a] in the past have done…it gives the kids exposure to a lot of material that they never would have seen before at this level," and later added that he loves teaching the "concept math" — he didn't think that he would be able to teach any other way. Elliot also spoke about the climate in his classroom, calling it "amazing." Here he particularly focused on the expectation that students will explain their solutions to each other, and the need for them to carefully listen to each other, since another student could have a "better way" or a "shorter way" to solve a problem.

Based on Elliot's comments, it would seem reasonable to assume that he would wholeheartedly embrace the CMP curriculum and carefully follow both the student curriculum and the suggestions for instruction outlined in the teacher's guide. This was not the case, however; in fact, the alignment between this curriculum and his vision of teaching was quite ambiguous. In the classes that were observed, Elliot did not teach from the CMP curriculum materials. Instead, he used more traditional materials that he had copied from another textbook to expose his students to the kind of material he believed would prepare them for their high school courses. In particular, Elliot was observed encouraging his students to model their solutions after the examples that were presented in the supplementary materials. When questioned about this practice, Elliot said that he wanted his students to learn to use printed resources rather than asking him how to solve problems. This suggests that although Elliot did not want to be the mathematical authority (Wilson & Lloyd, 2000) in his classroom, he also did not expect students to struggle to come up with ways to solve problems using their own thinking.

Note that Elliot's substitution of curriculum materials is quite different from the way that Nicole altered the curriculum by rewriting lessons. In Elliot's case, he did not just *adapt* the curriculum, but rather he *replaced* the curriculum with more traditional materials. He said that doing so allowed his students to see the "other side of the math spectrum," noting that it was a "nice way for them to evolve" by seeing that they can learn mathematics this way, too. When asked to elaborate, he said that he thought it was good for his students to see more traditional instruction and be exposed to drill and practice. Elliot's actions support Remillard's (2005) observation that a school's adoption of a single curriculum does not guarantee uniform instruction.

One might ask why Elliot felt that his students needed this exposure, given his enthusiasm about the CMP curriculum. In fact, it may be that Elliot did not feel as positive towards the CMP curriculum as his language would lead one to believe. Although when asked specifically about the curriculum, Elliot "talked the talk," possibly saying what he thought the interviewer wanted to hear, he made comments at other points in the interview that were in stark contrast to those that expressed a positive view of the curriculum. He said that students sometimes get bored with CMP, and that they needed an opportunity to "rise to the top" and show that they were ready for algebra. One concern that Elliot expressed was that he was preparing kids to fail by using too much cooperative learning when they were going to be subjected to a more traditional curriculum at the high school level. He added that his students got tired of explaining, having to go the extra mile. His top students, especially, were "just traditional math students…they need the drill and practice; that's how they want to learn." He felt that there was not enough of this type of learning in the CMP curriculum and thought that the students' basic skills were going to be weak in the long run. He justified the use of short procedural questions to "drill it into their brains," as compensation for what he saw as the lack of practice in the CMP materials. Elliot sums up his beliefs in the following dialogue:

> I think that for an advanced math class, for about 75% of the kids, it's not right for them. Because the real traditional, hard core math students can learn faster, can learn more, by doing it the traditional way. And that's one of the weaknesses, I think, of Connected Math.

The case of Elliot illustrates that using *Standards*-based curriculum materials is not sufficient on its own to ensure effective *Standards*-based instruction. Instead, the use of such curricular materials is mediated by teachers' beliefs about learning mathematics and the needs of their students (Wilson & Lloyd, 1995). This assertion is supported by other research. Ball and Cohen (1996) claim that how teachers enact a curriculum is influenced by what they think about their students and by what they perceive to be their students' views of the content, while Manouchehri and Goodman (1998) discuss the challenge that a teacher faces in balancing the development of conceptual knowledge of mathematics and the development of algorithmic knowledge. Lloyd (1999) adds that the relationship between the teacher and the curriculum can become strained when there is a conflict between the structure and practices outlined in the curriculum and the teacher's perceived need to change the curriculum in response to students' needs. As is the case with Elliot, many teachers in Manouchehri and Goodman's (1998) study felt an obligation to prepare students for algebra. They felt that the *Standards*-based curriculum was not adequately addressing this need, since it lacked skill-oriented exercises. The findings of Chavez (2003) sum up what we have observed in the case of Elliot and, to a lesser extent, of Nicole: "It is possible to 'adopt' a textbook and use it frequently without really espousing the epistemological assumptions that are attached to the textbook, and thus not change teachers' practices in ways that would better match the goals of the particular curriculum" (p.160).

Seeing it from the Outside

The final two participants, Beth and Sarah, were outspoken about how their curriculum materials hindered their ability to implement *Standards*-based practices. For them, there was an obvious mismatch between curriculum and stated vision. Beth felt overwhelmed by her perceived need to look for supplementary materials in other textbooks on a daily basis. Despite the significant effort this task required, she felt that it was necessary since her textbook was too traditional and offered limited opportunities for problem solving. To remedy this deficit, Beth wrote her own worksheets and investigations to include more open-ended problems in her instruction. Sarah also expressed frustration about the limitations of the curriculum that she used, but did not supplement it in the same way as Beth. Sarah said that she wished she could include more investigations and group work, but felt tied to the curriculum that her school had chosen.

Although Beth tried to adapt her curriculum to allow for discovery and student thinking, she was also concerned because doing so had caused her to fall two weeks behind the other instructors in her department. Given that her department had a common final examination based on the objectives for the course, she felt that she had to curtail some of her efforts in order not to disadvantage her students. She said that she "would love to go further in depth (working with cubic polynomials)…but I've got to get this chapter in." She added that activities were difficult to fit into the curriculum she was using and that teaching would be easier for her if she had a good curriculum to support her efforts.

A focus on following the curriculum and meeting objectives mandated by the district was also a driving force in Sarah's practice. She said that she tried to go as in-depth as possible by including some student investigations, but that both her textbook and her list of objectives were "huge." Sarah was worried about the potential consequences of not following the curriculum, saying "I do what I'm told so I can say, 'Well, I did what I was supposed to'." She closely followed her textbook to ensure that her students met all of the course objectives before the end of the school year so that she didn't "get blamed for certain things." Whether these fears were warranted or not, it was clear that they affected Sarah's practice. It is also possible that Sarah, like Elliot, "talked the talk" of *Standards*-based instruction while holding beliefs that would conflict with the goals of *Standards*-based curricula—such as that multiple solution methods would confuse students. Unlike Elliot, however, all of Sarah's comments that seemed to reflect such beliefs occurred as she was explaining the instructional decisions she made while using a non-supportive curriculum.

In a previous study, it was found that a teacher's experience with *Standards*-based materials allowed him to view his own traditional practices in a more critical way and to better articulate his need to make changes to his instruction (Lloyd, 1999). Through their teacher education program, Sarah and Beth developed a critical view of practice, as evidenced by their repeated talk about the ways in which they would like to change their practice. In particular, both expressed the desire to include more investigations, group work, and opportunities for student thinking. Without a curriculum that provided the necessary support to do so, however, neither was able to teach in the way she envisioned. Beth summed up her frustration by saying, "I felt like I was taught all these wonderful things and all these wonderful methods, but unless I have a curriculum to support it, it's hard. I mean, I try. I honestly do try." Despite her best efforts, however, Beth's instructional ratings indicate that her teaching fell short of the *Standards*-based instruction she experienced during her university methods courses and intern teaching.

Conclusions

It seems reasonable to expect that novice teachers whose university coursework and field experiences allowed them to think about and be involved in *Standards*-based practices would be better able to implement these ideas in their classrooms. Although the level of observed instruction was somewhat disappointing, it is not entirely surprising given the many challenges faced by new teachers and the difficulty even experienced teachers have meeting the high expectations of the *Standards* measured by the LSC instrument (Weiss, Arnold, Banilower, & Soar, 2001). This study suggests that an alignment between university coursework and field experiences is not enough. Even with such an alignment, the *Standards*-prepared beginning teachers in our study had difficulty implementing *Standards*-based instructional practices without access to curriculum materials supportive of such instruction. The teachers in our study who used CPMP materials in both their internship and their beginning teaching displayed the most elements of effective practice. This highlights the potential value of extending the alignment of curriculum to include university coursework, intern teaching and beginning teaching.

Similar to previous findings (Manouchehri & Goodman, 1998; Remillard & Bryans, 2004), however, we also found that using *Standards*-based curricula is not a panacea. Instead, a teacher's use of such materials is mediated by his or her beliefs about the materials and about the needs and capabilities of his or her students (Spillane, 2001). Our study supports Lloyd's (2002b) finding that a teacher's "receptivity to a particular innovation" depends on how well the innovation "fits" with the teacher's perceptions about teaching and learning. One of the challenges for those who work with prospective and beginning teachers, then, is to not only provide them with *Standards*-based materials, but also to address their beliefs about student learning and how these beliefs might support or inhibit their use of such materials. At the preservice level, this can be done by engaging preservice teachers in an explicit examination of the relationships among their past experiences, current beliefs and future teaching. Ongoing work at the inservice level can build on this foundation through professional development that requires teachers to examine their actions, and the relationship between those actions and their assumptions about teaching and learning. Existing professional development materials (e.g., Grant, Kline, & Van Zoest, 2001; Seago, Mumme, & Branca, 2004; Stein, Smith, Henningsen, & Silver, 2000) can provide a starting place for designing such work.

It appears that issues of fidelity to and adaptation of curricular materials also need to be addressed directly. The teachers in this study made changes to their curricula with the best of intentions, but they did not seem to have a clear sense of the stated goals of the curriculum and how their changes might affect the success of meeting those goals — that is, the difference between productive and fatal adaptations (Seago, 2007). For *Standards*-based curricular materials to be used to their fullest, teachers must be provided support in finding the balance between meeting the needs of their specific students and remaining faithful to the goals of the curriculum (Drake & Sherin, 2006). When *Standards*-based curricular materials are introduced in a university methods course, discussing the curriculum development process provides an opportunity to highlight the difference between the expertise of a beginning teacher and that of the curriculum authors. For example, a beginning teacher will know his or her students better than the authors and be able to judge whether or not a specific context will interfere with their learning. Adaptations that remove barriers, such as explaining or substituting a context, are likely to be productive. On the other hand, given the expertise of the curriculum author teams and the careful thought put into the sequencing of the mathematics topics, a beginning teacher's changes to the ordering of the lessons would more likely be fatal than productive. As the beginning teachers learn about their students and the specific mathematical goals of their schools and courses, conversations that examine potential adaptations—in light of their likelihood of meeting site-specific goals without undermining the goals of the curriculum materials themselves—can continue as part of ongoing professional development.

It is encouraging to note that even those participants who were hampered in their ability to implement the ideas from their teacher education program by their unsupportive curriculum were aware that there were other options, and expressed dissatisfaction with their current situation. Because of their experiences in the teacher education program, these teachers were able to view their practice in a more critical manner and to look at their curriculum in a way that might otherwise have been "invisible" to them (Lloyd, 2002a). Although this does not immediately result in the type of instruction envisioned in the *Standards*, it does seem to be a promising first step, especially if dissatisfaction leads to action. In fact, such dissatisfaction and a vision of a different way of teaching mathematics may position beginning teachers to join with colleagues in becoming change agents in their schools.

This research highlights the value of *Standards*-based curriculum materials in the development of classrooms reflective of the *Standards*. Not only does it point to the potential of using such materials in preservice teacher education, but also to the impact such materials can have on beginning teachers' ability to put the knowledge and skills they have gained as part of a *Standards*-based teacher education program into practice in their permanent teaching positions. Although not a solution in and of themselves, *Standards*-based curriculum materials are a critical piece in the complex puzzle of teacher preparation and the ongoing development of effective instructional practices. Further research into ways in which these materials can best support teachers, and conversely, the ways in which teachers need to be supported in order to implement such materials well, will inform the efforts of curriculum developers, teacher educators, and mathematics supervisors to improve learning at all levels.

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Gauging the Relative Effects of Reform-Based Curriculum Materials and Professional Development in Promoting Changes in Teacher Beliefs

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Following the publication of the *Curriculum and Evaluation Standards* (NCTM, 1989), three influences emerged as primary vehicles in promoting the mathematics reform movement: professional development, curriculum materials, and assessment (Smith & O'Day, 1991). Numerous research studies have examined the effects of two of these influences—professional development and curriculum materials--in assisting teachers to adopt or transition to a reform perspective in their classroom practice (Anderson, 1995; Cai, Watanabe, & Lo, 2002; Chavez, Reys & Reys, 2004; Herbel-Eisenmann, & Wagner, 2005; Herbel-Eisenmann, Lubienski, & Id Deen, 2006; Hirsch, Lappan, Reys, & Reys, 2005; Lloyd, & Herbel-Eisenmann, 2004; Remillard, 2000;). As the reform movement has gained momentum through the influence of such documents as *Principles and Standards for School Mathematics* (NCTM, 2000) and *Adding it Up* (Kilpatrick, Swafford, & Findel, 2001) many mathematics educators have become interested in the interplay between the influence of professional development and the influence of curriculum materials—both based on a reform perspective. Several studies have instigated an examination of the interactive relationship between these two influences.

Collopy (2003) suggests that reform-oriented curricula even without accompanying professional development possess a transformative potential, but Orrill and Anthony (2003) offer a different perspective demonstrating that curriculum materials based upon reform pedagogy are ineffective in promoting reform unless accompanied by professional development. Cohen & Ball (2001) concur stating that, ". . . curriculum materials can not determine the curriculum of the classroom and innovative curricula alone can not produce instructional improvement." (p. 74) This conclusion is due to the wide variation inherent in curricular implementation (Chval, Grows, Smith, Weiss, & Ziebarth, 2006) which implementation is in turn dependent upon teachers' orientations towards the curriculum materials (Remillard & Bryans, 2004). Hence, Ziebarth (2003) concludes that the wise use of both curriculum materials and professional development are needed in promoting real reform.

Bay, Reys, and Reys (1999) posit that one way to synergize the influences of professional development and curricula is to provide the opportunity to select curriculum materials as part of professional development. Reys and Bay-Williams (2003) later observed an interactive relationship between these two influences and Remillard (2005) has developed a framework for examining teacher interaction with curricula. Questions still remain, however, concerning the relative effects of professional development and curricula. For example, which has a greater effect? If the influence of curriculum materials is marginalized without accompanying professional development, is the opposite condition also true, i.e., is the influence of professional development marginalized without the support for implementation offered by curriculum materials?

In conjunction with our professional development work, we conducted a quasi-experiment regarding the role of curriculum materials in affecting reform. We were teaching two groups of elementary teachers in a two-year, school-wide professional development program consisting of 18 graduate level credits in mathematics content, curriculum, assessment, and pedagogy that leads to a license endorsement. One distinguishing characteristic between the two groups concerned the curriculum materials used by the teachers in each group. One group had been using a reform curriculum for five years, *Investigations in Number, Data, and Space* (TERC, 1998), while the other was using a more traditional text program. Many of the teachers in the former group also reported participating in district-sponsored workshops designed to support their use of Investigations. We therefore concluded that because both groups were receiving the same fundamental professional development, we were in a position to examine the relative effects of the use of two vastly different curricula in the context of professional development.

In order to examine the effects of these differing curricula, we decided to focus on teacher beliefs. Beliefs are frequently defined as dispositions to act (Cooney, Shealy, & Arvold, 1998) and have become a common way to examine the effects of mathematics teacher education practices (Civil, 1993; Mewborn, 2000; Pajares, 1992; Vacc & Bright, 1999).

Philipp, et. al. (2007) developed a set of seven beliefs that reflect a current reform perspective and have categorized them under three main headings: beliefs about mathematics, beliefs about learning or knowing mathematics, and beliefs about children's learning and doing mathematics. The beliefs are listed as follows:

Belief About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

Therefore, we were interested in how the use of different curricula could affect changes in the above beliefs among the teachers in our two professional development groups. Our research question became: How does the use of differing curriculum materials affect the belief changes of teachers involved in professional development? Specifically, we asked these questions in order to address our overall research question:

1. How did the teachers' beliefs in the two groups differ prior to the professional development?
2. How did those beliefs change in the course of the professional development?
3. How did the teachers' beliefs in the two groups differ after one year of professional development?

Method

Because the use of a reform curriculum was, in effect, a treatment of sorts, and because the subjects were not randomly assigned to groups, a Nonequivalent Pretest-Posttest Control Group Design (McMillan & Schumacher, 1984) was used to investigate our research question. There were 15 teachers who used a reform curriculum, referred to in this paper as the "Reform Curriculum Group" and another 15 teachers who used a more traditional curriculum, referred to as the "Traditional Curriculum Group." The teachers in each group taught in elementary schools within different school districts but within the same general intermountain area.

As mentioned previously, each group was engaged in essentially the same two-year, 18-credit hour professional development program leading to a license endorsement. This program consisted of two-hour, graduate level courses in pedagogy, assessment, curriculum, child development related to mathematics, technology, and two four-hour courses in mathematics content. Participants were given readings in mathematics education research and research-based materials (e.g., *Beyond Classical Pedagogy*, Wood, Nelson, & Warfield, 2001), engaged in numerous class-based activities (e.g., mathematical investigations, case studies, discussions, video analyses), and assigned reflective and research papers of various sizes and purposes, all focused on application to classroom practice. Many of the participants were also enrolled in a Master's Degree program for which the professional development courses served as credit.

The *Integrating Mathematics and Pedagogy (IMAP) Survey* (Philipp, et. al., 2007) was used to measure the degree or intensity to which respondents possess the seven beliefs identified. It presents written or video cases to which teachers are asked to respond. The responses are then analyzed via rubrics based on three-, four-, or five-point scales thus allowing for inferences to be made about the intensity of the beliefs held by those taking the survey. The following is an excerpt from the browse version of the survey available at http://www.sci.sdsu.edu/CRMSE/IMAP/pubs.html (IMAP: Integrating Mathematics and Pedagogy Publications/Presentations [Browse the Survey], retrieved September 21, 2007).

> (Respondents view a video in which a teacher uses a "teaching as telling" process to teach a child the procedure for dividing fractions.)
>
> 9.1 Please write your reaction to this videoclip. Did anything stand out for you?
> 9.2 What do you think the child understands about division of fractions?
> 9.3 Would you expect this child to be able to solve a similar problem on her own 3 days after this session took place? Explain your answer.
>
> (Respondents now view another videoclip in which the same teacher asks the same child to solve another dividing fractions problem and the child has no idea on how to do so.)
>
> 9.4 Comment on what happened in this video clip. (NOTE: This interview was conducted 3 days after the previous lesson on division of fractions.)
> 9.5 How typical is this child? If 100 children had this experience, how many of them would be able to solve a similar problem 3 days later? Explain.
> 9.6 Provide suggestions about what the teacher might do so that more children would be able to solve a similar problem in the future.

Of interest is the fact that the IMAP Survey was designed for use with preservice students. We are among the first researchers to use it in gauging belief changes in the context of work with inservice teachers (see Bahr & Monroe, 2008)

We invited the teachers to complete the survey twice — once after a year of professional development that served as a post-measure, and then a second time in a retrospective manner (i.e., to complete it as if they were doing so prior to our professional development work). Cantrell (2003) demonstrated the validity of retrospective pre-measures in assessing the beliefs of preservice students. These measures address the problem of response-shift bias (Aiken & West, 1990; Cronbach & Furby, 1970), and as a result, tend to produce gain scores with greater validity and greater statistical power (Bray, Maxwell, & Howard, 1984; Howard et al., 1979). Therefore, we felt justified in the use of the survey in our work with inservice teachers. We could have waited until the second year to administer the survey as a post-measure, but were anxious to examine the effects of our work after the first year in order to further inform our professional development work.

Results

This section will be organized according to the research questions previously outlined.

How did the teachers' pre-professional development beliefs in the two groups differ? We first analyzed the group means obtained from the retrospective pre-survey scores using an analysis of variance to determine whether or not there were pre-existing differences between the two groups relative to any of the seven beliefs. Inasmuch as the data obtained from the survey is not ordinal, but rather interval in nature, the use of distribution-dependent statistical procedures, such as t-ratios, would ordinarily be inappropriate tools for analysis. However, Philipp, et. al. (2003) demonstrated the validity of using these distribution-dependent procedures for analyzing data obtained from the Beliefs Survey. He did so by analyzing differences between groups via the Beliefs Survey using a polychotomous log-linear ratio method and then re-analyzing those differences using t-ratios. Both analysis procedures eventuated in discovering the same number of significant differences between the groups he studied. Therefore, because distribution-dependent analyses are more commonly used in quantitative studies and thus are more commonly understood, and because they have been shown to yield the same statistical conclusions with data obtained from the Beliefs Survey, we used them for purposes of our study.

Table 1 displays the results of those analyses, and not surprisingly, the means obtained from the reform curriculum group differed significantly from the traditional curriculum

Table 1. Comparisons of Pre-Survey Means for Reform Curriculum and Traditional Curriculum Groups

		Group Means				
Belief	Rubric Range	Reform Curriculum Group	Traditional Curriculum Group	df	F	*p*
1	0-4	1.067	0.188	29	8.96	.005
2	0-5	0.733	0.813	29	0.04	.843
3	0-4	2.800	0.313	29	100.4	<.001
4	0-4	2.667	0.500	29	45.17	<.001
5	0-5	2.333	0.5623	29	22.51	<.001
6	0-5	2.533	0.563	29	36.84	<.001
7	0-4	1.333	0.375	29	5.88	.022

group on six of the seven beliefs in favor of a more reform-oriented perspective.

How did those beliefs change in the course of the professional development? We compared the pre- and post-belief score means of each group using an analysis of variance procedure in order to determine if the teachers within each group experienced significant belief changes. The results are displayed in Tables 2 and 3.

As shown, a change in belief toward a reform perspective occurred in relation to three beliefs in the case of the reform curriculum group. We wondered about whether or not there might be some sort of ceiling effect that would preclude documenting actual belief changes since the pre-professional development means of this group were much higher than those of the traditional curriculum group. However, these means are far below the upper end of each scale, except in the case of belief 4, and a significant difference in relation to belief 4 was observed. In the case of the traditional curriculum group, significant differences between pre and post means were observed in relation to all seven beliefs. These observations lead us to wonder if our professional development work was more effective for teachers who are initially less-reform minded, or if greater belief changes might have been observed had we waited until the end of the two-year professional development program to administer the survey. We will discuss these issues more completely in the "Discussion" section of this article.

How did the teachers' beliefs in the two groups differ after one year of professional development? We then analyzed

Table 2. Comparisons of Pre- and Post-Means for Reform Curriculum Group

		Means				
Belief	Rubric Range	Pre	Post	df	F	*p*
1	0-4	1.067	1.733	28	2.32	.029
2	0-5	0.733	1.533	28	0.04	.028
3	0-4	2.800	3.067	28	.93	.361
4	0-4	2.667	3.533	28	3.13	.004
5	0-5	2.333	1.733	28	-1.46	.155
6	0-5	2.533	2.667	28	36.84	.698
7	0-4	1.333	1.533	28	5.88	.664

Table 3. Comparisons of Pre- and Post-Means for Traditional Curriculum Group

		Means				
Belief	Rubric Range	Pre	Post	df	F	*p*
1	0-4	0.188	1.563	28	5.69	<.001
2	0-5	0.813	2.000	28	3.28	.003
3	0-4	0.313	2.800	28	8.59	<.001
4	0-4	0.500	3.333	28	10.12	<.001
5	0-5	0.563	2.188	28	4.09	<.001
6	0-5	0.563	2.667	28	36.84	<.001
7	0-4	0.375	1.813	28	5.88	.003

the group means obtained from the post-survey scores using an analysis of variance to determine if there were differences between the two groups relative to any of the seven beliefs after the first year of professional development. The results are displayed in Table 4. As shown, none of the differences between the means obtained from the groups relative to each belief were significant. This is especially meaningful when we recall that the means obtained from groups on the pre-survey differed significantly on six of the seven beliefs, an issue that is discussed further in the next section.

How did belief changes vary across groups? To address this question, we re-examined the data and related analyses previously discussed and created graphs as displayed in Figures 1-7. These graphs pictorially display the pre and post-survey means of each group. They clearly show that although the means obtained from the two groups

Table 4. Comparisons of Post-Survey Means for Reform Curriculum and Traditional Curriculum Groups

		Group Means				
Belief	Rubric Range	Reform Curriculum Group	Traditional Curriculum Group	df	F	*p*
1	0-4	1.733	1.563	29	0.31	.580
2	0-5	1.533	2.000	29	2.58	.120
3	0-4	3.067	2.800	29	0.77	.387
4	0-4	3.533	3.333	29	0.64	.429
5	0-5	1.733	2.188	29	1.24	.275
6	0-5	2.667	2.667	29	0.00	1.000
7	0-4	1.533	1.813	29	0.38	.543

differed significantly prior to professional development in relation to six of the seven beliefs in favor of the reform curriculum group, those differences disappeared by post-survey administration. This observation suggests that the traditional curriculum group experienced greater belief changes towards a reform perspective than those experienced by the reform curriculum group even though both groups ended with similar belief intensities. In addition, the reform curriculum group actually experienced significant change themselves in relation to three beliefs.

Figure 1. Pre-Post Cross Group Comparison — Belief 1

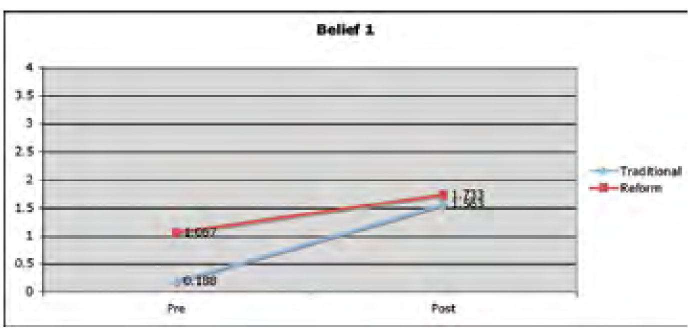
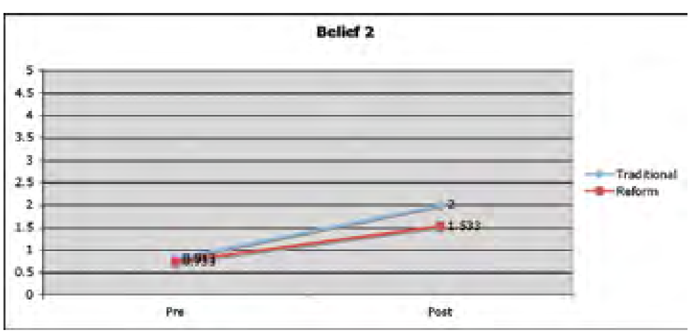


Figure 2. Pre-Post Cross Group Comparison — Belief 2



Conclusions

We will orchestrate this section of our article according to our questions and observations. To begin, our data confirms the work of many others about the effects of the use of reform-oriented curriculum materials. The beliefs possessed by the groups of teachers we engaged in professional development differed significantly prior to our work with them. Those who used reform curricula possessed beliefs that more closely approximated a reform perspective than those who used traditional curricula.

Figure 3. Pre-Post Cross Group Comparison — Belief 3

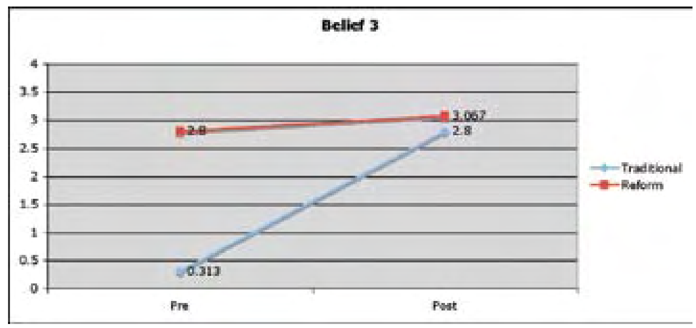


Figure 4. Pre-Post Cross Group Comparison — Belief 4

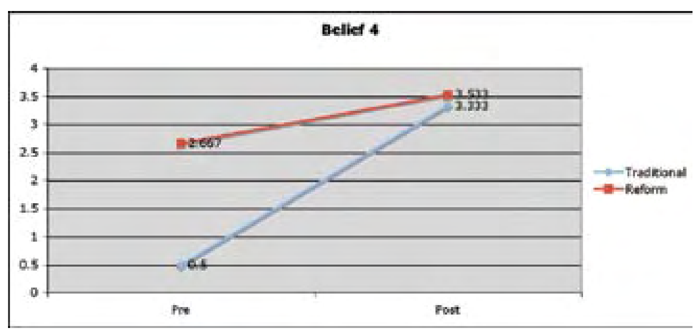


Figure 5. Pre-Post Cross Group Comparison — Belief 5

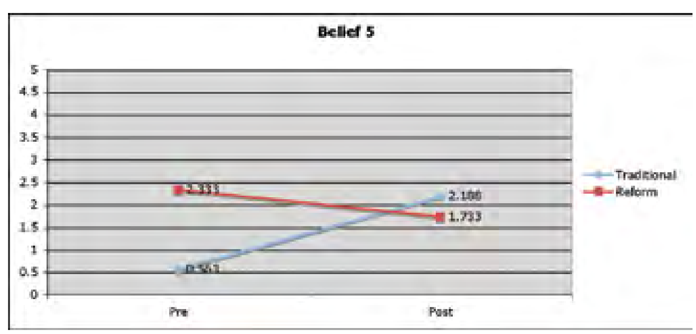


Figure 6. Pre-Post Cross Group Comparison — Belief 6

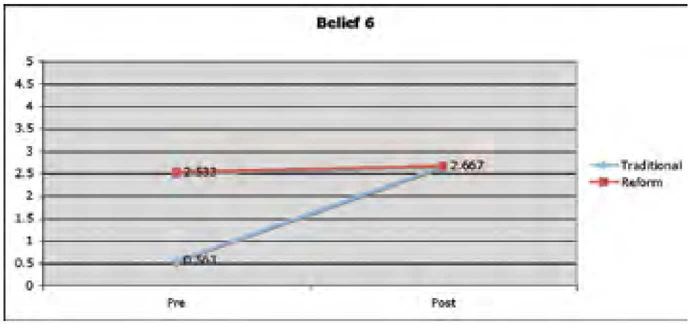
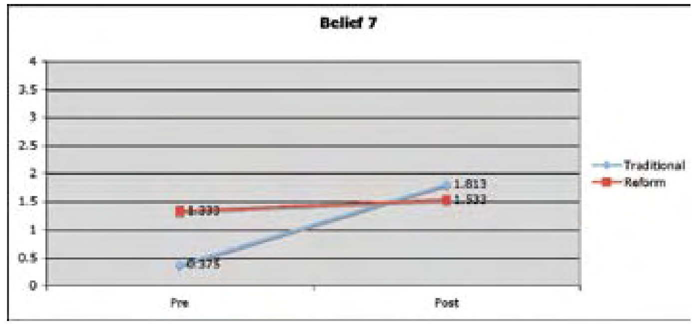


Figure 7. Pre-Post Cross Group Comparison — Belief 7



Second, because the teachers in both groups experienced belief changes, we conclude that our professional development efforts had some effect, which supports the conclusions of Orrill and Anthony (2003).

Third, inasmuch as both groups received the same basic professional development, and that the traditional curriculum group experienced greater changes in the intensity of their beliefs, we wondered about the relative influence of professional development together with curriculum materials. Despite the lack of reform curricula, the beliefs of the teachers in the traditional curriculum group "caught up" with those possessed by the teachers in the reform group. We find this result especially interesting in light of the previously-mentioned observation that several teachers in the Reform Curriculum Group reported participating in district-sponsored workshops designed to support their use of *Investigations*. It seems logical to conclude, therefore, that because pre-existing belief differences disappeared by the time of the post measure, professional development, or at least, our professional development work, might actually be more powerful in promoting belief changes than the use of reform curricula. Other alternative conclusions are also possible.

For example, these observations lead us to wonder, as previously mentioned, if our professional development work was more effective for teachers who are initially less-reform minded, particularly in the first year. It is entirely possible that differing belief changes may occur during the second year of the professional development program. Perhaps the reform curriculum teachers may experience an accelerated change as a result of the second year course work paralleling the change experienced by the traditional curriculum teachers during the first year. Then two additional scenarios may result. If the traditional curriculum teachers continue the same rate of change or even a greater rate than the rate experienced in the first year, then the change in beliefs experienced by both groups during the second year would parallel each other, again supporting the conclusion that professional development is more powerful than curriculum materials in promoting belief changes. If, on the other hand, the traditional curriculum teachers experience a slower rate of belief change while the reform teachers experience an acceleration, we might conclude that curriculum materials have a greater influence than we suspect. It is possible that the changes that potentially occur because of use of a reform curriculum have an end point—that is, that the use of a reform curriculum can only change teacher beliefs to a certain degree without professional development.

All in all, our research seems to support the conclusion of Ziebarth (2003) that the wise use of both curriculum materials and professional development are needed in promoting real reform.

Connecting to Future Research

This study and those which have preceded it have addressed questions regarding the reform-mindedness (RM) of teachers' beliefs, and the changes therein resulting from teaching with reformed curricula, RM professional development, and a combination of the two. Clearly, however, many options have not yet been considered in this research.

Nuanced time frames should eventually be investigated. For instance, will a teacher who receives no RM professional development, but uses a reformed curriculum eventually gain RM, and if so how long may it be expected to take? Or will a teacher who participates in RM professional development but uses non-RM curricular materials eventually gain RM, and if so how long may it be expected to take. Questions of this

Table 5. Duration of Transition to Reform-Mindedness

<table>
<tr><td colspan="4" rowspan="4"></td><th colspan="5">Reformed Professional Development</th></tr>
<tr><th colspan="4">Yes</th><th rowspan="3">No</th></tr>
<tr><th colspan="4">Years of Experience</th></tr>
<tr><th>x</th><th>y</th><th>z</th><th>...</th></tr>
<tr><th rowspan="5">Reformed Curriculum</th><th rowspan="4">Yes</th><th rowspan="4">Years of Experience</th><th>x</th><td colspan="4" rowspan="4">RM after A Years</td><td rowspan="4">RM after B Years</td></tr>
<tr><th>y</th></tr>
<tr><th>z</th></tr>
<tr><th>...</th></tr>
<tr><th colspan="2">No</th><th></th><td colspan="4">RM after C Years</td><td>RM after D Years</td></tr>
</table>

sort can be generated from the accompanying table and future research should attempt to answer some of these questions. An added dimension of the level of RM prior to, during, and at the end of certain curricular usage and professional development experiences can be integrated into time frame investigations (as was investigated in this study).

Professional development may take many forms. Among many others, these include investigating curricular materials, analyzing and selecting curricular materials, and developing new curricular materials. This can be extended into the investigation of RM professional development. Thus, future research should consider if one of these or other forms of RM professional development more quickly and deeply leads to participant RM.

In the same light, it should be determined if some RM curricular materials naturally lead a teacher to become RM even without the addition of RM professional development and the nature of such curriculum.

Future research should investigate recidivism of non-RM of teachers who are not continually involved in RM curriculum and/or professional development. Furthermore, significant research should continue to differentiate between teachers who claim to be RM and those who have instructional practices which are decidedly non-RM.

Summarily, many other issues warrant future investigation regarding the development and maintaining of RM among mathematics teachers. It is hoped that as this study has addressed some of these questions, research will continue to delve into the many other questions listed herein. Nevertheless, this study has demonstrated that teacher beliefs can be changed to RM when the professional development has such a goal.

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Teacher Knowledge and Student Achievement: Revealing Patterns

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University researchers and teacher facilitators implemented a state-funded professional development project during the 2005-06 academic year to help county middle school teachers improve student achievement in mathematics. In this paper, we discuss lessons and results from this innovative model, whose iterative cycle includes teacher content knowledge, item analysis from a high-stakes test, pedagogical content knowledge, big mathematical ideas behind test items, and designing-implementing-reflecting on lessons to address critical problem areas in student learning and understanding.

Theoretical Framework

Too often professional development focuses narrowly on changing teaching behaviors (e.g., on helping teachers learn how to use a new technology or new teaching strategy) with no attention to the impact of such tools on what students know and can do. Although teachers need to learn to use new techniques and tools, most importantly they need to step back from their own learning and consider the implications for the students' learning and achievement.

An intervention was driven by a professional development model designed by the project team. The model focused on a "teaching on evidence" approach, using item analysis as a main component. We used an iterative, cyclical model (see Figure 1) which included assessment of teacher content knowledge, item analysis from a high-stakes test, identifying (ID) and assessing low-performing items (LPI), teacher-driven discourse with pedagogical content knowledge (PCK) consideration, big mathematical ideas behind test items, vertical alignment of mathematical concepts, and evidence-based lesson study cycle (designing, implementing, and reflecting on lessons) to address critical problem areas in student learning and understanding. The model also included peer-observation and analysis of designed lessons. Teacher participants kept journal logs and regularly submitted their reflections on assigned items during the yearlong professional development.

We used data from our state's mandated standardized test (TAKS: Texas Assessment of Knowledge and Skills; see Texas Education Agency website: www.tea.state.tx.us) as a source to create cognitive pedagogical conflict for teachers. The conflict occurs between teachers' assumption of what happened in the classroom (i.e., "I taught this topic/objective") and actual student learning (i.e., "Did

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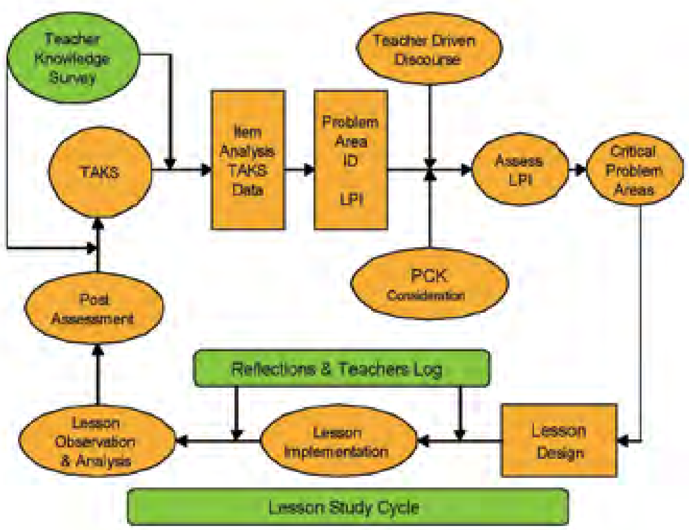
student learn what I taught?"). Usually, teachers have no ready explanations of why students would do poorly on a TAKS item based on a topic/objective that was taught in class. Our main professional development strategy was to identify error patterns in students' TAKS performance and use *item analysis* to engage teachers in conceptual discourse on how to overcome the disequilibrium and change their teaching practices in a way that would help student achievement. An example of a similar approach with elementary school teachers is discussed by Fisher and Kopenski (2007). During each professional development session, the workshop was launched and driven by the particular TAKS items and corresponding TEKS (Texas Essential Knowledge and Skills) competencies correlated with a content-specific objective — with most of the focus on the TAKS items on which student performance was the poorest.

The idea clearly is not to "teach to the test" and focus only on these particular items, but to unpack big mathematical conceptual ideas (e.g., reversibility, flexibility, and generalization) and effective teaching strategies that might help teachers to improve student achievement on a much larger collection of items, and to situate this understanding of big ideas in a greater set of curriculum objectives in the K-12 continuum.

Figure 1. Professional development model used in the study



An example of one of those big-picture teaching strategies was multiple representations (Lesser and Tchoshanov 2006). The professional development intervention included 13 three-hour workshop sessions (and a follow-up session during the summer), which was broken down into two sessions for each of the six objectives of the high-stakes middle school TAKS mathematics test: numbers, operations, and quantitative reasoning; patterns, relationships, and algebraic reasoning; geometry and spatial reasoning; measurement; probability and statistics; mathematical processes and tools. Sessions were driven mainly by analyzing low-performing TAKS items. Teachers worked on items individually, shared their work with group members, and then presented ideas from their group to the entire room, following the five-step *item analysis approach* designed by the project team:

(1) Solve the problem.

(2) What is/are the big mathematical idea(s) or core concept(s) of the problem? How does this connect to concepts from elementary school mathematics? How does this connect to concepts from high school mathematics?

(3) What TAKS objective and what TEKS knowledge and skill does this problem address?

(4) What do you think caused student low performance on this problem?

(5) What would you change/ modify in your teaching so students will be more successful solving this problem? What questions would you ask students during your teaching to prevent low performance with this problem? Make a list of two or three questions.

During professional development sessions, the project team used a *teaching on evidence* approach, which focused on the following main goals:

- Develop a culture of evidence by using TAKS data (item analysis approach) to improve teaching practices
- Develop an understanding of big mathematical ideas and vertical alignment of concepts across grade levels
- Develop mathematical habits of mind such as generalization (e.g., from a sequence of numbers to an algebraic rule) or reversibility and effective general strategies such as multiple representations and high-level questioning skills
- Investigate a topic not as a single item but as part of a set of connected ideas.

All six TAKS objectives were addressed before the TAKS test in an order informed by teachers' scope and sequence to maximize the opportunity for teachers to have a professional development session on a particular TAKS objective before they taught it.

One part of the model that was not fully realized was lesson study. While each teacher received feedback on her teaching, participated in regular reflection writing, etc., a true full-scale lesson study cycle was not possible to conduct within the time and resource constraints of the grant.

Methodology

In order to assess the impact of the intervention on teacher knowledge and student achievement, the project team used a *mixed-methods design* with the following measures.

1. **Texas Assessment of Knowledge and Skills (TAKS).** TAKS scores were collected to assess teacher impact on student achievement, which will be discussed in the Results section.
2. **Teacher Observation Protocol.** An existing standardized protocol used in a local district was used for documenting observations of teacher lessons. The form allows the observer to provide narrative comments as well as to choose among three rating levels for each indicator within each category. The categories are class structure, methods, teacher-student interaction, and content. (While each teacher had the benefit of being observed and receiving peer feedback from this form, the data was not rich and detailed enough to yield meaningful results with respect to the workshop's focus areas and this protocol is being revised accordingly for future use.)
3. **Teacher Knowledge Survey.** At the beginning of the series of workshops, this survey was used to assess teacher content knowledge and consisted of 33 multiple choice problems addressing corresponding TAKS objectives and using three different levels of cognitive demand. Further discussion of the construction of the survey appears later.
4. **Teacher Reflections.** Between professional development sessions, teacher participants submitted written reflections using the framework of applying the aforementioned five-step item analysis organizer (which includes addressing what you would change in your teaching) to new TAKS problems the authors supplied.

The research sample consisted of 22 in-service teachers from high-need (based on percentage of students at the school not passing the mathematics portion of the high stakes TAKS test) and low-SES schools (based on the percentage of students participating in free or reduced-price lunch programs). These schools' student bodies are about 80-90% Latino/Hispanic. One of the main variables and measures of the project was student achievement. Below, we provide statewide students' TAKS performance (Figure 2). The low pattern of achievement in the middle grades was a strong reason we targeted teachers of these grade levels.

Figure 2. Statewide Students' TAKS Performance

All Students, Percent Met Standard

	2003	2004	2005	2006
Elementary grades				
3rd Grade	74%	83%	83%	82%
4th Grade	70%	78%	81%	83%
5th Grade	65%	73%	79%	81%
Middle Grades				
6th Grade	60%	67%	72%	**79%**
7th Grade	51%	60%	64%	**70%**
8th Grade	51%	57%	61%	**67%**
High School Grades				
9th Grade	44%	50%	56%	**56%**
10th Grade	48%	52%	58%	60%
11th Grade	44%	67%	72%	77%

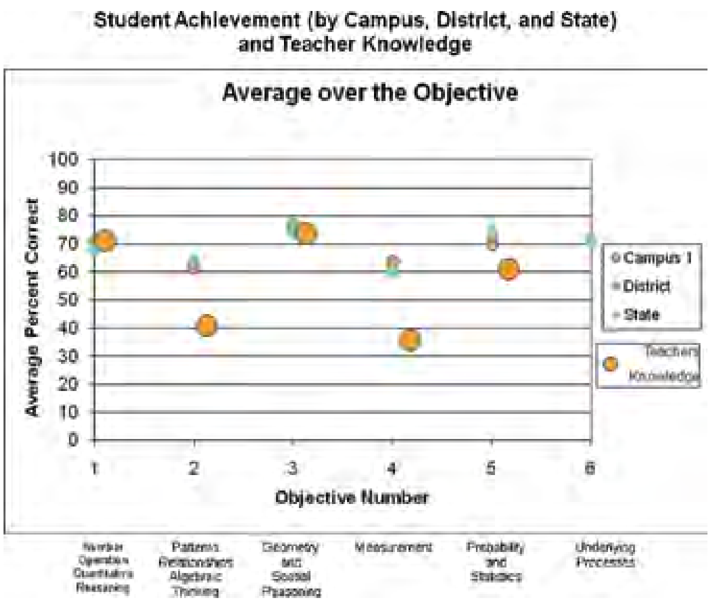
Results

Teacher Knowledge and Student Performance

A body of existing research claims that U.S. teachers lack essential knowledge for teaching mathematics and that teachers' intellectual resources affect student achievement (Coleman et al., 1966; Ball, 1991; Stigler & Hiebert, 1999; Ma, 1999; Hill et al., 2005). Our study supports this claim and shows that teacher knowledge and student achievement parallel each other (Figure 3). Teacher knowledge (as measured by the Teacher Knowledge Survey given at the beginning of the intervention) is denoted by the largest icon symbol in Figure 3.

Through the analysis of the TAKS data, the project team discovered how things are very similar from campus to campus within the feeder pattern. We believe that it makes sense since campuses in the same feeder pattern are supposed to work together and align their instruction. In Figure 3, we compare one of these campuses to the district as well as to the state. The same pattern occurs everywhere, which would seem to say that the teaching everywhere across the state is about the same. It seems that teachers across the state are about equally effective in conveying the same material to students, no matter what curriculum is used. Or, regardless of scheduling differences (e.g., scope and sequence of curriculum topics), teachers across the state teach relatively in the same manner.

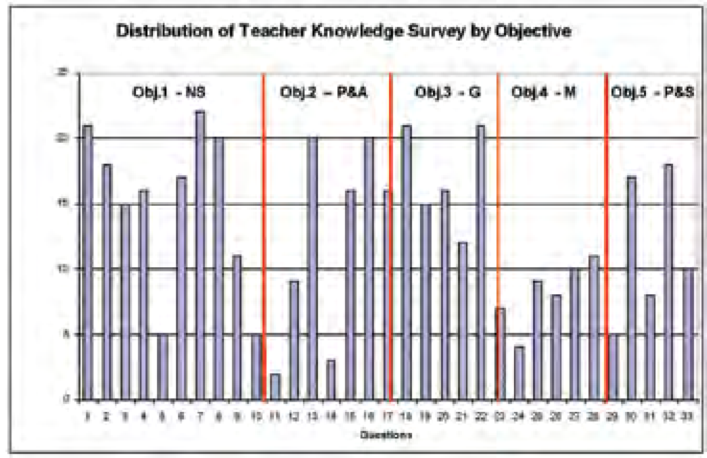
Figure 3. Student achievement (measured by TAKS) and teacher knowledge (measured by Teacher Knowledge Survey) for each objective



Another important observation is that student performance pattern by objectives mirrors teacher performance on the Teacher Knowledge Survey. In a sense, it means that if teachers have difficulty in mastering a particular objective then it impacts student achievement in the same objective. Figure 3 shows, for example, that low teacher knowledge on objectives #2 (patterns, relationships, and algebraic thinking) and #4 (measurement) is correlated with low student achievement on the same objectives compare to other objectives (e.g., objectives #1, 3 and 5). The project team did not collect data for the process objective #6 (underlying processes), focusing mainly on content specific objectives (#1 through #5). It does not mean that objective #6 is not important for the study, though, and we will talk about one of the process standards — problem solving — in more detail later.

Distribution of teacher performance by items on the Teacher Knowledge Survey is presented on Figure 4. The data shows the same pattern: teachers have a lack of knowledge on objective #2 (P&A=Patterns & Algebra) and objective #4 (M=Measurement) compare to objectives #1 (NS=Number Sense), #3 (G=Geometry), and #5 (P&S=Probability & Statistics).

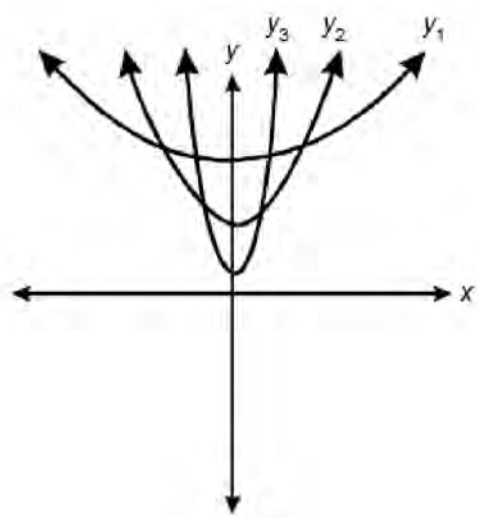
Figure 4. Teacher performance on the Teacher Knowledge Survey items by objective



Teacher Sample (N=22) was selected from Middle Schools:
(a) Performing Below State Average and (b) Economically Disadvantaged

The lowest-performing item (out of 33 items in the Teacher Knowledge Survey), item #11 from the Patterns and Algebra objective, is shown below (Figure 5):

Figure 5. Graph used in the lowest-performing item from the Teacher Knowledge Survey



(11) In the figure [Figure 5] above, the function y_3 is translated 4 units left and 7 units down. Which of the following equations best describes the new function?

A. $y = ax^2 + 11x + 28$
B. $y = ax^2 + 4x + 7$
C. $y = ax^2 + 8ax + c$
D. $y = x^2 + 28x + 11$

Only two out of 22 teachers were able to solve the item correctly. In contrast, item #7 (shown below) from the Number Sense objective was successfully solved by all teacher participants.

(7) What is the rule for fraction division?

A. $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$

B. $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$

C. $\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$

D. $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

As you see, there is a difference not only in content objectives but also in cognitive demand level between items #11 and #7. Whereas item #7 is addressing correctly identifying a mathematical procedure (fraction division rule) recorded with algebraic notation, item #11 focuses on applying a non-routine mathematical procedure with understanding. The problem can be viewed as a Level 2 cognitive demand (which will be explained shortly) in which a student has to choose the more appropriate form of a quadratic function to use — namely, the vertex form. After substituting the -4 and -7 numbers in the standard form of the quadratic function $y = a(x - h)^2 + k$ and expanding the equation, it becomes consistent with the choice C. The construct of cognitive demand provided guidance for developing the teacher knowledge survey.

The Roles of Task and Cognitive Demand in the Teacher Knowledge Survey

One indicator of teachers' conceptual understanding of mathematics is an ability to engage students in meaningful discourse in the classroom through selecting instructional and assessment tasks that embody learning goals (Shepard et al., 2005). Why are tasks important? Students learn from the kind of work they do during class, and the tasks they are asked to complete determines the kind of work they do (Doyle, 1988). Mathematical tasks are critical to students' learning and understanding because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). "The tasks make all the difference" (Hiebert et al., 1997, p. 17). Tasks provide the context in which students think about mathematics and different tasks place different *cognitive demands* on students' learning (Doyle, 1988; Henningsen & Stein, 1997; Porter, 2004).

Cognitive demands can be defined as the kind and level of thinking required of students in order to successfully engage with and solve the task (Stein et al., 2000, p. 11). Such thinking processes range from memorization to the use of procedures and algorithms (with or without attention to concepts, understanding, or meaning), to complex thinking and reasoning strategies that would be typical of "doing mathematics" (e.g., conjecturing, justifying, or interpreting) (Henningsen & Stein, 1997, p. 529).

Given the importance of tasks, the next issue is: "What do teachers need to know to select or make up appropriate individual tasks and coherent sequences of tasks? The simple answer is that they need to have a good grasp of the important mathematical ideas and they need to be familiar with their students' thinking" (Hiebert et al., 1997, p. 34). Similarly, Grossman, Schoenfeld, & Lee (2005) posed a critical question: "What do teachers need to know about the subject they teach?" (p. 201), and provided a fairly straightforward answer: "Teachers should possess deep knowledge of the subject they teach" (*ibid*, p. 201).

In constructing the teacher knowledge survey, we generated items for each of the following levels of cognitive demand. Note that our Level 1 includes Stein's first two levels (memorization; procedures without connections), our Level 2 is like Stein's third level (procedures with connections), and our Level 3 is close to Stein's fourth level (doing mathematics, which includes conjecturing, etc.). Further discussion addressing the reliability and validity of our instrument appears at the end of the article.

Level 1: Facts and Procedures

- Memorize Facts, Definitions, Formulas, Properties, and Rules;
- Perform Computations;
- Make Observations;
- Measure;
- Solve Routine Problems

Level 2: Concepts and Connections

- Justify and explain solutions to problems;
- Use and select multiple representations to model mathematical ideas;
- Transfer knowledge;
- Connect concepts to solve non-routine problems;
- Communicate "Big Ideas";
- Explain findings and results from analysis data

Level 3: Models and Generalizations

- Generalize;
- Make and test conjectures;
- Prove statements;
- Design mathematical models

To illustrate the difference between levels, we include an example using a fraction division problem under objective #1 "Number Sense."

Level 1. What is a rule for fraction division?

Solve the following fraction division problem

$$1\frac{3}{4} \div \frac{1}{2}$$

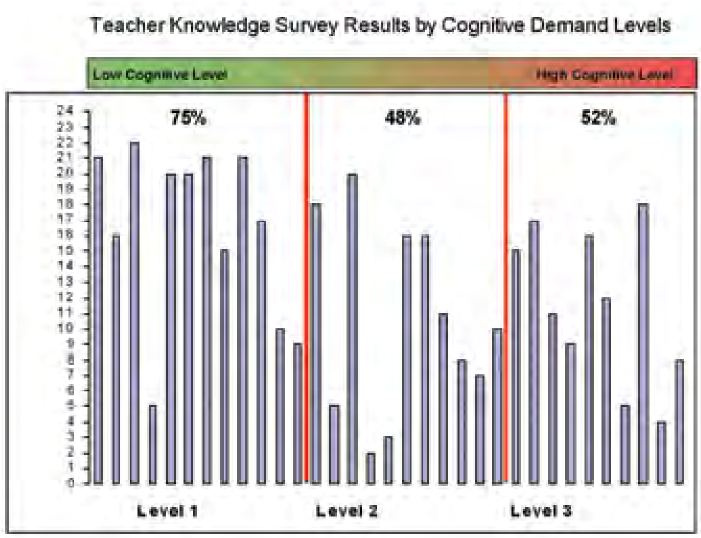
Level 2. Solve the same problem in more than one way, for example, draw a model or illustrate the problem with manipulatives.

Make up a story for the fraction division problem

Level 3. Is the following $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$ ever true?

Figure 6 consists of the same Teacher Knowledge Survey results (displayed in Figure 4) distributed by cognitive demand levels. Each bar represents a particular item.

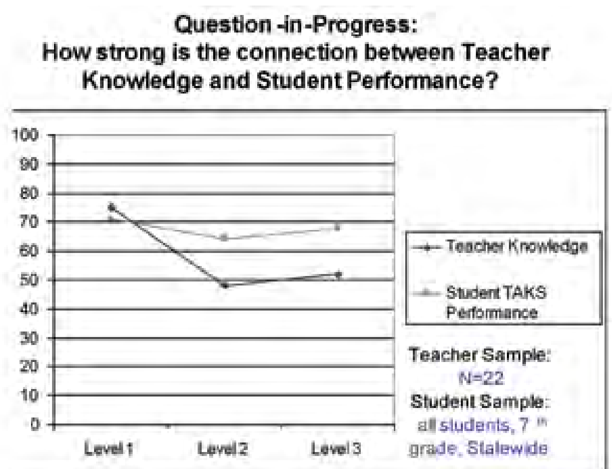
Figure 6. Teacher Knowledge Survey results by cognitive demand levels



Considering the selection criteria for teacher participants in the project, we expected that the percentage of correct responses would be highest for Level 1. However, the finding that Level 3 scores were higher than Level 2 scores was unexpected.

Following the teacher knowledge survey, we performed the same activity with TAKS items, classifying them by cognitive demand level. We triangulated the process by having three mathematics educators independently conduct the classifications, with 90% agreement. We distributed 7^{th} grade student statewide TAKS performance results by cognitive demand levels and compared it with teachers' data. The project team was surprised to observe a similar pattern between teacher and student performance (Figure 7). A Cohen's *d* effect-size calculation between the teachers' level 1 performance and the teachers' mean performance on levels 2 and 3 resulted in the large effect size of 2.3.

Figure 7. Teacher and student performance by cognitive demand levels

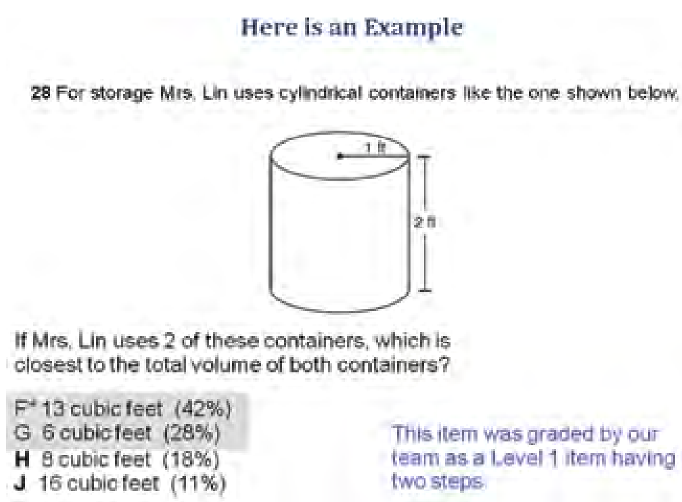


We wonder if the data in Figure 7 could add insight into the pattern we saw in Figure 2. In other words, the similarity on student TAKS achievement across the state might reflect similarities on cognitive demand level of tasks and assignments used in mathematics classrooms all over the state.

Does the Number of Steps in the Problem Matter?

It is well known that students do not perform well in solving word problems. Solving story or word problems is a challenging task at every level of schooling, including middle grades. Research shows that students' poor performance in word problem solving could be a result

Figure 8. Example of TAKS item sorted by cognitive demand and 'stepness' level



of their misunderstanding of the problem (Cummins et al., 1988). Among factors influencing misunderstandings are: difficulties in perception of mathematical language (Kane, 1967), insufficient subject matter knowledge (Mayer, 1992), problem posing (Butts, 1980), language deficiencies (Mestre, 1988), ineffective text processing (Nathan, Knitsch, & Young, 1992), and lack of effective reading strategies in problem solving (Shuard & Rothery, 1988). We examine a very specific component of this broad issue: does number of steps in solving problems (including word problems) impact student performance on standardized testing?

In order to answer this question, we conducted an analysis of TAKS items using the following criterion - number of steps required to solve the item. The project team called this criterion 'stepness.' We performed a similar triangulation process (as we did with sorting problems by cognitive demand level) for sorting the TAKS items by criterion of 'stepness.' Experts reached higher percentage of agreement in this sorting task — 97%. So, along with a cognitive demand level a particular item was assigned a 'stepness' level. We used the following scale for the 'stepness' criterion: 1, if an item requires one-step solution; 2, if an item requires two-steps for solution; 3, if the solution requires 3 or more steps. Figure 8 illustrates one of the items (item #28 from the April 2006 7th grade math TAKS test released to the Texas Education Agency website) by both cognitive demand and 'stepness' level. This item was sorted by experts as Level 1 item (e.g., performing procedure) having two steps: step one - to find/ estimate a volume of one cylindrical container, and step two – to multiply it by 2. We included the percentage of students' responses in parenthesis for each given choice. For instance, the right choice F (marked by *) was picked by 42% of students and the choice G was selected by 28% of students. Imagine that this problem was phrased as a one-step problem (asking to estimate a volume of one container), then it is not unreasonable to think that 42% + 28% = 70% of the students would answer this problem correctly. So introducing the second step would seem to reduce the student success rate by (.70-.42)/.70 = 40%!

After the project team sorted TAKS items by the 'stepness' criterion, the team looked for a connection between student TAKS performance and corresponding 'stepness' of the item. Figure 9 shows a negative correlation ($r = -0.34$) between student performance (black curve) and 'stepness' (scaled to 100% by dividing each mean rating for a problem's number of steps by the overall maximum number of steps) of the item (red curve). The black curve reflects the apparent tendency of the TAKS test-makers to put the least difficult items at the beginning (when students need that initial boost of confidence?) and end (when students are getting tired?) of the test. Inspired by a reviewer's suggestion, we wondered if the stepness connection to student performance was mirrored by stepness connection to teacher performance, in the same spirit as the pattern we saw in Figure 3. The first two authors independently coded each of the 33 problems on the Teacher Knowledge Survey (TKS) for number of steps involved. The authors agreed on the exact number of steps for the majority of the problems and the correlation between the two sets of ratings was significant ($p < .05$).

Figure 9. Connection between student performance (*black*) on item and its 'stepness' (*red*)

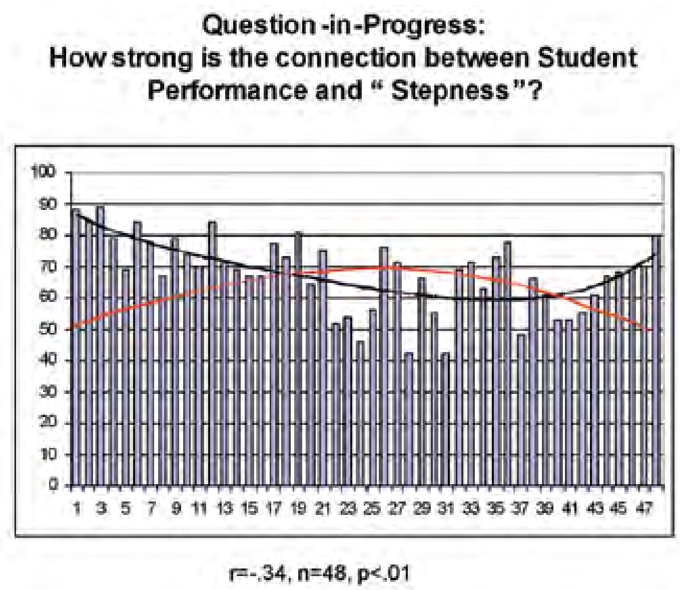
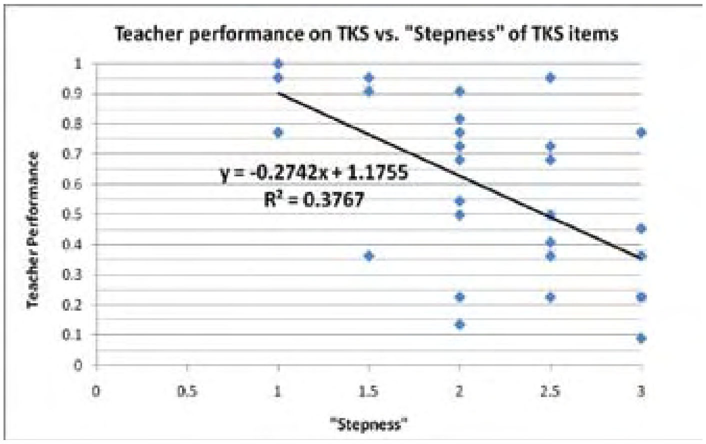


Figure 10. Connection between teacher performance on item and its 'stepness' (diamond icons with extensions denote multiple points at that spot)



The authors then took the mean stepness rating and found its correlation with the proportion of teachers who correctly solved the TKS problems (see Figure 10). This correlation was found to be r = -0.61, n = 33, df = 31, 1-tailed p-value = $7.3 \times 10^{-5} < 0.0001$. The R^2 value tells us that more than one-third of the variation in TKS scores can be explained by problem stepness. This result is an even stronger correlation than the one found for students.

Teacher Knowledge and Reflective Thinking

These quantitative findings are also supported by reviewing the teachers' reflection papers that used the five-question format mentioned near the beginning of the article. In particular, we see that teachers who were "high-high" (i.e., they scored higher than average on TKS and their students had higher than average improvement on passing rates, and this latter phrase will be discussed in the next section) approached their reflections qualitatively differently than teachers who were "low-low." To be more precise, we are comparing teachers from the upper right quadrant of the scatterplot in Figure 12 to those in the lower left quadrant.

As an example, let us look at a problem on the Teacher Reflection Paper assigned for January 9, 2006: "Which of the following is NOT true about similar figures?"

A. Similar figures always have the same shape.
B. Similar figures always have the same size.
C. Similar figures always have corresponding angles that are equal.
D. Similar figures always have corresponding sides that are proportional.

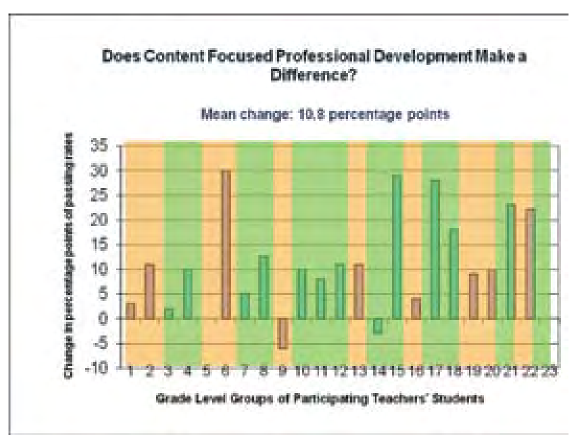
This question was problem #9 on the Spring 2004 7th grade mathematics TAKS test and the student response pattern was: A (8%), B (63%; correct answer), C (10%), D (18%). It was most revealing when teachers went through the five-step organizer (see p. 40) and addressed what they thought caused student low performance on this problem and what they would change (including questions to ask students) in their teaching. One of the high-high teachers' answers had a focus on mathematical concepts and terms, saying students had trouble with "Vocabulary of proportional, corresponding, etc. "and would need "practice on properties and scale factor." Typical answers from the low-low teachers, however, had more of a focus on test-taking or surface features, saying that "Students do not read the 'NOT' and therefore miss what the question is asking of them" and therefore would "stress reading the question."

Does Content-Focused Professional Development Matter?

We started the paper with the statement that too often professional development focuses narrowly on changing teaching behaviors with not enough attention paid to the impact of such tools on what students know and are able to do. The true focus of this project was on the effect of professional development on student achievement. In other words: does content-focused professional development make a difference?

We collected teacher participants' TAKS scores for two consecutive years: 2005 and 2006. By comparing the passing rate of teacher A's 6th graders in 2005 to the passing rate of teacher A's 6th graders in 2006, we were being consistent with the performance target specified in the grant application, which focuses on passing rates not for individual cohorts of students, but on a teacher or district level. Figure 11 below shows the change in TAKS passing rates for every participating teacher

Figure 11. TAKS passing rate gains for participating teachers from 2005 to 2006



by grade level assignment. In total, we have the results for 14 different teachers, each of whom taught either one, two, or three grades' worth of students. For instance, columns 5 and 6 are the data for the same middle school teacher who was teaching two different grade levels, one of which had no change in passing rates.

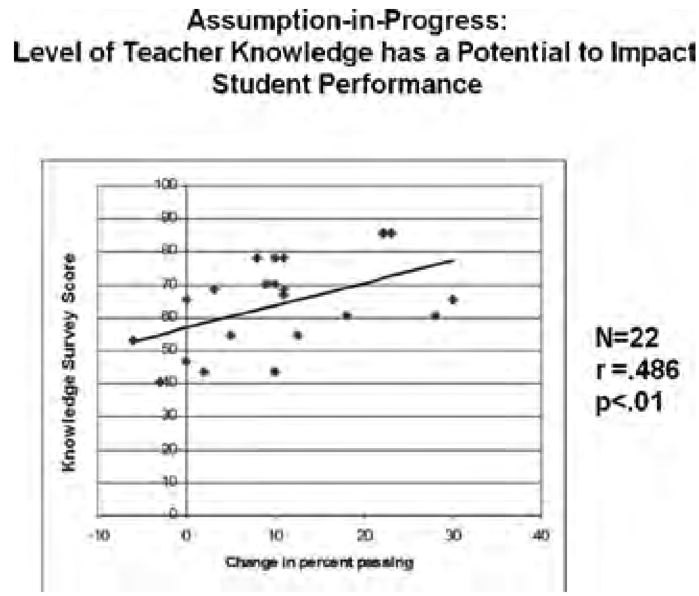
The mean gain in TAKS passing rates of participating teachers from 2005 test administration to 2006 was 10.8 percentage points, compared to 4.4 for the state's mean gain for middle grades students. It should be noted that 10.8 actually underestimates our teachers' improvements because two instances of negative change (#9 and #14) both involve teachers whose previous groups of students (including gifted and talented classes) had passing rates of 90% and 100%, respectively, which created a ceiling effect that limited further improvement. By removing these two groups from the dataset, the mean change actually would have been 12.2 percentage points. There may be much hope in these numbers: content-focused professional development based on the proposed model (Fig. 1) can make a difference in student achievement. Although it seems likely that the professional development may have also increased teacher knowledge, this is something we can only conjecture. The TKS was given only as a pretest because the direct goal of the grant (improving teachers' students' achievement) did not justify administering the time-consuming TKS survey again.

Last, but not least: could the level of teacher knowledge affect student achievement? We conducted a regression analysis using teacher participants' knowledge survey scores and correlated it with the teacher's students' TAKS gain over the period of the study. A promising finding here is that the level of teacher knowledge is highly related to improvement in students' passing rate (Figure 12), with $r = 0.486$ ($n = 22$, $p < .01$).

Further Discussion

The overall conclusion of this paper: there is a connection between teacher knowledge and student achievement in general, and there are revealing patterns in the connection with regard to specific mathematical domains, processes and levels of cognitive demand in particular. The Teacher Knowledge Survey (TKS) showed the lowest performance on the "patterns, relationships, and algebraic reasoning" and "measurement" objectives, which are precisely the lowest performing two out of the six TAKS objectives for students! Within each objective, items on the 33-problem Teacher Knowledge Survey were also sorted by levels of

Figure 12. Positive relationship between teacher knowledge and student performance



cognitive demand. Not surprisingly, teachers did the best on problems involving the lowest level of cognitive demand. Surprisingly, teachers did slightly better on problems at the highest level of cognitive demand than on problems at the middle level. The same pattern was observed in student performance on the state standardized test.

The TKS is itself an important accomplishment of this study, as its correlation with student performance is a measure of predictive validity and the instrument also had a respectably high level of reliability (Cronbach alpha = 0.76), especially considering the instrument's varying level of difficulty of problems. This is especially significant in light of the report of the National Mathematics Advisory Panel (United States Department of Education 2008, p. 37): "Evidence about the relationship of elementary and middle school teachers' mathematical knowledge to students' mathematical achievement remains uneven and has been surprisingly difficult to produce. One important reason has been the lack of valid and reliable measures. The literature has been dominated by the use of proxies for such knowledge, such as certification status and mathematics course work completed."

There appears to be much promise for this content-focused professional development model for identifying performance patterns and impacting some teacher variables on student achievement, and it will be interesting to explore how it might scale up or transfer to additional contexts.

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Curriculum Standards, Course Requirements, and Mandated Assessments for High School Mathematics: A Status Report of State Policies

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High schools across the country are under pressure to improve graduation rates, prepare students for an increasingly technical workforce, and raise student achievement outcomes as measured by college entrance, state-mandated, and international test scores. Unlike some school improvement initiatives sparked at the local level, current efforts are driven by changes in state policy that are responses to national policies (e.g. NCLB). In fact, many state departments of education are redefining what students are expected to learn, when and how students will be assessed on that knowledge, and the number of years students are required to study mathematics. This article summarizes current state policies regarding curriculum standards, course-taking requirements, and mandated assessments for high school mathematics. Specifically, we address the following questions:

- How many years of mathematics are required for high school graduation?
- Are specific high school mathematics courses required for graduation?
- How are state-level high school mathematics learning expectations (standards) organized?
- What is the nature of state-mandated high school mathematics assessments and how are they related to graduation requirements?

The information reported was gathered through searches of state[1] Departments of Education websites as of December 2006 (Reys, Dingman, Nevels, & Teuscher, 2007). Due to ongoing work of states, some changes may have occurred since this material was compiled. In fact, at the time of this study, at least six states were reviewing plans for updating or changing state standards, graduation requirements or state-mandated assessments. We recommend that those interested in particular state requirements consult the appropriate website for the latest and most complete set of information.[2]

How many years of mathematics are required for high school graduation?

A recent change by many states is to increase the number of years of mathematics required to graduate from high school (Achieve, 2004). In the United States, high school credit is earned during grades 9 through 12; however, current credit requirements vary across states (see Table 1). As noted in Table 1, five states do not mandate a minimum number of

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[1] In this report, the word "state" refers to the 50 states, as well as the District of Columbia (DC) and the Department of Defense Education Activity (DoDEA)
[2] Links to relevant information about state standards can be found at: http://mathcurriculumcenter.org/states.php

Table 1: Number of Years of High School Mathematics Courses/Credits Required for Graduation*

Specified at Local Level (5)	Colorado, Iowa, Maine, Massachusetts, Nebraska
1 year (0)	None
2 years (7)	Alaska, Arizona, California, Idaho, Montana, North Dakota, Wisconsin
3 years (24)	Connecticut, District of Columbia, DoDEA*, Hawaii, Illinois, Kansas, Kentucky, Louisiana, Maryland, Minnesota, Missouri*, Nevada, New Hampshire*, New Jersey, New Mexico, New York, Ohio, Oklahoma, Oregon*, Pennsylvania, Tennessee, Utah*, Vermont, Wyoming
4 years (11)	Alabama, Arkansas*, Delaware*, Florida*, Michigan, Mississippi*, Rhode Island, South Carolina, Texas*, Washington, West Virginia
Varies by Diploma (5)	Indiana (2-4 yrs), Georgia (3-4 yrs), North Carolina (3-4 yrs), South Dakota (3-4 yrs), Virginia (3-4 yrs)

* This information includes requirements that have been approved and are being phased in with a particular freshman class.

mathematics credits for high school graduation. Rather, the decision in these states is made at the district level. Seven states require two years of high school mathematics and the majority of states (24) require three years of mathematics for high school graduation. As of 2006, 11 states required students to take four years of mathematics.

Five states offer different graduation diplomas that are dependent, in part, on the number of years of mathematics students complete. For example, high school students in the state of Georgia who earn a "Technology/Career-Preparatory" diploma are required to complete three years of mathematics, while students receiving a "College Preparatory" diploma are required to complete four years of mathematics.

Table 1 reflects some changes that are being phased in over the next five years. For example, five states (Missouri, New Hampshire, Oregon and Utah, plus the DoDEA) are phasing in an increase in mathematics credits from two years of mathematics to three years. Five other states (Arkansas, Delaware, Florida, Mississippi and Texas) are increasing the requirement to four years of mathematics, up from the current requirement of three years.

Are specific high school mathematics courses required for graduation?

Twenty-five states (see Table 2) require students to complete at least Algebra I for high school graduation. Four of these states (Delaware, Louisiana, Michigan and Tennessee)

Table 2: Mathematics Courses Required for High School Graduation/Diploma

Required Courses	States	Number of States
Algebra I	California, District of Columbia*, Florida*, Georgia*, Indiana*, Louisiana, Mississippi, North Carolina*, North Dakota, New Hampshire, New Mexico*, Oklahoma*, South Dakota	13
Algebra I and Geometry	Alabama, DoDEA, Illinois, Kentucky, Maryland, Tennessee*	6
Algebra I and II	Minnesota	1
Algebra I, Geometry and Algebra II	Arkansas, Delaware*, Michigan, Texas, Virginia	5

* An equivalent course is permissible.

Table 3: Organization of State High School Mathematics Learning Expectations (as of Dec. 1, 2006)

Organization	States	Number of States
Grade-Band	Arizona, Colorado, Connecticut, DoDEA, Florida, Illinois, Iowa, Kansas, Maine, Michigan, Minnesota, Montana, Nebraska, Nevada, North Dakota, New Hampshire, New Jersey, New Mexico, Pennsylvania, Rhode Island, South Dakota, Vermont, Washington, Wisconsin, Wyoming	25
Grade-Level	Alaska, Delaware, Idaho, Louisiana[3], Missouri, Ohio	6
Course-Based	Alabama, Arkansas, California, District of Columbia, Georgia, Hawaii, Indiana, Louisiana, Maryland, Mississippi, North Carolina, New York, Oklahoma, South Carolina, Tennessee, Texas, Utah, Virginia, West Virginia	19
Grade-Band and Course-Based	Kentucky, Massachusetts	2
None	Oregon	1

specify Integrated Mathematics I as an appropriate substitute for Algebra I. Six states (Arkansas, Delaware, Michigan, Minnesota, Texas and Virginia) require courses equivalent in rigor to Algebra II for graduation. In addition to specifying a required number of high school mathematics courses, two states (Delaware and Michigan) require that students take a mathematics course in the senior year of high school.

How are state-level high school mathematics learning expectations (standards) organized?

Two approaches — course-based and grade-band/level — are used by states to organize and communicate high school mathematics curriculum standards. Course-based learning expectations define what students are expected to learn when taking a specific course (e.g., Algebra 1, Geometry, Algebra II, Integrated Mathematics I, II, III). Grade-band or grade-level expectations define what students across a set of grades (9-10, 11-12) or at a particular grade level (9, 10, 11, 12) are expected to learn. As noted in Table 3, 25 states organize learning expectations by grade-band, six by grade-level, and 19 by course. As noted, Kentucky and Massachusetts publish both course-based and grade-level learning expectations at the secondary level. Oregon is the only state that does not specify learning goals by course, grade-level or grade-band. Instead, it publishes a document called the *Certificate of Initial Mastery*. This document specifies content to be learned after eighth grade and before a student graduates but does not relate this content to specific courses or grades.

While Table 3 notes differences in the ways states organize high school mathematics standards, it does not specify differences in the learning expectations themselves. We encourage readers who are interested in particular state standards to review the source documents (see http://mathcurriculumcenter.org/states.php for links to these documents).

How do states assess high school students in mathematics?

NCLB requires that every student participate in assessments based upon state standards at least once during high school; however, some states require more than a single exam of all students. Three different types of mathematics exams (*end-of-course examinations, general high school examinations, and graduation exams)* are used by states at the secondary level, one of which is designated as fulfilling the NCLB requirement. *End-of-course exams* are generally state-developed assessments administered to students at the end of a specific mathematics course. In some states students take more than one end-of-course exam (one for each course designated by the state). *General high school examinations* are administered to students at a specific grade level, generally grade 10 or 11. *Graduation exams* are named as such and are required for a high school diploma. These three types of exams vary in content focus, the grade at which the exam is administered to students, and the stakes for students (e.g., some states require a passing grade on

[3] *Louisiana has grade-band standards for grades 9-10 and grade-level standards for grade 11.*

the graduation exam while others require only that students take the exam).

Table 4 summarizes, by state, mathematics assessments required at the high school level. Some of the state exams fulfill more than one purpose. For example, in Mississippi students are required to take and pass an end-of-course exam in Algebra 1. This exam satisfies NCLB requirements and students must pass it to receive a diploma. As noted in Table 4, 43 states administer a general high school exam, 15 states administer end-of-course exams, and 12 states administer a graduation exam. Eight states (Alabama, Alaska, Arizona, California, Florida, Louisiana, Nevada and New Mexico) administer both a general high school exam and a graduation exam to high school students. Eight states (Arkansas, DoDEA, Hawaii, Indiana, North Carolina, Tennessee, Utah and Washington) administer both end-of-course and general high school exams to high school students. Two states (Georgia and South Carolina) administer both end-of-course and a graduation exam to high school students.

The state-mandated assessments vary as to the level of consequence for individual students. In some states, students are required to pass the assessment(s) in order to receive a high school diploma (we classify these as *high stakes assessments*). In other states, while students are required to complete the state assessment, if they do not pass the exam they can receive a high school diploma by meeting alternative requirements. In some of these cases, the assessment score is factored into a course grade (we classify these as *low stakes assessments*). Finally, some states require students to complete the assessment, but the score does not have consequences for the student in their course grade and/or graduation eligibility (we classify these as *no stakes assessments*).

Table 5 provides a summary of our classification of the stakes of the state high school mathematics assessments. As noted, some states are included in more than one cell because they require students to take multiple exams. As shown, 24 states require high stakes assessments (students must pass an exam in order to receive a high school diploma). Six states mandate a low stakes form of assessment for high school students, whereas 40 states require a high school exam, but students are not required to take or pass this exam (no stakes).

Summary

States have initiated major changes since the passage of NCLB with regard to specifying standards for high school mathematics, increasing mathematics requirements for high school graduation, and developing assessments for accountability purposes. Although standards, course requirements, and assessments differ across states, efforts are directed at a common goal, namely to strengthen mathematics programs at the secondary level in order to provide opportunities for all students to learn important

Table 4: Types of High School Assessments Required in Mathematics

Type of State-Required Exam	States	Number of States
End-of-Course Exam(s)	Arkansas, DoDEA, Georgia, Hawaii[4], Indiana, Maryland, Mississippi North Carolina, New York, Oklahoma, South Carolina, Tennessee, Utah, Virginia, Washington[5]	15
General High School Exam	Alabama, Alaska, Arizona, Arkansas, California, Colorado, Connecticut, Delaware, DoDEA, District of Columbia, Florida, Hawaii, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Massachusetts, Michigan, Minnesota, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Mexico, North Carolina, North Dakota, Oregon, Pennsylvania, Rhode Island, South Dakota, Tennessee, Texas, Utah, Vermont, Washington, West Virginia, Wisconsin, Wyoming	43
Graduation Exam	Alabama, Alaska, Arizona, California, Florida, Georgia, Louisiana, Nevada, New Jersey, New Mexico, Ohio, South Carolina	12

[4] *An Algebra I exam is under development as of April 2007.*
[5] *An End-of-Course exam has been proposed and may be implemented April 2007.*

mathematics and to be prepared for continued study of mathematics as it relates to their future endeavors.

The variance of current state policy regarding high school curriculum standards, course requirements, and mandated assessments raises some interesting questions:

- What impact will raising course standards and graduation requirements have on student learning and graduate rates?
- Should all high school students study a common core of mathematics?
- Should high schools provide alternative mathematics course sequences beyond the core in order to prepare students for particular interests and career options?

Although answers to these questions are difficult to agree upon, preparing all high school students for an increasingly mathematics-intensive work environment requires rethinking traditional methods, exploring alternative course sequences and materials, and increasing the supply of "highly qualified" high school mathematics teachers and curriculum leaders (Steen, 2007).

Table 5: Summary of Level of Stakes and Type of Assessment for High School Students by State

	No Stakes	Low Stakes	High Stakes	Number of States
End-of-Course Exam(s)	DoDEA, Hawaii[6], Georgia, Indiana, North Carolina, South Carolina, Tennessee, Utah, Washington[7] (9)	Arkansas (1)	Maryland, Mississippi, New York, Oklahoma, Virginia (5)	15
General High School Exam	Alabama, Alaska, Arizona, Arkansas, California, Colorado, DoDEA, District of Columbia, Florida, Hawaii, Illinois, Iowa, Kansas, Kentucky, Louisiana, Maine, Michigan, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Mexico, North Dakota, Oregon, Rhode Island, South Dakota, Vermont, West Virginia, Wisconsin, Wyoming (31)	Connecticut, Indiana, Pennsylvania (3)	Delaware, Idaho, Massachusetts, Minnesota, N. Carolina, Tennessee, Texas, Utah, Washington (9)	43
Graduation Exam		Arizona, Ohio (2)	Alabama, Alaska, California, Florida, Georgia, Louisiana, Nevada, New Mexico, New Jersey, S. Carolina (10)	12
Totals	40	6	24	

[6] *An Algebra I exam is under development as of April 2007.*
[7] *An End-of-Course exam has been proposed and may be implemented April 2007.*

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