COMMON CORE STATE STANDARDS FOR MATHEMATICS

The *Standards for Mathematical Practice* is a document in the CCSS that describes different types of expertise students should possess and mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

The first of these are the National Council of Teachers of Mathematics, NCTM, process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). This paper combines information from those sources and lists what students will be doing when they demonstrate mathematical proficiency.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution.
- plan a solution pathway rather than simply jumping into a solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- monitor and evaluate their progress and change course if necessary.
- who are older might, depending on the context of the problem:
  - ✓ transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.
  - ✓ explain correspondences between equations, verbal descriptions, tables, and graphs.
  - ✓ draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- who are younger might:
  - ✓ rely on using concrete objects or pictures to help conceptualize and solve a problem.
  - ✓ check their answers to problems using a different method.
- continually ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems and identify correspondences between approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- bring two complementary abilities to bear on problems involving quantitative relationships:
  - ✓ *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
  - ✓ *contextualize* - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

- use quantitative reasoning that entails habits of creating a coherent representation of the problem at hand:
  - ✓ considering the units involved,
  - ✓ attending to the meaning of quantities (not just how to compute them), and
  - ✓ knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases.
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of plausible arguments.
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw in an argument, explain what it is.
  - ✓ Elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
  - ✓ Later, students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments.

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation.
  - ✓ In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- apply what they know to make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation.
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- routinely interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- who are in high school:
  - ✓ analyze graphs of functions and solutions generated using a graphing calculator.
  - ✓ detect possible errors by strategically using estimations and other mathematical knowledge.

- ✓ when making mathematical models, know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant external mathematical resources (e.g., digital website content) and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- try to use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- carefully specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ By high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
  - ✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
  - ✓ Older students, in the expression $x^2 + 9x + 14$, can see the 14 as 2 x 7 and the 9 as 2 + 7.
  - ✓ They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. (e.g., They see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.)

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

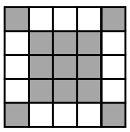
- notice if calculations are repeated.
- look both for general methods and for shortcuts.
  - ✓ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeated decimal.
  - ✓ By paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.
  - ✓ Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead high school students to the general formula for the sum of a geometric series.
- maintain oversight of the process of solving a problem while attending to the details.
- continually evaluate the reasonableness of their intermediate results.

Sidewalk Patterns

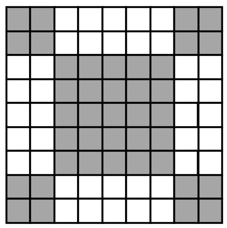
This problem gives you the chance to:
- work with patterns
- work out the n^th^ term of a sequence

In Prague some sidewalks are made of small square blocks of stone.

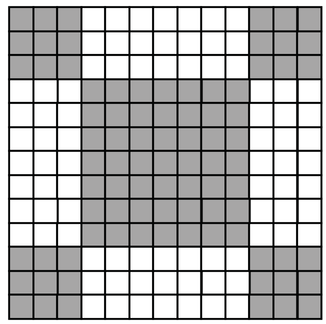
The blocks are in different shades to make patterns that are in various sizes.



Pattern number 1



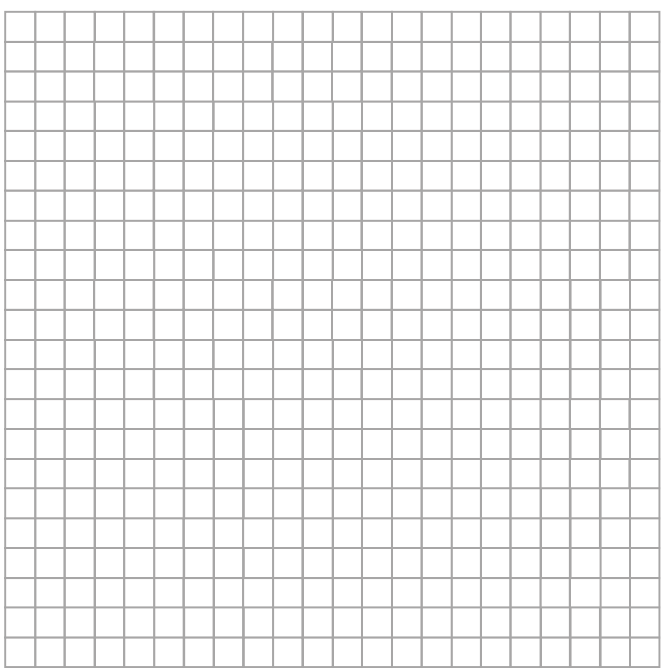
Pattern number 2



Pattern number 3

1. Draw the next pattern in this series.

You may not need to use all of the squares on this grid.



Pattern number 4

2. Complete the table below.

Pattern number, n	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

3. What do you notice about the number of white blocks and the number of gray blocks?

____

4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

   a. Fill in the blank spaces in this list.

   $25 = 5^2$ $81 =$ ______ $169 =$ ______ $289 = 17^2$

   b. How many blocks will pattern number 5 need?

   ____

   c. How many blocks will pattern number *n* need?

   ____

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

____

____

   b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

   ____

9

Student A

4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.
   a. Fill in the blank spaces in this list.

   25 = 5^2 81 = 9^2 169 = 13^2 289 = 17^2

   b. How many blocks will pattern number 5 need? 441 blocks

   c. How many blocks will pattern number n need? $(1+4n)^2$

Student C

4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.
   a. Fill in the blank spaces in this list.

   25 = 5^2 81 = 9^2 169 = 13^2 289 = 17^2

   b. How many blocks will pattern number 5 need? 441

   c. How many blocks will pattern number n need? n^2

Student D

4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.
   a. Fill in the blank spaces in this list.

   25 = 5^2 81 = 9^2 169 = 13^2 289 = 17^2

   b. How many blocks will pattern number 5 need? 441

   c. How many blocks will pattern number *n* need? $(\text{previous}+4)^2$

Student F

4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.
   a. Fill in the blank spaces in this list.

   25 = 5^2 81 = 9^2 169 = 13^2 289 = 17^2

   b. How many blocks will pattern number 5 need? 441

   c. How many blocks will pattern number *n* need? $(n+4)^2$

Student A

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

Divide the number of blocks by 2 and add half of one number to the other because the one less by 1 equals the number of white blocks.

b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

312 blocks

$312 + \frac{1}{2}$

$2\overline{)625}$

$312\frac{1}{2} - \frac{1}{2} \quad + \quad 3\frac{1}{2} + \frac{1}{2} = 625$

$312 + 313 = 625$

Algebra – 2008

Student B

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

you divide the total number of blocks by 2. Then add one to get the number of grey and that will make the white be one less than the greys.

b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

312 white blocks

$2\overline{)625}$ = 312.5

$312 + 313 = 625$

312 ↑ whites, 313 ↑ greys

9

Student C

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

You can figure out the number out white blocks because you know the grey blocks have one more you subtract and then see if that number is one less.

b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

~~625 − 301 = 324~~ ~~625 − 366 = 259~~ ~~625 − 256 = 369~~ ~~625 − 243 = 382~~ 625 − 312 = 313

625 ← total
−313 ← grey blocks
312 ← white blocks

312 blocks

Student E

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

All you would need to do is you subtract one from the total and divide by 2 to get White blocks.

b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

625 − 1 = 624
624 ÷ 2 = 312
312

312 white blocks

9