

COMMON CORE STATE STANDARDS FOR MATHEMATICS

The *Standards for Mathematical Practice* is a document in the CCSS that describes different types of expertise students should possess and mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.

The first of these are the National Council of Teachers of Mathematics, NCTM, process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy). This paper combines information from those sources and lists what students will be doing when they demonstrate mathematical proficiency.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution.
- plan a solution pathway rather than simply jumping into a solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- monitor and evaluate their progress and change course if necessary.
- who are older might, depending on the context of the problem:
 - ✓ transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.
 - ✓ explain correspondences between equations, verbal descriptions, tables, and graphs.
 - ✓ draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- who are younger might:
 - ✓ rely on using concrete objects or pictures to help conceptualize and solve a problem.
 - ✓ check their answers to problems using a different method.
- continually ask themselves, “Does this make sense?”
- understand the approaches of others to solving complex problems and identify correspondences between approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- bring two complementary abilities to bear on problems involving quantitative relationships:
 - ✓ *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
 - ✓ *contextualize* - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

- use quantitative reasoning that entails habits of creating a coherent representation of the problem at hand:
 - ✓ considering the units involved,
 - ✓ attending to the meaning of quantities (not just how to compute them), and
 - ✓ knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases.
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of plausible arguments.
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw in an argument, explain what it is.
 - ✓ Elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
 - ✓ Later, students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments.

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
 - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation.
 - ✓ In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
 - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- apply what they know to make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation.
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- routinely interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- who are in high school:
 - ✓ analyze graphs of functions and solutions generated using a graphing calculator.
 - ✓ detect possible errors by strategically using estimations and other mathematical knowledge.

- ✓ when making mathematical models, know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant external mathematical resources (e.g., digital website content) and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- try to use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- carefully specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
 - ✓ In the elementary grades, students give carefully formulated explanations to each other.
 - ✓ By high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
 - ✓ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
 - ✓ Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
 - ✓ Older students, in the expression $x^2 + 9x + 14$, can see the 14 as 2×7 and the 9 as $2 + 7$.
 - ✓ They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. (e.g., They see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .)

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated.
- look both for general methods and for shortcuts.
 - ✓ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeated decimal.
 - ✓ By paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.
 - ✓ Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead high school students to the general formula for the sum of a geometric series.
- maintain oversight of the process of solving a problem while attending to the details.
- continually evaluate the reasonableness of their intermediate results.

Polygons

Level D:

You work for a company that designs kites that fly in the wind with a long roll of string. The company wants a new line of kites in the shape of convex quadrilaterals. The kites are designed with two sticks to support and brace it in the wind. The sticks are cut from long pieces of wood.

Your boss wants to be efficient in designing the kites. She instructs you to figure out the design for ALL possible convex quadrilaterals.

“We need to cut the long stick so we have two sticks, and then we will fasten them together,” said your boss. “Each stick will be connected to the opposite corners of the kite to brace it against the wind. We need you to determine the size of each of these sticks, along with where and how they are fastened together,” she continued.

What does the boss mean?

- Explore the types of convex quadrilateral kites you can make using two sticks that intersect one another, so that each stick connects to opposite corners of the polygon. Determine how the arrangement of sticks will guarantee the polygon’s shape.
- List all the different quadrilaterals you can make into a flying kite. How do the sticks define the shape of the kites?
- Explain and justify, using geometry, your findings regarding the sizes and positioning of the sticks and the resulting quadrilaterals.

(Feel free to explore with the paper sticks provided.)

Performance Task

Polygons



Use these paper strips to simulate the sticks in the kite task. Cut out each stick, punch holes in the stick where indicated and fasten two together using brads to explore how the sticks can define the shape of the kite.

Definitions of Convex Quadrilaterals

A **parallelogram** is a quadrilateral with two pairs of parallel sides. Equivalent conditions are that opposite sides are of equal length; that opposite angles are equal.

A **rhombus** is a quadrilateral with all four sides of equal length. Equivalent conditions a parallelogram in which adjacent sides are of equal lengths.

A **rectangle** is a quadrilateral with all four sides right angles. Equivalent conditions a parallelogram with at least one angle a right angle.

A **square** (regular quadrilateral) is a quadrilateral with all four sides of equal length (equilateral), and all four angles are right angles. A quadrilateral is a square if and only if it is both a rhombus and a rectangle.

A **kite** is a convex quadrilateral with two pairs of adjacent sides equal length.

A **trapezoid** is a convex quadrilateral with exactly one pair of opposite sides parallel.

An **Isosceles trapezoid** is a trapezoid with the base angles equal in measure.

A **trapezium** is a convex quadrilateral with no sides parallel. (In British English this would be called an irregular quadrilateral, and was once called a *trapezoid*.)

Properties of Quadrilaterals Task

Inside Mathematics Website

The Core Ideas (standards) addressed by this task are:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.
 - Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
- Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures, and using counter examples and indirect proof.
 - Identify, formulate and confirm conjectures.
 - Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

TASK: A Kite manufacturer has the problem of making kites of a specific quadrilateral shape according to the lengths and position of the diagonal supports. They need a rule for how to measure and position the diagonal supports for all possible 7 convex quadrilaterals: parallelogram, rhombus, rectangle, square, trapezoid, isosceles trapezoid, and trapezium.

- 1) Working in groups with the given supports/brads and other available tools/materials; investigate how the two support diagonals of the 7 quadrilaterals should be measured and positioned in order to produce the desired quadrilateral. Convince first yourself, and then your group.
- 2) Draw your conclusions on chart paper and be prepared to explain your findings. Make your findings clear to the Kite Company engineering team.
- 3) Report your findings orally to the entire group.
- 4) In your group, for your assigned quadrilateral, write a formal proof to convince a skeptic.