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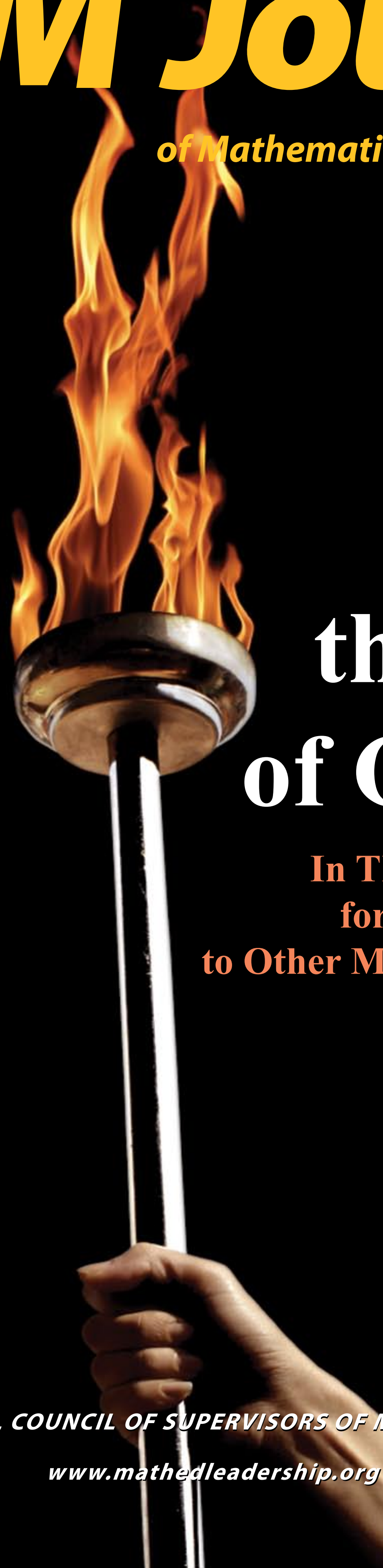

Fanning the Flames of Greatness

In This Issue, We Offer Ideas for Extending Your Passion to Other Mathematics Professionals

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS

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The *NCSM Journal of Mathematics Education Leadership* is published at least twice yearly, in the spring and fall.

The editors of the *NCSM Journal of Mathematics Education Leadership* are interested in manuscripts that address concerns of leadership in mathematics rather than those of content or delivery. Editors are interested in publishing articles from a broad spectrum of formal and informal leaders who practice at local, regional, national, and international levels. Categories for submittal include:

- Key Topics in Leadership
- Case Studies
- Research Report and Interpretation
- Commentary on Critical Issues in Mathematics Education
- Professional Development Strategies

Note: The last two categories are intended for short pieces of 2 to 3 pages in length.

Submission/Review Procedures

Submittal of items should be done electronically to the *Journal* editor. Do not put any author identification in the body of the item being submitted, but do include author information as you would like to see it in the *Journal*. Items submitted for publication will be reviewed by two members of the NCSM Review Panel and one editor with comments and suggested revisions sent back to the author at least six weeks before publication. Final copy must be agreed to at least three weeks before publication.

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CORRECTION FROM WINTER 2008 JOURNAL
The Winter 2008 *NCSM Journal* omitted co-author Daniel Clark Orey from the byline of the article, "It Takes A Village: Culturally Responsive Professional Development and Creating Professional Learning Communities in Guatemala." Dr. Orey is a professor of mathematics and multicultural education at California State University, Sacramento. We regret the omission.

Purpose Statement

The purpose of the National Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership

Comments From the Editor

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"... the question, Why try for greatness? would seem almost tautological. If you're doing something you care that much about, and you believe in its purpose deeply enough, then it is impossible to imagine not trying to make it great. It's just a given."

Jim Collins, Good to Great: Why some companies make the leap ... and others don't (p. 208)

NCSM leaders are people who strive for greatness for our students, and we recognize the vital role that learning mathematics plays in this goal. The members of NCSM are driven to continuously seek ways to lead teachers, administrators, community members, and colleagues toward ensuring that every child has access to meaningful and rigorous mathematics taught by highly qualified teachers. We believe in this purpose so deeply that we are constantly challenging ourselves to grow and learn professionally. We are acutely aware of the network of dedicated professionals from whose experience and expertise we can learn. With that in mind, the authors whose articles follow venture to share their knowledge and experiences to benefit each of our efforts in revealing the greatness in every student.

The *Teaching Gap* was the first introduction for many of us into the idea of lesson study. Since the publication of Stigler and Hiebert's book in 1999, much has been written about the concept. However, in their article, Mark, Gorman, and Nikula go beyond the structure of lesson study and instead focus on the role of the coach-leader in the lesson study process. They share strategies and give tangible suggestions how the leader as the facilitator can "maximize the learning" of teachers who collaborate about the teaching and learning of mathematics.

High quality instruction is at the heart of student learning, and one way to influence instruction is through the use of mathematics coaches. Moyer, Laughlin, and Cai suggest that given an appropriate tool for observing teachers, mathematics coaches can develop their craft knowledge and skills and thus have a greater impact on teacher instruction. In their article, these authors describe how the use of the LieCal observation instrument enabled mathematics coaches to strengthen and broaden their coaching skills while at the same time building trust with teachers and facilitating discussions around content in such a way as to change instructional behaviors.

The increasing numbers of students who are English language learners is a reality that we must address. Data too often reveals that ELL students are under performing in mathematics, and so the question becomes how do we help teachers who teach these students. In their article, Griffin and Barton describe how groups of teachers used video study groups to collaboratively examine and reflect upon classroom practices with ELL students. After participating in video study groups, teachers reflected on the changes in their practice.

The type of questions that teachers ask communicates to students the level of expectations the teacher has about the type and depth of knowledge and reasoning that is expected. Jacobbe's study seeks to inform us of the level of questions included in various textbooks and how this may or may not influence the questions posed in the classroom. First, Jacobbe looks to see if there exists a difference in the level of questions found in a standards-based textbook versus a traditional textbook. Secondly, he seeks to determine whether teachers who used standards-based textbooks ask higher level questions of their students.

The aforementioned article explores the level of questions in textbooks and the classroom. Kim and Kasmer delve more deeply into one specific questioning strategy. In their article, Kim and Kasmer suggest how the instructional strategy of asking students to predict can serve many purposes ranging from engaging students in the learning process to activating prior knowledge as well as serving as a means for the classroom teacher to assess what students know.

All of the above articles contain insights and strategies that directly connect to the classroom. Yet, leadership begins with a vision of what a mathematics classroom should look like. A shared vision takes the image of the ideal mathematics classroom and creates the foundation for necessary collaborative discussions focused on common goals around mathematics curriculum, teaching and learning. Kinzer and Bradley share how a school and university partnership brought together all stakeholders in creating the path that would guide decisions related to all aspects of mathematics in the district.

Each of us strives for greatness in our own ways that are uniquely meaningful to our stakeholders. Yet we recognize that we measure our "greatness" by the success of our students. Each of the authors in this journal has shared something they are passionate about. As a leader in mathematics education, what will you take from their stories and experiences to inform what you do? How might you extend all the good work you are passionate about and transform it into greatness?

References

Collins, Jim. (2001). *Good to Great: Why some companies make the leap…and others don't*. New York, NY: Harper Collins.

Stigler, J. W, & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. New York, NY: The Free Press.

Keeping Teacher Learning of Mathematics Central in Lesson Study

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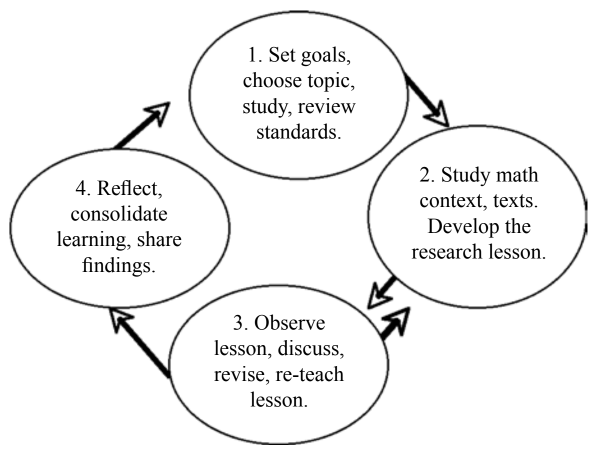
After being introduced through *The Teaching Gap* (Stigler & Hiebert, 1999) and the Third International Mathematics and Science Study (TIMMS), lesson study quickly gained attention in the U.S. as a promising strategy for creating long-term instructional improvement. The lesson study process that is emerging in the U.S. is a cyclical process, inspired by the Japanese model for lesson study. When participating in lesson study, small teams of teachers develop, test, and improve lessons for the purpose of building professional knowledge and improving the overall effectiveness of instruction. Teams of approximately 3-6 teachers meet regularly[1] over a one or two month period—including time for study, lesson development, team observation of the lesson with follow-up discussion, and reflection. The team's investigation of research lessons is an opportunity for teachers to open up a broad examination of mathematics, curriculum, teaching, and student learning.



In the National Science Foundation-funded *Lesson Study Communities in Secondary Mathematics* project[2], we have worked with dozens of schools and hundreds of teachers as they started lesson study in their schools. This work has shown us that lesson study provides many opportunities for teachers to deepen their understanding of and curiosity about mathematics, while also focusing on their students' learning of mathematics. Lewis, Perry, and Hurd (2004) argue that increased subject matter knowledge is one of seven key pathways to instructional improvement that underlie successful lesson study. Increasing teachers' mathematical knowledge has been a key goal of our project's lesson study activities. However, we have observed that novice lesson study teams often struggle to understand the multiple goals and new skills involved in the lesson study process, and so may not fully capitalize on opportunities to build their own understanding of mathematics.

In our experience, a coach or facilitator can play a very important role in helping lesson study teams recognize and utilize the opportunities for teacher content learning that are embedded in the lesson study cycle. This leadership role might be played by an instructional coach, lead teacher,

[1] *Usually team meetings occur once a week and last for an hour or hour and a half, depending on the teachers' schedules. Meetings may occur during team time, common planning time, after school, or on professional development release time. The research lesson is taught during the regular school day, and teachers and administrators arrange for coverage or substitutes to observe it.*

[2] *This material is based upon work supported by the National Science Foundation under the Lesson Study Communities in Secondary Mathematics project, grant number ESI-0138814.*

department chair, mathematics coordinator, or university professor. These individuals may work alongside the team attending all their meetings, while others might consult regularly with the team at key points during the process. The leader can generally support teachers in maximizing their learning by guiding the team's progress through the cycle, asking key questions that help focus the work, and offering appropriate resources or expertise about the content. In our work with teams, we have identified some specific powerful strategies for focusing lesson study work around teacher and student learning of mathematics. Some are most appropriate at specific moments in the cycle, and usually become structured into the team's process at those points. Others are woven through the whole fabric of the lesson study cycle. Leaders can play a major role in helping teams incorporate these strategies into their regular lesson study practice. These strategies include:

- Doing mathematics together with colleagues
- Analyzing the development of mathematical ideas in your curriculum
- Examining the development of mathematical ideas across grades
- Sharing expertise on teaching the selected topic
- Anticipating student responses
- Observing student thinking about mathematics
- Focusing the post-lesson discussion on mathematical learning
- Bringing additional content expertise into the team's work

In this article, we describe each of these strategies in greater detail. First, we situate the strategy in the context of the lesson study cycle, then share examples of the strategy in action from our experiences working with lesson study teams, and lastly, we offer suggestions that may be helpful to leaders in supporting lesson study teams to implement that strategy. It is important to note that in our experience, teams may use different strategies at different times in their lesson study cycle, or may really emphasize one or two strategies (e.g., sharing expertise, analyzing the development of ideas across grades) in particular cycles of lesson study work together.

Doing Mathematics Together With Collegaues

Doing mathematics together with colleagues is a key strategy for enhancing teacher content knowledge in lesson study. A critical activity for *all* teams is to explore problems related to the team's chosen mathematics topic, at some point early in the lesson study cycle. Most commonly, after choosing a topic focus for their research lesson, all team members bring one or two problems related to the topic to share with their teammates. Teachers spend time working together on these problems in order to develop their own understanding of the mathematics and to identify challenging, appropriate problems for the research lesson. Sometimes, teams explore extensions of the problems or related problems from their textbooks because such problems are a more appropriate level of challenge for the teachers participating in the group, yet still offer insight into student thinking of that mathematics.

One middle school team that was focusing on linear equations (in particular the difference between equations of the form $y=kx$ and $y=kx+c$) worked together on the following problem.

- A BIG party is being planned and everyone will sit at hexagon-shaped tables. Many tables will be pushed together to make one long table. If 57 tables are pushed together how many people could sit at the table? Keep in mind that tables are joined at a side (edge) not at a vertex, and that only one person can be seated at a side (edge) of the hexagon.

- Find a way to accurately predict how many people could be seated, given any number of tables?

By working on the problem, team members discovered together quite a variety of rules that could express this relationship, discussed the difficulties students have understanding the difference between directly proportional relationships and those that are linear but not proportional, and generated and worked on a series of extension problems like "what happens if the shape is a pentagon?" "What happens if the tables can be joined in any way, not just in a line?" They also debated whether the y-intercept was meaningful in this context.

Many teams also establish habits of practice that involve doing mathematics together *throughout the cycle*. For example, one of the teams that we worked with in the *Lesson Study Communities in Secondary Mathematics* project decided to launch each of its regular lesson study meetings by working together on a mathematics problem. For each meeting, one teacher would be responsible for bringing a problem that related to the team's research lesson topic and that would challenge the thinking of the team. As one teacher described it, "When we meet, one of the things that we like to do is to first individually solve a math problem and then share our strategies for solving it. ... Our variety of approaches has led us to think about all the strategies our students use."[3] Not all teams start every lesson study meeting with the exploration of a mathematics problem, but collaborative exploration of the mathematics through problems helps teachers to see how the key mathematical concept relates to the learning trajectory for a mathematical idea; to see connections across mathematical topics, and to see connections across grades, thus placing the topic in a broader mathematical context.

Suggestions for leaders: There are multiple opportunities within the lesson study cycle for teams to work on mathematics together. As a coach or facilitator, you might bring to the group engaging problems for the teachers to work on, or help them to create appropriate extensions to the problems from their textbook that they are exploring. By modeling and supporting this kind of exploration, you help teachers to build a connection between their own exploration of mathematics and their students' learning. Some guiding questions to keep in mind while teams are doing mathematics include: What is the important mathematics in this problem or these problems? What are we learning by working on problems together? What can help students learn that mathematics?

Analyzing the Development of Mathematical Ideas in Your Curriculum

In Japan, this analysis is called *kyozai-kenkyu*. We learned about this idea from Akihiko Takahashi, a leading expert on lesson study. *Kyozai-kenkyu* can be roughly translated as "instructional material research"[4] and in Japan is considered an important way for teachers to learn and improve their teaching. When a research lesson goes badly, the teachers may think "We need to do more *kyozai-kenkyu*." The idea is that teachers can learn by deciphering the curriculum writers' and other teachers' theories about how particular mathematical topics are developed. This idea grows from a belief that teachers can learn from and build upon high quality resources and research. Akihiko Takahashi describes the process as follows:

Kyozai-kenkyu[5]

A group of teachers usually does some ground work before actually developing a lesson plan. This investigation, called *"Kyozai-kenkyu"* in Japanese, includes studying:

- a variety of learning and teaching materials such as standards, textbooks, worksheets, and manipulatives
- a variety of teaching methods
- the process of student learning (students' typical ways of understanding, common misunderstandings and mistakes, etc.)
- research related to the topic

Teachers often begin *kyozai-kenkyu* by studying and comparing the teachers' guides published by various textbook companies.

One important aspect of curriculum analysis in lesson study is identifying the key mathematical goals of the lesson and unit. Lesson study teams need to ask themselves: What is it we really want students to learn or understand in this lesson and unit? What do students already know about the topic? What concepts are key to develop understanding? The focus of the curriculum analysis is on how students learn the content, and how the design of the lesson, unit, or chapter contributes to that student learning. If teachers work with high-quality instructional materials, this curriculum analysis strategy will help them to build on the best available lessons and knowledge for teaching the topic, thus allowing them to improve and learn from a quality lesson rather than trying to invent a brand-new lesson. Catherine Lewis (2002) has commented that "Lesson study is most productive when educators build on the best existing lessons or approaches, rather than reinventing the wheel […] Try to immerse yourself in others' lessons through whatever means you can…textbooks, research lessons, books, video […] If your group searches out and studies the best existing lessons, it will result in a better research lesson and help create a system that learns rather than one in which every group of educators reinvents the wheel." (p.62-63) One benefit of studying texts is that teachers become more aware of how well or poorly their textbooks and teachers editions are constructed to reflect a trajectory of learning.

[3] The quote here and the quotes on the pages that follow are from teachers who participated on lesson study teams in the Lesson Study Communities in Secondary Mathematics project.
[4] Definition from http://hrd.apecwiki.org/index.php/Glossary_of_Lesson_Study_Terms.
[5] The information about Kyozai-kenkyu shared here is from a presentation by Akihiko Takahashi in December, 2003 at a lesson study workshop for mathematics teachers hosted by Education Development Center, Inc.

Suggestions for leaders: For novice lesson study teams, we have found that the examination of teaching resources described above can be challenging because teachers are generally not accustomed to analyzing instructional materials in this way. As a coach or facilitator, you can provide support to teams as they begin to incorporate this strategy into their lesson study practice by offering good resources—particularly resources in which the intended trajectory of learning for students is clear—for teachers' examination. Sometimes, the study of national and state mathematics standards can be a valuable resource or entry point for investigating the development and connections between different mathematical ideas. You can also help in the analysis of these teaching resources by sharing your thinking about how the resource reflects particular ideas about student learning of the topic.

Examining the Development of Mathematical Ideas Across Grades: Vertical Integration Through Cross-Grade Teaming

Several of our lesson study teams found working on a cross-grade team to be particularly powerful in developing teachers' own mathematical knowledge, due to the diversity of mathematical experience in the group and the knowledge of the curriculum across grades. These teams consisted of grade six through eight teachers, or a mix of middle and high school teachers. We have also seen elementary cross-grade teams with two or three grades represented, and even one successful team that spanned grades K through 6. The expertise present in these groups enriched discussion of how particular mathematical ideas develop across the grades. Lesson study teams can consider questions such as: What mathematical ideas does this topic connect to in the previous grades? Where does this mathematical idea appear in the upper grades? What did students learn about this last year? Teams with experience at multiple grade levels will have a first-hand source of knowledge as they attempt to answer these questions.

> One district's cross-grades team was comprised of middle and high school teachers and chose topics that were important at both levels, such as probability. They always spent time together learning about the topic by sharing and solving problems related to that topic from the different grade levels. Then the team developed two research lessons related to the topic, one appropriate for their middle school students and a second one appropriate for their high school students. Both the middle and high school teachers observed both lessons. This experience of working across grades gave the teachers greater insight into their students' mathematical thinking across a larger grade span, illuminated how the topic ideas emerge in the curriculum, and helped them streamline their curriculum scope and sequence.

Suggestions for leaders: Coaches and facilitators can help their teams to include an analysis of how a topic develops across grades as part of their work together. The team can be encouraged to seek input from teachers (or textbooks/standards) at other grade levels when the knowledge is not already represented by teachers on the team. Teams also study state and national standards to get a sense for how topics develop across grades, but a greater level of depth can be developed by sharing of teachers' first-hand knowledge on a cross-grades team. Some teachers are skeptical that a cross-grades team will work—concerned that studying a lesson outside one's teaching assignment would be uninteresting or not worthwhile. This skepticism usually fades quickly when the knowledge sharing in a cross-grades team begins.

Sharing Expertise on Teaching the Selected Topic

Lesson study is based on the fundamental idea that teachers are keepers and seekers of content and pedagogical expertise, and that by sharing it with one another, everyone gains. Hence, sharing expertise goes on throughout the cycle, addressing various goals. Early in the cycle, the team shares expertise as they select a lesson topic, discussing questions like: What topics are difficult for our students to learn? What are the important mathematical concepts we want our students to understand? During the process of developing the research lesson, teachers share expertise to identify the important mathematical ideas students may encounter in learning the topic. Questions such as the following drive the discussion: What does it mean to understand this topic? What are common student misconceptions? What are the important ideas that contribute to developing students' understanding of the topic? These discussions are often a very energizing experience for lesson study teams. Teachers share instructional approaches that they have found effective in teaching the topic, or places where students typically get stuck in learning about the topic. They may debate what prior knowledge is needed for students to understand a topic, or whether you can teach a concept even without all the ideal prerequisite skills in place.

> For example, one team considering the broad topic of quadratic functions, learned through sharing their teaching experiences that a particular area of difficulty for their students was understanding the connections between solving equations by factoring, the quadratic formula, and zeros of the graph. This helped them narrow their focus for the research lesson.

Suggestions for leaders: One concrete way for a coach or facilitator to help lesson study teams focus on the mathematics through sharing of expertise is to make sure that formal time is set aside for this sharing. The team may benefit from time to write about what they know, or to talk about what they know, or some combination of these forms of communication. The coach or facilitator should consider the personal style of the team, as well as how the teachers on the team will be able to make sense of and use the knowledge that they share. Some important guiding questions include: What do we already know about effectively teaching the mathematics chosen for our research lesson? What has been difficult about teaching this topic? What are our open questions and what else do we want to know about the topic?

Anticipating Student Responses

Another key aspect of lesson study work is what teams usually call "anticipating student responses" to the problems or activities planned for the research lesson. The team will try to picture what methods students will use in solving problems as well as engagement and behavior. Anticipating students' responses can reveal predicted partial understandings, misconceptions, and a trajectory for student learning of a particular mathematical topic. Sometimes teachers find connections between their possible student responses and the methods they and their colleagues used to solve the same problems. Teachers can use their thinking about possible student responses to inform their lesson design by considering how to advance the thinking of students with particular responses and by increasing their own understanding of how students learn mathematical ideas. One teacher commented on her experience anticipating student responses: "It took us a while to make that leap into student learning. For a long time, we were still creating lessons…. Not just looking at student feedback, but analyzing student reactions made a big difference… I'd say I learned something about kids along the way, but I have a lot more to go….."

One elementary team worked on a problem for their research lesson that involved drawing on grid paper all rectangles that have whole number dimensions and an area of 24 square units, then making a table showing the rectangles' dimensions and perimeters as well as a graph of length and perimeter. When the teachers in this group had solved the problem themselves, they realized that different people computed the perimeters of the rectangles using different methods. They drew upon this experience to predict a number of different correct ways that their students might approach finding the perimeters for the different rectangles in this problem:

(1) Use values from the table to compute $2L + 2W = P$
(2) Look at each drawing and add the side lengths in order $[L + W + L + W = P]$
(3) Look at each drawing and add opposite sides then sum $[(L + L) + (W + W) = P]$
(4) Use values from table or drawings—add length and width then double $[(L + W) \times 2 = P]$

A next step for this team is to consider what incorrect or incomplete methods their students might employ. Making explicit possible *unexpected but desirable responses* from students can also help the team consider how they can develop students who are likely to extend their thinking in those directions. Another next step (and a critical one!) is to analyze each of the predicted responses, in order to determine what it reveals about student thinking about perimeter, and how the teacher can further the student's understanding from that starting point.

Suggestions for leaders: The team will focus intensively on anticipating student responses at a few key points in the process—generally after the lesson problems have been chosen and the team is refining their lesson plan pedagogy, and after the lesson has been taught as the team revises the lesson and considers what they have learned. You can help the team to consider the variety of ways that students might respond to the lesson problems, including correct, incorrect, and incomplete methods as well as questions or extensions to the problem that students might pursue. It is not enough for the team to stop at listing the solutions and solution methods they might see. To really explore the mathematics the team needs to take the next step of unpacking what these anticipated student responses might mean about how students are understanding the concept, and about how the teacher can further that understanding. Remember, also, that the team will gain new insights about student methods when they observe the lesson. It matters more that the team keep attending to this than that they produce an impressive list at the first try.

Observing Student Thinking About Mathematics

The observation and discussion of a research lesson are central to the lesson study process. Observation of the lesson allows teachers to see first hand how students think about and learn the mathematical ideas in the lesson, what understandings students bring to the lesson, and what ideas students are struggling with. Having multiple observers enables the team to collect a great deal of detailed data about students, from different perspectives, thus creating a stronger basis for understanding and interpreting student thinking, and for evaluating the research lesson. This body of data also contributes to a richer discussion about the effectiveness of the lesson in promoting student thinking and learning. Teachers have reported on the power of watching one student or group through the whole lesson. This observation strategy allows the observer to see the students' full process (including wrong turns, down time, role in the group, etc.) and is a stark contrast to attempts to determine what students are thinking based only on the final solutions that they present or write. This is something teachers rarely, if ever, are able to do in their regular teaching.

The observation is also a reality check for the team. They may realize that there are areas of the mathematics that their students don't fully understand, or that their expectations of students' abilities or knowledge are unrealistic. What contributes greatly to the power of the observation is that the team has been fully immersed in the mathematics of the lesson and has formed hypotheses about how students learn this mathematics. This observation is a culminating moment in their research.

Suggestions for leaders: Observing student thinking with a researcher lens at the forefront rather than a teacher lens is one of the many "new" practices that teachers experience as part of lesson study, and therefore one that can benefit from the support of coaches or facilitators. You can help the team to determine particular questions to focus their observation, so that the team will be well equipped to collect useful data for the discussion after the lesson. You can also help the team to stay focused—the observation is about student learning of mathematics, not the teacher. Finally, share your excitement about the power of the observation with the team. The lesson study observation provides a unique opportunity for teachers to learn together about their research lesson, especially because the teachers planning the lesson don't know how it is going to go, and likely, the students will do or say things that surprise the team. Those moments of surprise are opportunities for understanding how students learn the mathematics, and you can help the team focus on those moments.

Focusing the Post-Lesson Discussion on Mathematical Learning

When the lesson is taught, a lot of observational data is usually collected. In order to provide focus when using those data, a key question to consider in the post-lesson discussion is: What did we learn about the mathematics and students' learning of the mathematics? The purpose of the post-lesson discussion is to share observations, discuss what those observations mean, and discuss what the team has learned from the teaching and observation of the research lesson. It can be helpful at the beginning of a post-lesson discussion, when everyone is eager to share what they have seen, to take a few minutes for some reflection on the key question, "What did we learn about the mathematics and students' learning of the mathematics?" A successful strategy we have used is to provide a prompt for a brief period of individual reflection and writing. Maintaining this focus on what is learned about the mathematics of the lesson makes it possible to have an evidence-based conversation, and ground interpretations and emerging theories in the collected data. The goal is to refine ideas about how students think about and understand the mathematics of the research lesson, which can then inform revision of the research lesson. Examples of questions that might be helpful to guide the discussion include: What was the effect of asking a particular question or posing a particular problem? What mathematics did students learn in this lesson or in a particular part of the lesson? What activities or questions helped to keep students' focus on the mathematics? When in the lesson did we observe the most student learning?

Suggestions for leaders: A coach or facilitator can play the role of moderating the post-lesson discussion, ensuring that the discussion stays focused on the data from the lesson about how students are understanding and learning the mathematics. Questions such as those described above can be helpful in this regard. The coach or facilitator will also have to attend to the particular needs of their group in order to determine the best way to keep a strong focus on the mathematics and students' learning in the post-lesson discussion.

One example of unpacking the mathematics in a post-lesson discussion comes from this entry in a log written by the coach for a lesson study team:

The teachers [on this lesson study team] have had trouble articulating what they think the math of their research lesson is, beyond stating the standards for probability or saying that the math is "understanding probability." During the teaching of the research lesson, students were able to correctly write some numbers in ratios but were having trouble connecting those ratios to verbal descriptions of the likelihood of particular outcomes. Teachers noted this trouble, so I asked them to try to describe what they thought students did and did not understand. The teachers responded that they thought the students understood the ratio of possible outcomes, but that they didn't completely understand what probability is because they couldn't connect it to those ratios. During the lesson, all of the small groups of students collected data and then pooled that data into one class set of data. The ratios for the class set of data were closer to the probability-driven predictions than the small-group data sets had been, but still did not match the probability-based prediction. (... Although the class data set drew upon data from several small groups, it still didn't include a very large number of trials). I asked how the results of the pooling of data might relate to the homework questions students would be answering that night about "why use probability?" I think in our next meeting I'll ask the teachers to predict possible answers that students might write for "why use probability?" as well as what answers they'd ultimately like to see from students, because it's a question we haven't explored as a group."

Leaders also need to keep in mind that the team will have two post-lesson discussions if the team is able to schedule time for revisions and a second teaching. In this case, there is a chance to revisit important discussion topics, or to focus more heavily on data relevant to revisions in the first discussion, and on larger mathematical themes in the second.

Bringing Additional Content Expertise Into the Team's Work

One of the lesson study teams with which we worked was participating in an NSF-funded Mathematics and Science Partnership program that brought together university mathematicians and study groups of secondary mathematics teachers. Because this school was already actively engaged in lesson study, the university mathematician joined the lesson study team. In one of their lesson study cycles, the teachers identified combinatorics as a topic that many of them were interested in learning more about for themselves, as well as for teaching their students. The university mathematics professor supported this team's learning by offering problems for the group to work on together and by making connections between different ideas such as combinations and permutations and how the formulas for combinations and permutations can be developed. This mathematics professor also participated in a lesson study open house[6] that the team hosted at their school, and following the observation and discussion of the research lesson, he led a session for the teachers who attended the open house to extend the mathematics from the student lesson. This idea of drawing on a content expert, sometimes referred to as a "knowledgeable other," is common in the Japanese practice of lesson study.

Suggestions for leaders: Often, the coach or facilitator is a source of outside expertise for the team—sometimes bringing knowledge of the lesson study process, or group facilitation, or of the mathematics and pedagogy. It is important that the coach or facilitator help the team determine what needs they have for expertise beyond what is represented on the team, and how they might access that kind of outside expertise. Outside expertise often comes in the form of invited guests to participate in certain parts of the lesson study process, in particular the study of the content or the lesson observation and post-lesson discussion. However, content expertise can also be brought to the team in written form, by finding articles and books about relevant research. Inviting in outside experts also brings some challenges in that the teachers may have a difficult time relating the outside expertise to their work, or may feel inadequate in their own knowledge or understanding. An important role for a coach or facilitator is to familiarize the visitor with the goals of the lesson study process, and work to help teachers think about how best to incorporate this new knowledge or perspective into their current thinking.

[6] *A lesson study open house is a form of professional sharing and learning during which a lesson study team invites outside visitors to observe the teaching of their research lesson and participate in the post-lesson discussion based on that research lesson observation.*

Conclusion

All of the strategies described here are an integral part of the lesson study cycle, but in practice, there is often quite a bit of variation in how lesson study is implemented and in some cases, key elements of the lesson study process are missed or addressed quickly or superficially. Within the lesson study process, there are plentiful opportunities for teacher learning of mathematics. However, it can be challenging for teams to take advantage of these opportunities, especially new teams, because they can be overwhelmed with learning all the parts of the lesson study cycle, or are focused primarily on developing their lesson. They may also be unaccustomed to learning mathematics in the ways offered by lesson study because teachers do not usually have the time to examine the mathematics and their teaching practice in the ways the lesson study process allows. However, we have seen in our work that once introduced to these strategies, teachers welcome and embrace the kinds of opportunities available through lesson study to develop their own understanding of mathematics.

As we mentioned above, we have found in our work that a coach or facilitator can be tremendously helpful to lesson study teams by leveraging opportunities for mathematics learning in the process. An important leadership role for coaches or facilitators is to help the team recognize the opportunities for their own learning of mathematics embedded in the lesson study cycle. In addition, mathematics leaders can think strategically about the fit between the various strategies described in this article and the needs and contexts of the team of teachers with whom they're working. A mathematical leader can also model good mathematical discussions and ask thought-provoking questions at critical junctures. Similarly, a coach or facilitator can support individual teachers on lesson study teams as they take on responsibility for attending to mathematics learning, and eventually, over time, these strategies can become core elements of the teams' lesson study practice. Helping the team see why mathematical explorations are important, and the connection between learning the mathematics for themselves and the improvement of their teaching and their students' learning is a critical contribution that a coach or leader can make in their work with a lesson team.

We close by offering a number of reflection questions for coaches and leaders engaged in assisting teams to keep the learning of mathematics central in their lesson study work.

- Are teams having substantive discussions about mathematics? How does the content teachers are exploring extend their own understanding? How does it help them better understand students' thinking about the mathematics?
- What evidence is there of a strong focus on mathematical thinking and on how mathematical ideas develop in the team's work?
- Does the team bring a learning stance to their lesson study work? How open are teachers to developing new understandings of the mathematics and their students' learning?
- What connection do teachers see between their own understanding and learning of mathematics and their students' understanding of mathematics?
- Are teachers on the team posing questions about mathematics and their students' learning of mathematics that they are interested in exploring?

In addition, there are some resources available to assist leaders who are working with teachers using the lesson study process. A few of these resources include:

- Lesson Study Group at Mills College, *www.lessonresearch.net*. This group offers resources for new lesson study teams and maintains a library of articles and research related to lesson study. They have developed research-based toolkits for proportional reasoning, area of polygons, and fractions that enable mathematics lesson study groups to access and use content knowledge effectively.
- EDC Lesson Study Center, *www.edc.org/lessonstudy*. This group is developing a ten-session professional development course for teams new to lesson study in mathematics, and a leadership guide for coaches, facilitators, and other leaders. These materials will be published by Heinemann and available in 2010. The group also offers services in support of lesson study implementation, research, and lesson study in mathematics workshops. The website also includes information about the *Lesson Study Communities in Secondary Mathematics* Project.
- Global Education Resources, *www.globaledresources.com*. GER provides materials, workshops, and services to teachers, schools, and districts. Resources include English translations of widely used Japanese mathematics textbooks for grades 1–6.

We encourage you to try some of the strategies and guiding questions described in this paper, and access the resources listed above as you work to improve mathematics teaching and learning through lesson study in your own school or district.

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High Quality Coaching Using the LieCal Observation Instrument

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In the LieCal[1] Project, virtually every teacher and every principal we interviewed emphasized the importance of posing classroom tasks that require conceptual understanding, making connections among mathematical ideas, and problem solving. Yet, despite this avowal of the importance of teaching higher order thinking, our observations showed that only slightly more than one-third of the 496 instructional tasks we observed in sixth grade mathematics classrooms involved thinking at higher levels. Similar results were reported in other projects (e.g., Stein et al., 1996). What can be done to remedy such a widespread disconnect between intent and practice?

Some educators believe that one answer may lie in the use of mathematics coaches. The recent coaching movement in the United States is an attempt to help teachers become more effective so that students learn at higher levels. However, too often, schools implement school-based coaching too simplistically, underestimating the complexity of implementing change initiatives. As a result, one concern is that coaching will not live up to its promise without more strategic and systematic development (West et al., 2007).

The purpose of this article is to discuss how the use of the LieCal observation instrument could lead to high quality coaching, and thus high quality teaching. We begin with a brief review of the recent literature about coaching competencies that are crucial to the success of mathematics coaches. Then, we present a framework used in the LieCal Project to design part of the LieCal observation instrument. Finally, using a classroom vignette from the LieCal Project, we discuss how the observation instrument from the LieCal Project can be used to help coaches attain these crucial competencies.

Coaching

The term "coaching" can be defined broadly to mean any job title that includes assisting teachers with improving mathematics instruction as part of their responsibilities (West et al., 2007). Across America today, hundreds of instructional coaches are being hired to improve professional practice in schools (Knight, 2007). Some coaches are in roles that are poorly articulated, are not trained in the complexities of adult learning, or face a school culture that hasn't been adequately prepared for this form of professional development (Sweeney, 2007). Therefore, professional development is needed to develop effective coaching skills. In designing professional development for coaches, three components are crucial: establishing trusting relationships, using content knowledge as the focus of coaching, and using influence skills to change behavior (Driscoll, 2005; Knight, 2007).

Establishing Trusting Relationships in Coaching

It is most important that coaches not be perceived as critics of teachers' practices. Effective coaches have to learn to discuss instructional issues with teachers in ways that enlighten without threatening or offending the teachers. For that reason, most advocates of the coaching movement agree that effective coaching begins with the establishment of

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a trusting relationship and open communication between the coach and the teacher (Brady, 2007; West & Staub, 2003).

An effective way for coaches to establish trusting, open relationships with teachers is to collaboratively analyze student work to determine the students' understandings and misconceptions. The key to being an effective coach is listening and asking questions to develop the teacher's own capacity during the analysis (Silicon Valley Mathematics Initiative, 2007). When coaching questions are grounded in student work and student learning, the dialogue between teacher and coach takes on a collaborative spirit, with the common goal of improving student learning. If the coach phrases genuine questions for reflection rather than questions with a single correct answer (Feger et al., 2004), the teacher will see the coach's questions as prompts for reflection, not critical judgments that put the teacher on the defensive.

Using Content Knowledge as the Focus of Coaching

Too few coaches pay attention to the specific mathematical content of a lesson. In response, the concept of "content-focused coaching" (West & Staub, 2003) was developed by researchers at the University of Pittsburgh. A content-focused coach helps teachers deepen their content knowledge of the mathematics being taught and broaden their repertoire of pedagogical strategies to help students access important mathematical concepts and skills (West, 2006).

The focus of content coaching is on students' thinking, understandings and work products. Of particular interest to content-focused coaches is the question: "How does this lesson engage students in thinking that moves them toward the teacher's stated goals?" (West & Staub, 2003). To answer this question, coaches must help teachers gather and interpret evidence of student understanding. The purpose is to link evidence of understanding to teaching so teachers can decide whether they need to modify instruction. Together, coaches and teachers analyze students' thinking by discussing questions like: "What is the mathematics?" "What does this piece of student work, or this student's response, tell us about what the child understands?" "What might you do next?"

Using Influence Skills to Change Behavior

In order to influence or persuade teachers, coaches must apply skills that are similar to those of effective leaders. When coaches lead teachers through difficult change, they challenge what teachers hold dear, and often teachers' first reaction is to resist. To help overcome such resistance, coaches need to understand and capitalize on one of the principles of effective leadership, which is to persuade, rather than dictate (West, 2006).

Coaches find that teachers' actions are frequently incongruent with their espoused intents. In leadership theory, a typical intervention is to call attention to a gap between espoused theory and theory-in use. The intervention first involves the coach presenting a challenge by pointing out gaps between intentions and actions. Then, the coach provides support by helping the teacher understand the source of the gap so that new ways of thinking and acting can be integrated into their teaching practice. By acting in this way, the coach holds out an implicit vision of congruence between aspirations and actions (McGonagill, 2000).

Task Framework and Observation Instrument

Mathematical Task Framework

In the Mathematical Tasks Framework, Stein and Smith (1998) define a task as a segment of classroom activity that is devoted to the development of a particular mathematical idea. The Mathematical Tasks Framework distinguishes four levels of cognitive demand found in tasks: memorization, procedures without connections, procedures with connections, and doing mathematics. Tasks categorized as "memorization" or "procedures without connections" are considered low-level cognitive tasks. Tasks categorized as "procedures with connections" or "doing mathematics" are considered high-level cognitive tasks.

Besides distinguishing the four levels of cognitive demand, the task framework also differentiates three phases through which tasks pass: first as they appear in the instructional materials; next as they are set up or announced by the teacher; and finally as they are actually implemented by students in the classroom (Stein and Smith, 1998).

Realizing that a focus on the cognitive demand of mathematical tasks and on the way they are implemented in classrooms can assist teachers in the reflection process, we used the Mathematical Tasks Framework as the basis for designing part of the classroom observation instrument in the LieCal Project. As a tool for reflection, the Mathematical Tasks Framework draws attention to what students are actually doing and thinking during mathematics lessons. The focus on student thinking, in turn, helps the teacher adjust instruction to be more responsive to, and supportive of, students' attempts to reason and make sense of mathematics.

LieCal Observation Instrument

The LieCal Project compares the relative effectiveness of a National Science Foundation funded middle school mathematics curriculum with curricula that was not funded by NSF. An important part of the LieCal Project is the examination of instructional practice using classroom observations. During 2005-2006, two trained research specialists observed 195 lessons. While in the classrooms, the observers made minute-by-minute records of the lessons as they unfolded. Afterwards, they filled in LieCal observation forms by reflecting upon and coding important aspects of the mathematical tasks that were used during the lessons.

Among other things, the observers coded how the LieCal teachers selected and used tasks to maximize student opportunities to learn important mathematics. The Appendix shows the form used to code one task. The first column of each table on the form distinguishes among three phases of each task: as intended by the author, as set up by the teacher, and as implemented by the teacher and/or students. If the cognitive demand of a task changes as it unfolds--from curriculum intent, through setup, to implementation--the completed form captures that change.

Creating Coaching Opportunities Using the Observation Instrument

In this section, we present a classroom vignette to show how the observation instrument can be used to create coaching opportunities. The classroom vignette is taken from a LieCal 6th grade mathematics class that uses a non-NSF funded curriculum. Mr. A spent the previous day's class teaching two different approaches (factor trees and lists) to finding the greatest common factor of two or three numbers. The students had been given a homework assignment of 20 exercises to practice finding the greatest common factor, and at this point, most students in the classroom were comfortable with the procedures.

As in all the LieCal schools, Mr. A's school values problem solving. The teachers are encouraged to have the students work problems that encourage higher-order thinking skills. In his pre-observation interview, Mr. A himself professed to value problem solving and higher-order thinking. Consequently, Mr. A spent a second day on the topic of greatest common factor, devoting most of the class period to having the students solve an application problem in groups.

Presenting the Task

The Plants Problem: The table lists the number of tree seedlings Emily has to sell at a school plant sale. She wants to display them in rows so that the same number of seedlings is in each row.

Seedlings for sale

Type	Amount
Pine	32
Oak	48
Maple	80

8. Find the greatest number of seedlings that can be placed in each row.
9. How many rows of each tree seedling will there be?

This first excerpt captures how Mr. A set up the task. After a student reads the problem aloud, Mr. A leads the following discussion:

Mr. A: *Now, what do you think we can use to find that answer? Hint, hint, it's something we've been working on for the last few days. Shamika.*
Shamika: *GCF.*
Mr. A: *GCF, OK, so that would help define number 8; let's also look at number 9. Anna, read 9.*
Anna: *How many rows of each tree seedling will there be?*
Mr. A: *OK, after you find your GCF in 8, you should be able to use that to help you find the answer for 9. All right, now we're going to work as a group on this. I'm going to give us, oh, about 5, 6, 7, 8 minutes or so. I'll watch to see how long it takes. And I'll be back with you in a little bit to see how the groups did on this. OK?*

Analyzing the Cognitive Demand of the Task

Task as intended by the author. In terms of the Mathematical Tasks Framework, this task, as intended by the author, was coded as *Procedures With Connections* because it "… focuses students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas," (Stein et al., 2000, p.16). It therefore fits into the school's priority of focusing on problem solving and higher order thinking skills. Mr. A correctly identified this task as one that could lead to a high level of cognitive demand, because it asks students to engage with the conceptual ideas that underlie the procedures for finding GCF in order to successfully complete the task and develop understanding. He hoped that it would stimulate students to make connections between the concept of GCF and the application of the idea to a real-life situation.

Task as set-up in the classroom. A decline in cognitive demand occurred in the set-up phase for the task. The observer coded the task set-up as *Procedures Without Connections* because the "Use of the procedure is … evident based on instruction…" (ibid, p. 16). During this phase, the teacher reduced the level of cognitive demand by the way he set up the task. Before the students had time to think about the solution to either of the problems, Mr. A. told them how they should proceed. Furthermore, Mr. A. described how to use the answer from problem 8 to solve problem 9. His directions took away the challenge introduced by the unstructured nature of the task, and hence reduced the cognitive demand.

Task as implemented by the teacher and students. The actual implementation of the problem was also at a reduced level of cognitive demand. In the following excerpt, the students have computed the GCF of 32, 48, and 80 to be 16. However, they have done so only because they were directed to do so by Mr. A. What is evident in this second excerpt is that the students did not know why they had found the GCF, nor did they realize that 16 was the answer to problem 8.

Shamika: *Because you can't divide 3 by 16.*
Duane: *It only goes to 15…trying to figure out.*
Shamika: *I know you can't divide it, 3 divided by 16, though. If there's 3 trees and we got 16 for the greatest common factor, and there's 3 trees, how can you divide 3 by 16?*
Carlos: *I don't know. But, um, wouldn't, wait, if there's how many rows of tree seedlings will there be, and there's 3 trees, wouldn't it just be 3 rows, one for each tree?*
Shamika: *3 trees.*
Duane: *3 rows.*
Shamika: *3 rows of seedlings.*
Duane: *But no, she wants to even 'em out on each row.*
Shamika: *It don't say even 'em out.*
Duane: *Yes, it does. Look it. She wants to display them in rows with the same number of each type of seedling in each row.*
Shamika: *Oh. Mr. A? But it won't be even though.*
Duane: *It can't be 3 rows.*
[At this point, Mr. A walks up to the group for the first time.]
Shamika: *We got 16 for our GCF.*
Mr. A: *That, that sounds great.*
Duane: *What's number 9?*
Shamika: *16 divided by 3.*
Mr. A: *No, 16 is your GCF. 16, 16 is the amount of the greatest number of seedlings that can be placed in each row.*
Duane: *Well, what about number 9?*
Mr. A: *Well, number 9, now we have to use the answer from here to find the answer to number 9 by using division.*

Rather than asking the students to explain what they knew about the task, Mr. A provided brief answers that did not lead to any conceptual understanding about the solutions. Specifically, when he walked up and heard the number 16, he said, " …16 is your GCF. 16, 16 is the amount of the greatest number of seedlings that can be placed in each row." Then he proceeded to give them a procedural hint to get the answer for problem 9. Mr. A's interactions with other groups were also procedurally oriented.

This third excerpt took place during the whole group discussion at the end of class. In it, Mr. A continues to focus the students' attention on procedures, rather than the underlying concepts.

Mr. A: *OK. Now, what did we agree that the GCF or the most seedlings in a row could be? This is problem 8. What did we agree? Karl? How much?*
Karl: *16.*
Mr. A: *16. And most groups ended up with 16. Look, you guys, I wrote it out earlier. I used the tree method. Look, please.*
[Here, Mr. A refers to a tree diagram on the overhead.]
2 x 2 x 2 x 2 = 16 seedlings in a row. It ended up being the prime factorization that they had in common was 2 times 2 times 2 times 2. And for some reason, some of you were going, that equaled 8. Well, look. 2 times 2 is 4. 4 times 2?
Karl: *8.*

Mr. A: *8 times 2?*
Karl: *16.*
Mr. A: *All right. So, it ended up being 16 seedlings in a row. Now, let's use that answer to divide to find the answer to 9. Someone read 9. Read 9. Tony.*
Tony: *How many rows of each tree seedlings will there be?*
Mr. A: *OK, how many rows of each tree seedlings will there be? Very good. Now, go back to the table. How many seedlings were there for pine? How many? Go to the table. How many, Bonita?*
Bonita: *2 rows.*
Mr. A: *Listen to my question. How many seedlings were there for pine?*
Bonita: *32.*
Mr. A: *32. What can we divide 32 by to find out how many rows? Bonita?*
Bonita: *Divide it by 16.*

From the observer's point of view, it appeared that most groups did not understand the reason to use the GCF even after the whole group discussion. In terms of the Mathematical Tasks Framework, Mr. A's implementation of the task would be classified as *Procedures Without Connections*.

This vignette illustrates how a teacher can miss an opportunity for students to solve a task with a high level of cognitive demand at several critical junctures during the class. First, during his set-up, Mr. A reduced the level of cognitive demand when he provided hints that essentially told the students how to solve the problems in the task. Next, in responding to student requests for help during group time, he eliminated the sense-making aspects of the task and deprived the students of the opportunity to develop meaningful mathematical understanding. Finally, during the summary discussion with the whole class, Mr. A continued to question students about procedures rather than about their understanding of why they should use the GCF to solve the problem.

Building Coaching Competencies

Although Mr. A was hoping to engage his students in a high-level task, the coach observed that he intervened very early to reduce the cognitive demand. Why did this happen, and what can the coach do about it?

In this section, we discuss how the coach can use the task analysis portion of the LieCal observation instrument to affect Mr. A's future set-up and implementation of high level tasks. This discussion draws on the three components of a successful coaching relationship. That is, we will show that the coach can change Mr. A's future behavior by focusing on content, and by using persuasion, while maintaining a trusting relationship.

From past experience, the coach knows that there are several reasons why teachers might lower the cognitive demand of problem situations.

- The teacher thinks the students do not have the necessary background (e.g. number sense, operation sense, basic facts, algorithm skill, the connections among them) to solve the problem.
- The teacher has an underlying belief that students learn best when shown.
- The teacher's students do not behave when they are put into open-ended structured problem solving situations.
- The teacher assumes that once a student gets an answer, the student understands what the answer means. As a result, the teacher does not probe to find out if a student really understands the conceptual basis for the answer.
- The teacher has a closely held vision of an effective teacher as a "sage on the stage."

An effective coach realizes that Mr. A may not even be aware that his beliefs led him to lower the cognitive demand of the task. A thoughtful conversation about student thinking during the task can help the coach influence the teacher's pedagogy, and at the same time cement a trusting relationship and focus on the mathematical content of the task.

During the post-lesson discussion, the coach must be careful to maintain the trusting relationship between the coach and the teacher. This can be done effectively by concentrating on the students' work and the students' learning. The coach can begin by having a discussion with Mr. A about how the students' work shows whether the students learned how to use the GCF to solve the Plants problem.

C: *While you were observing the groups working on the task, I was doing the same.*
A: *I was really pleased with the behavior of all the groups, weren't you?*
C: *Yes, I agree that they were very well behaved. It is obvious that they know what is expected of them when they are working in groups.*

A: Thank you. We have worked hard on that.

C: I was observing Shamika and Duane's group just before you walked up to them. I thought they were really struggling with the problem. They had found that the GCF was 16, but their conversation indicated that they were trying to figure out what the 16 meant. They kept asking whether they could divide 3 into 16. And I wondered if they even knew what the 3 meant, because Shamika kept talking about 3 trees and Duane talked about 3 rows. Did you observe other groups having the same type of difficulty?

A: Most of the groups I observed got 16.

C: I wonder if they knew what the 16 meant in relation to the problem, or how to use it to solve problem 9.

A: Well, I think so, because when I set up the problem I told them they needed to find the GCF to get the answer to problem number 8.

C: Why did you tell them that they needed to find the GCF?

A: I wanted them to understand that this problem was related to the work we were doing yesterday, and I think they did understand because they got the right answer to problem #8.

At this point in the conversation, many thoughts are going through the coach's mind. The coach wants to get Mr. A to realize there is a disconnect between Mr. A's goals for the lesson and the way the lesson was set up and implemented. On the one hand, teachers should be a main source of mathematical information and actively help students make sense of mathematics. On the other hand, teachers should not intervene too much and so deeply that they cut off students' initiative and creativity. It is essential for teachers to balance between allowing students to pursue their own ways of thinking and providing important information that supports the development of significant mathematics (Ball, 1993; Ball & Bass, 2000).

How will the coach get Mr. A to realize the balance has not been established? The coach steered the conversation as follows.

C: How do you interpret the difficulty that Shamika was having just before you walked up?

A: She had the answer, she just didn't know it.

C: I wonder why she didn't know she had the answer.

A: Maybe because she doesn't understand what GCF means for this problem. … … ***[Realizing what he just said--]*** *Oh, I hadn't thought about it like that.*

C: Like what?

A: I guess I just assumed that if she had the answer, she knew what it meant, but I never really asked her that. So maybe I didn't know for sure. Like they said in our last in-service about the importance of probing students' thinking. I guess I didn't do that.

C: Yes, I remember that in-service, too. Thinking back, what was your initial reaction to it?

A: At that time, I wasn't sure about the whole idea. Maybe there is something to it, after all. Thinking back on what happened today, I may have made some poor assumptions by not asking for my students to explain their answers. Maybe I did too much thinking for them.

C: Thinking back on the lesson, what would you change?

A: Well…, I'm not sure. I need to be sure that they really understand what they are doing. Like Shamika. Until you told me, I didn't realize that the students had problems knowing what to do with the GCF once they found it. Maybe I should spend more time explaining what the problem is asking for before they start.

C: I think it's a good idea to make sure they really understand the problem before they start. I saw them work well in groups. Do you think they could work in groups to make sense of the problem before they begin to solve it?

A: Well, ... maybe. But what would I ask them to do, and wouldn't most of them be floundering?

C: Ahhhh, don't underestimate them. You could give them a short time, say five minutes, to read and understand the problem. After five minutes, have a class discussion centering on how the display could be laid out so that the same number of seedlings are in each row.

A: But when I gave them the hint to use the GCF, didn't that do the same thing?

C: Judging from what I saw in Shamika's group, they found the GCF but had no idea what to do with it. If the students came up with any way, say four trees in each row, at least you could direct the conversation to the fact that they are using common factors.

A: Yes, and from there they would have to realize that they need to use the greatest common factor, not just any old factor.

In the course of this conversation, the coach was able to help Mr. A realize that he holds some underlying beliefs about teaching that guide his actions. Specifically, Mr. A believed (1) that his students did not have the necessary background to use the GCF and (2) once a student gets an answer, the student understands what the answer means. So, he acted in total harmony with his underlying beliefs, and literally told the students to find and use the GCF. This diminished the challenge of the task from the beginning, but he was able to justify this move to himself because of the strength of his other underlying beliefs. The coach helped Mr. A realize that, at least in this situation, his beliefs interfered with his goal of having students use high level thinking to solve problems. As a result, the next time Mr. A has his students work on a high level task, he will be more aware of the need for students to spend time understanding the problem and its solution, and how his teacher moves can inhibit or enhance that understanding.

Conclusion

The LieCal observation form is a lens for reflecting on teacher instruction. By using the form, a coach is guided to reflect upon and decide whether the evolution of mathematical tasks during the lesson matches the teacher's goals. The form helps the coach know what to look for during the lesson and what to talk about with the teacher afterwards.

To give insight into the evolution of the task, the LieCal observation form requires that the coach record a minute-by-minute account of the events of the lesson, including questions asked, answers given, teacher moves, and student moves as they unfold during the lesson. Ideally, the coach has scheduled a meeting to discuss the lesson. Prior to that meeting, the coach refers back to the minute-by-minute log to analyze the goals of the tasks as intended by the textbook author, how the tasks were set up by the teacher, and how the tasks were implemented by the teacher. The minute-by-minute log also helps the coach think about the examples the teacher used and the questioning strategies that enhanced (or not) the development of students' conceptual understanding and problem solving.

The LieCal observation form, with its minute-by-minute log and its task analysis form, helps focus the coach's attention on important topics that he/she should discuss with the teacher. For example, "Did Mr. A realize that the cognitive demand of the task had declined?" "Did Mr. A realize that his students stayed at level of memorized procedures that are disconnected from underlying ideas?" Perhaps not. The post-lesson conversation may be the first time that Mr. A realizes that his actions do not match his goals.

In this article we have examined how the LieCal observation instrument can be used to help coaches foster the three components of good coaching: establishing trusting relationships, using content knowledge as the focus of coaching, and using influence skills to change behavior.

When the coach discusses student actions, reactions, or work with the teacher, as recorded on the LieCal observation form, the coach is strengthening their trusting relationship, not jeopardizing it, because the focus is on helping students, rather than on correcting teacher shortcomings.

By using the LieCal observation form, the coach helps the teacher (1) focus on setting up tasks so that they foster the goals of the lesson, (2) organize the implementation of tasks to foster the goals of the lesson, (3) formulate questions that challenge students to meet the goals of the lesson. By focusing on students' thinking, understandings and work products the coach helps the teacher link evidence of understanding to the teaching that occurred in the lesson. This content coaching, which uses mathematics as a focus for discussion, can be used to help teachers meet their goals.

At the same time, the coach is influencing the teacher to change behavior because the coach's questions help the teacher himself realize that there is a disconnect between the teacher's stated goal and the teacher's actual actions.

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Appendix

Analysis of Mathematical Tasks

Each mathematical task should be analyzed from four categories: Task as Intended, Task Set-up, Task Implementation, and Factors Associated with Decline or Maintenance of High-Level Cognitive Demands.

Task as Intended by Author	Solution Strategy	Representation (Circle all that apply)	Explanation (Circle all that apply)	Cognitive Demand (Circle **one** level only)
	1 Single 2 Multiple	1 Symbolic 2 Written words 3 Pictorial 4 Table 5 Graph 6 Verbal 7 Manipulatives	1 Not required 2 Show work 3 Explain/justify	1 Memorization 2 Procedures w/o Connections 3 Procedure w/ Connections 4 Doing Mathematics

Task Set-up by Teacher	Solution Strategy	Representation (Circle all that apply)	Explanation (Circle all that apply)	Cognitive Demand (Circle all that apply)
	1 Single 2 Multiple	1 Symbolic 2 Written words 3 Pictorial 4 Table 5 Graph 6 Verbal 7 Manipulatives	1 Not required 2 Show work 3 Explain/justify	1 Memorization 2 Procedures w/o Connections 3 Procedure w/ Connections 4 Doing Mathematics

Task Implementation by Teacher and/or Students	Solution Strategy	Representation (Circle all that apply)	Explanation (Circle all that apply)	Cognitive Demand (Circle all that apply)
	1 Single 2 Multiple	1 Symbolic 2 Written words 3 Pictorial 4 Table 5 Graph 6 Verbal 7 Manipulatives	1 Not required 2 Show work 3 Explain/justify	1 Memorization 2 Procedures w/o Connections 3 Procedure w/ Connections 4 Doing Mathematics

Factors Associated with the <u>Decline</u> of High-Level Cognitive Demands: (Circle all that apply)

1 Challenging aspects for students routinized 2 Emphasis shift 3 Too much/little time
4 Class management problems 5 Inappropriate task 6 Lacks accountability for high level

<u>OR</u>

Factors Associated with the <u>Maintenance</u> of High-Level Cognitive Demands: (Circle all that apply)

1 Scaffolding 2 Self-monitoring 3 Model performance
4 Sustained press for meaning 5 Build on Prior Knowledge 6 Draw Frequent Connections

Improving Mathematics Instruction for ELL Students

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Performance of English language learner (ELL) students in mathematics is a growing national concern—not only because of the persistent achievement gap between ELL students and their non-ELL counterparts, but also due to the increasing ELL population. Ten percent of all public school students received ELL services in the 2005–2006 school year (NCELA, n.d.) and those numbers are expected to grow dramatically. According to a study by the Pew Hispanic Center, "the projected number of school-age children of immigrants will increase from 12.3 million in 2005 to 17.9 million in 2020, accounting for all the projected growth in the school-age population" (Fry, 2008, p. iii).

ELL students consistently trail their non-ELL peers on the National Assessment of Educational Progress (NAEP). The 2007 NAEP data reveal that 44 percent of ELL students in fourth grade scored "below basic" in mathematics and that number rose to 70 percent for eighth-graders (Lee, Grigg, & Dion, 2007). In comparison, only 16 percent of non-ELL fourth-graders and 27 percent of non-ELL eighth-graders had below basic scores in mathematics.

Addressing the needs of ELL students in mathematics classrooms is a key component of promoting equity in mathematics. The National Council of Supervisors of Mathematics' *PRIME Leadership Framework* (NCSM, 2008) identifies equity as the first leadership principle, calling on leaders to close gaps in mathematics achievement expectations and in access to high-quality mathematics learning for every student. Equity is also the first of six principles of the National Council of Teachers of Mathematics, stating, "Excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 11).

One School's Approach

How does a school with a small and diverse ELL population put that principle into practice? The Northwest Regional Educational Laboratory (NWREL) in Portland, Oregon, worked with one elementary school in southwest Washington on a pilot program focusing on achieving equity for ELL students in mathematics. NWREL professional development provider Linda Griffin met with a cadre of six teachers at Dorothy Fox Elementary over a period of two semesters to apply research-based strategies to promote the success of ELL students in mathematics classrooms. The aim was to train these teacher leaders so they could, in turn, transfer their learning to the rest of the faculty.

Dorothy Fox Elementary is a 540-student school in rural/suburban Camas, Washington, where 15 percent of students go home to families that do not speak English. Fourteen different languages can be heard in the school's hallways with Russian and Spanish predominating. "We've outgrown the pull-out system, but we're not big enough to do a magnet program," explains Principal Cathy Sork. "We have a couple of ELL kids in every class, so every teacher was becoming an ELL teacher but they really didn't have the skills and training they needed."

Sork and her faculty had identified low proficiency in mathematics as a problem for both ELL students and a number of their non-ELL counterparts. Of the 47 students who did not pass the mathematics portion of the Washington

Assessment of Student Learning in 2008, more than 20 percent were ELL students. To address the issue, the school—with district support—applied for and received a two-year ELL demonstration grant from the Washington Office of Superintendent of Public Instruction (OSPI). According to Sork, the state was interested in sharing Dorothy Fox's lessons on how focused staff development might improve mathematics achievement of a multi-language population spread throughout the school.

Both school district and building leaders agreed that an important first step in structuring the grant activities was to conduct an external review of Dorothy Fox's ELL program with an emphasis on mathematics instruction. "We wanted to be sure our limited resources were going to be used very intentionally and focused on where the need was," says Sork. The school contracted with NWREL's Griffin to perform a program review that described the experience of an ELL student in math class and across the span of a school day. Griffin, who holds a doctorate in educational leadership and a master's in mathematics education, came to the task with 13 years of experience as a mathematics professional development provider and a dozen years as a middle and high school mathematics teacher. In addition, Griffin is trained in SIOP (Sheltered Instruction Observation Protocol) and has a personal interest in issues of equity as applied to the mathematics classroom.

During the course of two days, Griffin and a colleague collected data by observing mathematics lessons in all classrooms; surveying teachers, administrators, and ELL program staff; interviewing Sork, teachers, and staff members; holding focus group discussions with students and parents to find out about learning experiences; and examining documents related to the ELL program, such as the school improvement plan and the ELL handbook.

The classroom observations were particularly telling for school leaders. "One of the pieces that came out was the teachers were doing most of the talking and thinking about math and students were doing a lot of listening, and it should be the other way around," says Sork. "[Students] need to be able to use language to show their thinking and express their thoughts in lots of different ways. That was a big 'aha' that we hadn't noticed ourselves."

Best Practices for ELL Students

The program review yielded research-based recommendations for effective ELL practices and for effective practices in mathematics. The intersection of these two bodies of work centers on improving the quality and quantity of communication in the classroom. NCTM emphasizes the role of communication in ensuring support for the mathematical development of ELL students. "It is important for all students, but especially for ELL students, to have opportunities to speak, write, read, and listen in mathematics classes, with teachers providing appropriate support and encouragement" (NCTM, 2008). The Center for Research on Education, Diversity, & Excellence (2002) also stresses communication in two of its Five Standards for Effective Pedagogy:

- Developing Language and Literacy Skills across all Curriculum—Develop competence in informal, problem-solving, and academic language through conversation and through reading and writing across the curriculum
- Emphasizing Dialogue over Lectures—Engage students in instructional conversations rather than through lectures

To apply those standards to the classroom and pave the way for ELL students' language development, Echevarria (1998) suggests some basic steps for teachers. These include understanding students' language needs, explicitly planning lessons to meet those needs, delivering lessons, and then conducting assessments to see if students understood the lessons.

Capturing Student Conversations on Video

With the research and program review results in hand, the school asked all teachers and paraeducators to commit to a professional development activity focused on ELL students or mathematics. Options, which were funded by the OSPI grant, included such activities as participating in a book study led by a Washington State University professor, serving on a parent communication committee, or joining the school leadership committee. A group of six teachers volunteered to form a video study group (VSG), facilitated by Griffin, that would allow them to capture and reflect on the participation of ELL students during their mathematics lessons. When signing up, none of the group members had a clear picture of what a VSG was, but they shared good relationships with one another and all had an interest in improving their practice.

At the outset, the group read about the principles behind VSGs (Linsenmeier & Sherin, 2007; Sherin, 2004; Sherin & Han, 2004) and then collaboratively analyzed how the process could be used to improve mathematics outcomes for their ELL students. They determined that videos would give them added insights into the interactions among ELL and non-ELL students. They also hoped to investigate how different instructional strategies impacted ELL students' use of language in the mathematics class.

The group met monthly for 90 minutes during school time, with substitutes covering their classrooms. Prior to each VSG meeting, Griffin visited the classrooms of two or three group members and taped the class period. While taping, the camera was trained on students, rather than the teacher. The aim was to focus on students' work and their interactions, questions, and responses to instruction. The tape was then given to the teacher, who chose a short clip to share and use as a springboard for facilitated discussion about improving instructional practice.

While viewing the clips, the VSG developed group norms and used the following protocol to guide discussions:

- The teacher of the videotaped class provided background information about the class and the lesson.
- The teacher explained why he or she chose a particular clip to share.
- The teacher put the clip into the context of the whole lesson.
- The group viewed the clip.
- Each group member shared one or two observations that were factual statements (for example, "I noticed that…").
- The group discussed how the clip related to other teachers' experiences.

At one session, the group watched Mary LeFore's third-graders search for patterns in the hundreds chart. The teachers noted that students failed to use mathematical vocabulary as they worked in pairs to solve the problem and LeFore admitted she was "floored" by what her students were missing. Everyone agreed with Griffin's comment that "the crux of the matter is if we don't listen to what students are saying, how do we know what they're learning?"

Although this particular video highlighted a third-grade classroom, the feedback touched on issues common to all classrooms. Kathy McConnell, a first-grade teacher-leader, observed that the VSG's focus on students makes teachers more willing to participate in the tapings. "They realize, no one is counting your 'umms,'" she said. In fact, there's an understanding that the tapes are not used as teacher evaluations. "[The group] is looking to see if kids are talking, do they know how to work in groups, are they using their vocabulary, and are they showing their thinking by using language?" said Principal Sork.

After taking part in the VSG for two semesters, members of the group reflected on the changes to their practice. One teacher commented, "[The VSG was a] huge learning experience, [it] advanced my teaching quicker than with just classroom experience alone." Another participant remarked, "This process helped focus my instruction, gave me practical ideas to try and reflect on, offered encouragement and feedback, and helped me think of extensions or modifications to help clarify or differentiate." A VSG member felt the video allowed her to observe more students. "As I played it back I could consider what happened and how to improve," she said. "The debriefing with others' tapes reinforced my understanding and brought new ideas to consider."

In an evaluation at the end of the trainings, group members said that the experience has made them more intentional about how they hand out materials and give instructions during mathematics lessons. For example, the VSG teachers agreed to distribute or post cards with "sentence stems" for students to use in classroom discussions. These included prompts such as "The pattern I noticed is" or "I agree with you because." Teachers also said they became more aware of the need for students to have opportunities to use mathematical vocabulary and were more intentional about how to group students for collaborative problem-solving activities.

Sustaining and Expanding the Professional Development

At the beginning of 2009, the teacher-leaders planned to roll out VSGs school-wide. The teachers who participated in the original video study group were poised to serve as facilitators for new VSGs at Dorothy Fox Elementary with the goal of involving about half the faculty initially and eventually ramping up the practice to include all faculty. However, the economic downturn in early 2009 led OSPI to cut the project's funding. The school is now grappling with how to cover the cost of substitutes so more teachers can participate in the video groups; according to Principal Sork, VSGs are on the priority list for funding for the 2009–2010 school year. This situation points out the need for dedicated funding to broaden and sustain embedded professional development initiatives.

Prior to the funding cutback, Dorothy Fox was able to complete other parts of its multipronged ELL strategy, including school-wide training in Guided Language Acquisition Design (GLAD). The staff has continued to monitor student progress on classroom mathematics assessments and will track ELL students' achievement on the state proficiency tests held in late spring. With these strategies, Principal Sork hopes that Dorothy Fox teachers will be better able to serve the diverse students that fill their classrooms, rather than "teaching the kids we used to have." At the end of the 2008–2009 school year, she'll ask NWREL to conduct a second ELL program review that will show how far the school has come, and where it still needs to go to improve learning for all children.

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The Influence of Standards-Based Curricula on Questioning in the Classroom

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This article explores the influence of nationally funded textbooks on the levels of questions posed in the classroom. Evaluations were made of the questions asked in courses taught with either a traditional text or a textbook from the *Core-Plus Mathematics Project* (CPMP). Analysis revealed that higher levels of questions occur more frequently in CPMP textbooks and the courses corresponding to their use. However, upon further exploration, it was evident that although textbooks may be the driving force for the classroom, they are not the sole factor in determining what transpires in the classroom. The results of this study may help guide other national and international organizations in their attempts to transform the levels of questions that are posed in typical classrooms.

Introduction

In the United States, the National Council of Teachers of Mathematics (NCTM) has published numerous standards documents in order to guide the reform of mathematics education. The most recent document that includes secondary level mathematics is the *Principles and Standards for School Mathematics* (NCTM, 2000). One of the primary roles and purposes of the *Principles and Standards for School Mathematics* (PSSM) is to guide the development of curriculum frameworks, assessments, and instructional materials (NCTM, 2000). However, it is up to the textbook publishers and authors to incorporate the concepts and methods introduced in those standards.

In response to NCTM's vision outlined in each of the standards documents, the National Science Foundation (NSF) challenged organizations to develop curricula materials that follow the framework for mathematical instruction set forth by the NCTM. One of the fundamental objectives of the NSF-funded projects was to improve the quality of learning and teaching of mathematics in classrooms (NSF, 1991). Several submissions were received by the NSF and eleven textbook series were created. The NSF had several reasons for choosing textbooks to assist in the implementation of the new standards. This paper will shed light on whether or not "standards-based" textbooks have an influence on the types of questions posed in the classroom. The interaction between the teacher and the textbook may be the most important factor influencing what transpires in the classroom. For this very reason, it is important to consider whether or not higher levels of questions are posed in classrooms where standards-based textbooks are utilized.

Textbooks have a profound impact on what takes place in the mathematics classroom. Senk has reported that student learning has been found to be more influenced by the text rather than the teacher (2003, p. 4). If textbooks have such a large impact on the way students learn mathematics, then it appears as though NSF responded to the standards set forth by NCTM in a productive manner. Without adequate textbooks to meet the standards, teachers would lack adequate resources to enact the vision of the standards. Teachers need assistance to create worthwhile tasks for their students to complete. One of the fundamental ways teachers can influence what transpires in the classroom is to ask appropriate questions. It is equally important to ask a wide-range of questions. The research of Bloom developed a taxonomy for classifying the levels at which questions are posed (1956). Bloom identified six categories in his taxonomy.

The six categories are knowledge (K), comprehension (C), application (AP), analysis (AN), synthesis (S), and evaluation (E). There have been some minor revisions to the order and names of categories, but the six listed date back to Bloom's original work. Bloom's original work served the purposes of this study in that it allowed the researchers to differentiate among the questions posed. Table 1 provides definitions as well as example questions for each of the categories.

As one might imagine, it is easier to construct questions at the lower levels (Knowledge and Comprehension), and more difficult to create higher-level (Application, Analysis, Synthesis, and Evaluation) questions. One would hope that NSF-funded textbook series were designed to provide teachers with a resource for selecting a range of question types, with an emphasis on higher-level questions. According to Moyer & Milewicz, "A good question may mean the difference between constraining thinking and encouraging new ideas, and between recalling trivial facts and constructing meaning" (2002, p. 293). Mathematics teachers in the United States are more prone to constrain students' thinking by asking questions that only allow for the practice of basic skills (Stigler & Hiebert, 1999). This pattern was documented in the Third International Mathematics and Science Study (TIMSS) videos. Lower-level questions that only require one word answers are frequent throughout the video samples provided of American classrooms (Moyer & Milewicz, 2002). According to Stigler & Hiebert, "the nature and tone of teachers' questions often give away the answer..." (1999, p. 45)

Research clearly shows there is room for improvement when it comes to the levels of questions posed in the mathematics classroom. NSF-funded, standards-based textbooks have been constructed in part to address this shortcoming. The foundation of these textbooks is providing tasks for students to develop high levels of mathematical thinking. Good tasks are those that provide an appropriate level of challenge and support for the students as well as lead students to the discovery of important concepts and problem solving techniques (Hirsch et. al, 1995). Tasks designed to develop students' higher-order thinking skills are provided by NSF-funded, standards-based curricula materials, but it is up to the teacher to implement those materials (Cai, 2003).

Research Questions

- How do the levels of questions posed in a standards-based textbook compare to the levels of questions posed in a traditional textbook?

Table 1

Category	Definition	Example Question
Knowledge	require students to recall previously-learned material	What unit do you use to measure an angle?
Comprehension	ask students to demonstrate understanding of a concept	In your own words, explain how an obtuse angle differs from an acute angle and from a straight angle?
Application	involve students using methods, concepts and theories in new situations	[Provide students with a set of angles – including right, straight, obtuse, and acute] Measure the following angles; classify the angles as acute, right, obtuse, or straight.
Analysis	require students to break down information into parts and support their decomposition	Suppose you were asked asked to determine if a given angle, A, could be formed by adding some number of copies of another angle, B. How would you determine if this were possible?
Synthesis	require students to put ideas together	Show that the sum of the measures of the interior angles in a triangle is 180°.
Evaluation	involve students making judgments about information based on a set of criteria	Person A showed that the sum of the interior angles of a triangle is 180° by measuring angles in several triangles and finding that the sum was always 180°. Person B showed that the sum of the interior angles of a triangle was 180° by cutting off the corners of a "random" triangle and putting the corners together, vertex to vertex and edge to edge, to show that a straight angle was formed. Which of these methods is a more valid demonstration of the interior angle sum of a triangle? Explain.

- What is the influence of the use of a standards-based textbooks on the level of questions posed in the classroom?
- What is the influence of the use of standards-based textbooks on the level of questions posed on teacher-constructed assessments?

Methodology

Participants

The participants in the study were seven high school teachers at a suburban high school in northwest Ohio. The school is located in the only school district within northwest Ohio that is using an NSF-funded textbook series at the secondary level. However, this school district is still teaching many course sections using traditional textbooks.

Courses and Textbooks

Since this school district implemented the use of CPMP, two years ago, only Course 1 and Course 2 were being taught at the time of this study. Course 1 is a freshmen level mathematics course that the district is using to correspond to the traditional Algebra Course. Course 2 is a sophomore level mathematics course that the district is using to correspond to the traditional Geometry course. Although two newer editions have been published since 1998 (2003, 2008), this edition was used for the purposes of this study since that edition was being used by the district at the time of this study. The text *Algebra 1* (Holiday et al., 2003) was used in the traditional Algebra course and *Geometry* (Boyd et al., 2004) was used in the traditional Geometry course. Since CPMP is an integrated mathematics curriculum, careful consideration was made in choosing units where the content of the lessons was similar.

Five of the seven teachers involved with this study teach traditional courses with the assigned text. Three of the five teach algebra, while the other two teach geometry. The remaining two teachers teach the non-traditional courses with CPMP. Each of these two teachers attended weeklong training seminars provided by the textbook company prior to the start of the academic year.

Instrumentation

The levels of questions posed in a CPMP textbook were compared with those posed in a traditional textbook. Comparable sections were selected. A Questioning Levels Evaluation Form developed by the researcher was used to evaluate the levels of questions posed in each textbook. This evaluation form was based on the six levels of questions identified by Bloom (1956).

The author/investigator observed each class involved with this study a total of five times. The date and time of each observation was chosen by the individual teachers. All classes lasted a duration of 42 minutes. The levels of questions posed during each class were transcribed and later evaluated using a Questioning Levels Evaluation Form developed by the researcher. This evaluation form was based on the six levels of questions identified by Bloom (1956). The categorization of the questions were corroborated by an independent reviewer. In the event there was disagreement between the researcher and the independent evaluator, a discussion was held to classify the question in the appropriate category.

Each teacher was asked to provide two representative tests for their course. These tests were collected to provide a sample of questions to determine what levels of questions were being used on assessments. These tests also provided insight into whether or not the levels of questions during observations were consistent with the levels of questions posed on assessments.

Each teacher was also interviewed to ascertain the individual teacher's values and beliefs in regard to mathematics education, as well as to determine their level of professional development associated with mathematics education.

Results

Textbook Question Evaluation

The first comparative analysis involved one investigation from the CPMP series and two sections from the traditional text. The focus of the investigation from the CPMP was on slope and direct variation (Coxford et al., 1998, Course 1, Part A, pp. 182-194). This investigation involved 100 questions over 12.5 pages. There were no worked out examples in this investigation. One section from the traditional text focused on slope (Holiday et al., 2003, pp. 256-262). This section posed 65 questions over 7 pages, and there were 10 worked out examples. The second section centered on the concepts of slope and direct variation, and asked 62 questions over 7 pages (Holiday et al., 2003, pp. 264-270). There were also 10 worked out examples in this section.

The second comparative analysis also involved one investigation from the CPMP series and two sections from the traditional text. The central theme of this investigation from CPMP was on point-slope form and finding linear equations in that form (Coxford et al., 1998, Course 1, Part A, pp. 194-199). This investigation asked students 33 questions over 5 pages, with no worked out examples. The first section from the traditional text discussed the concept of point-slope form (Holiday et al., 2003, pp. 272-277). This section posed 55 questions over 6 pages, with 8 worked out examples. The second section focused on writing linear equations in point-slope form (Holiday et al., 2003, pp. 280-285). There were 47 questions asked over 6 pages, with 4 worked out examples. The overall proportion of questions posed at each level is displayed in Table 2.

Table 2: Comparative Textbook Evaluation

Textbook	Traditional	CPMP
Knowledge	0.83	0.5
Comprehension	0.06	0.05
Application	0.09	0.40
Analysis	0.02	0.40
Synthesis	0.00	0.05
Evaluation	0.00	0.00

Classroom Observation and Test Evaluations

Teachers 1 through 5 were using a traditional textbook whereas teachers 6 and 7 were using a standards-based textbook. The mean number of questions posed per observation for each teacher is displayed in Table 3. The mean number of questions posed per assessment for each teacher is displayed in Table 4.

Tables 5 and 6 display the mean proportion of questions posed during observations and on representative assessments, respectively.

Table 3: Mean Number of Questions Per Observation

Teacher	Questions
1	32
2	24
3	23.4
4	29.6
5	34
6	20
7	17

Table 7 displays the overall mean proportion of questions posed at each level during observations and on assessments.

Table 4: Mean Number of Questions Per Assessment

Teacher	Questions
1	25.5
2	27
3	23
4	24
5	5.5
6	17
7	10

Discussion

Table 1 conveys a clear distinction between the levels of questions posed in a CPMP series textbook versus those

Table 5: Mean Proportion of Questions Posed During Observations

Teacher/Question	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
1	0.47	0.25	0.16	0.13	0	0
2	1	0	0	0	0	0
3	1	0	0	0	0	0
4	0.86	0.07	0.03	0.03	0	0
5	1	0	0	0	0	0
6	0.60	0.20	0.05	0.15	0	0
7	0.21	0.41	0.15	0.12	0.06	0.06

Table 6: Mean Proportion of Questions Posed on Representative Assessments

Teacher/Question	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
1	0.65	0.16	0.08	0.06	0.02	0.04
2	0.96	0.04	0	0	0	0
3	1	0	0	0	0	0
4	0.94	0.06	0	0	0	0
5	1	0	0	0	0	0
6	0.44	0.41	0	0.12	0.03	0
7	0.45	0.20	0.05	0.25	0	0.05

Table 7: Overall Mean Proportion of Questions Posed

Teacher/Question	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
1	0.55	0.21	0.12	0.10	0.01	0.02
2	0.98	0.02	0.00	0.00	0.00	0.00
3	1.00	0.00	0.00	0.00	0.00	0.00
4	0.90	0.07	0.02	0.02	0.00	0.00
5	1.00	0.00	0.00	0.00	0.00	0.00
6	0.53	0.30	0.04	0.02	0.01	0.00
7	0.30	0.33	0.11	0.06	0.04	0.06

posed in a traditional textbook. The traditional textbooks examined for this particular study displayed a trend that as the level of question went up, the frequency with which a question was posed at that level went down. Although the same trend can be seen with the CPMP series textbook, it is far more gradual.

The traditional methods associated with mathematics teaching involve the instructor demonstrating how to perform a certain task (Senk, 2003). These methods are mirrored by the traditional textbooks in their format of showing examples for the majority of problems students will encounter (Cai, 2003). As you can see from the results of this study, there were 32 worked-out examples in the traditional text, and not a single worked-out example in the CPMP series textbook. In a traditional text, students often must simply work individually to replicate what was performed by their teacher or the text in order to be successful.

As discussed in the introduction, one of the primary goals of the NSF-funded project was to improve the quality of learning and teaching of mathematics (NSF, 1991). According to Cai, the problems posed in standards-based curricular materials are constructed in such a way that they aid in the development of students' higher-order thinking skills (2003). Teachers can encourage higher levels of thinking by asking questions that stimulate thought (Cooney, 1975). Techniques can be implemented to stimulate students' thinking by posing questions at different cognitive levels (Beamon, 1997; Brahier, 2000). Teachers have different styles and strategies for developing students' higher-order thinking skills, but effective teachers know how to ask questions (NCTM, 2000). It must be noted that this study did not explore the level of student learning that takes place in the classroom. Simply because higher levels of questions are posed, does not imply students develop higher levels of thinking. However, students will only respond to a question to the depth at which it is posed.

Examining Tables 4 and 5 one can see that standards-based curricular materials may increase the levels of questions posed in the classroom and on assessments. The largest differences can be seen in the knowledge, comprehension, and analysis. The frequency with which application questions were asked was comparable in both courses and on assessments.

Examining Tables 4 and 5 one can see that standards-based curricular materials may increase the levels of questions posed in the classroom and on assessments. The largest differences can be seen in the knowledge, comprehension, and analysis. The frequency with which application questions were asked was comparable in both courses and on assessments.

An interesting observation can be made by further examining the individual breakdown of questions for each teacher. One traditional teacher (Teacher 1) stands out from the rest. The number of questions posed in each category was very different from the other four traditional courses. The textbook may not be the only factor in determining the levels of questions posed in a particular classroom.

What is it about this particular teacher that influences the questions posed to students? One may think that this teacher has more experience than the other traditional teachers. However, Teacher 1 is the most junior of the traditional course teachers with 5 years experience. The other four teachers have a mean number of 14.25 years experience. This teacher is currently working on a master's degree in mathematics education, whereas the remaining four traditional course teachers have not taken a mathematics education course in a mean number of 15.25 years. Those who have pursued graduate degrees have focused their studies on school administration and supervision. One other distinct difference is that of all the teachers involved in this study, only Teacher 1 is a member of any professional organization. It is difficult to pin down the one factor that distinguishes this teacher from the rest. The difficulty may be due to the fact that there is not just one factor that guides teaching practice.

Implications

In general, it appears as though there are more high-level questions posed in a CPMP course versus a traditional course. The increased frequency with which higher levels of questions are posed seems to spread from the textbook to the classroom and to the assessment measures used. However, there are several concerns that still exist in relation to what transpired in the classrooms involved in this study.

One thing that must be mentioned is that although the CPMP textbooks seem to pose higher levels of questions, the actions of the teacher truly determine the level of a question. This study simply examined the question that was posed and did not explore the actions of the teacher in responding to students. One CPMP teacher, Teacher 6, would answer the questions for the students without allowing them to struggle with the problem at hand. Even further concern arose when one student asked what types of questions would appear on an upcoming assessment. Teacher 6 responded by informing the students of what specific examples provided in the notes would appear verbatim on the test. These actions confirm that although a textbook is used which inspires teachers to ask higher levels of questions, it is up to the teacher to implement the textbook in an appropriate manner. National and international efforts may be better spent on providing professional development programs focused on the importance of questioning techniques.

An important implication from this study can be seen in the actions of Teacher 1. Clearly what transpired in the classroom of this teacher did not depend on the textbook in use. This particular teacher made sure that everyone responded to questions and called on students at random (selecting names at random from a stack of index cards). The teacher also provided ample time for the students to think about their answers before moving onto another student for a response. There are exceptional teachers in school systems that will succeed no matter what type of textbook they use.

The analysis of the data resulting from this study results in more of an introduction to a new study than a conclusion. As with most studies, more questions have arisen during the course of the study than could have been imagined. There are several questions that could extend this study in the future.

(1) How have national and international efforts to create standards and curriculum documents influenced the types of questions posed in textbooks? What impact do these efforts have on the types of questions teachers pose in the classroom and on assessments?

(2) What teacher variables influence the activities in the classroom the most? Namely, what qualities do successful mathematics teachers, like Teacher 1, possess which relate to better methods of instruction?

(3) Do students in a course that utilizes a textbook based on the national and international reform efforts actually develop higher-levels of thinking versus students in a traditional course as a result of the increased frequency with which questions are posed at higher levels?

Conclusion

Textbooks may be seen as the driving force for what types of questions are posed in the classroom. Since the United States does not have a national curriculum, the effort to transform textbooks at a national level may have a tremendous impact on the way mathematics is taught. More specifically, it may have an impact on the types of questions posed in the classroom. However, textbooks are clearly not the sole factor in determining what transpires in the classroom. Other national and international organizations may come together in an attempt to create a set of standards and expectations for students to become successful. In order for those efforts to have a more profound impact on what truly takes place in the classroom, professional development programs should be created to help teachers increase the depth of questions posed in the classroom. Additionally, further research should be conducted on what variables influence the levels of questions posed in a classroom. If research can shed some light on this issue, then the international mathematics education community can learn how similar collaborative efforts at the national and international level will help teachers make strides toward increasing the levels of questions posed in all mathematics classrooms.

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Prediction as an Instructional Strategy

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In various disciplines, using prediction has been investigated and incorporated into an instructional sequence in order to facilitate teaching and learning, and research has reported the effectiveness of using prediction (e.g., Gunstone & White, 1981; Palincsar & Klenk, 1991; Battista, 1999). However, using prediction in the mathematics classroom is a relatively new idea, and practitioners have been provided limited guidance of how prediction can be used to help mathematics instruction.

In this article, we address using prediction as an instructional strategy to enhance classroom practices. Researchers emphasized the importance of effective teaching practices on student learning (Marzano, Pickering, & Pollock, 2001; Wenglinsky, 2002; Wright, Horn, & Sanders, 1997). For example in his evaluation of data from the National Assessment of Educational Progress (NAEP), Wenglinsky indicated that teaching practices seemed to have more of an influence on student learning than socioeconomic status on NAEP student outcomes. Using prediction as an instructional strategy can lead to classroom practices where students actively engage in the meaningful learning of mathematics. Some immediate questions that arise are: What does prediction mean? How can using prediction create desirable pedagogical practices? What are some effective ways of using prediction? We address these questions in terms of the role that prediction can play in the teaching and learning of mathematics.

What is Prediction?

Prediction can be understood as reasoning about the mathematical ideas of the lesson at the launch by using prior knowledge, patterns, or connections. Prediction does not necessarily mean a simple premature guess. Rather, prediction is a sophisticated reasoning process connecting relevant ideas. In order to make a plausible prediction, students have to activate their prior knowledge and connect concepts from previous explorations. For example, students may be asked before exploring a problem to predict what effect increasing walking rates will have on the table, the graph, and the equation as they examine the relationship between distance and time. When making such predictions, students have to look back on what they already know (i.e., what walking rates mean, and how those rates are represented in a table, a graph, or an equation) and use that to reason about what will happen when a rate is increased. Such an opportunity helps students build a better understanding of key ideas based on the connections they make.

Advance Organizer

Predictions can be used as advance organizers. Advance organizer is one of the instructional strategies that Marzano, Pickering, and Pollock (2001) suggest. Originally, Ausubel (1968) introduced advance organizers as “relevant and inclusive introductory materials” presented in advance of learning. According to Ausubel, “advance organizers are … at the same level of abstraction as the material to be learned, [however] are designed to bridge the gap between what the learner already knows and what he needs to know before he can successfully learn the task at hand” (p. 148). Lesh (1976) conjectured that these advance organizers are valuable tools for learning new material. According to Kim and Kasmer (Kim & Kasmer, 2007a, 2007b; Kasmer, 2008), posing prediction questions prior to students exploring the mathematical ideas of a problem helps invoke prior knowledge and bridge between mathematical concepts (e.g., previous concepts and a new one). Prediction helps make sense of the problem context and identify related mathematical concepts. How is this problem similar to and different from previous ones? What mathematics are embedded in the problem? As a result, engaging in prediction activities prompts students to make connections of mathematical ideas which helps set the foundation for future learning. Overall, prediction encourages students to engage in mathematical sense making.

Consider a classroom episode below (Kim & Kasmer, 2007a). In this example, middle school students solved a problem involving a race between two brothers:

> Emile's walking rate is 2.5 meters per second and his little brother Henri's walking rate is 1 meter per second. Henri challenges Emile to a walking race. Emile gives Henri a 45-meter head start. How long should the race be so that Henri will win by just a bit? (Adapted from Lappan, Fey, Fitzgerald, Friel, & Philips, 1998)

Prior to solving the problem, students were asked to make several different types of predictions related to this problem, write their predictions and supportive reasoning on paper, and then discuss which of the predictions seemed reasonable. First, they predicted whether graphs, tables, and equations of this problem would look similar to what they had done before. One student said that this problem would produce lines with constant rates, which made them "linear." Other students agreed with him and said that some of work they had done previously were constant rates and some were not.

Next, they predicted which line would be steeper if they graphed the situation. One student said, "Henri's got steeper because he has a 45-meter head start." Another student offered, "I think Emile because he goes further and faster in a shorter time." When the teacher asked how she knew he went further and faster in a shorter amount of time, the student answered, "Because he's a lot faster." Some students agreed with her. One student said, "Because he goes 2.5 meter per second and he travels faster, so his line will be steeper."

Last, students predicted how long they thought the race should be. Students offered various predictions ranging from 50m to 250m (50m, 100m, 200m, 60m, 55m, 75m, 70m, 250m, 175m, and 150m were their predictions, in order of being offered). One student whose prediction was 250m said, "There should be a longer distance so Emile can catch up." As soon as he finished, another student said, "I disagree with him because Emile's walking rate is double Henri's. So, it's not going to be 100 and up." Many agreed with her and said, "100 is too high" and changed their prediction for the race distance. A couple of students were attempting to determine the distance that each could make in a certain amount of time, for example, 10 seconds. At that point, the teacher asked students in pairs to solve the problem using the ideas that they had generated.

In this example, predictions were made about three ideas: a) if the problem looked familiar with respect to the graph, table, and equation; b) which of the two lines would be steeper; and c) what would be the optimal distance of the race. As advance organizers, predictions related to the first two ideas enabled the students to make connections between what they had previously explored and the problem they had been given to solve. In making such connections, the key concepts were constant rates and steepness of lines. Using these concepts, students also predicted some characteristics of the lines that the problem would produce. Particularly interesting is that even though students knew Emile would win eventually since he was faster, they had not yet built the connections between the problem context and its graphical representations at this point. In fact, this was challenged by the first two prediction questions.

Predictions related to the last idea certainly invited students' interest in the problem and provided motivation to solve the problem. It also made them consider the relationship between the two lines in terms of two different constant rates as well as a head start. When predicting the brothers' race distance, students provided random guesses at first, but revised their predictions in more sophisticated manners once they began to discuss the reasoning behind the predictions. When a girl pointed out the brothers' walking rates in relation to one another ("Emile's walking rate is double Henri's"), many students were convinced and changed their predictions. Having this discussion prior to solving the problem enabled students to actively engage in the problem, to agree or disagree with each other's predictions based on their reasoning, and to make sense of the problem situations by visualizing the two linear relationships.

To summarize, in the above classroom example, making predictions and discussing related mathematical ideas served as an effective advance organizer.

Aids for Visualization

A number of researchers expressed the importance of visualization in learning (Bishop, 1988; Brown & Wheatley, 1990; Clements, 1982; Cuoco, Goldenberg, & Mark, 1996; Presmeg, 1991, 2006; Skemp, 1987; Suwarsono, 1982). For example, Dunham and Osborne (1991) suggested that students learn "how to see" in order to promote conceptual understanding. Kim and Kasmer (2007a) and Kasmer (2008) found that using prediction invoked visualization. The classroom example provided earlier illustrates that prediction can encourage the visualization of problem situations and related concepts. Through predictions they made, the students were encouraged to imagine what was happening in the problem context and with the given condition what would happen in the end. They also were thinking about pictorial images of graphs and tables that the problem context would produce. According to Presmeg (1991), the first type of visualization (i.e., mentally picture the problem situation) is considered as "concrete imagery" and the second (i.e., utilizing graphical images of the problem situation) as "pattern imagery." When visualizing graphical representations of the problem situation, students had to utilize the fact that the problem context involved linear relationships. More specifically, they had to connect rates of walking distance with steepness of lines and the head start with a y-intercept. Eventually, such visualizations helped the students make sense of the problem situation and relate the problem context with various representations (e.g., what the graph of the situation would look like).

Tool for Informal Assessment

Using predictions allows teachers to assess students' thinking prior to the investigation of a task (Kim & Kasmer, 2007a, 2007b; Kasmer, 2008). When students pose their predictions, teachers have an opportunity to establish what students have understood from prior explorations and what connections they are able to construct with reference to the current problem. In addition, teachers can determine students' misunderstandings and misconceptions through their predictions. Such informal assessments enable teachers to adjust their plans taking into consideration the students' predictions to further develop and focus on the mathematics of the lesson. Prediction also allows individual students a chance to assess their own thinking as they prepare to begin a new problem.

In the classroom example above, the teacher could see where his students were before the exploration of the problem. Students were able to see the linearity of the problem context. However, the specific aspects of the problem, such as different walking rates and the head start, were not clearly connected back to the characteristics of linear relationships. Some thought that the 45-meter head start would yield a steeper line: others thought the faster walking rate would produce a steeper line. While discussing their ideas, students were able to see some agreements and disagreements with their own thinking. Such arguments and reasoning would be resolved and pursued through the exploration of the problem.

Means to Promote Student Engagement and Classroom Discussion

Earlier we illustrated that predictions invokes students' prior knowledge and engagement. Using prediction also helps guide classroom discussion. In a recent study (Kasmer, 2008) prediction was found to be an effective tool to engage students as well as assist teachers in focusing the classroom discussion. That is, the prediction questions provided a vehicle to begin or focus classroom discourse where the teacher was able to organize discussions based on the students' responses. These discussions in turn, afforded the teacher with an impetus to promote classroom interactions where students can justify their thinking and listen to and make sense of others' thinking.

Kasmer also found that students in an algebra classroom where prediction questions were routinely posed prior to the explore segment of a problem demonstrated a higher level of engagement compared to a similar class where prediction questions were not used. When prediction questions were posed and students responded with supportive reasoning, first in writing, then sharing their responses in whole group discussions, it was noted students were engaged in sustained conversations that were created by a culture precipitated by the inherent free virtue of prediction and its absence of certitude. Once students have had an opportunity to consider the question and record their predictions, they are more confident in their responses.

Furthermore, the prediction questions presented both the teacher and students a focus for discussion. This deliberate discourse is often difficult for teachers to orchestrate as they juggle both the complexities of the mathematics and the discourse. However the prediction questions along with the student responses, which were prevalent during the prediction phase of the lesson, provided both the teacher and students a direction for discussion. Moreover, permitting all students with purposeful time devoted to activating and consolidating their individual thinking prior to class discussion created richer discussions of mathematical ideas.

Suggestions for Using Prediction Questions

While using prediction as an instructional strategy provides benefits, it is not trivial to create appropriate prediction questions and use them effectively. Drawn from previous work (Kim & Kasmer, 2007b; Kasmer & Kim, under review), we provide some suggestions.

Create an appropriate classroom culture. At the beginning of the school year, it is important that teachers create a classroom environment where students feel comfortable taking risks and making predictions. The teacher must develop a culture that establishes the norms of interaction where students are reassured that all prediction responses will be valued and supportive reasoning should follow all predictions. Students should approach their prediction responses as plausible ideas and not merely a random guess. Also, students need to be encouraged to share ideas with one another and constructively evaluate each other's ideas.

Make a deliberate plan to include prediction questions. Teachers should examine the key ideas of the lesson when deciding to use prediction questions. Prediction questions should implicitly reflect the mathematical ideas of the main problem without revealing the essence of the problem. These questions should be presented to students as they potentially generate opportunities to engage students in the mathematics of the lesson. The teacher presents the prediction questions in conjunction with the launch of the investigation. Students would record in writing their individual responses to each prediction question the teacher poses, as described later. After students record their predictions, the teacher then elicits student responses without commenting on the accuracy of the prediction or the appropriateness of reasoning.

Have students write their prediction prior to class discussion. Individual student written responses are necessary to provide evidence of each student's thinking as time constraints do not allow each student the opportunity to share their predictions during the launch of the lesson. Furthermore, writing individual responses also affords students the occasion to organize their thinking about the mathematics of the problem before verbalizing their thinking to the entire class. Requiring students to respond in writing to the prediction questions helps students utilize their own reasoning, rather than those of classmates. Such writing also helps students prepare to engage in discussion and feel more confident.

Revisit the students' prediction responses. It is important to revisit the prediction questions and student responses during the summary segment of the lesson through which students can reconcile any discrepancies between their initial prediction and the outcome of the problem. Exploring elementary students' 3D geometry, Battista (1999) found that discrepancies between student predictions and actual results helped build a useful mental model to solve problems. Noticing the differences and examining "why" will encourage students to engage in careful thinking and thorough reasoning.

Final Remarks

Hiebert and Grouws (2007) suggest student engagement and students' entry knowledge are two aspects of opportunities to learn. The National Research Council (2001) reports that the "opportunity to learn is widely considered the single most important predicator of student achievement" (p. 334). Predictions made and discussed before exploring the main task of a lesson create learning opportunities for students by playing a role of advance organizer and enhancing students' engagement. When classroom teachers use prediction as an instructional strategy, they are creating a learning environment where students can activate their prior knowledge, make connections of mathematical ideas, make sense of what they explore through visualization, and actively engage in problem solving and discussion. This instructional strategy also allows teachers to informally assess students' on-going thinking. Therefore, we suggest that mathematics classrooms use prediction as an instructional strategy to promote students' mathematics learning.

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Developing a Shared Vision for Mathematics

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"We will learn, understand, and use mathematics concepts and processes as critical thinkers and effective problem solvers."

Much has been written in education about developing and implementing a shared vision for school districts (Lambert, 2003; Senge, 1990, 1994; Weiss, Miller, Heck & Cress, 2004). As Senge (1990) states, "Few, if any, forces in human affairs are as powerful as shared vision" (p. 206). A shared vision is a mental image of what is important to the organization and its individuals. It reflects the beliefs and shared values (Hord & Sommers, 2008) and captures what the group wants to create together through collective commitment.

How can a school district create a shared vision for all students' mathematics learning that includes a broad range of stakeholders in the development process? This story belongs to a school district that is a convergence of cultures living in the southwest. As a school district, we faced numerous challenges with our fifth superintendent in five years and new standards-based mathematics curricula that were adopted in grades K-8. The district did not embrace a common view of what mathematics teaching could look like and each campus had diverse approaches and outcomes.

The district had a history of school-based decisions. Each school taught its own math curriculum. Educators in the school district asked how we could develop a supportive and cohesive structure that provides a focus and direction for mathematics learning for all students. When a few administrators and mathematics educators gathered in a room to develop a five-year implementation plan for mathematics, they realized the district had many components working independently, and the district was not functioning coherently as a learning organization. We thought about the educational community and concluded the mathematics plan must be developed with the voices of the parents, teachers, mathematicians, and administrators. From this meeting of a few educators, the District Leadership Team (DLT) emerged.

As the DLT worked together, a group of dedicated people representing a microcosm of the school system, community, and university partnership developed. The team included mathematics educators, mathematicians, administrators, curriculum specialists, teachers, state legislators, business people, school board members, parents and mathematics specialists. The team agreed to work collaboratively to learn about the current state of mathematics teaching, and what could be done to improve or refine the mathematics learning for all stakeholders in the district.

The team began by developing norms and building a safe learning environment. The superintendent addressed the group at the first meeting and stated that the purpose of the DLT was to study the data (student achievement, demographic, and surveys) together so as to learn about

Mathematically Connected Communities project is a partnership between New Mexico State University, the New Mexico Public Education Department Math and Science Bureau, and school districts to improve mathematics teaching and learning.

the current status of mathematics teaching and learning and over time to develop a comprehensive five-year math plan to guide the district.

The DLT used several resources in its work to achieve the aims outlined by the superintendent, including the following: the Southwest Educational Development Lab's Working Systemically Model (Buttram et. al. 2006); data from the Scaling up Mathematics Achievement (SUMA) Research Team; and the ideas of purposeful distributed leadership (Hargreaves & Fink, 2006). These resources were used to think about the district mathematics program and student achievement. The DLT studied the current state of the system and conducted a gap analysis using the Working Systemically Model (SEDL). The team learned to work together through dialogue, reflection, inquiry-based thinking, and building both the culture and capacity for learning. DLT members became an effective learning group over time through doing mathematics, analyzing data, and evaluating research within a collaborative inquiry process.

One of the most important resources for the DLT was the use of the Scaling Up Mathematics Achievement (SUMA, 2007) research data. The National Science Foundation SUMA research project is a partnership between the public school district and the local university to study the mathematics implementation process in relation to student achievement. SUMA played an integral part in providing data to district stakeholders, including the DLT, through both quantitative analysis and a design-based research process. The SUMA researchers attended the DLT sessions to provide data for the group throughout the year.

The SUMA research data was based on the SUMA Building Capacity Model. The SUMA model included three primary elements: 1) quality aligned and learned curriculum; 2) teaching quality and purposeful collaboration; and 3) leadership/policy/community support for learning. In an effort to align with the Building Capacity Model, three learning groups were formed as each DLT member chose one element of the SUMA model to focus on. These learning teams engaged in ongoing conversations, data analysis, and study.

Through many hours of discussion, inquiry, and debate the DLT began a process of developing a shared vision for mathematics that was clear, compelling, and connected to articulated goals for learning mathematics.

The DLT Process for Creating the Mathematics Vision included:

1. Creating a vision statement based on personal and professional mathematics learning experiences from stakeholders including parents, teachers, administrators, university mathematicians, mathematics educators, curriculum specialists, business people, state senators, and mathematics coaches.
2. Comparing our vision statement with national and state visions for mathematics learning.
3. Revising the vision based on research, data, and national and state documents.
4. Developing a succinct statement of the vision to clearly communicate to all stakeholders. (Ex. *Succinct District Math Vision: We will learn, understand, and use mathematics concepts and processes as critical thinkers and effective problem solvers.*)
5. Sharing the complete vision statement and the "succinct" version with stakeholders across the district.
6. Articulating and documenting throughout the system what the vision will look like in classrooms, schools, in district policy, resources, and professional learning activities.
7. Considering the vision dynamic — one that can grow over time as it is enacted and refined and as data provides evidence of results.

We have learned from this process the importance of bringing together diverse stakeholders as partners to create a district vision that focuses attention on what is important in mathematics teaching and learning. A collectively shaped vision provides a clearer sense of direction for all stakeholders and can guide decisions related to professional learning opportunities, curriculum, and instruction. These actions can be systemically coordinated and aligned to take measured steps towards attaining the vision. Data will indicate the ways that administrators, children, parents, and teachers understand and support the vision.

As a District Leadership Team, we have engaged in purposeful collaboration to co-construct a five-year math plan and a vision for mathematics. The DLT recognizes that the vision will come to life as it is lived in the classrooms and schools. The academic year provides time for pursuing the vision, providing feedback, and reflecting on the journey. These efforts contribute to realizing a shared vision for mathematics.

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