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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Brief commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We want to hear about your reactions, questions, and connections you are finding to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Linda Ruiz Davenport, at ldavenport@boston.k12.ma.us. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

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***Note:** Information for manuscript reviewers can be found on the inside back cover of this publication.

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editor

Linda Ruiz Davenport
Boston Public Schools, Boston, Massachusetts

These [Common Core State] Standards are not intended to be new names for old ways of doing business . . .. (CCSS, p5)

These are interesting times for mathematics education leaders. Many states have adopted the Common Core State Standards (CCSS) for Mathematics, and other states, even if not adopting these standards, are looking closely at them to consider any implications for mathematics teaching and learning across their states. In addition, everyone is wondering about the new CCSS assessments being developed by the Partnership for Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium (SBAC). How will these measure what students will be expected to know? How might they shape what actually happens in classrooms? What are the implications for district administrators, mathematics coaches and specialists, teacher leaders, teachers, students, and parents? What are the implications for our work as mathematics education leaders?

None of the articles in this issue of our journal specifically reference the CCSS for Mathematics, but as you read through them and consider connections to your own leadership work, it might also be useful to look for connections to any future leadership work that may be coming.

For instance, in *Moving Beyond the Word Wall: How Middle School Mathematics Teachers Use Literacy Strategies*, we have opportunities to consider the kinds of literacy strategies that can help support mathematical reasoning and sense making in middle school mathematics classrooms. Some teachers in their study seemed to be able to successfully integrate a range of literacy strategies into their mathematics instruction while some did this in more superficial ways that did not as substantially advance their mathematics learning goals for students. The authors raise questions about how teachers of mathematics might be better prepared to successfully integrate literacy strategies into their mathematics instruction as a way to support and deepen mathematics learning. What are implications for preservice programs for mathematics teachers? What about professional development opportunities for practicing mathematics teachers?

> *Just as students need to learn to read, write, speak, listen, and use language effectively in a variety of content areas, so too must the Standards specify the literacy skills and understandings required for college and career readiness in multiple disciplines.* (p4)

Similar notions about the literacy demands of the mathematics classroom are also explored in *Content Matters: A Disciplinary Literacy Approach to Improving Student Learning* (McConachie & Petrosky, eds., 2010) where the chapter "Disciplinary Literacy in the Mathematics Classroom" (Bill & Jamar, 2010) describes how to integrate literacy development with efforts to engage students in authentic mathematical inquiry. These discussions resonate strongly with the goals of the CCSS Mathematical Practice Standards.

In *An Activity-Based Approach to Technology Integration in the Mathematics Classroom*, we have opportunities to consider what it means to help teachers make good technology choices as they plan and enact their mathematics lessons—choices that contribute to cohesive interactions across content, pedagogy, and the technology itself. Many of the conversations about the CCSS include discussions of the importance of integrating technology into the reasoning and sense making we want to see happening in classrooms (e.g., Shaughnessy, 2010; CSMC Summary Report, 2010). This is something that many agree needs serious consideration as we move into the implementation of the CCSS, and the "activity types" discussed in this article may be very helpful as we undertake this work.

In *Observing Mathematics Lessons: What Does it Mean for Principals to be Up-to-Speed?* we learn about how principals make judgments about the mathematics lessons they observe. What kind of "leadership content knowledge" is important for principals to have in order to make thoughtful assessments of teachers' mathematics teaching practice? What will principals need to know and understand in order to support teachers as they begin to move their practice in the direction of the CCSS? Will they be able to recognize the Mathematical Practice Standards as teachers engage their students in exploring important mathematics content? Will they be able to offer thoughtful feedback as they supervise their teachers? There is much that needs to be learned about how we might help get more principals "up to speed" for mathematics classrooms of the future.

Finally, in *Using Professional Development Materials Productively: The Role of Adaptations*, we explore what it can mean to offer practice-based professional development in a range of contexts and within a range of constraints, with attention to how facilitators of this kind of professional development might be supported in making adaptations that do not constrain what we hope teachers will learn. These are important questions to consider, both in our ongoing work to strengthen mathematics teaching and learning, and in any future work as many of us begin to move toward our implementation of the CCSS. What kind of professional development materials might need to be developed? What supports for facilitators might be included in these materials and why? How can we help ensure that facilitators thoughtfully use these materials in ways that help achieve the identified learning goals for participants? And finally, might some existing materials be "productively adapted" to explicitly address the expectations of the CCSS?

As you read through these articles, we hope you will find stories that resonate with your own mathematics education leadership work, both as it currently exists and where it may be heading in the future. We hope these articles have something to offer your thinking and your practice. Please let us hear back from you about connections you are finding, things you are now thinking about more deeply or in a different way, or steps you plan to take because of something you read here. We are eager to include any of your reflections in our Comments from Our Readers section of the journal.

We are also seeking reviewers for the manuscripts submitted to this journal. Becoming a reviewer gives you some input as to whether manuscripts have something important to say to our mathematics education leaders, and how their contributions might be even stronger with some editorial guidance, and perhaps some revision. It is important for us to be sure what we publish in the journal meets the needs of our readers and your voice in this is important.

Finally, we hope we will see many of you at the NCSM Annual Conference in Indianapolis on April 11th through April 13th where we will be exploring the theme "On Track for Student Success: Mathematics Leaders Making a Difference." There will be sessions addressing STEM education, sessions of interest to emerging leaders, and sessions designed to address the work of being a mathematics coach. You will also see that many who have published in this journal are also presenting sessions.

We hope you enjoy this issue of the journal! We also hope we see you in Indianapolis!

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Letters to the Editor

Please submit letters to Journal editor Linda Ruiz Davenport at ldavenport@boston.k12.ma.us

To the Editor:

A round of applause for the innovative model of collaboration detailed in *A District Mathematics Leadership Team: Deepening Collective Focus* in the last issue of the journal in the Spring of 2010.

As I read the article, it was clear that engaging a diverse group of stakeholders in creating and sustaining a five-year mathematics plan through a District Mathematics Leadership Team (DMLT) was a central and organizing principle of the work. This included district leaders, principals, teachers, school board members, university mathematicians and educators, parents, a business member, and a state representative. What a rich and interesting collection of perspectives that must have been!

Given the complex task of identifying and enacting systemic changes within the district along with the presence of so many diverse perspectives, it was very helpful to hear from the authors about all the steps that were taken even before launching the DLMT, and in particular, how a strong collaborative culture was created within the group. I appreciated being able to see some sample agendas for this work together and learn how doing mathematics together, observing classrooms, and examining data intentionally supported deep and productive conversations.

I also found myself becoming curious about what kinds of challenges emerged and how conflicting perspectives within the group were resolved, even if over time. Perhaps this is a story to tell in yet another article?

There is no one approach to foster involvement of all the different stakeholders in a community. But as different \- districts across the country begin to craft their future plans, particularly in transitioning to the Common Core State Standards, the article offered one model to consider and helped me think about whether there might also be another similar model that could work in our own district.

Connie Henry, Elementary Math Program Director
Boston Public Schools

To the Editor:

I was pleased to see an article discussing the transition from teacher to math coach in the recent edition of the NCSM journal.

As someone who has made that transition, I appreciated both the acknowledgement of the challenges involved as well as the recommendations for how administrators might support teacher leaders in making that move.

Being "isolated from other teacher leaders" and potentially having an "undefined role in the school" are real challenges for math coaches. The support I received through regular district-wide coach meetings and the structures that principals put into place for my work in their schools were invaluable for both my growth and effectiveness.

Districts that want to develop a successful program of math coaching should consider the recommendations of this article.

Many thanks to Fran Arbaugh, Kathryn Chval, John Lanin, Delinda Van Garderen, and Liza Cummings for their many helpful thoughts and acknowledgements in their article *Supporting the Transition from Experienced Teacher to Mathematics Coach*.

I look forward to seeing other articles in the journal that address the complexities of being an effective math coach in a school or district.

Peter Thorlichen, Math Coach
Boston

To the Editor:

In my position, I work closely with principals, both in their professional learning sessions that are organized by our district as well as when I make visits to their schools. The article *Principals' Views of Mathematics Teacher Learning* felt very connected to what I am seeing and thinking about in my own work.

It has certainly become clear to me as I work with principals that their vision of math teaching and learning can make all the difference when it comes to the kinds of supports that are important in order the help math teaching and learning become strong across an entire building. Of course that leaves me with questions about what kinds of professional development might be helpful to principal and other school administrators.

Without a strong belief in the importance of unpacking student ideas, probing deeply into students' misconceptions, and listening attentively to students even when they may struggle to express mathematical ideas that are challenging, principals might be persuaded to accept a day of "skill and drill" because the teacher makes such a strong case for that kind of student practice. How do we expand principals' ideas about the complexities of teaching and learning mathematics?

In our district, our math office continues to think about strategies for supporting principals. What I find myself wondering about is whether we can do this in bits and pieces through a small chunk of time on the agenda at a professional learning session, through a conversation in the hallway as we move in and out of classrooms together during a school visit, or in our phone conversations with a principal calls with a question. It would be lovely to find a more cohesive way to do this work, even though I recognize that everyone is very busy and way over committed, and principals find it very hard to be out of their buildings.

Perhaps it makes sense to think of cultivating principal leadership in mathematics among a small group of interested principals, at least as a place to start?

Thanks again for a great article!

Sherry Sajdak, Elementary Math Program Director
Boston Public Schools

To the Editor:

The article Supporting the Transition from Experienced Teacher to Mathematics Coach was particularly salient to me as a former math coach in my district about to make the transition to math teacher leader in my current school.

I greatly appreciated the article's point that a shift in perspective is necessary for a novice math coach, and that such a shift also requires a math coach to become familiar with not only a greater amount of content than they may have needed while teaching in the classroom, but also an understanding of how that content fits together across grade levels. This resonated with my experience as a district math coach several years ago, and now after being in my own classroom for some time, I am getting ready to make this shift again.

Developing the kind of understanding that math coaches or math teacher leaders need takes time. I think it is important for school administrators to build the kind of support structures that allow these understandings to develop while we taking on these challenging roles.

Thank you for bringing the challenges facing math coaches and others in similar roles so clearly into focus!

Kathy Simpson, Teacher Leader, Kilmer K-8 School
Boston, MA

Moving Beyond the Word Wall: How Middle School Mathematics Teachers Use Literacy Strategies[1]

Ellen S. Friedland, Susan E. McMillen, and Pixita del Prado Hill
Buffalo State College, Buffalo, New York

Middle school mathematics teachers were asked the following question during a post-lesson interview: "What role does literacy play in your math instruction?"

> *Justine[2]: "I guess it kind of guides my instruction; it is my instruction. Because that's how I'm instructing [students] — using those tools."*

> *Linda: "I think it needs to play a bigger role. I think I need to be more aware of the strategies, and I would definitely use more of them because it [seems] to help [students]."*

> *Kelly: "I think it should be integrated especially with the state testing having a lot of reading and word problems and [students] struggle with that so I think that it should be in my everyday lessons, but I feel I struggle as a teacher to integrate it."*

The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000) and the *Curriculum Focal Points* (NCTM, 2006) documents espouse a view of mathematics instruction as reasoning and sense making as a means to strengthening one's mathematics proficiency. This, importantly, includes problem solving, reasoning and proof, making connections, building and using representations, as well as communicating:

> Mathematical communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Explanations should include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. (NCTM, 2000, p. 60)

In addition, talking, reading, and writing about mathematics broadens one's view of the subject and its connections to other subjects and real life. For all of these reasons, it is increasingly the case that teachers are being encouraged to explore the role of literacy strategies in their mathematics instruction. However, the comments by these three teachers point to the challenges associated with doing so on an ongoing basis.

Proponents of mathematics reform believe that mathematics teachers should be aware of their responsibility to incorporate literacy components into their teaching to facilitate their students' mathematical understanding (Borgioli, 2008; Carter & Dean, 2006; Zollman, 2009). After all, "…language is the primary medium through which any discipline is negotiated, constructed, and

[1] We wish to thank the teachers who participated in this study toward the goal of improving mathematics instruction for all students.

[2] All names used are pseudonyms.

learned" (Borgioli, 2008, p. 189). However, despite NCTM's stance and the International Reading Association's (IRA) efforts in support of adolescent literacy instruction across the curriculum (Moore, Bean, Birdyshaw, & Rycik, 1999), convincing middle and high school mathematics teachers of the learning possibilities associated with literacy strategies has been challenging (Frykholm, 2004; Siebert & Draper, 2008). Mathematics teachers may feel it is someone else's job to teach literacy strategies. Mathematics teachers may be unfamiliar with the literacy strategies that might be useful. Finally, even mathematics teachers who are familiar with literacy strategies may feel that focusing explicitly on them during their mathematics instruction might take important time away from a focus on the mathematics content itself (Darvin, 2007; Draper, Smith, Hall, & Siebert, 2005).

Because Departments of Education across the United States acknowledge the importance of literacy in content area teaching, most middle and high school teacher certification programs require literacy courses (Come Romine, McKenna, & Robinson, 1996). In these courses, a generally accepted definition of content area literacy is "The ability to use reading and writing for the acquisition of new content in a given discipline" (McKenna and Robinson, 1990; p. 184). These courses are designed to provide teachers with strategies that can facilitate their students' comprehension of and communication about specific content. Even so, many preservice and practicing mathematics teachers do not always find these courses useful or relevant to their content (Darvin, 2007; Muth, 1993). Furthermore, many practicing teachers who are familiar with literacy strategies and believe they are effective may not necessarily employ them (Barry, 2002; Siebert & Draper, 2008; Silver, 1999; Spor & Schneider, 2001; Sturtevant, 1996; Wedman & Robinson, 1988).

If mathematics teachers are going to successfully engage all students in mathematical reasoning and sense making, and if all of the NCTM process standards are to be taken seriously, it is important for mathematics teachers to be better prepared to use a wide range literacy strategies in their mathematics instruction beyond a few add-on strategies such as the "word wall," which is often where mathematics teachers begin in their efforts to address the literacy needs of their content area. In this article, we discuss a study designed to examine and support the use of literacy strategies among a small group of middle school mathematics teachers.

As teacher educators in literacy and mathematics, we were interested in learning how we might provide preservice and inservice mathematics teachers with experiences that would lead them to attain a perspective consistent with reforms in mathematics education and beliefs about content area literacy instruction. In order to do this, we realized we needed to be more aware of which literacy strategies mathematics teachers find effective and the factors that impact their decisions to integrate them into mathematics instruction. Therefore, our study addressed the following questions:

1. Which literacy strategies do mathematics teachers use (and not use)?
2. How do mathematics teachers explain their use (and non-use) of literacy strategies?
3. What resources do mathematics teachers use for finding strategies and incorporating them into their mathematics instruction?
4. How do content area literacy courses affect mathematics teachers' attitudes toward and use of literacy strategies?
5. What factors within the school environment (school culture, program demands, assessment, district requirements, students) affect mathematics teachers' use of literacy strategies?

We were particularly interested in learning if and how mathematics teachers who had completed literacy course requirements as part of their certification programs were integrating literacy strategies into their mathematics instruction once they had begun teaching in their own classrooms.

Our Study of Literacy Strategies Used During Mathematics Instruction

Our study focused on six full-time middle school mathematics teachers (one male, five female) who had completed teacher education programs that included two content area literacy courses as required by law in New York State although only two of these teachers had participated in a field experience component associated with any of these literacy courses. The mathematics teaching experience of this group, at the onset of the study, ranged from 8 months to 3 years. Half were teaching in urban school districts and the other half were teaching in suburban districts. Three of the teachers taught from *Connected Mathematics Program* (CMP), a reform-oriented curriculum, while the other

Table 1

PARTICIPANT INFORMATION					
Participant	**Male/female**	**Urban/suburban**	**Teaching**	**Curriculum experience**	**Field experience as part of content area literacy course**
Allen	Male	Urban	1 year	Traditional	no
Jane	Female	Urban	1 year	Reform	yes
Justine	Female	Suburban	2 years	Reform	no
Kelly	Female	Suburban	3 years	Traditional	no
Linda	Female	Urban	1 year	Reform	no
Rebecca	Female	Suburban	2 years	Traditional	yes

three taught from more traditional curricula. Table 1 summarizes this information about our participating teachers.

We described the study to the participating teachers and told them we would like to observe two different mathematics lessons in which they used literacy strategies. The teachers then invited us to observe lessons in which they *believed* they were using literacy strategies as part of their mathematics instruction. There was no initial discussion as to what we meant by "literacy strategy" or any exploration of how they defined "literacy strategy."

However, in order to obtain information about teachers' prior knowledge of specific literacy strategies before we observed these lessons, we asked teachers to respond to a *Literacy Strategy Awareness Checklist*. This was a checklist we had created that included 37 literacy strategies commonly found in middle and high school content area literacy textbooks addressing vocabulary, comprehension, study skills, or writing.[3] We asked teachers to check one of the following for each strategy: "I have heard of this strategy," "I know how to apply this strategy in math instruction," "I would use this strategy," "I have used this strategy and would use it again," "I have used this strategy and would not use it again," "I would never use this strategy." We also provided additional space for teachers to list any additional strategies not included on the checklist or for us to name any additional strategies we observed in their mathematics lessons or were mentioned in the post-lesson interviews with teachers. The *Literacy Strategy Awareness Checklist* and teacher responses to it are summarized in Table 2.

Three of us observed each teacher during two different math lessons either through a visit to the classroom when the lesson was being taught or by viewing a videotape of the lesson. We each took detailed notes on any literacy strategies used, when it was used, how students responded, and any other details relevant to the use of literacy strategies during mathematics instruction. Each of us then wrote a summary of our observations.

Following each lesson, we interviewed and audiotaped each teacher to obtain their reflections on their use of literacy strategies during their lesson. The interview questions were designed to explore the teacher's working definitions of "literacy strategy," why the teacher selected specific literacy strategies, what the teacher thought about the effectiveness of the literacy strategies used during the lesson, where the literacy strategies were learned, and what resources were used to find literacy strategies more generally.

Our data analysis of both the lessons observed and the interviews involved an iterative process of comparative analysis (Glaser & Strauss, 1967) in which we each wrote, exchanged, and discussed theoretical memos that highlighted emerging themes including similarities, contrasts, divergent findings, and questions from each data set.

How Did These Middle School Mathematics Teachers Define "Literacy Strategy"?

Since these middle school mathematics teachers were asked to incorporate literacy strategies into the lessons we observed, we believed it was essential to determine how these teachers defined a literacy strategy. In response to

[3] This paper describes only the strategies used by the middle school mathematics teachers participating in the study. For further explanations and more examples of these and other strategies, see Alvermann, Phelps, & Ridgeway Gillis, 2010; Barton & Heidema, 2002; Buehl, 2008; and Readence, Bean, & Baldwin, 2008.

Table 2

FEATURES OF "PROVIDE RESOURCES" ROUTINE

Literacy Strategy	Focus of the Literacy Strategy: Vocabulary (V), Comprehension (C), Study (S), Writing(W)	Number of participants self-reporting that they have used this strategy and would use it again
Analogical Study Guide	VCS	0
Anticipation Guide	C	1
B-D-A (Before-During After Reading)	C	0
Concept Definition Mapping	V	2
Cloze	VC	1
Cornell Method (Split-Page Notetaking)	S	2
Cubing	W	0
DRTA (Directed Reading-Thinking Activity)	C	0
Elaborative Interrogation	C	1
Embedded Questions	C, S	1
Fact Pyramids	C	1
Semantic Feature Analysis	V	0
Frayer Model	V	4
Graphic Organizers	VCS	6
Guided Listening Procedure	C	0
Guided Writing Procedure	W	0
Inquiry Charts (I -Chart)	CSW	0
Interactive Reading Guides	C	1
Journal Writing	W	3
Knowledge Rating	V	0
KWL (Know- Want to Know- Learned)	C	2
Learning Logs	W	2
Math Reading Keys	VC	0
Mind Mapping	VC	0
Possible Sentences	V	0
QARs (Question-Answer-Relationships)	C	0
Quick Writes	W	1
RAFT (Role, Audience, Format, Topic)	W	2
Reciprocal Teaching	C	1
Semantic Map	V	0
SQRQCQ (Survey-Question-Read-Question-Compute-Question)	S	1
Three Level Study Guide	CS	0
Verbal and Visual Word Association	V	1
Vocabulary Overview Guide	V	1
Vocabulary Self Collection Strategy	V	1
Word Family Trees	V	1
Word Sorts	V	1

our questions about the role of literacy strategies in mathematics instruction during our interviews, they said the following:

Allen: " I think it means ways to help students understand what they are reading and being able to retain the information more efficiently."

Jane: "…tool or method I can use to help my students read and write better than they otherwise would. Something to improve reading and writing skills at the same time as using whatever content we are using that day."

Justine: "Any type of like organizational tool for the kids along with reading and writing."

Kelly: "I think it means anything with reading, writing, getting the kids to write their thoughts down, vocabulary, comprehension, anything."

Linda: "A way to get students to understand, retain, and then recall and use information."

Rebecca: "…Anything to help the kids to understand…."

These data suggest these mathematics teachers perceived literacy strategies as vehicles to help students comprehend, explain, and learn information. It is interesting to note that none mentioned using literacy strategies to uncover what students were thinking or monitor their progress in learning important mathematics content.

Which Literacy Strategies Did These Mathematics Teachers Use During Mathematics Instruction and How Did They Use Them?

As indicated earlier in Table 2, a total of 22 literacy strategies were in use by at least one of these mathematics teachers. All six mathematics teachers indicated that they use graphic organizers during mathematics instruction. The other strategies used by more than one teacher included:

- Frayer model, which is a graphic organizer focusing on a vocabulary word, and the template for which can be see in Figure 1;
- Journal writing;
- Concept definition mapping, which is a graphic organizer focusing on ways to describe a concept including what is it, what is it like, and some examples;
- Split-page methods of notetaking, using a divided page, with students writing key terms on the left side of the page and explanations of the terms on the right side of the page;
- K-W-L, which is a comprehension strategy asking students what they know, what they want to know, and what they learned about a topic;
- Learning logs; and
- RAFT (Role, Audience, Format, Topic), which is a strategy that includes writing about a topic from a perspective other than their own, and can be seen applied to an exploration of prime and composite numbers in Figure 2.

During the interviews, some of these middle school mathematics teachers mentioned using other strategies they had not listed on the Literacy Strategy Awareness Checklist such as highlighting important information and using "exit tickets" to check for understanding of key ideas.

FIGURE 1: Template of Frayer Model.

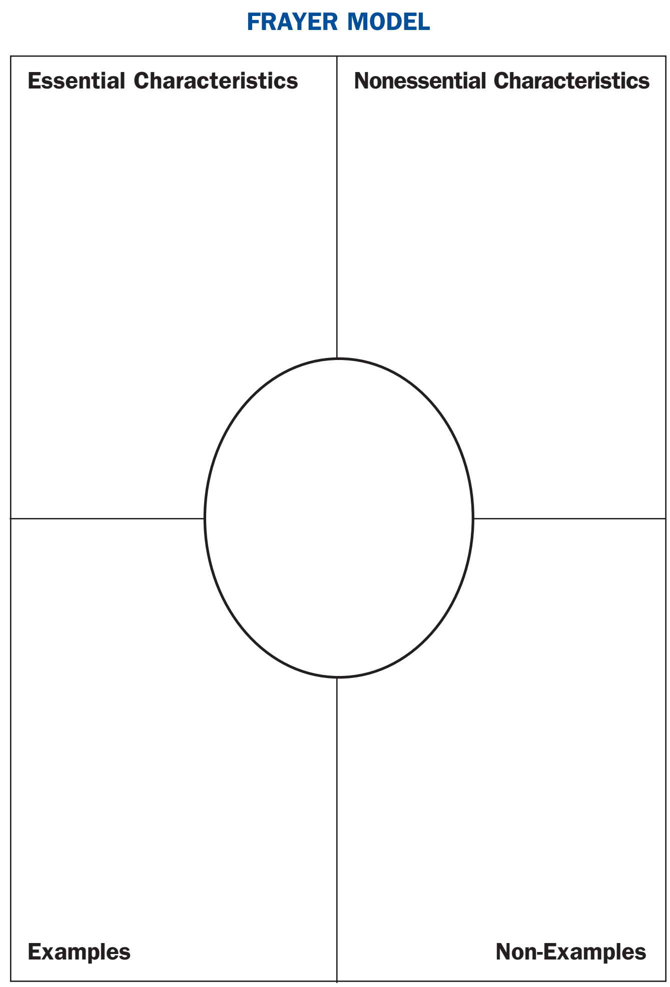


FIGURE 2: RAFT writing assignment on prime and composite numbers.

Name:

Primes/Composites

Directions: Choose ONLY 1 assignment. Please keep in mind the role that your are taking on and the audience that you are talking to. Discuss the topic given in the correct format. The final assignment should be typed. All assignments will be graded based on the given rubric.

Role	Audience	Format	Topic
Composite Number	Prime Number	Conversation	Discuss why you are better
Prime Number	Composite Number	E-mail or Text Messaging	Compare/Contrast yourself to the composite number
Prime Number	All other numbers	Diary Entry	Discuss why you are special
Manager	All numbers	Job Ad (Help Wanted)	How can you be as hired as a factor

For many of the literacy strategies, teachers frequently indicated they did not know how to apply the strategy in mathematics even if they had heard of it. At the same time, it was only through discussion during the interview that some teachers even became aware that their mathematics instruction included literacy strategies. For example, during the interview Allen described a graphic organizer vocabulary strategy he uses when introducing new concepts which includes definition, characteristics, examples, and nonexamples but he was unaware that it was named the Frayer model and had not indicated knowledge of the strategy on the *Literacy Strategy Awareness Checklist*.

In our observations of mathematics lessons, we found that each teacher used multiple literacy strategies during their mathematics instruction, all of which are identified in Table 3.

However, we found a striking range in how teachers used literacy strategies during their mathematics instruction, ranging from a high degree of literacy strategy integration to a limited degree of literacy strategy integration. The teachers who developed lessons with a high degree of literacy integration seemed to be more at ease with both the mathematics content of the lesson and with the literacy strategies themselves, and effectively used these literacy strategies to introduce new material, reinforce previous learning, and monitor comprehension. In contrast, the teachers who developed lessons at the limited integration end of the spectrum seemed to add a literacy strategy because they were expected to use one, appeared to be uncomfortable with the lesson, and used activities often did not seem to relate to each other. Furthermore, there were differences in the way the students responded to the lessons; the students were consistently engaged in the highly integrated lessons, while students who participated in lessons that were at the limited integration end of the spectrum were not on task as often. This finding was apparent in both urban and suburban settings using both reform and traditional curriculum materials and was not dependent on level of teaching experience.

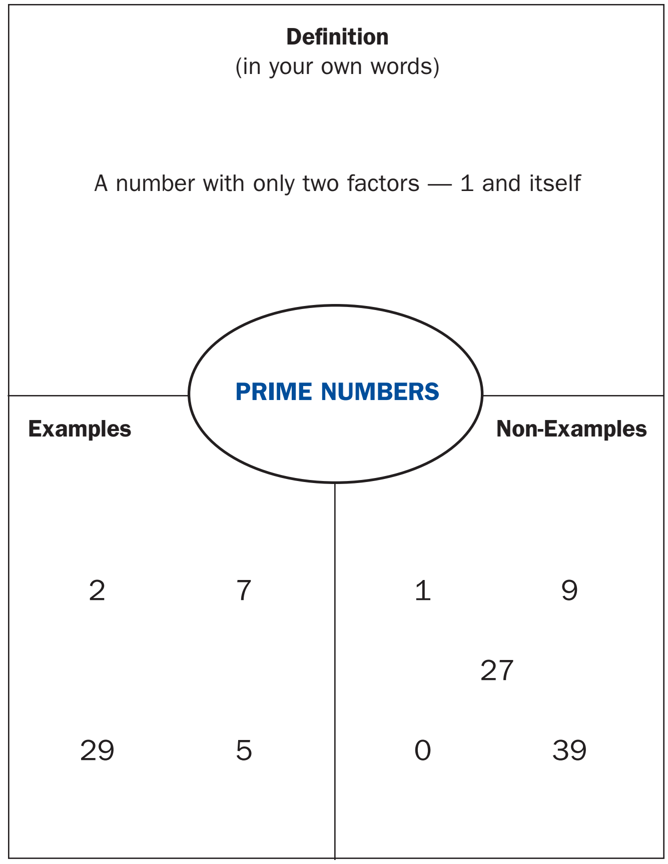
Vignettes of Literacy Strategy Integration Based on Observations of Lessons and Interviews

The following vignettes, based on observations of mathematics lessons and interviews, are intended to capture the range of literacy strategy integration across the classrooms we observed, starting with full integration and ending with limit integration.

Table 3

SUMMARY OF OBSERVATION AND INTERVIEW DATA				
Teachers	**Literacy Strategies Observed During Mathematics Lessons**	**Reasons Given for Using Literacy Strategies**	**Sources of Literacy Strategies**	**Curriculum Materials and District Context**
Allen	Guided discussion/ questioning; summarizing; alternatives to text; activation of prior knowledge (vocabulary); vocabulary study guide; word-meaning-example; summary writing; post-reading comprehension questions; think-pair-share	• Help students retain information • Provide students with something to refer to when they study • Make math easier for struggling students and, consequently, help them enjoy math • Introduce new terms at the beginning of a lesson • Promote student discussion and understanding of content	• Promote student engagement • ELA teacher • Undergraduate and graduate mathematics education courses	Traditional; Urban
Jane	Guided discussion/ questioning; interactive word wall; word-meaning-example; vocabulary categorization; modified Frayer model; journal writing	• Help students learn to communicate math in their own words • Help students solve problems • Required by the school and the district • Facilitate students' understanding of mathematical concepts	• Content area literacy coursework • Textbooks from literacy courses	Reform; Urban
Justine	Guided discussion/ questioning; notetaking from multiple sources; modified Frayer model; graphic organizers; reciprocal teaching	• Help students learn to communicate math in their own words • Facilitate students' understanding of mathematical concepts • Help students organize information	• College coursework • Internet • Other teachers, e.g., the special education teacher • Professional development programs • Professional journals	Reform; Suburban
Kelly	Guided discussion/ questioning; vocabulary notebook; song writing; definition-picture-example; vocabulary categorization; graphic organizers; RAFT; exit tickets	• Help students organize their thoughts • Monitor students' understanding • Promote student engagement • Help prepare students for the state assessments • District focus on improving vocabulary • Help students learn to communicate math in their own words	• Cooperating teacher during student teaching • ELA teacher • Internet • Professional journals • School Math Department	Traditional; Suburban
Linda	Read aloud (trade book); prereading introduction to vocabulary (multiple choice activity); post-reading comprehension questions; split-page notetaking; exit ticket	• Monitor students' understanding • Facilitate students' understanding of mathematical concepts • Help students "do well" and make concepts easier to learn • Part of the math program (journal writing)	• ELA teacher • Noted that she hadn't been looking for literacy strategies until participating in this study	Reform; Urban
Rebecca	Guided discussion/ questioning; exit tickets; analogies; split-page notetaking; graphic organizers; modified cloze activity	• Does not want to teach to the test • Help students go beyond rote memorization of information • Motivate students to learn the content	• Content area literacy coursework • Internet • Other teachers • Professional development programs • Professional journals • Textbooks from content area literacy courses	Traditional; Suburban

FIGURE 3: Example of a modified Frayer Model organizer.

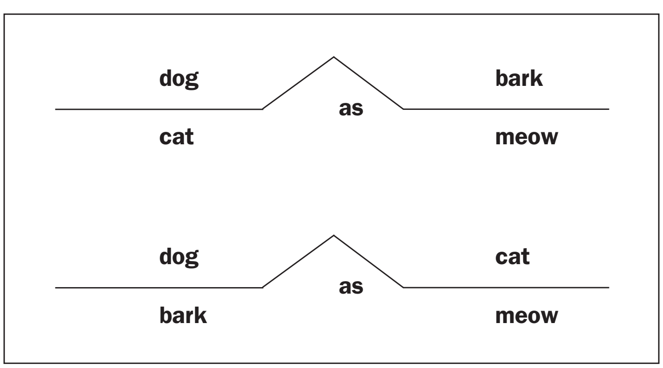


Justine fully integrated literacy strategies into her mathematics teaching. For example, she felt comfortable using the Frayer model and modified it to suit her purpose (see Figure 3). She set up the organizer to include creating a definition in one's own words and providing example and non-examples. She then had her class complete it for prime numbers, using jigsaw grouping (expert groups learn information and then return to their original group to teach the group members what they learned) and with students consulting textbooks, trade books, and other resources to explore the meaning of prime number. As the lesson unfolded, she frequently related any specialized vocabulary to their more general meaning in an effort to help her students understand new terms. She noted that she had learned the Frayer model in a college course but had modified it for use in introducing new concepts to her students.

Jane also seamlessly integrated literacy into her mathematics teaching. She consistently asked the students higher-level thinking questions to monitor their comprehension of mathematical concepts. Her "think-pair-share" activities also engaged the students in using the language of mathematics. Jane was also observed using an interactive word wall, where students were asked to actively categorize words on a large chart, as well as using journals that involved reflective writing using this vocabulary. Jane explained that she decided to "combine" these two strategies—the word wall and journals—that she had learned in her literacy courses.

Rebecca also effectively integrated a variety of literacy strategies into her lessons. For example, she used graphic organizers in several ways and incorporated a non-mathematics analogy to help students understand how to set up a proportion, displaying the analogy "dog : bark" as "cat : meow" (see Figure 4), asking them for another way the analogy could be written, and then relating the example to a mathematics problem. During the interview, Rebecca noted that she believes that using words without numbers helps students understand mathematical concepts. She ended class with an exit ticket, a proportion word problem to see how the students could use the organizer to represent the proportion. Rebecca explained that she had learned this analogy strategy at a conference, and though it had not been applied to a mathematics context, she believed it could be an effective strategy for facilitating students' comprehension of mathematics content.

FIGURE 4: Example of an analogy graphic organizer.



Allen's lesson fell more toward the middle of the literacy integration spectrum, using literacy strategies in his lesson, but not fully integrating them into his teaching. For example, at the beginning of a lesson focused on connecting mathematics with real life, Allen wrote on the board, "When do you use math in everyday life? Give an example of how you would." The students offered responses such as baking, banking, building, and construction, and Allen reinforced these responses by eliciting comments about how each response connected to mathematics, asking questions to ensure comprehension and encourage discussion. Allen then passed out a mathematics magazine, asked students to read the article and then discuss the article with another students, and then distributed a sheet with instructions asking students to summarize the article—but students were not guided through the writing process nor did Allen model or explain what he meant by a summary. He allowed each student to discuss the article with another student. At the end of this lesson, one student presented her summary while Allen asked questions to check her comprehension, but never returned to the question of how the real-life scenario in the article related to mathematics. During the interview, Allen stated that when he began teaching at the school he began receiving the mathematics magazines and "didn't know what to do with them." This was the first time he had used the magazine in class and he stated that he did so because he was asked to use a literacy strategy for this study.

Kelly acknowledged that she was uncertain about how to integrate literacy into her mathematics teaching. She informed the interviewer that she had sought help from the language arts teacher who told her about RAFT (role, audience, format, topic) but this language arts teacher had been unable to help her apply it to mathematics content she was teaching. Because we requested that she use a literacy strategy during her mathematics instruction, Kelly decided to use RAFT as a guided practice activity, but she did not make time during the lesson to introduce the RAFT strategy to her students or explain how it could be used to address the mathematics content they were learning, nor did she provide adequate time for her students to use the RAFT strategy effectively even if it had been appropriately introduced. Kelly did not have the prior knowledge needed to fully integrate the RAFT strategy into her instruction.

Linda's lesson also fell toward the limited literacy integration end of the spectrum. The purpose of the mathematics lesson was to facilitate her students' understanding of different kinds of angles since they had not done well on a test that required students to classify and name angles based on particular attributes. Linda had planned a lesson that involved the use of a trade book hoping that this text "…make real-world connections for them to help them understand." (Linda acknowledged that this was the first time she had used a trade book in her mathematics instruction.) Students began by completing a multiple choice vocabulary activity containing isolated non-mathematical vocabulary words from the story (e.g., mounted, cautiously, abrupt). The book was then projected using a document camera so that the entire class could view the book while it was read aloud. No reference was made to the words on the pre-reading vocabulary list when they appeared in the book, no picture clues or references designed to help students understand key vocabulary were referenced, and no questions were asked to monitor the students' comprehension while the book was read. It was only after finishing the reading and completing a multiple-choice comprehension check that Linda closed the lesson by making explicit connections between different kinds of angles and the concepts discussed in the book.

All of these middle school mathematics teachers were concerned about teaching mathematics effectively to all their students. They all noted that their main objective was to use techniques that facilitated their students' understanding of the mathematics content of their lessons. In particular, they indicated that the use of literacy strategies during mathematics instruction was intended to help their students understand, organize, and retain the mathematics content of their lessons. Even more specifically and with regard to the observed mathematics lessons, teachers indicated they used literacy strategies to teach vocabulary (Jane, Kelly and Rebecca), to engage students (Allen), to reteach something that students struggled with (Linda), to organize a large quantity of information (Allen, Kelly and Rebecca) and to improve on a lesson as it is presented in the math book (Jane). However, our observations and interviews showed important differences in how efforts to integrate literacy strategies into their mathematics instruction actually played out in practice.

What Resources Did These Middle School Mathematics Teachers Use To Find Literacy Strategies?

Some of the middle school mathematics teachers in the study sought advice on literacy strategies from colleagues, including special educators and ELA teachers. However, according to these mathematics teachers, the ELA teachers knew some literacy strategies but not how to apply them in mathematics. Interestingly, we found that none of the mathematics teachers approached their school's literacy support specialist for support with the use of literacy strategies during mathematics instruction. The teachers were either unaware of who that person was or stated that there was no literacy specialist in the building, yet further inquiry on the part of the authors found that there was a literacy support specialist in each participant's school. This finding could be due partially to a lack of clarity in the school regarding the role and responsibilities of the literacy support specialist. In the schools studied, the literacy specialist held different titles such as "Reading Specialist Coach," "Reading Teacher," "Reading Coach," and "AIS (Academic Intervention Skills) ELA Teacher." In addition, the literacy support specialist in several schools worked only with younger students, and therefore did not seem to be a resource for the middle school teachers in the building. As noted in Table 3, these middle school mathematics teachers also referred to other sources for information: professional journals, the Internet, professional development programs, textbooks, and coursework. Jane and Rebecca —the teachers who participated in a field experience associated with their content area literacy courses—were the only teachers who mentioned using materials from their content area literacy classes.

What Did We Learn About the Use of Literacy Strategies During Middle School Mathematics Instruction?

The middle school mathematics teachers in our study had all completed teacher education programs in which they were exposed to literacy strategies and how to apply them in mathematics. Graphic organizers, some form of the Frayer model, and journal writing were frequently used strategies. However, responses on the *Literacy Strategy Awareness Checklist* showed a lack of knowledge of many of the strategies that are commonly presented in content area literacy textbooks. Even if they were aware of the strategies, they did not necessarily use them in their mathematics teaching. Furthermore, some teachers were unaware when they did use literacy strategies in their mathematics teaching.

While all these middle school mathematics teachers believed that using literacy strategies engaged their students and helped them learn mathematics content, our study suggests that content area literacy courses may not provide enough background and support to promote the consistent integration of the strategies they learned. An associated literacy field experience during the teacher certification program suggests promise for greater literacy integration into mathematics instruction, as evidenced in Rebecca's and Jane's mathematics lessons, but the support of ongoing professional development and opportunities for partnerships with literacy specialists are also needed if mathematics teachers are to find effective ways to use literacy strategies to strengthen the mathematics learning of their students.

Implications: Crossing the Mathematics/Literacy Divide

For the teachers in our study, a wide gap appears to exist between their preservice preparation and their inservice practice. Two of the three teachers who were able to successfully integrate literacy strategies into their mathematics teaching had participated in field experiences as part of their content area literacy courses. During those field experiences they had developed mathematics lessons that integrated literacy strategies and then actually taught those lessons in middle mathematics classrooms. In fact, there is some evidence to suggest that teachers who initially were resistant to the idea of incorporating literacy strategies into mathematics instruction begin to reconsider their views, developing an appreciation of the role these strategies play in mathematical reasoning and sense making. Without this kind of literacy field experience, it appears to be more difficult for mathematics teachers to make a commitment to the use of literacy strategies during mathematics instruction and successfully plan for and enact the integration of these strategies into their mathematics teaching practice.

In addition, mathematics methods courses are often taught through mathematics education departments and may not incorporate literacy strategies into the coursework, either through modeling the use of literacy strategies during instruction or in the assignments given during the course. Yet these mathematics methods courses are strong predictors of the strategies mathematics teachers use in their mathematics teaching practice (Gagnon & Maccini, 2007) and have been shown to be effective in changing preservice teachers' beliefs about what it means to teach mathematics (Wilkins & Brand, 2004). Our study points to the need for further research to explore the potential benefit of increased collaboration between literacy instructors and mathematics instructors to provide preservice teachers with additional opportunities to both use literacy strategies in their mathematics instruction and to deepen their understanding of the value of doing so.

We also know that building a strong collaboration between literacy instructors and mathematics instructors can be a complex undertaking. It may involve examining and discussing the similarities and differences in instructional goals and practices that are represented by mathematics and literacy educators. It may require the creation of a common ground where literacy and mathematics educators can simultaneously consider literacy and mathematics issues that arise in mathematics classrooms. For an interesting discussion of the complexities of this kind of collaboration between two university educators—one a literacy educator and one a mathematics educator—and the potential for a shared perspective, see *Different Goals, Similar Practices: Making Sense of Mathematics and Literacy Instruction in a Standards-Based Mathematics Classroom* (Draper and Siebert, 2004). The creation of these kinds of shared perspectives are essential to the creation of collaborations that contribute to the integration of literacy strategies that strengthen both the practice of teaching mathematics and student learning of mathematics.

At the inservice level, we know that ongoing professional development is an important vehicle for strengthening mathematics teaching practice (Cady, Meier, & Lubinski, 2006; Heck, Banilower, Weiss, & Rosenberg, 2008). Our literature search provided a plethora of descriptive work that provides mathematics teachers with tools (ideas, examples, applications) for integrating literacy strategies into mathematics instruction but few research-based articles examine what the use of these strategies looks like in practice or how they impact mathematics learning. These are important questions to address as we consider that nature of the professional development that might be designed.

Our study points to the need for research to better understand what forms of support will help teachers learn and integrate literacy strategies described in the literature in their own classrooms and the impact of these strategies on student learning. If teachers are exposed to strategies that other mathematics teachers have used successfully, they may be more likely to try them. In addition, working together in professional learning communities with colleagues to explore what works and how it works can be an important source of support. Teachers need opportunities to see that "…mathematics learning and literacy are inseparably intertwined…and that every mathematics learning event is also a literacy event, and every literacy event in a mathematics classroom is a mathematic learning event" (Draper & Siebert, 2004, p. 953). Finally, we know that the support of administrators in their roles as instructional leaders in mathematics, also plays a key role in supporting teachers as they attempt to take on these new instructional practices (Burch & Spillane, 2003). Even while there is much that needs to be explored further, the results of our study suggest the following for mathematics leaders:

- Explore resources that identify literacy strategies that might be used to strengthen mathematics instruction and begin to establish shared visions of what this might look like in practice.
- Create opportunities for mathematics teachers and literacy specialists to work together with the mathematics leader serving as "translator" between mathematics and literacy concepts.
- Build on the work of classrooms teachers who are already integrating literacy strategies to strengthen the mathematics learning of their students by holding best practice professional development sessions where teachers can share ideas they have used effectively.
- Examine what it might mean to use literacy strategies to assess student understanding and monitor student progress in mathematics.
- Review school/district textbooks/curriculum packages for literacy strategies included and highlight these for teachers during professional development sessions.
- Examine the involvement of the school and district in innovative standards-based efforts such as literacy integration across the curriculum and how these might be used to strengthen mathematics instruction in a systemic way.
- Include interview questions about literacy integration to signal to new teachers that literacy integration is valued and expected.

But in order to move forward with these recommendations, we need a clearer understanding of the ways in which literacy and mathematics specialists might collaborate to develop effective professional development programs that support teacher learning and practice (Shanahan & Shanahan, 2008). It is important for literacy specialists to develop an understanding of mathematics as a discipline so that they can develop a shared perspective with mathematics educators on the teaching of mathematics content, just as it is important for mathematics specialists to learn how literacy strategies can serve to deepen the focus on mathematical reasoning and sense making and help students learn important mathematics content (Draper & Siebert, 2004). It is also important to achieve greater clarity about the role and responsibilities of literacy support specialists and mathematics support specialists so that collaboration with each other can lead to the kind of professional development program that could help effect change at the school level. As a result of such partnerships, we hope that more mathematics teachers will move beyond using limited literacy strategies such as the word wall in order to take on the kinds of literacy strategies that result in a richer and deeper mathematics teaching practice and contribute to the mathematics learning of all students.

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An Activity-Based Approach to Technology Integration in the Mathematics Classroom

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Tim was so learned, that he could name a horse in nine languages. So ignorant, that he bought a cow to ride on.

Benjamin Franklin, 1914, p.54

Becoming a mathematics teacher today can be a challenging endeavor, requiring teachers to learn difficult content and specialized pedagogies as well as becoming fluent with new technological tools and techniques. Pre-service teachers at the secondary level are faced with programs of study that often begin with Calculus and include abstract topics such as non-Euclidean geometry, discrete mathematics, and modern algebra (NCTM, 2000; U.S. Dept. of Education, 2005). With increased federal mandates for mathematics instruction, today's pre-service elementary teachers are also faced with significant mathematics content involving topics such as number sense, geometry, and probability (NCTM, 2006). At the same time, both pre-service and practicing teachers of mathematics at all levels are encouraged to consider relatively sophisticated strategies for instruction such as problem-based learning, student-centered teaching, and scaffolding (Davis, Maher, Noddings, 1990; Fuson, Kalchman, & Bransford, 2005).

Technologies such as graphing calculators, symbolic processing programs, mathematical simulations and cross-discipline instructional tools such as robotics kits are becoming ever more available to teachers as an aid to instruction (Heid, 2005). For many teachers, these are new tools to consider in their instruction, but it is important for these tools to be used thoughtfully and strategically to strengthen instruction (NCTM, 2003, 2006). As Mr. Franklin admonished (albeit indirectly), how do we help teachers make good technology choices as they plan and enact instruction? How do we ensure they do not “buy a cow to ride on,” but rather, consider mathematics content and pedagogy along with their choice of technology to make decisions that are instructionally sound and promote student learning?

One answer may lie in the ways we help teachers to consider how mathematics content, pedagogy and technology might be combined to plan effective instruction in today's quickly evolving mathematics classroom, keeping in mind that the field of mathematics is changing rapidly as mathematics educational technologies evolve, and these technological changes often have implications for mathematics content and pedagogy (Heid, 2005; Peterson, 1988; Sinclair & Crespo, 2006). For example, computational technologies have helped mathematicians explore the use of fractal geometry to model and examine real-life phenomena such as lightening strikes, plant growth, cloud formation, coastline erosion, and blood circulation yet the integration of fractals into mathematics textbooks and coursework is comparatively new and often requires the use of technology and appropriate pedagogy (Falconer, 2003). Technology use is similarly changing the mathematics of statistics, graphing, plane geometry, matrices, and probability, to name just a few and teachers of mathematics are encouraged to use a wide range of educational technologies to help their students to learn about such topics (Heid, 2005; Rosen, 1999). Given the proliferation of new mathematics content, new instructional strategies, and new mathematics-based educational technologies, how can we help these teachers make optimal choices when so much is changing so quickly?

Technological Pedagogical Content Knowledge

Technological Pedagogical Content Knowledge or TPACK (Koehler, & Mishra, 2008) is a framework that describes the interconnected and interdependent content, pedagogy, and technology knowledge that teachers must have to make good instructional choices when planning and enacting a mathematics lesson. TPACK is built upon Shulman's (1986, 1987) notions of pedagogical content knowledge, which is the knowledge necessary to teach particular curriculum content, including both disciplinary and general pedagogical knowledge. TPACK designates knowledge of educational technologies—especially how to use these rapidly proliferating tools and resources instructionally in varying educational contexts—as pedagogical content knowledge that requires deliberate examination and development. Teachers who have well-developed TPACK demonstrate this knowledge by incorporating a strategic mix of mathematical content, appropriate pedagogies, and well-chosen technologies within their lessons (Grandgenett, 2008).

In some ways, mathematics educators have a bit of a head start on TPACK development, since the profession's integration of instructional technology to date, when compared with other disciplines, has been relatively strong. For example, the use of graphing calculators in high school mathematics classes has been suggested by several authors to be one of the most successful integration of technologies into teaching and learning (Fuson, Kalchman, & Bransford, 2005; Kaser, Bourexis, Loucks-Horsley, & Raisen, 1999; Reece, Dick, Dildine, Smith, Storaasli, Travers, Wotal, & Zygas, 2005). Technology-based applications like *Geometer's Sketchpad* and *Excel*, and Web-based resources like the *National Library of Virtual Manipulatives* are relatively common and well embraced in today's mathematics classroom (Heid, 2005). However, current integrations of digital technologies, such as graphing calculators or Excel, only scratch the surface of the educational opportunities that these tools and resources make possible in mathematics instruction. Innovative software programs such as *Inspire Data* or the newly enhanced *Mathematica*, or new technologies such as robotics and global positioning systems (GPS) are providing exciting opportunities for the learning of mathematics.

On one hand, when considering the possibilities for effective technology use in the mathematics classroom, there appear to endless possibilities. On the other hand, if one considers the learning activities that a teacher of mathematics might *typically* plan, a more limited list of possibilities would probably be generated. We suggest that providing teachers with a comprehensive list of possible mathematics learning activities along with some specific educational technologies that might be considered useful tools to support that activity might help better integrate educational technologies into mathematics instruction. We believe that such a resource could contribute significantly to the TPACK development of teachers and strengthen their mathematics teaching practice overall.

Supporting the Integration of Technology into Mathematics Instruction

We are attempting to generate and categorize a comprehensive taxonomy of mathematics learning activities and useful technologies to support each activity so a teacher planning a lesson for a particular mathematical topic or concept can review the taxonomy, select several learning activities to combine in a lesson, unit, or project plan, and consider any technology tools that might be useful to incorporate into their instruction. The taxonomy was generated through a careful review of the technology-based activities published during the past five years in the three teaching-related journals published by the National Council of Teachers of Mathematics: *The Mathematics Teacher, Mathematics Teaching in the Middle School*, and *Teaching Children Mathematics*. In all, more than 180 journal issues were examined. We have identified 31 distinct mathematics learning activity types and acknowledge that some may need to be edited or combined, some may need to be added, and some may need to be removed. Our taxonomy is presented as a beginning point for others to consider and the list will no doubt grow and evolve along with advancements in the discipline. We have set up a wiki at http://activitytypes.wmwikis.net/ to help to facilitate this ongoing process of identifying mathematics learning activity types and we invite the readers of this article to contribute to this effort.

By creating and sharing this taxonomy of mathematics learning activity types, and by including any associated technologies, we hope we can support the development of "TPACK in action" for teachers of mathematics so they are better able to select instructional strategies and technology tools to help students meet particular curriculum content standards. The mathematics learning activity types are intended to represent possibilities for instruction, conceptualized primarily in terms of student actions, and focused on what students might actually be *doing* during a mathematics lesson. For example, a teacher planning to address the concept of algebraic slope might use the taxonomy to consider using a "interpreting a phenomenon mathematically" mathematics learning activity that has students driving an electronic car up different sloped ramps and then using an interactive graphing program to represent the changing equation of the slope of the ramp.

Mathematics Learning Activity Types

The purpose of presenting a learning activity types taxonomy for mathematics is to introduce a full range of possible learning activities for teachers to consider when building lessons that effectively integrate technology, pedagogy, and content. In doing so, we attempt to scaffold teachers' thinking about how to best structure learning activities, how to best support those activities with educational technologies, and how to creatively engage their students in learning mathematics. The mathematics learning activity types are designed to be catalysts for thoughtful and creative instruction by teachers.

We have conceptualized seven genres of activity types for mathematics that are derived from NCTM's process standards. These activity types are expressed using active words to represent the pursuit of a dynamic and student-centered learning environment: *Consider, Practice, Interpret, Produce, Apply, Evaluate*, and *Create*. Many of the student actions embedded within the activity types are drawn directly from the NCTM standards themselves. Each of the seven genres is presented in a separate table below that names the activity types included in that genre, defines them briefly, then provides some example technologies that could strategically be used to support students' learning within each activity.

THE "CONSIDER" ACTIVITY TYPES

When learning mathematics, students are often asked to consider and make sense of new information. This request is a familiar one to both students and teachers. Yet, although such learning activities can be very important contributors to student understanding, the "Consider" activity types also often produce some of the lowest levels of student engagement, and are manifested typically using a comparatively direct presentation of foundational knowledge.

THE "PRACTICE" ACTIVITY TYPES

In learning mathematics, it is often important for students to be able to practice computational techniques or other algorithm-based strategies so that fluency with these skills can be developed for later and higher-level mathematical application. Some educational technologies can be used

Table 1

THE "CONSIDER" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Attend to a Demonstration	The student gains information from a teacher or student presentation, videoclip, animation, interactive whiteboard or other display media.	Powerpoint, YouTube, document camera, interactive whiteboard, videoconferencing, or other display media
Read Text	The student extracts information from textbooks, or other written materials, in either print or digital form.	Electronic textbooks, websites (i.e. the Math Forum), informational .pdfs
Discuss	The student discusses a concept or process with a teacher, other students, or an external expert.	Ask-an-expert sites (e.g., Ask Dr. Math), online discussion groups, videoconferencing
Recognize a Pattern	The student examines a pattern presented and attempts to more fully understand the pattern.	Graphing calculators, virtual manipulative sites (e.g., the National Library of Virtual Manipulatives), spreadsheets
Investigate a Concept	The student explores or investigates a concept (such as fractals), perhaps by use of the Internet or other research-related resources.	Web searching, informational databases (Wikipedia), virtual worlds (Second Life), simulations
Understand or Define a Problem	The student strives to understand the context of a stated problem or to define the mathematical characteristics of a problem.	Web searching, concept mapping software, ill-structured problem media (i.e., Jasper Woodbury)

Table 2

THE "PRACTICE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Do Computation	The student undertakes computation-based strategies using numeric or symbolic processing.	Scientific calculators, graphing calculators, spreadsheets, Mathematica
Drill and Practice	The student rehearses a mathematical strategy or technique and perhaps uses computer-aided repetition and feedback in the practice process.	Mathblaster drill and practice software, online textbook supplements, online homework help websites (WebMath).
Solve a Puzzle	The student carries out a mathematical strategy or technique within the context of solving an engaging puzzle that may be facilitated or posed by the technology.	Virtual manipulatives, Web-based puzzles (magic squares), brainteaser Web sites (CoolMath)

to assist these processes. The table above offers both the range of practice-based learning activities and example technologies that can assist their implementation.

THE "INTERPRET" ACTIVITY TYPES

In the discipline of mathematics, concepts and relationships can be quite abstract, and can sometimes seem to be a bit of a mystery to students. Students often need to spend time exploring these relationships in order to understand them more deeply. Educational technologies can be used to help students investigate concepts and relationships more actively and assist with interpretation of what they observe. Table 3 displays activity types that can support such interpretive processes and provides examples of available technologies that can be used to support their formation.

Table 3

THE "INTERPRET" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Pose a Conjecture	The student poses a conjecture, perhaps using dynamic software to display relationships.	Dynamic geometry software (Geometer's Sketchpad), widgets (Explore Learning), e-mail
Develop an Argument	The student develops a mathematical argument related to why they think that something is true. Technology may help to form and to display that argument (e.g., a proof).	Concept mapping software (Inspiration), presentation software, blogs, specialized word processing software (Theorist), e-mail
Categorize	The student attempts to examine a concept or relationship in order to place it into a set of known categories.	Database software (Microsoft Access), online databases, concept mapping software, drawing software
Interpret a Representation	The student explains the relationships apparent from a mathematical representation (e.g., table, formula, chart, diagram, graph, picture, model, animation).	Data visualization software (Inspire Data), 2D and 3D animations, video (iMovie), Global Positioning Devices (GPS), engineering visualization software (MathCad)
Estimate	The student attempts to approximate some mathematical value by further examining relationships using supportive technologies.	Scientific calculator, graphing calculator, spreadsheets, student response systems (Clickers)
Interpret a Phenomenon Mathematically	The student, assisted by technology as needed, examines a mathematics-related phenomenon (e.g., velocity, acceleration, the Golden Ratio, gravity).	Digital cameras, video, computer-aided laboratory equipment, interactive graphing program, specialized word processing, robotics, electronics kits

THE "PRODUCE" ACTIVITY TYPES

When students are actively engaged in the study of mathematics, they can become motivated producers of mathematical documents rather than just passive consumers of prepared materials. Educational technologies can serve as excellent "partners" in this production process, aiding in the refinement and formalization of student products as well as helping students share the fruits of their mathematical labors. The activity types listed include suggestions for technology that can assist these efforts.

Table 4

THE "PRODUCE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Do a Demonstration	The student demonstrates a topic or concept to show their understanding of a mathematical idea or process. Technology may assist in the development or presentation of the product.	Interactive whiteboard, video (YouTube), document camera, presentation software, podcasts
Generate Text	The student produces a report, annotation, explanation, journal entry or document, to illustrate their understanding.	Specialized word processing (Math Type), collaborative documents (Google docs), blogs, online discussion groups
Describe an Object or Concept Mathematically	Technology may assist in the description or documentation process, as the student produces a mathematical explanation of an object or concept.	Engineering visualization software, concept mapping software, specialized word processing, Mathematica
Produce a Representation	The student develops a mathematical representation (table, formula, chart, diagram, graph, picture, model, animation, etc.) using technology for production assistance, if necessary.	Spreadsheet, virtual manipulatives (digital geoboard), spreadsheets, Inspire Data, concept mapping software, graphing calculator
Develop a Problem	The student poses a mathematical problem that is illustrative of some mathematical concept, relationship, or investigative question.	Word processing, online discussion groups, Wikipedia, Web searching, e-mail

THE "APPLY" ACTIVITY TYPES

The utility of mathematics in the physical world can be found in its authentic applications. Educational technologies can be used to help students apply mathematics in the world and link mathematical concepts to real-world phenomena. The technologies essentially become students' assistants in their mathematical work, helping them connect mathematical concepts to the realities in which they live.

Table 5

THE "APPLY" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Choose a Strategy	The student reviews or selects a mathematics related strategy for a particular context or application.	Online help sites (WebMath, Math Forum), Inspire Data, dynamic geometry/algebra software (Geometry Expressions), Mathematica, MathCAD
Take a Test	The student applies their mathematical knowledge within the context of a testing environment, such as with computer-assisted testing software.	Test-taking software, Blackboard, survey software, student response systems
Apply a Representation	The student applies a mathematical representation to a real life situation (table, formula, chart, diagram, graph, picture, model, animation, etc.).	Spreadsheet, robotics, graphing calculator, computer-aided laboratories, virtual manipulatives (algebra tiles)

THE "EVALUATE" ACTIVITY TYPES

When students evaluate the mathematical work of others or reflect on their own work, they have an opportunity to develop more sophisticated understandings of mathematical concepts and processes. Educational technologies can become valuable allies in this effort by assisting students in the evaluation process, helping them compare concepts, test solutions or conjectures, and integrate feedback from other individuals into revisions of their own work. The following table lists the range of evaluation-related mathematics learning activities.

Table 6

THE "EVALUATE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Compare and Contrast	The student compares and contrasts different mathematical strategies or concepts to determine which might be more appropriate for a particular situation and why.	Inspiration, Web searches, Mathematica, MathCad
Test a Solution	The student systematically tests a solution and examines whether it makes sense based upon systematic feedback, and which might be assisted by technology.	Scientific calculator, graphing calculator, spreadsheet, Mathematica, Geometry Expressions
Test a Conjecture	The student poses a specific conjecture and then examines the feedback of any interactive results to potentially refine the conjecture.	Geometer Sketchpad, statistical packages (e.g., SPSS, Fathom), online calculators, robotics
Evaluate Mathematical Work	The student evaluates a body of mathematical work through the use of peer- or technology-aided feedback.	Online discussion groups, blogs, Mathematica, MathCad, Inspire Data

THE "CREATE" ACTIVITY TYPES

When students are involved in some of the highest levels of mathematics learning, they are often engaged in very creative and imaginative thinking processes. Albert Einstein once implied that imagination was as important as knowledge in mathematics (Priwer & Phillips, 2003). It is said that this idea was consistent with his strong belief that mathematics is a very inventive, inspired, and imaginative

Table 7

THE "CREATE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Teach a Lesson	The student develops and delivers a lesson on a particular mathematics concept, strategy, or problem.	Presentation software, interactive video, video, podcasts
Create a Plan	The student develops a systematic plan to address some mathematical problem or task.	Concept mapping software, collaborative writing software, MathCad, Mathematica
Create a Product	The student imaginatively engages in the development of a student project, invention, or artifact such as a new fractal, tessellation, or other creative product.	Word processor, animation tools, MathCad, Mathematica, Geometer Sketchpad
Create a Process	The student creates a mathematical process that others might use, test, or replicate, essentially engaging in mathematical creativity.	Computer programming, robotics, Mathematica, MathCad, Inspire Data, iMovie

endeavor. Educational technologies can be used to help students to be creative in their mathematical work. The activity types following represent these creative elements and processes in students' mathematical learning and interaction.

Combinations of Activity Types

A creative lesson or learning plan by a teacher usually combines two or more activity types into a varied and engaging learning experience. In fact, when learning activities are combined and integrated, they may better resemble the complexity of real-life applications of mathematics, creating opportunities for students to encounter and solve rich mathematical problems that are often more realistic than the often more artificial problems often found in textbooks (Checkly, 2006; Fuson, Kalchman, & Bransford, 2005). Combining activity types may also provide opportunities for students to develop more divergent ways of thinking (Aris, 1994; Gershenfeld, 1998). Below are a several examples of how activity types might be combined, including a simple combination and two more complex combinations of mathematics activity types.

Example 1: Recognizing and Researching the Fibonacci Series

A common mathematics topic for teachers to assign for student research in middle school is the remarkable Fibonacci Series. This series, where each term is created by summing the two terms that appear before it (e.g., 1, 2, 3, 5, 8, 13, 21, 34) is found quite commonly in such items as the spiraled skin of pineapples, the stems of conifer trees, the curved edges in sea shells, and even the family trees of honeybees (Cook, 1979). A simple combination of activity types that might be used to build student understanding of the Fibonacci Series involves first asking students to "recognize [the] pattern" (from the "Consider" activity types) by asking them to display it on a chalkboard or spreadsheet in order to ensure that students are constructing the sequence correctly and then asking students to "investigate [the] concept" (also from the "Consider" activity types) by doing a Web search on the Fibonacci series to explore where it might be represented in the physical world. Students are often amazed at the many diverse examples of this series that can be found. These activities can become a context for exploring patterns within the Fibonacci sequence and how they can be expressed.

Example 2: Defining, Representing, and Solving a Paper Folding Problem

An interesting problem that is sometimes posed to elementary students who are studying exponential numbers to explore what happens to the thickness of a piece of paper if it is folded in half a total of 10 times. The increasing thickness of the folded paper soon creates an impossible situation and students find that they need to move to computational strategies to solve the problem. At this point, a teacher might encourage students to use a spreadsheet to mathematically "represent" (from the "Produce" activity types) what is happening in the problem and look for patterns. Students might then be encouraged to "test a conjecture" (from the "Evaluate" activity types) about these patterns having to do with powers of 2 and the notion of exponential growth. If the teacher realizes this same problem has been showcased on the television program "MythBusters," where the hosts jokingly use a sheet of paper the size of a football field and modern construction equipment to see if the size of the paper makes any difference in how many folds they are able to make while exploring this problem, students could be asked to "attend to" (from the "Consider" activity types) the related Mythbusters clip and then discuss it in relation to their explorations.

Example 3: Interpreting, Producing, and Testing a Garbage Pickup Model

In high school discrete mathematics, mathematical modeling activities often include the question of how a garbage truck might efficiently move through a system of streets to pick up the garbage each week. A teacher might encourage students to "interpret a representation" (from the "Interpret" activity types) by examining maps of local streets, or perhaps viewing a satellite image of their area using Google Maps. The students could then be asked to "understand or define a problem" and decide upon the parameters for efficient garbage pickup, such as the need to conserve gas by not retracing a route once the truck has already traveled a street. The students could then be encouraged to "produce a representation" (from the "Produce" activity types) of the streets as a network of line segments for the streets and nodes for the street intersections. They could then be asked to "create a plan" (from the "Create" activity types) for an efficient garbage pickup route using this mathematical representation of their neighborhood. Often students prefer to use some sort of computer-assisted drawing program, such as the drawing utilities in Microsoft Word or the more sophisticated MathCad, to depict a system of nodes and connecting line segments and to formalize their planning. Soon students realize that "odd or even" nodes (named according to the number of line segments coming together at a street intersection) are important considerations for planning the most efficient route. Finally, students might be asked to "compare and contrast" their routes (from the "Evaluate" activity types) by creating some sort of numerical index for their route (perhaps with a spreadsheet chart) that might compute the number of miles traveled or the amount of gasoline used. As they do so, they are encouraged to "evaluate [their own and others'] mathematical work" (from the "Evaluate" activity types) to create maximally efficient routes. This particular mathematical challenge illustrates the use of mathematical modeling while also entailing a combination of mathematical learning activity types that encourage flexibility, creativity, and pedagogically appropriate technology use.

Final Thoughts

As leaders of mathematics education, we know that "doing mathematics" is a very creative, exciting and dynamic endeavor. It "involves observing, representing, and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (Steen, 1998, page 16). We hope the mathematics learning activity types presented in this article might help teachers better engage and motivate students in their classrooms, involving them more fully in the creativity of doing real-life mathematics, acquainting them with the growing number of technology tools available to explore that mathematics, and helping them appreciate the role of mathematics in understanding our natural world.

If we are to help teachers to develop their TPACK so they might be better prepared to integrate mathematics content, pedagogy, and technology successfully in their classrooms, we will no doubt need a range of instructionally sound strategies and examples. When we separate mathematics content, pedagogy, and technology instruction in our preservice teacher education programs or in our professional development efforts with practicing teachers, we risk giving teachers a very superficial understanding of the instructional power of their successful combination, resulting at times in less-than-optimal mathematics lessons. Instead, we need to carefully and consciously scaffold the development of teachers' TPACK, so they can make thoughtful and maximally effective instructional choices that combine mathematics content, pedagogy, and technology and more authentically engage students in "doing mathematics" together in classrooms.

Such integration can be done, and done well, if we give teachers the support and encouragement they need to be creative designers of classroom instruction. Ben Franklin would no doubt be in agreement with this approach, since he had an uncommon passion for applying knowledge to the world in which he lived, as well as generating new knowledge from the successes and failures of the experiments he conducted. The taxonomy of mathematics learning activity types shared here is an attempt to provide a vehicle to support teachers who are trying to generate similar passions among their students, building interest and motivation through diverse, engaging, and technology-rich learning activities while also deepening and extending their learning of important mathematics. If we are successful in such efforts—in the words of Mr. Franklin—the students in our schools will be less likely to "buy a cow to ride on" and will instead be well prepared to do and see mathematics in the world around them.

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Observing Mathematics Lessons: What Does It Mean For Principals To Be Up-to-Speed?

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The 2008 release of the PRIME Leadership Framework has refocused our attention on what leaders in mathematics education can do to support and improve teacher and student learning. A central idea in this framework is that mathematics education leaders need to make certain that teachers have the knowledge of mathematics and pedagogy to ensure a high quality mathematics education for all students.

Such knowledge on the part of teachers enables them to make sense of the content of their students' mathematical thinking in order to assess what they know. Research by Carpenter, Fennema, Peterson, Chiang and Loef (1989) and Cohen and Hill (2000) reports that students' achievement in mathematics is affected positively when teachers pay attention to students' mathematical ideas. The information teachers can gain when they listen carefully to their students is critically important because it enables teachers to adapt their instruction to the levels of understanding across the class; when teachers probe their students' thinking to determine where the soft areas are and where their thinking is robust, they position themselves to make informed decisions about which instructional steps would most effectively strengthen their students' grasp of the concepts. As mathematics education leaders, principals of elementary schools need to be able to recognize when teachers are paying close attention and responding appropriately to their students' mathematical thinking and to recommend what kinds of additional support teachers may need when they aren't.

One of the most important opportunities principals have to influence classroom instruction in mathematics is through the process of classroom observation and teacher supervision. When principals observe mathematics lessons, they make judgments about the effectiveness of the instruction and use this information to decide what to feature in their evaluations of teachers and in their post-observation conferences with them. They also may use what they learn to formulate improvement plans and priorities for teachers' professional development.

At Education Development Center[1] we have been researching the supervisory practices in mathematics of a national sample of principals with various degrees of *leadership content knowledge*. Stein and Nelson (2003) define *leadership content knowledge* (LCK) as a combination of mathematics knowledge, views of how mathematics should be learned, and views of what high-quality mathematics instruction should look like. A primary goal of our research has been to understand how principals' LCK affects their supervisory practice. We measured the LCK of approximately 500 elementary and middle school principals using a survey to collect information about their professional histories, their views about mathematics learning and teaching and their mathematics knowledge for teaching.[2]

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[1] The Education Development Center in Newton MA has a 6-year grant from the National Science Foundation for a national study of elementary school principals' leadership in the area of mathematics instruction. The work reported in this paper was supported by a grant from the RETA program of the National Science Foundation, grant # EHR 0335384. Any opinions, findings, conclusions, or recommendations expressed here are those of the authors and do not necessarily reflect the views of the Foundation.

[2] Refer to http://www2.edc.org/tmi/tmi_survey.html for more information about this survey.

We then selected a sub-sample of 13 principals with a range of LCK profiles to be case study subjects. We studied the supervisory practices in mathematics of these 13 principals, making three site visits to each of them.[3]

We found that principals' LCK greatly influences what they focus on when they observe mathematics classes and what they discuss with teachers in post-observation conferences. The principals in our study fall into nine groups according to their LCK; each group has its own distinct LCK profile that combines different amounts of mathematics knowledge with different views about the learning and teaching of mathematics. For example, one group of principals *(Profile A)* has strong mathematics knowledge for teaching, compared to other principals in our sample, and views of effective instruction that are aligned with reform teaching practices. These views consist of the teacher paying attention to the content of students' mathematical thinking and using this information to plan next instructional steps. When these principals observe in mathematics classrooms, they look for evidence that the teacher's actions are directed toward obtaining a detailed understanding of students' mathematical thinking. In post-observation conferences, these principals are in a position to judge the extent of the teacher's understanding of her students' thinking and the quality of her plans to further her students' mathematical development.

LCK	Mathematics knowledge for teaching	Views of effective instruction
Profile A	High	A focus on students' thinking

At the other end of the spectrum is another group of principals *(Profile C)* whose LCK reflects a modest amount of mathematics knowledge for teaching, compared to other principals in our sample, and traditional views of effective instruction where the teacher presents information and closely guides students' thinking. When these principals observe, they look for whether the teacher clearly demonstrates how students should solve the assigned problems and whether she quickly corrects any mistakes students may make. In post-observation conferences, these principals are in a position to comment on the teacher's clarity and the extent to which students successfully executed the mathematical procedures. However, these principals are not well positioned to work with the teacher on what might be impeding the progress of students who are having difficulty and what teaching steps would further the understandings of these struggling students as well as the students whose understanding of the lesson's mathematical concepts is already strong.

LCK	Mathematics knowledge for teaching	Views of effective instruction
Profile A	High	A focus on students' thinking
Profile C	Low	A focus on executing correct procedures

A third group of principals *(Profile B)* are those whose LCK can be characterized by mathematics knowledge for teaching that is in the middle range of principals in our sample and views of teaching that are associated with commonly accepted forms of instruction such as having students develop and share their own problem-solving strategies, dialogue with each other, and explain the thinking underlying their problem-solving approaches. Because these principals pay attention to these forms of instruction when they observe in classrooms and in post-observation conferences, they are often considered "up-to-speed" in

LCK	Mathematics knowledge for teaching	Views of effective instruction
Profile A	High	A focus on students' thinking
Profile B	Medium	A focus on students' doing
Profile C	Low	A focus on executing correct procedures

[3] During these site visits, we observed and audio-taped their pre- and post-observation conferences with teachers. We also observed and took ethnographic field notes of the mathematics lessons themselves. We transcribed all audio-tapes and analyzed them as well as the ethnographic field notes for what they revealed about how these principals used their LCK in their supervisory practices.

their understanding of mathematics instruction. However, principals with this kind of LCK do not closely examine the mathematical thinking these practices support students to do; they are more focused on what students are doing than on the content of their thinking. This limited focus places significant constraints on these principals' ability to judge whether this important area of a teacher's practice—the capacity to understand and work with the content of students' thinking—requires further development.

In this paper, through the use of dialogue excerpted from post-observation conferences, we show how much more a *Profile A* principal can achieve in a post-observation conference with a teacher than is possible for a *Profile B* principal.

Focusing on What Students Do Rather than the Content of their Thinking

We begin with Ms. Fordham[4] whose LCK puts her into the *Profile B* group. At the time of our study, she was a third year principal of a K – 5 school located in a middle class suburb of a mid-western city. The lesson we observed in her school took place in the kindergarten classroom of Ms. Mantle. It was about learning to add and subtract small numbers through solving story problems.[5] Students worked in groups, drawing pictures to help them solve the problems. As they worked, the teacher moved around the room and talked to students about the different approaches she observed.

During their post-observation conference, most of Ms. Fordham's comments focused on Ms. Mantle's classroom management and general pedagogical practices. She praised Ms. Mantle for how well her students knew the routines of the classroom, how engaged they were in the activities, how little time was wasted during class, and for the rigorous pace she had set.

In addition, at several points during their post-observation conference, Ms. Fordham and Ms. Mantle turned their attention to particular students' problem-solving approaches. The exchange below about the solution strategy of one of Ms. Mantle's students, Orrin, is illustrative of how Ms. Fordham used her LCK in her practice. It demonstrates Ms. Fordham's capacity to move beyond the limited focus on classroom management and teachers' actions to a consideration of how the student interacted with the mathematics. However, this exchange also suggests that Ms. Fordham did not appreciate the importance of Ms. Mantle developing an understanding of this student's mathematical thinking in order to use what she learned to plan her next teaching steps.

The story problem Orrin was working on was: *A toad ate 22 dragonflies. A snake ate 12 more dragonflies than the toad. How many dragonflies did the snake eat?*[6] For his visual representation, Orrin made 22 marks on his paper, followed by 12 marks and then struggled with how to use the representation he had drawn to solve the problem. Ms. Fordham and Ms. Mantle discussed the pros and cons of giving smaller numbers to Orrin, but they did not consider what Orrin was thinking and what might be getting into his way of solving the problem.

> ***Ms. Mantle:*** *…And then even my little Orrin who is very brilliant, is drawing out 22 and drawing out 12. He didn't know even how to solve that. But when I gave him the problem, the same exact problem but with three and one, he knew right away, four…*
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> ***Ms. Fordham:*** *Watching how quickly Orrin did the three plus one or four plus one or whatever that was, I thought, I kind of had a little wondering thinking what would have happened if he had done the easy one first and then the 22 and 12… He's such a smart thinker when it comes to that stuff. But I don't know how that happens.*
>
> ***Ms. Mantle:*** *It might have. But part of the reason I did it the way I did it is, in that group, if I give them those simple problems, two of them will immediately write the answer down. Won't show me how they solve it… And I will say, "How did you get that answer?" And we've really fought this all year, "I just knew it," or "I did it in my head."*

This small piece of Ms. Fordham's practice demonstrates the preliminary nature of the mathematical issues she raised for discussion with Ms. Mantle. Ms. Fordham made a good start by talking to Ms. Mantle about her choice of numbers. When Ms. Fordham asked about this, she

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4 The principals, teachers, and students have been given pseudonyms.

5 Ms. Mantle formed small groups for her students and developed a list of Cognitively Guided Instruction-based story problems for each of the groups.

6 These numbers, unusually large for kindergarten age children, were chosen deliberately by Ms. Mantle to encourage students in this particular group to demonstrate their solution strategies visually.

demonstrated she understood that the choice of numbers makes a difference in how accessible a problem is to students and that teachers must give careful consideration to the numbers they select. Ms. Mantle explained that she used the larger numbers to push Orrin, someone who can manipulate smaller numbers in his head, to create a visual representation to demonstrate his problem-solving approach.

In addition to discussing the choice of numbers, there is an important set of mathematical and pedagogical questions related to how Orrin interacted with the mathematics that Ms. Fordham and Ms. Mantle did not address such as: What did the representation he created indicate about where the boundaries of his mathematical understanding were? What might be interfering with his ability to use his visual representation to solve the problem? Would he have benefitted from using cubes before he drew a representation on paper? Did he understand that 12 and 22 could be broken apart and put back together? What would next best steps be for him?

Ms. Fordham might also have explored with Ms. Mantle what next best steps for the class as a whole might be. For example, Ms. Mantel and Ms. Fordham might have looked across the range of ways students approached the story problems with the goal of analyzing the mathematical thinking each approach reflected. They then might have ordered students' approaches in terms of their level of sophistication and considered which ones would be best to feature in a whole class discussion and in what sequence. Through the deliberate selection and sequencing of several students' approaches for the whole class to make sense of and through the shaping of the ensuing discussion, Ms. Mantle would have given everyone access to the significant ideas that emerged from students across the class. Ms. Fordham's LCK did not position her to address these important practices with Ms. Mantle. The capacity to do this is at the heart of what it means for principals to be up-to-speed.

What does the supervisory practice of a principal who takes these additional steps with their teachers look like? One of the principals whose practice we examined provides such as image. Harriet Umsel was principal of a K-5 school located in a suburb of a large East Coast metropolitan area. Her LCK placed her in the *Profile A* group, and as such, when observing in classrooms, she focused her attention largely on students' mathematical ideas and how the teacher worked with their ideas. This capacity allowed her to provide valuable input as she worked with teachers on several important fronts during post observation conferences:

- Making sense of students' problem-solving strategies and of what their strategies revealed about their understanding of the mathematical concepts.
- Categorizing and ordering students' strategies from less to more sophisticated.
- Using what has been learned about students' thinking to inform next teaching steps.

Making Sense of Individual Students' Problem-Solving Strategies

The lesson we observed was in the first grade classroom of Ms. Harvey. For this lesson, Ms. Harvey presented several subtraction word problems orally to her students. The children worked on each problem using cubes, counting on fingers, drawing pictures, or drawing on known math facts. They then wrote out their solutions and explained their answers and the strategies they had used to solve the problem.

Like many principals, when Ms. Umsel observed in a mathematics class she attended to a range of classroom management and general pedagogical practices, such as students being on task, the pacing of the lesson and transitions between activities. In addition, she attended very closely to the particulars of students' mathematical thinking. As such, she was in a position to contribute to the knowledge her teachers were acquiring about their students' mathematical thinking as the following comments reflect:

> ***Ms. Umsel:*** *So the first thing [Micha] started with is really knowing his doubles stack. And then he started to decompose…It's interesting the way Micha represented this because he represented it as an addition problem instead of seven minus two.... Intuitively he knew that he took away from one seven and he needed to add it to the other seven.*

> ***Ms. Umsel:*** *…and Sohn…he's using what he considers friendlier numbers. Instead of doing a double digit number minus a single digit number, he's changed it to single digit numbers because somehow it's easier for him to compute. And he knows it in his head.*

Insights such as these that connect what students did with what they seem to understand about the mathematics are very important pieces of information for teachers to have as they work to get a sense of the skills and understandings of individual students and of the class as a whole.

Categorizing and Ordering Students' Strategies

Once Ms Umsel and Ms. Harvey had analyzed students' individual approaches, they started a process of grouping, categorizing and ordering their approaches from least to most sophisticated. As they engaged in this analysis, Ms. Umsel and Ms. Harvey built on each other's thinking about what was going on mathematically for students. Through the course of the discussion, Ms. Umsel and Ms. Harvey developed the insight that some students saw the numbers as whole solid entities while others saw them as having smaller numbers nested within them and, therefore, it was possible to decompose and manipulate them in order to find a solution. As is clear from the excerpt below, Ms. Umsel was very much involved in helping to develop a picture of the problem-solving approaches of the class as a whole.

> ***Ms. Umsel:*** *So that's somewhere in here. So you have quite a range here. And it looks like we have kids who represented with pictures or counters. Counters seem to be the first pass. And then being able to represent what they did with counters with pictures.*
>
> ***Ms. Harvey:*** *Right.*
>
> ***Ms. Umsel:*** *That sort of next part of the spectrum is those kids who use pictures but didn't need the counters could create a more abstract way of representing it. That they could pretend that this picture is a kid. They didn't need the actual thing to do it.*
>
> ***Ms. Harvey:*** *…It feels to me that there is a space here. There's something in the middle and I'm not sure what it is. Maybe it's that they start to use the cubes... That they start to say, "Okay, I've got 14, I'm going to take 10 away. Or take 4 away to get to my 10, and then take one more away." …Began to decompose the numbers.*
>
> ***Ms. Umsel:*** *It seems to me these kids are looking at the whole as one thing and these kids seem to be able to see the numbers inside of it…*

Ms. Umsel's LCK positioned her to engage in such an analysis of students' mathematical thinking with Ms. Harvey. What they learned about students' thinking in turn laid the groundwork for planning the next steps Ms. Harvey would take with her students.

Using What has been Learned about Students' Thinking to Inform Next Teaching Steps

In the latter part of their post-observation conference, Ms. Umsel and Ms. Harvey turned their attention to how to help students move to more efficient strategies. Ms. Umsel played an important role in this discussion when she suggested to Ms. Harvey that she organize a whole class discussion around what she learned about her students' thinking when they were working individually and in small groups.

> ***Ms. Harvey:*** …. I think that's the part I struggle with is helping kids finding the strategy that's more efficient for them in that whole group setting. I find that a tricky thing to do.
>
> …Maybe pull over the kids who did counters and pictures…and say, "Okay, let's share." And then I can push the counter kids to think about what do the picture kids do. And then maybe pull the picture kids and the kids who started to think a little more abstractly about number over so they can hear the sort of next big jump. But I do think it's so individual that it's hard, in that whole group, to say this is a really efficient strategy. Well for some kids it is, but for some kids it's not. And that's tricky I think.
>
> ***Ms. Umsel:*** You know what I'm wondering? ...When you were going around working with kids if you had some children at different levels in mind to share and spend more time sharing three or four examples, what that would have done for the math congress?
>
> ***Ms. Harvey:*** Sharing three or four examples—like sharing one, sort of each type of thinker?
>
> ***Ms. Umsel:*** Yeah, to be able to say, "Alright, well I'm seeing several models from problem-solving." And what if you had chosen one from essentially each pile? Three or maybe four. And then explored them in depth…

Here again, Ms. Umsel demonstrated how well-served she was by her LCK, not only in terms of her capacity to understand the mathematical content of students' thinking, but also because of her ability to devise a teaching plan to help students become more efficient problem solvers. In contrast, Ms. Fordham's LCK did not position her to help Ms. Mantle work with her students' problem-solving approaches; Ms. Fordham's understanding of mathematics was not strong enough for her to understand the mathematics in students' thinking; nor did she demonstrate the capacity to help Ms. Mantle to design a lesson that would have further developed students' understanding of the mathematics by providing opportunities for them to explore and make sense of each other's problem-solving approaches.

Getting Up to Speed

Most principals are not mathematics specialists trained to take on the challenging role of mentoring their teachers. Given this, what do principals need to know in order to help their teachers, and how can they learn it? Importantly, they need to understand the value of furthering their own professional development, both mathematically and pedagogically, and to look for opportunities to do so.

One way for principals to begin building the requisite knowledge is through professional development programs for principals that help them learn to both focus on students' mathematical thinking and assess the effectiveness with which teachers are able to work with students' mathematical ideas. *Lenses on Learning* by Grant et al is one such program. Attending professional development programs in mathematics content and pedagogy along with their teachers is another. In addition, principals can learn from the expertise of others, such as math coaches or teachers whose mathematical and pedagogical content knowledge are already very strong. These coaches or teachers can support principals' own efforts to improve their understanding of what is happening in mathematics classrooms throughout their schools with the goal of ascertaining what teachers need to learn and what types of professional development would best facilitate that learning.

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Using Professional Development Materials Productively: The Role of Adaptations[1]

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Sara, a district coach, was planning a workshop for teachers focused on helping them learn how to identify, describe and foster students' algebraic thinking. She decided to use a particular professional development program because it matched her goals. The program's six sessions consisted of a series of core activities, each important in achieving the program goals. Although the materials called for three-hour sessions, the district only allowed her two hours for each. Sara was faced with a problem—the professional development is designed for eighteen hours, but she only has twelve.

Frank, a regional supervisor, used the same set of materials in a workshop for middle grade teachers preparing to implement new standards-based instructional materials. Unlike Sara, he was able to conduct the full program of six three-hour sessions. In session three, as the discussion unfolded one teacher brought up a mathematical idea that could be pivotal to discussions in later sessions. This idea was not the topic for the particular activity at this time so Frank had to decide whether to take it up now or set it aside for the later sessions.

The decisions Sara and Frank face are among the many that leaders of professional development confront on a regular basis. Although they are each using a set of professional development materials carefully designed to achieve specific learning goals, each leader is faced with decisions that may be considered adaptations to the original program. In Sara's case the adaptations were forced—a result of time constraints imposed by her district. To use the materials she will need to make decisions that impact its overall design. What gets modified? What changes can she make and still adhere to the program goals? Should Sara try to shorten the time allocated to each activity, or does she omit some? If, so, what? Unlike Sara, Frank is faced with a situation that unfolded during a specific session. Does he take up of the opening[2] to discuss the ideas now? How does this impact the flow and timing of the session? How does it impact future sessions? How well does either choice further the goals? Both Sara and Frank are faced with making choices about what will best support teachers' attainment of their goals. Both cases involve consideration of making adaptations (both large and small) to professional development materials.

This article aims to shed light on the issues related to adapting professional development (PD) materials. Nanette Seago, in her article on fidelity and adaptation identified issues related to modifying PD materials (Seago, 2007).

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[2] "Openings" are when unanticipated questions, challenges, observations, or actions on the part of participants in professional development that require facilitators to make on-the-spot decisions about how to guide the discourse, and when successfully navigated, provide facilitators with opportunities to foster learning by capitalizing on mathematical or pedagogical issues as they arise and connecting these to the learning goals of the professional development(Remillard & Geist, 2002).

We build on this earlier article to further explore adaptations that facilitators make, identify additional considerations in the use of professional development materials to effectively support teacher learning and end with some advice to facilitators on making adaptations. We hope this article will help facilitators consider how they might use published professional development materials effectively by carefully considering the importance of the overall goals of the materials, reflecting on the purposes of specific activities within these materials, and making adaptations in light of these goals and purposes.

Some History

There has been a tradition of facilitators of professional development creating their own activities for teachers. For the most part this had been due to the fact that there were few carefully constructed programs of professional development curricular materials available. Over the years facilitators have worked hard to find and create activities to use with their teachers. Sometimes this involved seeing or experiencing a great activity themselves and then turning around and using it with their teachers. Sometimes it meant scouring books or articles to find worthwhile activities that related to their specific needs and context. This often resulted in cobbling together sets of activities, which unfortunately translated into fragmented, disconnected experiences for teachers. Akin to teachers creating their own curriculum, developing a carefully constructed program of professional development is beyond the reach of many facilitators, either in terms of available time or required experience.

Research has demonstrated that practice-based professional development that utilizes artifacts such as samples of student work, video and/or narrative records of classrooms, provide powerful contexts for teacher learning (Borko, 2004; Smith, 2001). The recent arrival of published practice-based mathematics professional development materials, many of which have been developed through grants from the National Science Foundation, provides facilitators of professional development with more coherent, well-specified programs that target specific learning goals and provide carefully sequenced activities to achieve those goals. (Driscoll, M., 2003; Driscoll, et. al., 2008; Miles Grant, et. al., 2003, 2009; Schifter, Bastable, & Russell, 1999-2008; Seago, Mumme, & Branca, 2004; Smith, Silver, & Stein, 2005). Significantly, these high quality materials are the result of years of development and field-testing by educators with a depth of mathematical background and vast experience in supporting teacher development. More than a collection of disconnected efforts, developers of many of these materials have been learning from and building on the work of one another.

In the opening scenario both Sara and Frank were using a published professional development program and although the materials were well-designed, each leader needed to consider adaptations. "Adaptation is inevitable because it means to take seriously the context (i.e., setting, participants, facilitator) in which materials are used" (Seago, 2007). Adaptation is not synonymous with unproductive professional development and, given attention to the goals and intent of materials, adaptations can lead to worthwhile experiences for teachers. Indeed, professional development takes place in complex situations and should be shaped to address the needs of the teacher group involved. However, regardless of contextual needs, some facilitators of professional development may not feel they "own" published materials and see the need to make adaptations to the materials, either to personalize them or fit them to their specific contextual requirements. In addition, some "professional developers may be more likely to place a premium on creativity and attention to context that is only possible with adaptation" (Seago, 2007).

In her 2007 article Seago outlines and describes categories of adaptation—ranging from those that are productive, to those that produce no impact, to those considered fatal. With this work in mind, we interviewed and observed several facilitators, many using one of two sets of professional development materials. We report on what we found so that we can expand on Seago's ideas to provide further insights into how facilitators might effectively adapt and use professional development materials. Our purpose in observing professional development sessions was to further understand the types of adaptations that facilitators make in using professional development materials and to examine the relationship between adaptations and fidelity to the intent of those materials.

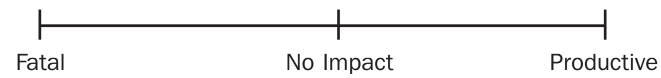
Professional Development Materials Considered in the Study

Our primary observations involved facilitators using one of two published PD materials—*Fostering Geometric Thinking Toolkit* (Driscoll, et. al., 2008) and *Learning and Teaching Linear Functions* (Seago, Mumme, & Branca, 2004). We chose these materials for two reasons: 1) the authors of this article are also the developers and authors of these materials and therefore intimately familiar with the goals and intent; and 2) each of these materials represent a genre of PD materials that are well specified, i.e., each carefully specifies learning goals, makes explicit its design characteristics and provides extensive supports for facilitators.

The *Fostering Geometric Thinking Toolkit* (FGTT) is a comprehensive professional development program involving 20 two-hour sessions designed for middle school and high school mathematics teachers. (There is the option of pairing sessions such that groups would meet for 10 four-hour workshops instead of 20 two-hour sessions.) The materials focus on the key topics addressing geometric properties, transformations, and measurement, with the following overall goals: strengthening teachers' understanding of geometry; enhancing teachers' capacity to recognize and describe geometric thinking; increasing teachers' attention to students' thinking; enhancing teachers' understanding of students' geometric thinking; and preparing teachers to advance students' geometric thinking. The materials contain two guiding structures designed to address the goals of FGTT. The first guiding structure is a cycle of activities that, over the course of a pair of sessions, takes teachers through the exploration of mathematics activity that teachers do together, reflecting on their own learning as a result of the activity, and then considering student work related to the mathematics of the activity. The second structure is the Geometric Habits of Mind (GHOMs) framework that provides a lens for teachers to use when analyzing their own geometric thinking, colleagues' geometric thinking, and students' geometric thinking. The facilitator materials provide clear instructions for use including agendas, facilitator notes and tips, and other helpful resources. Its accompanying DVD contains an array of tools, including video clips for use in particular sessions, PowerPoint® slideshows that summarize existing research on students' geometric thinking, printer-ready participant handouts and geometry applets for use by both participating teachers and their students.

Learning and Teaching Linear Functions (LTLF) consists of five modules designed to help teachers deepen their understanding of mathematics content, students' mathematical thinking, and instructional strategies as well as develop norms and practices for learning about teaching. The first of five modules, *Conceptualizing and Representing Linear Relationships*, is a sequential set of eight 3-hour sessions designed to enrich teachers' ability to teach linear relationships including the various representations that capture these linear relationships and connections among them. Each session has at its core one of two video clips that

Categories of Adaptation



capture students doing important mathematics. These video clips reflect a range of grade levels, different geographic locations, and a diverse student population. The agenda for each session addresses four basic components: framing the goals of the session, exploring a mathematics activity related to the mathematics of the video clip, viewing and discussing the video clip, and making connections to practice. The facilitation guide for these materials offers explicit and well-specified support including a complete overview of the materials, explanations and rationale of the underlying principles and specific goals, sample agendas and guidelines for sessions, lists of references and useful resources, tips for facilitation including caution points, mathematics commentaries, and excerpts from a composite facilitator's journal chronicling the experiences of others having used these materials.

In addition to the two sets of materials described above, we draw upon data and observations from two professional development leadership projects—*Learning to Lead Mathematics Professional Development* (Carroll & Mumme, 2007) and *Researching Mathematics Leader Learning*.[3] The *Learning to Lead Mathematics Professional Development* collected data from a number of practice-based professional development programs. Facilitators were observed, videotaped and interviewed. The materials developed in this project have been published and are being used to support K-12 mathematics education leaders, whether novice or experienced. Data was also gathered from *Researching Mathematics Leader Learning* where we engaged approximately 70 mathematics education leaders in a series of leadership seminars that were videotaped. We interviewed and observed several leaders as they conducted professional development, mostly in school-based settings.

Professional Development Leaders Involved

3 An NSF project (ESI-0554186) directed by Mumme investigating how professional development leaders create mathematically rich environments in professional development.

in the Study

Six facilitators were interviewed, observed, and videotaped using the ***LTLF*** and *FGTT* materials. We also drew on hundreds of hours of video of professional development sessions and interviews we gathered from the *Learning to Lead Mathematics Professional Development* project and data from the *Researching Mathematics Leader Learning Project*. Facilitators in these projects represented a broad range of levels of experience in teaching and in facilitating professional development. In the *LTLF* and *FGTT* sessions, four of the facilitators had participated in facilitator institutes to learn about the materials and two were "off-the-shelf" users. Three of the facilitators were new to this role but had taught for at least five years as secondary mathematics classroom teachers, one was a secondary mathematics teacher with over 30 years of teaching experience and 8 years of facilitating professional development, and one was an elementary teacher of 10 years of teaching experience and five years of experience facilitating professional development. In most settings, sessions were cofacilitated, and all of the facilitators always planned ahead of time—doing the mathematics beforehand, previewing videos if applicable and, when cofacilitating, dividing the session work amongst each facilitator.

Findings

As a result of our observations and interviews we have identified a number of reasons why facilitators make adaptations to materials, the types of adaptations they make, and the impact of those adaptations. As we saw in the opening scenarios, Sara's situation was externally imposed. She was required to constrict the time allocation for her sessions. Frank, on the other hand, saw an opportunity to pursue his goals through a situation that arose during a session. We decided to chunk the adaptations into three types: those due to contextual or external constraints; those that facilitators chose based on their knowledge, assumptions, and beliefs; and those that were the result of situations that arose during a session.

ADAPTATIONS RESULTING FROM CONTEXTUAL OR EXTERNAL CONSTRAINTS

Adaptations resulting from contextual or external constraints included situations where there were time constraints, where participants were absent from sessions, and where participants didn't complete homework assignments. Each is discussed below.

Time Constraints. In one LTLF professional development offering, the district was only able to provide time for six 3-hour sessions, rather than the eight sessions prescribed in the materials. The facilitators were faced with the decision as to what to cut. Since the LTLF materials are carefully sequenced, the facilitators chose to keep the first five sessions intact, and then bring things together with *linking to practice* activities in session six. The facilitators explained, "I guess we are trying to make the most of it. We know that mathematically, and probably in other ways too, they obviously aren't getting what you would get from a complete eight session PD." The facilitators reasoned that it was better to keep the integrity of the first five sessions, rather than trying to squeeze everything into the shortened schedule. Given the time constraints imposed, the facilitators appear to have made a productive adaptation, especially considering the fact that additional LTLF sessions were scheduled for the following school year.

In another example from a professional development session videotaped for LLMPD during which novice facilitators were using FGTT materials designed to help teachers examine student thinking, the facilitators only had an hour for what was designed as a two-hour session. They chose to engage teachers in working on some math tasks, which didn't leave time for watching and discussing the video clip of students discussing their thinking about these tasks, clearly missing the point of the activity and limiting what teachers were able to learn from the session.

Time constraints were the most frequent cause of adaptations. Most professional development materials suggest optimal timing per session and are organized to address specified content over a specified timeframe. Not every professional development contexts fit neatly into these schedules and often facilitators are required to make adjustments to fit their circumstances. Most often this involves a shorter time than desired. The FGTT facilitators indicated they would have appreciated guidance on what to do, explaining, "the reality of it is, we have had to shorten something almost every time... Because we're making that decision but we're not, we didn't write this, so how would you know…how does that affect what's to come?" This presents a dilemma for developers of professional development materials. If facilitators are offered suggestions for cutting, the fear is that facilitators will be more apt to do so, and participants might miss out on what the full program has to offer. However, by failing to offer these suggestions, facilitators who need to make cuts are left to decide what to omit or shorten on their own. Although the facilitator materials describe the purposes of each session, without a clear understanding of the goals and purposes of the professional development program, it can be difficult to make productive adaptations in relationship to time constraints.

Participant Absenteeism. In professional development programs where ideas are designed to build from one session to the next it can be problematic if people miss a session. In the *Learning to Lead Mathematics Professional Development* project we observed a facilitator taking some extra time at the beginning of a session to have the group bring returning absentees up to date. The facilitator asked participants to reflect on key ideas from the last session in small groups, then in the whole group, with some prompts to hone in on key points. This served to bring absentees up to speed, give other participants an opportunity to reflect on their own learning and to provide valuable information to the facilitator on where people were with their thinking. When this wasn't a built-in feature of the professional development agenda, this added discussion took away from time in the regular agenda, but was made up for in the benefits of the time to reflect. When these reflection times didn't consume too much of the planned agenda, they were often very productive adaptations.

It is rare that PD materials explicitly mention the issue of absenteeism, yet most facilitators recognize this as an issue they often face. While some facilitators may choose to begin the session with a small group reflection time, others may choose to begin with a whole group review of the previous session, while still others may choose to send out an email summarizing the previous session for absentees. The important point is that facilitators plan for the possibility of absenteeism.

Participants Failing to Complete Homework. Many professional development programs ask teachers to complete assignments in advance of a session, like trying out a mathematics task with their students and bringing back student work, or reading a case ahead of time as a way to prepare for discussion. FGTT follows this practice in many of its sessions, where teachers are asked to try a task with their students, and then submit student work to facilitators in advance of the session itself so facilitators can preselect student work that will best advance the key ideas of the session. In one FGTT session we observed, most teachers didn't try the task with their students, and no one submitted student work to the facilitators as requested. This forced facilitators to make some adaptations to their plans for the session. While they had collected some student work that they could have shared, they decided it would be more valuable if the discussion focused on work that participants brought, even though none of it had been submitted in time for them to review it ahead of time, and asked one participant to share what she had brought. The quality of the discussion about these pieces of work was less than what the facilitators had hoped. They wondered if they had made the best choice about how best to proceed. They had been hoping for a greater variety of student work, to discuss and also felt handicapped by the fact that they had not seen the student work ahead of time. This adaptation was, in a sense, forced upon the facilitators and was not particularly productive. In this case, the facilitators had samples of student work available but chose to use some a teacher supplied. Where this may have added credibility to the student work itself, some key points of the session were missed because of this choice, and using the student work previously collected by the facilitators might have better served the session goals. Given that homework tasks are an important component in many professional development programs, facilitators would be wise to talk explicitly with participants about the role of homework tasks and the importance of homework deadlines. They would also be wise to consider contingency plans for how to proceed if homework is not submitted.

ADAPTATIONS BASED ON FACILITATORS' KNOWLEDGE, ASSUMPTIONS, AND BELIEFS

Decisions based on facilitators' knowledge, assumptions, and beliefs was also an important category of adaptations. This includes facilitator knowledge of the materials, their perceptions of the needs of participants, and their own additional goals for the professional development sessions themselves. Each of these is discussed below.

Knowledge of the Materials. Facilitators need to understand how all of the design elements that support the use of a set of professional development materials are important. For instance, during an LTLF discussion of a video clip, the facilitators needed to be able to go back into specific footage because participants had different perceptions as to what had happened in the segment. The materials are set up to allow facilitators to use links from the on-line transcript to go to specific portion of the video clip, but the facilitator didn't know how to do this, and precious time was wasted while she attempted to find that portion of the video clip. This is a crucial feature in the design of the materials, as it allows the facilitator to bring different perspectives out on the table, using evidence from the video clips. While this may not be clearly an adaptation, it had an impact on how the materials were used in the sense that it didn't allow for full use of what the materials were intended to offer, and is an example of unproductive use of the materials. How the technological aspects of professional development materials can hinder or support opportunities for teacher learning.

Even when sessions involve two or more facilitators, it is important for each facilitator to fully know the materials and understand the goals of each session. In the FGTT sessions we observed, we found that the facilitators seemed to have specific roles. One FGTT facilitator, Alice, explained, "I'm the Do Math girl." Another facilitator, Phyllis, added, "I'm the time keeper." One of them added, " Fred is the GHOMy (Geometric Habits of Mind) . . . that's what we call him." As she gestured to another one of the facilitators, she explained, "You're really the 'analyze the student work' person…so we've kind of broken it into those three sections [based on] our strengths." One of the facilitators added, "And our personalities." This might be viewed as resulting in stronger learning experiences for teachers, and in some cases this might be true. But Alice also said, "It's hard to answer on the Geometric Habits of Mind part because that was Fred's. Fred's baby I guess, is what we were calling it… he leads that discussion." Since the Geometric Habits of Mind are such an integral part of the FGTT materials, it raises questions as to how well the facilitators other than Fred were equipped to make decisions as to what to take up in discussions during their turn at leading.

Another example highlighting the importance of each facilitator fully understanding the materials and the goals for each session appeared during an observation of an LTLF session. During her time to lead, one facilitator asked a question that was aimed at the purpose outlined in the materials. When there was prolonged silence the other facilitator jumped in with a different, unrelated question, taking the group off target. Silence can be difficult to accept, and what one does as a result needs to be tied to purpose.

Each of these examples points to the fact that facilitators need to have deep knowledge of the materials, including the technology associated with the materials, as well as the learning goals for participants—whether they are facilitating alone or cofacilitating with partners. Knowing the purposes of each element of the materials individually and collectively allows for a more coherent well-orchestrated learning experience

Perceptions of the Needs of Participants. Another important issue that influences the adaptations of professional development materials has to do with how facilitators respond to what participants believe about whether the professional development program is meeting their needs. In one LTLF series, participants complained that things were moving too slowly. They had seen enough tasks and video involving linear functions and wanted to move on. In response, the facilitator modified sessions to omit the video so she could add in mathematical tasks that addressed quadratics and other more complex functions, thus changing the focus of the professional development program which centered on the teaching of linear relationships — including the use of mathematics tasks and video clips addressing this content. These participants spent the remainder of their sessions doing and discussing mathematics tasks that addressed other kinds of mathematics functions, one after another, and did not have opportunities to continue to deepen their understanding of linear relationships, examine and compare representations for linear relationships, and consider the implications for teaching—a fatal adaptation. An interview with this facilitator, an "off-the-shelf" user of the materials, revealed that she apparently didn't understand the storyline that LTLF was developing and didn't communicate its value to the participants.

A similar situation was observed in FGTT. A facilitator determined that participants were getting "restless" looking at student work, so she omitted this aspect of the professional development in favor of simply "doing the math"—another fatal adaptation. Whereas facilitators need to be responsive to the needs of participants, they also have to know the value of each of the design elements of a particular professional development program, and determine ways to build and communicate the value of each of these components to participants.

In the *Learning to Lead Mathematics Professional Development Facilitators* we observed several facilitators weighing teachers' mathematical strengths. We saw serious attempts to slow down the mathematics to insure that teachers developed a deep fundamental understanding. We observed facilitators adding in content, believing that this would help teachers' fragile knowledge. We also observed facilitators making decisions to skip over activities assuming that the teachers already knew the content.

Evidence should guide perceptions of teacher knowledge, needs, and contentment. Assumptions should be checked. Facilitators need a repertoire of strategies for gathering evidence of what teachers bring to the professional development. Facilitators can use teacher reflections, surveys, and their work on mathematical problems to gain insights into how and what teachers are thinking. In addition, they can use probing questions to gain more information about teacher reasoning. This evidence then needs to be weighed against how the program materials were designed to address these issues. Many of the practice-based professional development materials referenced earlier are the result of years of development and field-testing across multiple contexts. Making adaptations should be considered in this light. On the other hand, being responsive to genuine teacher needs is important. Adhering rigidly to an agenda and ignoring teacher needs can be fatal. Facilitators need to know the value of a professional development program and determine ways to build and communicate its value to teachers.

Identifying Additional Goals. Adhering to the goals of the professional development program was often mediated by facilitators' attention to cultivating particular orientations toward mathematics. For example, in one professional development session we observed in *Researching Mathematics Leader Learning*, the facilitators were using professional development materials designed to strengthen teachers' mathematical knowledge. The facilitators also decided they wanted to help teachers understand the constructivist learning philosophy and how it was the foundation of their new curriculum adoption. Doing the mathematics in professional development was thus situated within this major aim. One facilitator explained, "what we were looking at with the staircase problem was to engage them in that struggle as well as to provide some modeling about work within the constructivist model." The facilitators wanted the mathematical task to generate willingness to persevere with problem solving in the face of difficulty, to be comfortable sharing vulnerabilities, and to cultivate the curiosity to question each other and engage in the task. Whereas these are laudable goals, this could be done at the expense of what is to be learned mathematically. It must be noted that sometimes additional goals are mandated externally, such as a district requirement that all professional development include a certain goal or address a district priority. Taking up these additional goals must be balanced with the goals of the program and considered in light of what is feasible within the time available.

ADAPTIONS RESULTING FROM SITUATIONS THAT ARISE DURING SESSIONS

Often, adaptations result from situations that arise during particular professional development sessions. These include running out of time during a session or having to negotiate an "opening" that arises during a session.

Running Out of Time in a Session. When time was a crunch, and it often was in sessions we observed, the final "pulling ideas together" or reflection activity was often omitted. In one LTLF session facilitators ran out of time to finish the activities. Before time was up, however, they made time for teachers to write reflections in their journals. They indicated that without time for reflection teachers would lose an opportunity for making sense of their experience. Interviews with facilitators indicated that they made contingency plans, outlining what they would do if an activity took longer than anticipated. They identified key points not to be missed to help guide decisions about time. In some sessions in FGTT and LTLF sessions we saw student work or video omitted from sessions when time was an issue. In many instances these were the centerpiece of the session. In some professional development sessions when time was limited we saw facilitators omit activities that asked teachers to apply ideas to practice. In general, in carefully constructed professional development programs, selectively leaving out portions of the activities (unless cited as optional) will result in lost learning opportunity and may destroy the integrity of the program. Carefully monitoring time is important, but sometimes adaptations can't be avoided, often for reasons that are about supporting teacher learning. Recognizing that even with the best intent, time can get away from you, facilitators can make contingency plans in advance for how they will "make up" for key lost pieces.

Navigating through Openings. "Openings" are created when unanticipated questions, challenges, observations, or actions on the part of participants in professional development require facilitators to make on-the-spot decisions about how to guide the discourse. When these are successfully navigated, openings can provide facilitators with opportunities to foster learning by capitalizing on mathematical or pedagogical issues as they arise and connecting these to the learning goals of the professional development (Remillard & Geist, 2002). During our observations of

LTLF and FGIT sessions we saw numerous openings. For example, during the discussion of a video clip in a LTLF session, one teacher talked about how he noticed that students seemed to be talking about every other odd number —an issue that would come up in the next clip—but the facilitator chose not to highlight this mathematical moment even though doing so might have helped teachers prepare to focus on this important idea in the upcoming discussion. Successful navigation of openings requires a deep understanding of the specific purposes of an activity, the goals of a session, and the overall goals of the program—all of which well-specified professional development materials can provide

Discussion

In our findings we discuss only a few of the adaptations facilitators make with professional development materials. Reasons for adaptations were many—some were the result of external contextual issues, some were driven by facilitators' knowledge, assumptions and beliefs, while others were the result of issues that arose during a session. Some adaptations, even those that arose during a session, were planned in advance, while others were in-the-moment decisions. Many of the facilitators were aware of the adaptations they were making and could provide a rationale for the modifications, while others appeared to be unaware of any adaptations being made.

In *Fidelity and Adaptation of PD Materials*, Seago (2007) identified "categories of adaptation." These categories are arranged on a scale that ranges from fatal adaptations at one extreme to productive adaptations at the other, while in the middle lies the types of adaptations that do not impact the design of the materials negatively or positively. She describes these categories as follows:

> ***Fatal Adaptations.*** Adaptations that violate the goals and intent of a program can be considered fatal errors and seriously undermine critical components of the materials. For example, in our data we saw a facilitator choose to omit the video and replace it with more math tasks, a "fatal adaptation." In this case we believe it revealed misconceptions the facilitator held about the intended use of the professional development materials. Sticking with the program design and communicating its value to participants may have benefitted teachers more by addressing the goals that lead the facilitator to select these materials. We do not want to suggest, however, that fatal adaptations are necessarily unproductive for participants, even though they violate the goals of the professional development materials.
>
> ***No Impact Adaptations.*** Some adaptations seem relatively neutral in that they don't have a big impact on use with fidelity. For example, in the situation cited under time constraints where the facilitators had to reduce the number of sessions from eight to six, they chose to keep the first five session intact and use session six to pull ideas together. Whereas this wasn't ideal, their adaptation was neither fatal nor productive given the situation. These "no harm, no foul" adaptations are categorized as no impact because they don't undermine the basic design or values of the materials, nor do they make better use of them.
>
> ***Productive Adaptations.*** Some adaptations by facilitators are productive in that they make better use of the materials given the circumstances in which they are being used. For example, the strategy used by facilitators to bring absentees up-to-speed, beginning with a discussion of key ideas from the previous session, was an example of a productive adaptation. It not only allowed those who had not been present to gain a sense of what had happened in the last session, but also served to rekindle ideas for those who had been present. Productive adaptations are those that relate to particular participants in particular contexts, while at the same time keep an eye on the learning trajectory of the materials.

An adaptation in itself is neither necessarily productive nor fatal. It all depends on the degree to which the adaptation helps participants achieve the goals addressed in the professional development materials. A thorough understanding of these goals allows facilitators to weigh an adaptation in light of its impact on teacher learning. When you consider the years of development work that go into the design of these materials, and their productive use in a wide range of contexts, facilitators should take time to consider whether adaptations that deviate from the identified agendas are wise.

The extent to which professional development materials contain supports for facilitator of the professional development can influence the kinds of adaptations facilitators are inclined to make as they plan and facilitate sessions, as they are then better equipped to make adaptations that do not compromise the integrity of the learning goals. Seago

(2007), in her earlier paper, suggested, "Well-specified professional development materials make it possible to use materials with fidelity because they explicitly communicate the underlying principles."

In 2008, Horizon Research convened a meeting of several invited mathematics educators to examine issues in the design, development, and use of practice-based mathematics teacher professional development materials. A draft report issued from this meeting outlined several components that should be included in published professional development materials in order to support effective use by facilitators of the professional development (Heck, Markworth, & Weiss, 2008). The components include the following:

- An *overview* that explicates the pedagogical and mathematical learning goals overall and of each session.
- *Logistical information* about the program and its implementation, including timing suggestions, participant grouping, recommendations for structuring activities, etc.
- *Resources* for each session's implementation such as masters for handouts, PowerPoint slides, transcripts, posters, and prepared student work samples.
- *Prompts* that provide guidance for starting, continuing, concluding tasks and discussions, and for getting things back on track when necessary.
- Material-specific *facilitation techniques* and instructions for any relevant features such as setting the desired intellectual and social climate, how to facilitate discussion effectively, how to react to participants' responses, etc.
- *Links to practice* that describe how concepts, issues and activities are likely tied to the teachers' school or classroom contexts and how the facilitator can use state/district standards or instructional materials to create tighter links to the context.
- A variety of potential *answers* and solution approaches to mathematics tasks, along with commentaries on their uniqueness and connections, common incorrect solutions or interpretations, along with suggestions about how to respond to various solutions.
- *Samples and examples* to illustrate how a task or session might progress.
- A means for *assessment* of the progress of participants that could include embedded formative assessments, scoring rubrics, exit card questions, what to look for in group work and other evidence to watch for to determine if the pedagogical and mathematical learning goals are being met.
- *Support notes* that provide guidance for using recommended technology, acknowledgement of issues or concerns that might arise and ways to respectfully deal with them, and commentaries on mathematics content of tasks.
- Suggestions for *alternatives and extensions* to consider that extend the experiences.

Having these components available in published professional development can also help make it more likely that those responsible for identifying professional development materials can make good choices based on the learning goals they wish to achieve.

Conclusions

We saw from our observations of professional development sessions and our interviews with facilitators that constraints resulting from the scheduling of professional development or the challenges that arise from unexpected events in the professional development itself often require adaptation in how professional development materials are used with participants. However, the design features of professional development materials identified above are only of use if facilitators thoughtfully attend to what these design features offer as they plan and facilitate their sessions. Thoughtful use of the design features of professional development materials can also be supported by facilitator participation in training on the use of the professional development materials that is often available where a range of constraints and challenges can often be discussed and explored with other facilitators and the authors of the materials themselves.

Our findings suggest that even when using professional development materials that include all the recommended design features, facilitators might be advised to develop contingency plans during their planning sessions, thinking ahead about what issues might arise in each session and what kinds of adaptations might be considered to address those issues—all the while keeping in mind the core principles of the materials. Some contingency plans might include adaptations to address the following:

- Participants have been absent from an earlier session
- Participants haven't done the homework
- A key point doesn't come up in a discussion
- A solution method you want isn't created by someone from the group
- Participants struggle with the mathematics beyond what is planned
- Participants don't seem to see the value in the activities
- Part of a session takes longer than anticipated

We believe that facilitators who prepare for these contingencies are more likely to be prepared to make productive adaptations that address them.

Creators of professional development materials can help support facilitators by purposely designing for adaptations.The Learning and Teaching Geometry Project[4] is designing for potential adaptations as they are developing video case materials for use in professional development. In an effort to create well-specified materials aimed at supporting facilitators to use the materials in accordance with the core principles, they are measuring adaptation and fidelity in their pilot tests to examine adherence to and focus on the mathematical and pedagogical storylines of the materials. The data collected will be used to inform the content of the facilitation materials—to better design supports for using the materials as they are designed to be used. Developers who take seriously the importance of helping facilitators adapt professional development materials productively can impact the scaling up of high quality professional development learning opportunities for teachers.

We all believe that high-quality professional development is key to improving mathematics teaching and learning. Well-designed professional development materials are crucial to this effort, but how these materials are used to promote teacher learning, including what adaptations are made as they are used, ultimately determines the effectiveness and impact of the professional development. For these reasons, thinking carefully about how to support facilitators of professional development as they use well-designed professional development materials is an important question for our mathematics education leadership community.

[4] An NSF-funded professional development materials project (ESI- 0732757) intended to produce video-based professional development materials for grades 6 through 10 due to be published in 2012.

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