



The Future of Math Education:

A Panel Discussion of Promising Practices






Distinguished Panel

- **Francis 'Skip' Fennell**, professor of education, McDaniel College, past NCTM, AMTE president
- **Cathy Fosnot**, professor emeritus of childhood education, City College of New York, Founding Director of Math in the City
- **Valerie L. Mills**, president, National Council of Supervisors of Mathematics; supervisor and mathematics education consultant, Oakland Schools, Michigan


Moderator

- Tim Hudson, Sr. Director of Curriculum Design, DreamBox Learning
- 

Agenda

- Formative Assessment
 - Success for All Students with Common Core & Learning Resources
 - Selecting & Implementing Digital Learning Resources
 - Q & A
- 

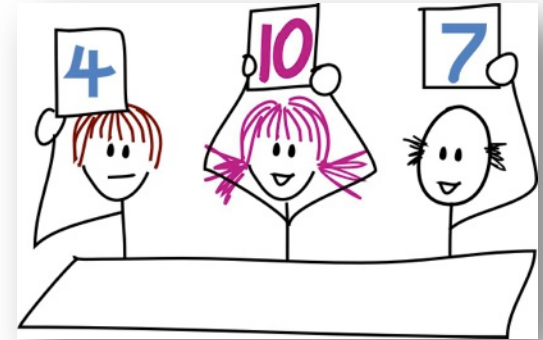
Formative Assessment

- How do we ensure it's not just “another thing” to do?
 - How do we ensure it's an integral component of learning rather than as another approach to assessment?
- 

Valerie Mills



Embedded Formative Assessment

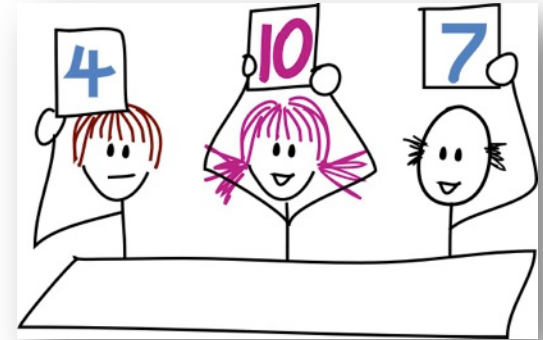


Three key elements:

- 1. elicit evidence** about learning to close the gap between current and desired performance,
- 2. adjust** the learning experience to close the performance gap with useful feedback, and
- 3. involve students** in the assessment learning process

Adapted from Margaret Heritage, 2008

Formative Assessment and Productive Goals



Goals and lessons need to...

- focus on the mathematics concepts and practices (not on doing particular math problems)
- be specific enough that you can effectively gather and use information about student thinking
- be understood to sit within a trajectory of goals and lessons that span days, weeks, and/or years

Complete problems #3 -18

SIMPLIFYING EXPRESSIONS Simplify the expression. Write your answer using exponents.

3. $\frac{5^6}{5^2}$

4. $\frac{2^{11}}{2^6}$

5. $\frac{3^9}{3^5}$

6. $\frac{(-6)^8}{(-6)^5}$

7. $\frac{(-4)^7}{(-4)^4}$

8. $\frac{(-12)^9}{(-12)^3}$

9. $\frac{10^5 \cdot 10^5}{10^4}$

10. $\frac{6^7 \cdot 6^4}{6^6}$

11. $\left(\frac{1}{3}\right)^5$

12. $\left(\frac{3}{2}\right)^4$

13. $\left(-\frac{5}{4}\right)^4$

14. $\left(-\frac{2}{5}\right)^5$

15. $7^9 \cdot \frac{1}{7^2}$

16. $\frac{1}{9^5} \cdot 9^{11}$

17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12}$

18. $4^9 \cdot \left(-\frac{1}{4}\right)^5$

Revise directions to focus students on mathematical goals that describe important concepts as well as skills.

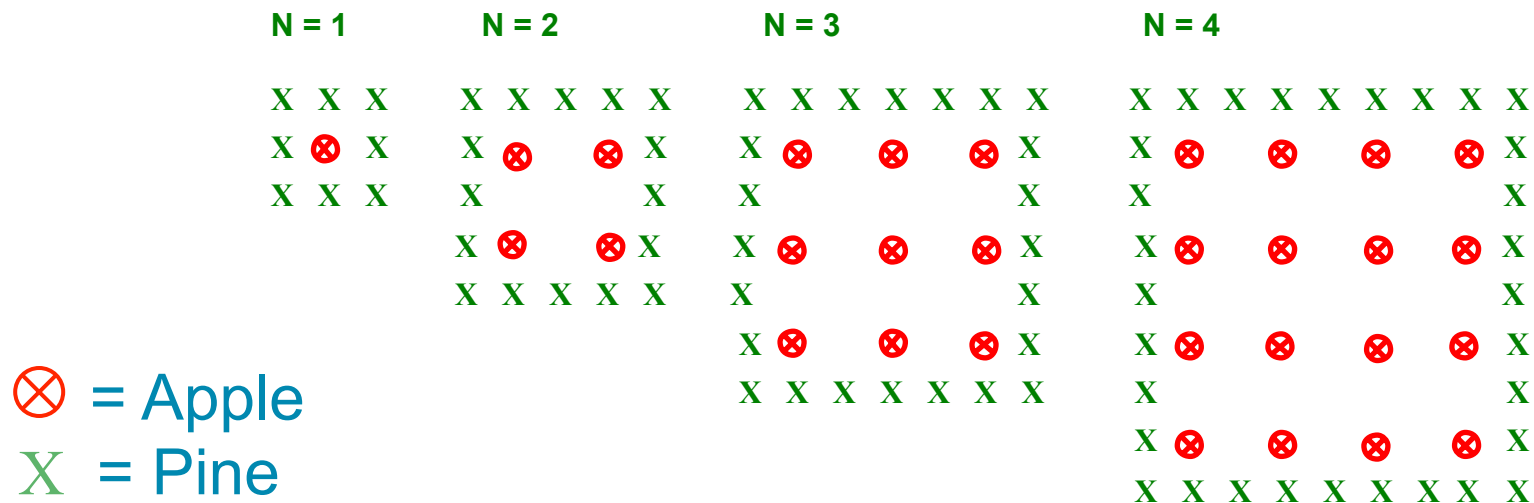
Look closely at this problem set to identify the solutions that will be positive and those that will be negative without fully simplifying each task. Describe the important features of an expression that help you make this decision.

| | | | |
|----------------------------------|----------------------------------|---|---|
| 3. $\frac{5^6}{5^2}$ | 4. $\frac{2^{11}}{2^6}$ | 5. $\frac{3^9}{3^5}$ | 6. $\frac{(-6)^8}{(-6)^5}$ |
| 7. $\frac{(-4)^7}{(-4)^4}$ | 8. $\frac{(-12)^9}{(-12)^3}$ | 9. $\frac{10^5 \cdot 10^5}{10^4}$ | 10. $\frac{6^7 \cdot 6^4}{6^6}$ |
| 11. $\left(\frac{1}{3}\right)^5$ | 12. $\left(\frac{3}{2}\right)^4$ | 13. $\left(-\frac{5}{4}\right)^4$ | 14. $\left(-\frac{2}{5}\right)^5$ |
| 15. $7^9 \cdot \frac{1}{7^2}$ | 16. $\frac{1}{9^5} \cdot 9^{11}$ | 17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12}$ | 18. $4^9 \cdot \left(-\frac{1}{4}\right)^5$ |

Revise instructional goals and directions to focus students on important mathematical concepts and relationships as well as skills.

Apple Orchard


A farmer plants apple trees in a square pattern. In order to protect the apples trees against the wind he plants pine trees all around the orchard. Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees.



Apple Orchard


Question 3.1

Complete the table:

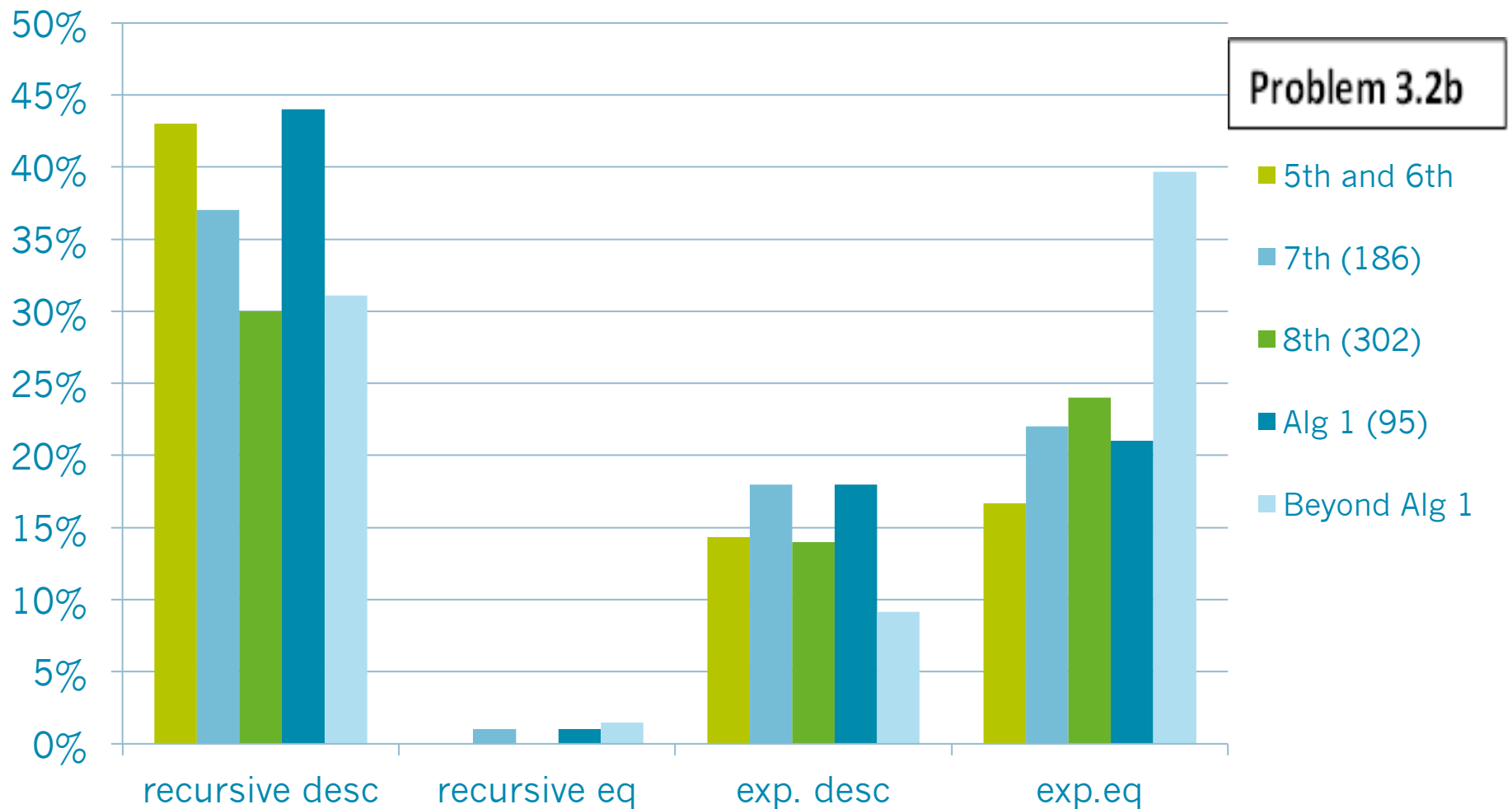


| n | Number of apple trees | Number of pine trees |
|-----|-----------------------|----------------------|
| 1 | 1 | 8 |
| 2 | 4 | 16 |
| 3 | 9 | 24 |
| 4 | 16 | 32 |
| 5 | 25 | 40 |

Question 3.2

- 
- A** Describe the pattern (words or symbols) so that you could find the number of apple trees for any stage in the pattern illustrated on the previous page: *Number of apple trees = n^2*
- B** Describe the pattern (words or symbols) so that you could find the number of pine trees for any stage in the pattern illustrated on the previous page: *Number of Pine Trees = $8n$*
Next number of Pine Trees = Current number of pine trees plus 8
- C** For what value(s) of n will the number of apple trees equal the number of pine trees. Show your method of calculating this.

Apple Tasks – Linear Pattern




Cathy Fosnot





Jennifer James, Anthropologist

“Tapestry is that body of assumptions, beliefs, customs, and practices that we accept as foundational. They define who we are. In this time of great change, the tapestry is being torn rapidly and everywhere, and we begin to fall apart, becoming anxious and losing belief in who we are. We look backward. We become pessimistic about the present and the future because we can’t envision a new tapestry.”






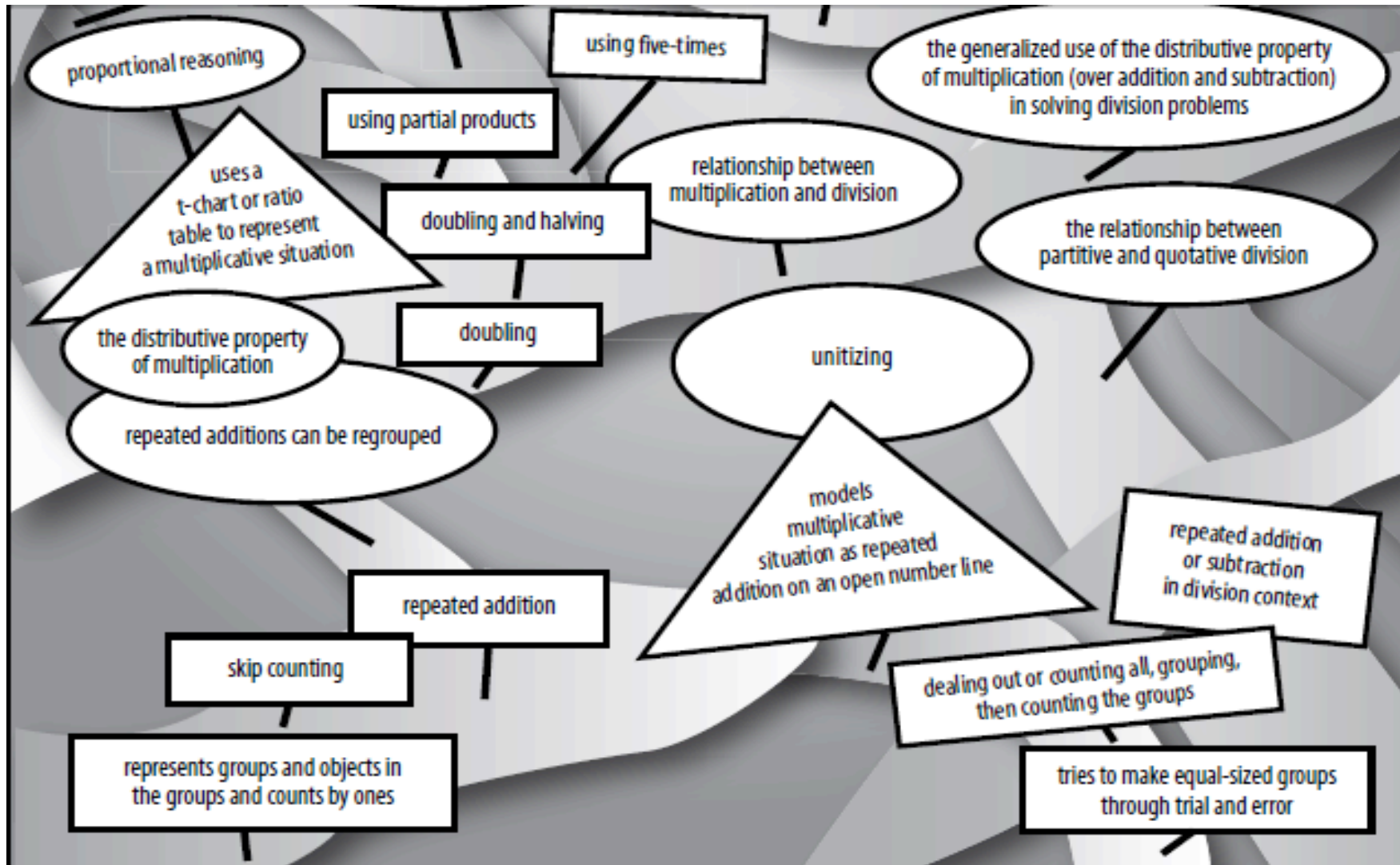
Heraclitus

“You cannot step twice into the same stream. For as you are stepping in, other waters are ever flowing.”

Assessment should guide teaching. It should be continuous and provide information about the “zone of proximal development” (Vygotsky 1978). To do so, it needs to foresee where and how one can anticipate that which is just coming into view in the distance (Streefland 1985). It needs to capture genuine mathematizing: children’s strategies, their ways of modeling realistic problems, and their understanding of key mathematical ideas. Bottom line, it needs to capture where the child is on the landscape of learning—where she has been, what her struggles are, and where she is going: it must be dynamic (Fosnot and Dolk 2001; van den Heuvel-Panhuizen 1996).




The Landscape of Learning





Getting continuous data

- In the moment
 - when conferring
 - analyzing children's work
 - kidwatching as they work
 - From digital technology: DreamBox
 - Formal items designed to capture more than answers
- 

Two-pen assessment

$4 \times 25 =$



$40 \times 25 =$

$16 \times 25 =$

$10 \times 100 =$

$27 \div 3 =$

$10 \times 13 =$

$2 \times 13 =$

$12 \times 13 =$

$3 \times 9 =$

$3 \times 90 =$

$12 \times 9 =$



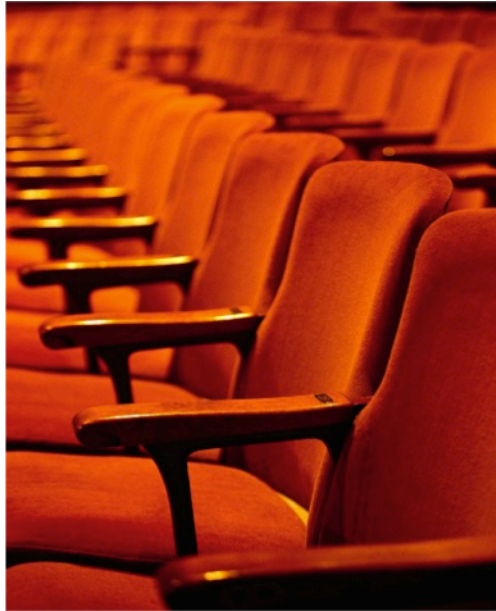
$6 \times 18 =$

$12 \times 12 =$



$6 \times 24 =$

Open-ended



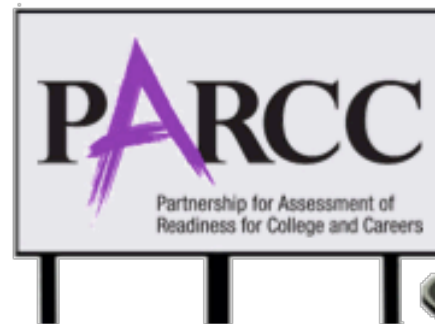
The Auditorium: 16 seats in each row

Each row in the auditorium has 16 seats. The first and second grade classes come in to watch a performance and they fill up 4 rows. How many people are in the audience so far?

Workspace:

Skip Fennell





FORMATIVE ASSESSMENT CONSIDERATIONS





Formative Assessment: Pathways

Observing

Interviews
Show Me

Hinge Questions

Exit Tasks


How can this be communicated and shared with others – teaching teams, parents, links to SMARTER/PARCC?



Success for ALL Students



In the Common Core era...

- How can educators wisely choose and create resources?
 - What can be learned from past initiatives about standards and resource implementation?
- 

Skip Fennell





Understanding

4.NBT

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NF

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions and compare decimal fractions.

4.MD

- Geometric measurement: understand concepts of angle and measure angles.



Representation

3.NF.2 – Understand a fraction as a number on the number line; represent fractions on a number line diagram.

4.NBT.5 – Multiply a whole number...Illustrate and explain...by using equations, rectangular arrays, and/or area models.

5.MD.4 – Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

6.RP.3 – Use ratio and rate reasoning...by reasoning about tables of equivalent ratios, tape diagrams, double line diagrams or equations.



Here's the Point

**Conceptual understanding is
NOT an option,**

*It's an expectation! AND, it's about
time!!!*

Valerie Mills





CCSS Curriculum Materials Analysis Tools

Financial support for this project was provided by


- Brookhill Foundation (Kathy Stumpf)
- Texas Instruments (through CCSSO)

Development team lead by William S. Bush (chair),
University of Louisville, KY

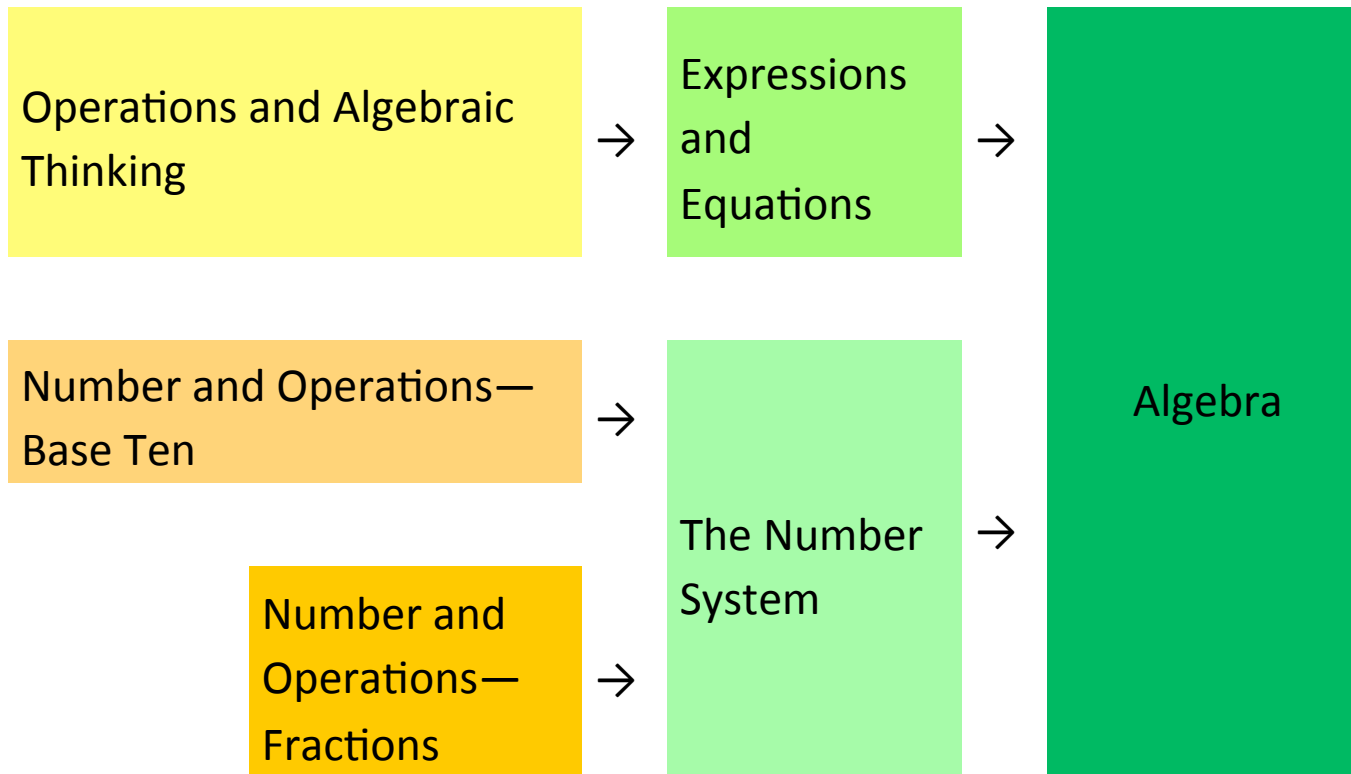
The toolkit can be downloaded from the NCSM website at:
<http://www.mathedleadership.org/ccss/materials.html>



CCSS Curriculum Materials Analysis Tools

- Overview
 - User's Guide
 - Tool 1: Content Analysis
 - Tool 2: Mathematical Practices Analysis
 - Tool 3: Overarching Considerations
 - Equity
 - Assessment
 - Technology
 - Professional Development Facilitator Guide
 - PowerPoint Slides
- 

Content Progression



K 1 2 3 4 5 6 7 8 High School

Tool 1 – Standards for Mathematics Content

| CCSSM Curriculum Analysis Tool 1—Number and Operations in Base Ten for Grades K-2 | | | | | | | | | | | |
|--|-------------|----------------|---|--|-------------|----------------|----------------------|---|-------------|----------------|---------------|
| Name of Reviewer _____ | | | School/District _____ | | | | Date _____ | | | | |
| Name of Curriculum Materials _____ | | | Publication Date _____ | | | | Grade Level(s) _____ | | | | |
| Content Coverage Rubric (Cont) Not Found (N) - The mathematics content was not found. Low (L) - Major gaps in the mathematics content were found. Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. High (H) - The content was fully formed as described in the Standards. | | | Balance of Mathematical Understanding and Procedural Skills Rubric (Bal): Not Found (N) - The content was not found. Low (L) - The content was not developed or developed superficially. Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. Acceptable (A) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. High (H) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed. | | | | | | | | |
| CCSSM Grade K | | | CCSSM Grade 1 | | | CCSSM Grade 2 | | | | | |
| K.NBT/CC Counting and Cardinality/ Number and Operations in Base Ten | Chap. Pages | Cont N-L-M-A-H | Bal N-L-M-A-H | 1.NBT Number and Operations in Base Ten | Chap. Pages | Cont N-L-M-A-H | Bal N-L-M-A-H | 2.NBT Number and Operations in Base Ten | Chap. Pages | Cont N-L-M-A-H | Bal N-L-M-A-H |
| Work with numbers 11-19 to gain foundations for place value | | | | Understand place value | | | | Understand place value. | | | |
| 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. | | | | 2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). | | | | 1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). | | | |
| Counting and Cardinality | | | | Extend the counting sequence | | | | Understand place value | | | |
| 1. Count to 100 by ones and tens 2. Count forward beginning from a given number within the known sequence. 3. Write number from 0 to 20. Represent a number of objects with a written numeral 0-20. | | | | 1. Count to 120, starting at any number less than 120. In this range read and write numerals and represent a number of objects with a written numeral. | | | | 2. Count within 1000; skip count by 5s, 10s, 100s. 3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. | | | |

Tool 2 – Standards for Mathematical Practices


| CCSSM Mathematical Practices Analysis Tool 2 | | Page 1 |
|---|---|--|
| Name of Reviewer _____ School/District _____ Date _____ | | |
| Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____ | | |
| Tool 1 Domain Considered <i>Understanding Place Value</i> | | |
| Opportunities to Engage in the Standards for Mathematical Practices Found Across the Content Standards | | |
| Overarching Habits of Mind | 1. Make sense of problems and persevere in solving them. | 6. Attend to precision. |
| Evidence of how the Standards for Mathematics Practice were addressed (with page numbers) | | |
| Reasoning and Explaining | 2. Reason abstractly and quantitatively. | 3. Construct viable arguments and critique the reasoning of others. |
| Evidence of how the Standards for Mathematics Practice were addressed (with page numbers) | | |

Cathy Fosnot






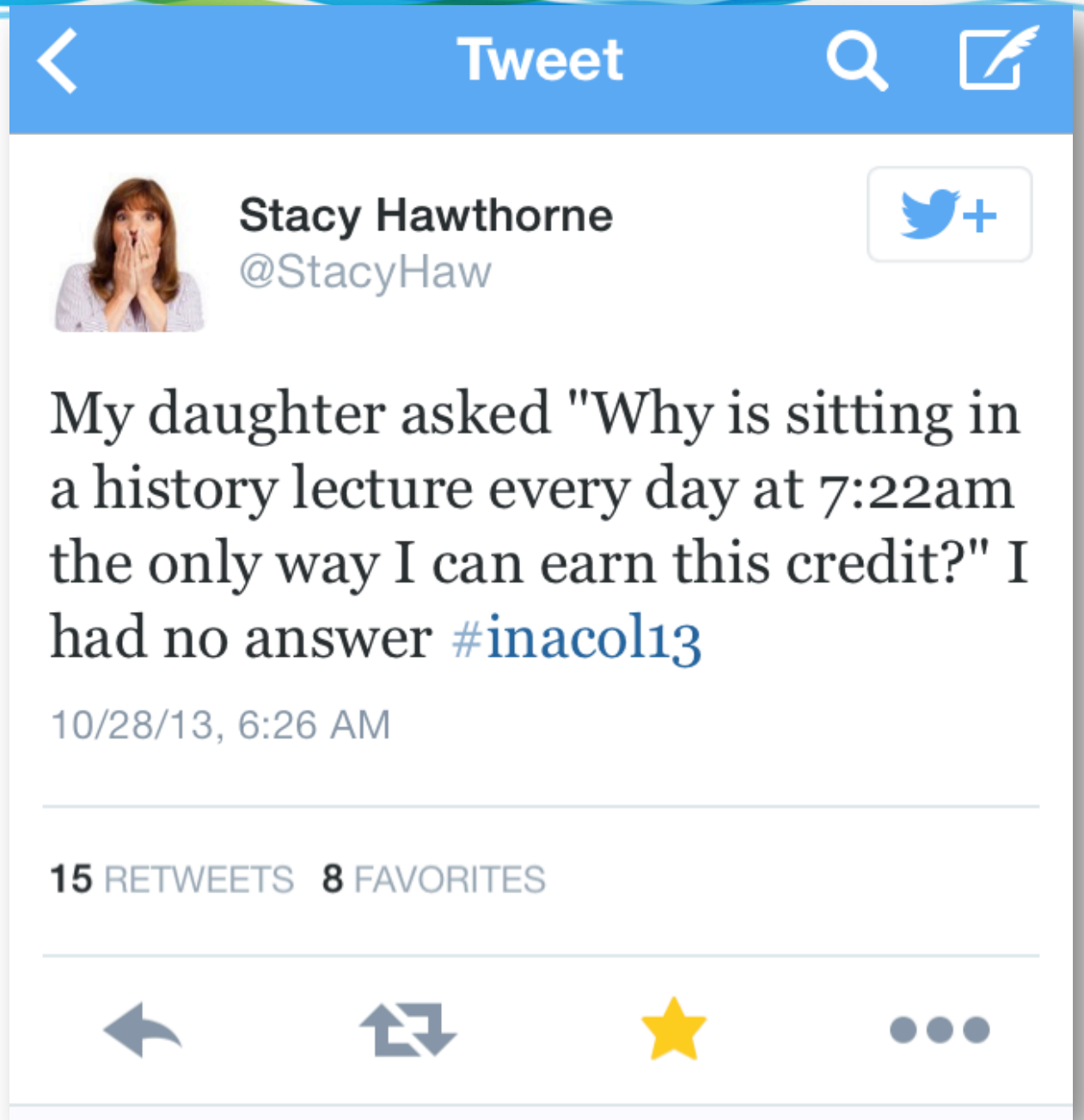
Criteria

- Takes the Standards of Practice seriously
 - Provides professional development *within...teachers learn as they use the materials*
 - Not just a bunch of activities but *crafted sequences* to support progressive development: learning trajectories (landscapes)
- 

Digital Learning

- How can technology meet the needs of every child?
 - How can educators wisely select and implement digital learning resources and technologies?
 - When does learning benefit from the inclusion of digital instructional resources and when might it undermine learning?
 - What do teachers and administrators need to learn about effectively facilitating learning using digital learning resources and what will it take for educators to develop this expertise?
- 

Student Voice





Fullan: Alive in the Swamp

“Technology-enabled innovations have a different problem, mainly **pedagogy and outcomes**. Many of the innovations, particularly those that provide online content and learning materials, use basic pedagogy – most often in the form of introducing concepts by video instruction and following up with a series of progression exercises and tests. **Other digital innovations are simply tools that allow teachers to do the same age-old practices but in a digital format.**” (p. 25)

@sjunkins



**I WASN'T MADE
TO BE A WORKSHEET.**



AND NEITHER WAS I.

@fnoschese



Frank Noschese @fnoschese

30 Nov

The biggest shift in teaching will not happen by replacing textbooks with iPads, but by replacing textbooks with experiences and questions.

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75

RETWEETS

32

FAVORITES



11:51 AM - 30 Nov 13 · Details

Cathy Fosnot






Criteria

General

- Takes the Standards of Practice seriously
- Provides professional development within...
teachers learn as they use the materials
- Not just a bunch of activities but *crafted sequences* to support progressive development: learning trajectories (landscapes)

Digital environments


- Intelligent adaptive learning
 - Seamless formative assessment
 - Seamless home/school connection
 - Choice/personalized learning
- 

Skip Fennell





Using Appropriate Tools Strategically

1. What types of tools are we and will we be using?
 2. The role of manipulative materials?
 3. General tools (drawings, number lines, others)
 4. Technological Tools – what, and when?
 5. The Flipped Classroom – for all?
 6. Transmedia?
- 


Valerie Mills






How do educators evaluate digital learning resources?

Ten Design Considerations

1. Topics are developed with multiple representations (graphs, tables, and equations) and students are asked to use multiple representations in sense making.
 2. Students are engaged in constructing mathematical understanding through substantive tasks that maintain a high level of cognitive demand.
 3. Mathematical discourse is valued.
 - a. Some tasks require written responses.
 - b. Electronic forums and the like promote interaction between peers and instructor.
 - c. Teacher-student and student-student conversations within the confines of the physical classroom.
- 



How do educators evaluate digital learning resources? (Continued)

4. **Online tools and resources support the learning environment. (i.e., Online calculators, graphing tools, journals, hotlinks, etc.)**
 5. **Mathematical content is delivered or available in a variety of formats (i.e., Teacher lecture, demonstrations and applets, games, audio, cooperative problem solving, etc.)**
 6. **Mathematical experiences are provided to build conceptual understanding in conjunction with procedural fluency.**
 7. **Online tutors are available, accessible, and mathematically competent.**
 8. **Program takes advantage of technology (animation, color, movement, links).**
 9. **Program offers suggestions to the teacher for monitoring student learning, adjusting instruction, and providing possible interventions.**
 10. **Program offers supplemental activities (online and offline) to the teacher that support students in developing mathematical reasoning.**
- 



The SAMR Model


S
A
M
R

Q & A





Contact Information

- **Francis 'Skip' Fennell**
 - @SkipFennell
 - **Cathy Fosnot**
 - www.NewPerspectivesOnLearning.com
 - **Valerie L. Mills**
 - <http://www.mathedleadership.org/>
 - **Tim Hudson**
 - @DocHudsonMath
- 

DreamBox Combines Three Essential Elements to Accelerate Student Learning

Rigorous Elementary Mathematics

- Common Core State Standards
- Standards for Mathematical Practice



Motivating Learning Environment

- Student directed, empowering
- Gaming fundamentals, rewards

Intelligent Adaptive Learning[™] Engine

- Millions of individualized learning paths
- Tailored to a student's unique needs

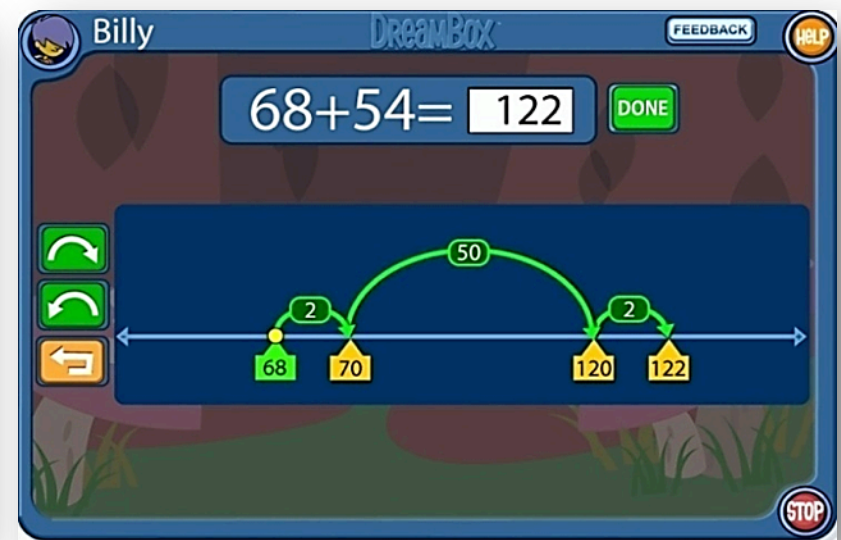
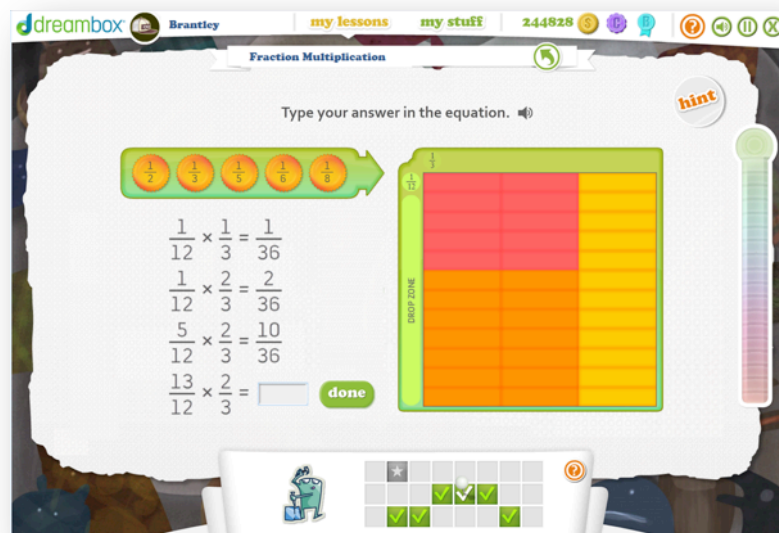


DreamBox Lessons & Virtual Manipulatives



Intelligently adapt & individualize to:

- Students' own intuitive strategies
- Kinds of mistakes
- Efficiency of strategy
- Scaffolding needed
- Response time



Robust Reporting



Classroom Summary Report

School:

Class:

Teachers:

Date:

Symbol Legend

- Skipped based on initial placement
- Passed in unit pretest
- Completed curriculum
- Pending assessment
- Needs assistance
- Working inefficiently

| Student | Grade | Kindergarten Curriculum | 1st Grade Curriculum | 2nd Grade Curriculum | 3rd Grade Curriculum | Time on Task (HH:MM) | Notifi-cations | Student Reports | |
|-------------|-------|-------------------------|----------------------|----------------------|----------------------|----------------------|----------------|------------------------|------------------------|
| Alexander F | 1 | | | | | 17:55 | | Weekly | Detail |
| Alexi K | 1 | | | | | 14:04 | | Weekly | Detail |
| Billy R | 1 | | | | | 14:14 | | Weekly | Detail |
| Brianna S | 1 | | | | | 51:43 | | Weekly | Detail |
| Cassandra H | 1 | | | | | 18:02 | | Weekly | Detail |
| Erinne N | 1 | | | | | 20:42 | | Weekly | Detail |
| Jayce D | 1 | | | | | 28:13 | | Weekly | Detail |
| Josephine J | 1 | | | | | 15:59 | | Weekly | Detail |
| Kevin M | 1 | | | | | 18:18 | | Weekly | Detail |
| Kylee P | K | | | | | 10:24 | | Weekly | Detail |
| Linda C | 1 | | | | | 36:10 | | Weekly | Detail |
| Marianne I | 1 | | | | | 15:58 | | Weekly | Detail |
| Mario E | 1 | | | | | 23:44 | | Weekly | Detail |
| Michael B | 1 | | | | | 28:40 | | Weekly | Detail |
| Ramona G | 1 | | | | | 11:00 | | Weekly | Detail |
| Renee Q | 1 | | | | | 11:02 | | Weekly | Detail |
| Rilee L | 1 | | | | | 13:18 | | Weekly | Detail |
| Roberta A | 1 | | | | | 21:04 | | Weekly | Detail |
| Sakurah P | 1 | | | | | 00:16 | | Weekly | Detail |
| Solomon O | 1 | | | | | 09:57 | | Weekly | Detail |

Strong Support for Differentiation

Concept: Multiplication: Double & Halve

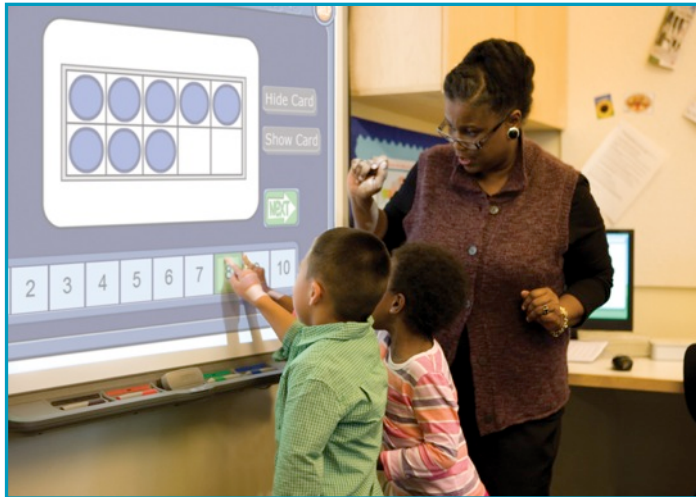
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Students use known basic facts and double one factor and halve the other to determine the product of a more challenging problem.

| # Completed with Proficiency | # In Progress | # Not Started |
|-------------------------------|-------------------|---------------|
| 7 students | 10 students | 9 students |
| John P (about 1 month ago) | Avaneesh S (71%) | Anthony P |
| Jacob C (about 1 month ago) | Charles K (71%) | Brittany B |
| Rebecah D (about 1 month ago) | Emmanuel M (71%) | Christina P |
| Julian B (about 1 month ago) | Luke R (71%) | Emily C |
| Edgar H (about 1 month ago) | Alanna M (64%) | Karly H |
| Pedro S (2 months ago) | Domenic G (64%) | Leah P |
| Daniel C (3 months ago) | Daniel S (57%) | Michael D |
| | Dominique S (28%) | Samantha S |
| | Suna C (28%) | Vanessa C |
| | Caitlin S (21%) | |



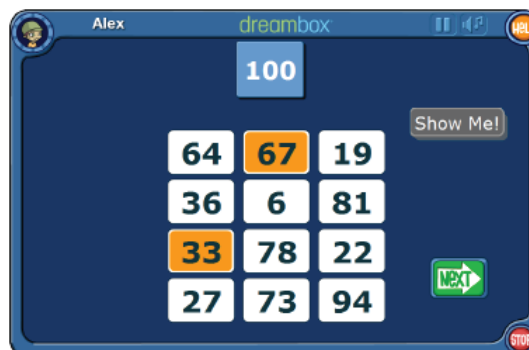
DreamBox supports small group and whole class instructional resources



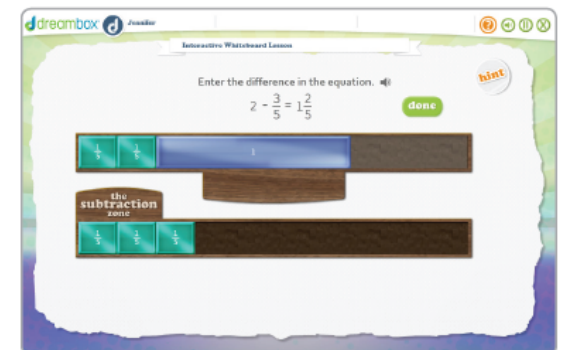
- Interactive white-board teacher lessons
- www.dreambox.com/teachertools
- Tutorials for virtual manipulatives
- Concept video introductions



Subtraction on the Number Line Using Constant Difference



Matching Number Pairs for One Hundred



Subtracting Fractions

Free School-wide Trial!

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DreamBox Learning Math

The rigorous and adaptive online program proven to lead students to success



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[Teachers](#) ►

[Parents](#) ►



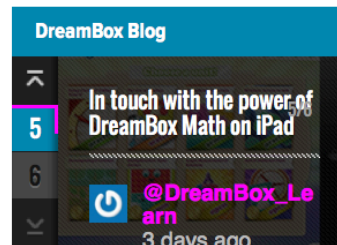
[Data-Driven Education](#) ►



[K-8 Blended Learning](#) ►



[Intelligent Adaptive Learning](#) ►



Thank you!

