Diving Deeper into the Common Core State Standards for Mathematics: Leading with the Mathematical Practices

Today’s Webinar will begin shortly

Please download the webinar handout at:

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A recording of today’s webinar will be available at:
National Council of Supervisors of Mathematics

Diving Deeper into the Common Core State Standards for Mathematics:
Leading with the Mathematical Practices

www.mathedleadership.org
Presenters

Diane J. Briars, NCSM President
Valerie Mills, NCSM Central 1 Regional Director
Suzanne Mitchell, NCSM President-Elect

A recording of today’s webinar will be available at:
The critical first steps will be to help educators interpret and understand the CCSS and to support the development and implementation of comprehensive, coherent instruction and assessment systems...we plan to work with our local, state, and national affiliates to feature the CCSS in our professional development opportunities, including annual and regional conferences, academies, and seminars...

NCSM Joint Public Statement with NCTM, AMTE and ASSM, June 2010
Today’s Goals

• Provide a close look at the Standards for Mathematical Practices
• Consider how you can support student engagement in the mathematical practices through your mathematics program.

A recording of today’s webinar will be available at:
What is NCSM?

International organization of and for mathematics education leaders:

- Coaches and mentors
- Curriculum leaders
- Department chairs
- District supervisors/leaders
- Mathematics consultants
- Mathematics supervisors
- Principals
- Professional developers

Publishers and authors
Specialists and coordinators
State and provincial directors
Superintendents
Teachers
Teacher educators
Teacher leaders

A recording of today’s webinar will be available at:
Common Core State Standards for Mathematics

Two types of standards:
• Content Standards
• Standards for Mathematical Practice

*Please have a copy of the Standards for Mathematical Practice. They are in the webinar handout at*


A recording of today’s webinar will be available at:
Standards for Mathematical Practice

“The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.”

(CCSS, 2010)

A recording of today’s webinar will be available at:
Underlying Frameworks

National Council of Teachers of Mathematics

5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations


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http://www.carnegielearning.com/webinars  
http://www.mathedleadership.org/events/webinars.html
Underlying Frameworks

Strands of Mathematical Proficiency

- Strategic Competence
- Conceptual Understanding
- Adaptive Reasoning
- Productive Disposition
- Procedural Fluency


A recording of today’s webinar will be available at:

Strands of Mathematical Proficiency

- **Conceptual Understanding** – comprehension of mathematical concepts, operations, and relations
- **Procedural Fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic Competence** – ability to formulate, represent, and solve mathematical problems
- **Adaptive Reasoning** – capacity for logical thought, reflection, explanation, and justification
- **Productive Disposition** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

A recording of today’s webinar will be available at:
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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http://www.carnegielearning.com/webinars
http://www.mathedleadership.org/events/webinars.html
The Standards for Mathematical Practice

Take a moment to examine the first three words of each of the 8 mathematical practices… what do you notice?

Mathematically Proficient Students…
The Standards for [Student] Mathematical Practice

Consider the *verbs* that illustrate the student actions each practice.

For example, examine Practice #3: Construct viable arguments and critique the reasoning of others.

*Highlight the verbs.*

*Discuss with a partner:*

*What jumps out at you?*

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http://www.carnegielearning.com/webinars  
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Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students:

• understand and use stated assumptions, definitions, and previously established results in constructing arguments.
• make conjectures and build a logical progression of statements to explore the truth of their conjectures.
• analyze situations by breaking them into cases, and can recognize and use counterexamples.
• justify their conclusions, communicate them to others, and respond to the arguments of others.
• reason inductively about data, making plausible arguments that take into account the context from which the data arose.
• compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.
• construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
• determine domains to which an argument applies.
• listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

A recording of today’s webinar will be available at:

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students:

- **understand** and **use** stated assumptions, definitions, and previously established results in constructing arguments.
- **make** conjectures and **build** a logical progression of statements to **explore** the truth of their conjectures.
- **analyze** situations by **breaking** them into cases, and can **recognize** and **use** counterexamples.
- **justify** their conclusions, **communicate** them to others, and **respond** to the arguments of others.
- **reason** inductively about data, **making** plausible arguments that take into account the context from which the data arose.
- **compare** the effectiveness of two plausible arguments, **distinguish** correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-**explain** what it is.
- **construct** arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
- **determine** domains to which an argument applies.
- **listen** or **read** the arguments of others, **decide** whether they make sense, and **ask** useful questions to clarify or improve the arguments.

A recording of today’s webinar will be available at:

The Standards for [Student] Mathematical Practice

On a scale of 1 (low) to 6 (high), to what extent is your school/district promoting students’ proficiency in Practice 3?

Evidence for your rating?

A recording of today’s webinar will be available at:
The Standards for [Student] Mathematical Practice

SMP1: *Explain and make conjectures* …
SMP2: *Make sense of* …
SMP3: *Understand and use* …
SMP4: *Apply and interpret* …
SMP5: *Consider and detect* …
SMP6: *Communicate precisely to others* …
SMP7: *Discern and recognize* …
SMP8: *Notice and pay attention to* …

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Standards for Mathematical Practice

• Describe the thinking processes, habits of mind and dispositions that students need to develop a deep, flexible, and enduring understanding of mathematics; in this sense they are also a means to an end.

SP1. Make sense of problems

“….they [students] analyze givens, constraints, relationships and goals. ….they monitor and evaluate their progress and change course if necessary. …. and they continually ask themselves “Does this make sense?”

A recording of today’s webinar will be available at:
Standards for Mathematical Practice

AND….

• Describe mathematical content students need to learn.

**SP1. Make sense of problems**

“……. students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.”

A recording of today’s webinar will be available at:

Buttons Task

Gita plays with her grandmother’s collection of black & white buttons. She arranges them in patterns. Her first 3 patterns are shown below.

1. Draw pattern 4 next to pattern 3.
2. How many white buttons does Gita need for Pattern 5 and Pattern 6? Explain how you figured this out.
3. How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out.
4. Gita thinks she needs 69 buttons in all to make Pattern 24. How do you know that she is not correct? How many buttons does she need to make Pattern 24?

A recording of today’s webinar will be available at:
Button Task

1. Individually complete parts 1 - 3.

2. Then work with a partner to compare your work and complete part 4. (Look for as many ways to solve parts 3 and 4 as possible.)
Buttons Task-Solutions

Gita plays with her grandmother’s collection of black & white buttons. She arranges them in patterns. Her first 3 patterns are shown below.

1. Draw pattern 4 next to pattern 3.
2. How many white buttons does Gita need for Pattern 5 and Pattern 6? Explain how you figured this out. **15 buttons and 18 buttons**
3. How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out. **34 buttons**
4. Gita thinks she needs 69 buttons in all to make Pattern 24. How do you know that she is **not** correct?
   How many buttons does she need to make Pattern 24? **73 buttons**

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- [http://www.carnegielearning.com/webinars](http://www.carnegielearning.com/webinars)
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Buttons Task

How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out.

Pattern #1     Pattern #2             Pattern #3                Pattern #11

• 3+3+3+3+3+3+3+3+3+3+3+1
• 11 groups of 3 plus 1
• (11 x 3) + 1

• 11 + 11 + 11 + 1
• 3 groups of 11 plus 1
• 3 x 11 + 1

A recording of today’s webinar will be available at:
Button Task

Which mathematical *practices* are needed complete the task?

Indicate the primary practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Standards for [Student] Mathematical Practice

“Not all tasks are created equal, and different tasks will provoke different levels and kinds of student thinking.”

Stein, Smith, Henningsen, & Silver, 2000

“The level and kind of thinking in which students engage determines what they will learn.”

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

A recording of today’s webinar will be available at:
The Nature of Tasks Used in the Classroom …

Will Impact Student Learning!

Tasks as they appear in curricular materials → Student learning

A recording of today’s webinar will be available at:
But, WHAT TEACHERS DO with the tasks matters too!

The Mathematical Tasks Framework

Tasks as they appear in curricular materials → Tasks as set up by teachers → Tasks as enacted by teachers and students → Student learning

Stein, Grover & Henningsen (1996)
Smith & Stein (1998)
Stein, Smith, Henningsen & Silver (2000)

A recording of today’s webinar will be available at:
www.Inside Mathematics.org

A reengagement lesson using the Button Task

Francis Dickinson
San Carlos Elementary
Grade 5


A recording of today’s webinar will be available at:
3. How many buttons in all does Gita need to make Pattern 11?

34 buttons

Explain how you figured this out.

I x (11 x 3) + 1 = 34 buttons

I added one for the black button in the middle.
Learner B

3. How many buttons in all does Gita need to make Pattern 11?

\[ \frac{2}{3} \times 5 \times 6 = 78 \times 3 \times 2 \]

\[ 34 \checkmark \]

Explain how you figured this out.

I added \[ 4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 \]

\[ = 34 \text{ which is the sum of buttons} \]

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Button Task Revisited

• What might a teacher get out of using the same math task two days in a row, rather than switching to a different task(s)?
  – Address common misconceptions
  – Support students in moving from less to more sophisticated solutions

A recording of today’s webinar will be available at:
Button Task Revisited

Which of the Standards of Mathematical Practice did the students engage in when they revisited the task?

Indicate the primary practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
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But, WHAT TEACHERS DO with the tasks matters too!

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Smith & Stein (1998)
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Triangles Task

In triangle $ABC$, the point $P$ is one third of the way from $A$

1. Explain why the area of the shaded triangle is $2/3$ of the area of triangle $ABC$.

Source: Insidemathematics.org: MARS, 2008

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http://www.carnegielearning.com/webinars
http://www.mathedleadership.org/events/webinars.html
Two new points, Q and R, have been added to triangle ABC. Q is one third of the way from B to C and R is one third of the way from C to A.

2. Explain why the area of shaded triangle PBQ is 1/3 of 2/3 of the area of triangle ABC.

Source: Insidemathematics.org: MARS, 2008

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Standards for [Student] Mathematical Practice

The 8 Standards for Mathematical Practice – place an emphasis on student demonstrations of learning…

Equity begins with an understanding of how the selection of tasks, the assessment of tasks, the student learning environment creates great inequity in our schools…

A recording of today’s webinar will be available at:
Implementation Issue

To what extent do all students in your class, school or district have the opportunity to engage in tasks that promote attainment of the mathematical practices on a regular basis?

*Please rate on a scale of 1 (low) to 6 (high).*
Leading with the Mathematical Practices

• Build upon/extend work on NCTM Processes and NRC Proficiencies
• Phase in implementation
• Consider relationships among the practices
• Analyze instructional tasks in terms of opportunities for students to regularly engage in practices.

A recording of today’s webinar will be available at:
A recording of today’s webinar will be available at:
Insidemathematics.org

MARS Tasks
Buttons

- Task
- Rubric
- Core Mathematical Ideas and Challenges
- Questions for Teacher Reflection
- Discussion of Successful Examples of Student Work
- Discussion of Student Misconceptions
- Graph and Analysis of the MARS Task Data
- Summary of Student Understandings and Misunderstandings
- Implications for Instruction
- Entire "Buttons" Toolkit Packet (large file)

- Video

A recording of today’s webinar will be available at:
Forthcoming Resources and Tools

**NCSM**

- *Illustrating the Standards for Mathematical Practice* professional development materials.
  - NCSM Annual Conference session

- Tool for analyzing instructional materials in light of CCSS and related professional development activities.
  - Under development in collaboration with Bill Bush, University of Louisville, and CCSSO.
  - Target release date: June 1, 2011.

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[http://www.mathedleadership.org/events/webinars.html](http://www.mathedleadership.org/events/webinars.html)
AMTE, ASSM, NCSM, NCTM

Priority Activities

1. Advancing the Vision of High Quality Mathematics Education: Supporting Implementation of CCSS.
   a. Toolkit
   b. Regional meetings of leadership teams

2. Appoint a Joint Committee of AMTE, ASSM, NCSM and NCTM to serve as an ongoing advisory group regarding CCSS.

3. Convene a panel of professional development experts to develop a conceptual framework for teacher professional development systems to support CCSS at the school, district and state levels.

A recording of today’s webinar will be available at:
4. Convene an Assessment Working Group to coordinate the field’s best guidance on assessment development and ensure that new student assessments address the priorities (e.g., mathematical practices) articulated in CCSS.

5. Develop and launch a research agenda focused on implementation of the CCSS that includes systematic study of the instantiation and implementation of the standards, monitors the impact on instruction and student learning and informs revisions of CCSS.
Forthcoming CCSS Companion Resources

• Technical Manual
  – Highlights structural features in the standards but not highly visible, e.g., how particular ideas connect and grow across grades.

• Standards Progressions documents

A recording of today’s webinar will be available at:
The Illustrative Mathematics Project

- Will develop a complete set of tasks for each standard
  - Range of difficulty
  - Simple illustrations of single standards to complex tasks spanning many standards.
- Provide a process for submitting, discussing, reviewing, and publishing tasks.
- Launch Team: Phil Daro, William McCallum (chair), Jason Zimba

[link] illustrativemathematics.org

A recording of today’s webinar will be available at:
NCSM Professional Development Opportunities

A recording of today’s webinar will be available at:
http://www.carnegielearning.com/webinars
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NCSM Professional Development Opportunities

- NCSM Summer Leadership Academy
  - June 21-23, 2011, Atlanta, GA

- Fall One-Day Seminars
  - October 19, 2011, Atlantic City
  - October 26, 2011, St. Louis
  - November 2, 2011, Albuquerque

A recording of today’s webinar will be available at:
Reflection: Now What?

What actions will you take based on what we discussed today?

– What do you need to learn?
– Who will you work with?
– What do you need to integrate into your practice?
– Who will support you?

A recording of today’s webinar will be available at:
Thank you for joining us!

Additional questions? Contact:
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Valerie Mills: Valerie.Mills@oakland.k12.mi.us
Suzanne Mitchell: suzmitc@comcast.net

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