COMMON CORE STATE STANDARDS FOR MATHEMATICS

The *Standards for Mathematical Practice* is a document in the CCSS that describes different types of expertise students should possess and mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.

The first of these are the National Council of Teachers of Mathematics, NCTM, process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy). This paper combines information from those sources and lists what students will be doing when they demonstrate mathematical proficiency.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution.
- plan a solution pathway rather than simply jumping into a solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- monitor and evaluate their progress and change course if necessary.
- who are older might, depending on the context of the problem:
  - ✓ transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.
  - ✓ explain correspondences between equations, verbal descriptions, tables, and graphs.
  - ✓ draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- who are younger might:
  - ✓ rely on using concrete objects or pictures to help conceptualize and solve a problem.
  - ✓ check their answers to problems using a different method.
- continually ask themselves, “Does this make sense?”
- understand the approaches of others to solving complex problems and identify correspondences between approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- bring two complementary abilities to bear on problems involving quantitative relationships:
  - ✓ *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
  - ✓ *contextualize* - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

- use quantitative reasoning that entails habits of creating a coherent representation of the problem at hand:
  - ✓ considering the units involved,
  - ✓ attending to the meaning of quantities (not just how to compute them), and
  - ✓ knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases.
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of plausible arguments.
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw in an argument, explain what it is.
  - ✓ Elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
  - ✓ Later, students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments.

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation.
  - ✓ In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- apply what they know to make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation.
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- routinely interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- who are in high school:
  - ✓ analyze graphs of functions and solutions generated using a graphing calculator.
  - ✓ detect possible errors by strategically using estimations and other mathematical knowledge.

- ✓ when making mathematical models, know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant external mathematical resources (e.g., digital website content) and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- try to use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- carefully specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ By high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
  - ✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
  - ✓ Older students, in the expression $x^2 + 9x + 14$, can see the 14 as 2 x 7 and the 9 as 2 + 7.
  - ✓ They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. (e.g., They see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.)

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated.
- look both for general methods and for shortcuts.
  - ✓ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeated decimal.
  - ✓ By paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.
  - ✓ Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead high school students to the general formula for the sum of a geometric series.
- maintain oversight of the process of solving a problem while attending to the details.
- continually evaluate the reasonableness of their intermediate results.

Illustrating the Standards for Mathematical Practice: *Look for and express regularity in repeated reasoning* and *construct viable arguments*

Video Clip 6 Transcript – Slide Number 28

Teacher – Here's the question, it's about changing numbers around. It's, we have the equation 23 and 2 more and someone gave us that equation and said it was 25. So, we are wondering if you take the 2 and you say and you put it first and you say 2 plus 23, do you still get 25?

Children – Yes, it doesn't matter –

Teacher - It doesn't matter. Why?

Girl in white shirt and black jeans – Because if you keep on just switching it around, it will still make 25. Cause you are just changing one. You're not taking away or adding nothing to it. Cause then it will still be the same number.

Teacher - OK

Narrator – Although these students are using an example with the particular numbers 23 and 2, they're making general statements that can apply to any number, that because nothing has been added or taken away, the order of the addends doesn't matter. Now the teacher presents some problems with numbers that are too large for these second graders to add up easily. She wants to see if they can extend their arguments when they can't actually carry out the addition.

Teacher – So I'm going to go back to this one though, what if I take the 266 and put that first and then put the 175 second? Let's just take a little survey, how many people think that if I put the 266 first and the 175 second that my answer is still going to come out to this 441? How many people think that will happen? OK, does anyone want to say what they are thinking? Then we are going to move on to something else. OK, someone I haven't heard from, Anam, over here.

Anam – You see the numbers are still going to be the same because . . .

Teacher – Say it louder so everyone in the room can hear.

Anam – You see because the numbers are gonna be the same, the numbers are the same, so it must be the same answer.

Teacher – Alright. So why don't you use the marker, cause I noticed you kind of pointing to things and explain what you meant about the numbers are the same.

Anam – This number and that number are the same, and this number and that number are the same, so that it's gonna be the same answer.

Teacher – So maybe you might draw lines and kind of match them up to show which numbers are the same.

Anam – These two and these two.

Teacher – And so what else is going to be the same?

Anam – This one.

Teacher – So alright, maybe you want to fill that in so that you think that the total or the sum will be the same.

Children – It is, because you're not, you're not, you're just switching it around. Yeah, cause nothing changes. Only the other number goes first.

Narrator – Later in the discussion the students think about switching numbers in subtraction. Some young students assume that if they can change the order of the numbers in addition they can change the order in other operations. Here, students present their arguments about why subtraction is different.

Girl with white shirt and black jeans – That if you have 3 take away 7, but 3 doesn't have 7 so you can only do 7 and 3 because you don't, because 3 is not a 7.

Teacher – There's not enough in 3 to take away 7, is that what you are kind of saying?

Girl with white shirt and black jeans - Yes.

Teacher – So, how many could I take away?

Girl with white shirt and black jeans – Um, if you have 7 then you can do that one.

Teacher – What if I had 3 and I wanted to take away 7, how many could I take away?

Girl with white shirt and black jeans – You could only take away 3.

Teacher – I could take away 3.

Girl with white shirt and black jeans – To make 0, but if you didn't want you could take away 4 or you can take away more.

Teacher – So there is something about what Regis was saying about this 4. So, Ashanti?

Ashanti – I think that um, you can’t use the 3 because after you use the 3, you 3,2,1,0,0,0, it’s going to keep on repeating itself until it gets to 7.

Teacher – Say the last part, it’s going to keep on . . .

Ashanti – The 0 is going to keep on repeating itself when it gets to 7.

Teacher – Oh, so when I go back I get to the 0 and I’m still at 0 and it keeps repeating? OK. So we have done a lot of good thinking here. This is a really difficult question. Last night when I was with some teachers, and we were talking about this, it was a tricky question for all of us to think about, but you know what? You have done some really hard thinking and come up with some ideas. Did you want to say one thing because your hand was up Laguar?

Laguar – Well, that wouldn’t be 0. That would be negative 4.

Teacher – Why are you thinking that? What’s negative 4?

Laguar – That means you’re going lower. If you are going lower than 0 that means negative 1, negative 2, negative 3.