NCSM Journal

of Mathematics Education Leadership

FALL 2011 VOL. 13, NO. 2

National Council of Supervisors of Mathematics www.mathedleadership.org

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Brief commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We want to hear about your reactions, questions, and connections you are finding to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Linda Ruiz Davenport, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM)

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***Note:** Information for manuscript reviewers can be found on the inside back cover of this publication.

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education

- Fostering inquiry into key challenges of mathematics education leadership

- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice

- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Linda Ruiz Davenport, *Boston Public Schools, Boston, Massachusetts*
Angela T. Barlow, Ph.D., *Middle Tennessee State University Murfreesboro, Tennessee*

These continue to be very interesting times for mathematics education leaders. Many states and districts are taking their first transitional steps into the Common Core State Standards (CCSS) for Mathematics, the assessment consortia responsible for helping us monitor how well our students are learning these practice and content standards continue to move forward with their efforts, and we are all thinking hard about the implications for teachers, teacher leaders, administrators, and other mathematics educators. What new challenges await us as we consider the complexities of these transitions in so many different arenas and at so many different levels?

For instance, there are important questions about what families may need to learn about the CCSS for Mathematics and any new expectations for their children. In *Organizing a Family Math Night*, Tim Jaccobe includes a description of handouts he used with each family math activity that explained how to play the game, identified the mathematics addressed by the game, and discussed how the game could be played at home. Might it be possible to consider similar handouts during a family math night that reference particular Standards for Mathematical Practice and any Standards for Mathematical Content that might be relevant, perhaps in a form that was friendly to families, as a way to begin acquainting families with what these mean?

The CCSS for Mathematics also raises important questions about appropriate use of technology for teaching and learning mathematics. In *Improving Student Achievement by Systematically Integrating Effective Technology*, Jeremy Roschelle and Steve Leinwand make a number of recommendations for how technology can enhance students' mathematical thinking and learning. It will be important to keep these recommendations in mind as we consider the technology implications of both the Standards for Mathematical Practice and the Standards for Mathematical Content, particularly as schools and districts begin to invest in curriculum materials and tools that support the transition to the CCSS for Mathematics and any professional development associated with those materials and tools.

We all recognize that the CCSS for Mathematics has significant implications for teacher knowledge and teacher practice. Mathematics coaches and mathematics specialists will be important resources as we consider how to support teachers during the transition to this new framework for mathematics teaching and learning. John Sutton and his colleagues, in the article *Mathematics Coaching Knowledge: Domains and Definitions*, propose a set of domains for mathematics coaching knowledge and what those domains might include. The more robust a mathematics coach's knowledge base, the more prepared he or she will be to provide the supports needed by teachers working to strengthen their mathematics teaching practice. At the same time Janet Herrelko, in *Reflections on Creating Strong Mathematics Coaching Programs*, reminds us that a strong mathematics coaching knowledge base may not be enough to have an impact on mathematic teaching and learning in schools and districts. Structural features of mathematics coaching programs make an important difference as well. Who supervises mathematics coaches? How do we ensure they have access to teachers and classrooms? What kind of professional development support do they receive as they take on their very challenging work? Each of these considerations can make an important difference in whether well-prepared mathematics coaches are able to provide effective supports for mathematics teaching and learning. Skip Fennell, in *We Need Elementary Mathematics Specialists Now, More Than Ever: A Historical Perspective and Call to Action*, reminds us of the very real challenges of strengthening mathematics teaching and learning at the elementary grades, where teachers' mathematics knowledge for teaching may not be sufficiently deep, and there is much to do if we are to take the Standards for Mathematical Practice and the Standards for Mathematical Content of the CCSS seriously at the elementary grades. He also points out that it is often these elementary mathematics specialist who are expected to help schools and districts develop plans for the transition to the CCSS for Mathematics and what this means in terms of both policy and practice.

We also all recognize that support from mathematics coaches or elementary mathematics specialists may not be sufficient to address all that teachers may need to learn in order to strengthen their mathematics teaching practice. Well-designed professional development is also an important source of support for teachers. Janet Lynne Tassell and her colleagues describe a four-year professional development effort consisting of summer institutes and school day sessions designed to strengthen mathematics teaching and learning in their region. One can imagine similar professional development experiences that focus on important aspects of the CCSS for Mathematics, digging deeply into the mathematics content and the mathematical practices associated with these standards and considering implication for planning for instruction, what it means to thoughtfully teach these lessons, and how to determine what students are learning through a rich set of formative assessment strategies.

We all know the CCSS for Mathematics cannot mean "business as usual." But what *are* the implications, given prior state frameworks across the country? Barbara Reyes and Amanda Thomas compare standards for computational fluency in state frameworks prior to the adoption of the CCSS for Mathematics to what will now be expected by this new set of standards and find "a rather ambitious roadmap for changes in the K-8 mathematics program." Making these changes will mean bringing to bear all we know and understand about the design of strong professional development, the roles of mathematics coaches and mathematics specialists and the contexts in which they work, strategies for engaging families, and thoughtful recommendations for fully leveraging technology. We hope the thoughtful articles in this issue of the *Journal for Mathematics Education Leadership* will have something to offer mathematics leaders in all these areas.

But we also want to hear from you about any efforts you are involved in to strengthen mathematics teaching and learning in schools and districts, whether you are working in a state that has chosen to adopt the CCSS for Mathematics or whether there are other motivations for your efforts. How did you plan these undertakings? What were your goals? Are you working with preservice or inservice teachers? Are you working with school administrators? Are you involved in a state-led effort? What are you learning that the rest of us in mathematics education leadership roles might learn from as well? Please consider writing about your efforts and submitting them to the journal.

And if any of the articles in this issue resonate with some of what you are thinking about or working on, we also want to encourage you to let us know through a letter to the editor. That letter can be a brief acknowledgement to the authors of a particular article, it can be a discussion of how the work in a particular article connected to yours, and it can even be a way to share a small bit of what you are working on in relation to a similar undertaking. It would be a lovely way to generate conversation within our mathematics education leadership group.

One important advantage to the fact that so many of us now are beginning to share the same standards for mathematics teaching and learning is that it opens up possibilities of collaboration and discussion across these shared pieces of work, regardless of our particular geographic locations. We encourage you to think of our journal as a context for these kinds of collaborations and conversations, whether it involves sharing your work through an article in the journal, a letter to the editor, or even through discussions with colleagues about what you reading in our issues. Both of us—Linda Ruiz Davenport in the role of Editor and Angela Barlow in the newly created role of Associate Editor—sincerely hope you enjoy this issue! Please let us hear from you!

Improving Student Achievement by Systematically Integrating Effective Technology

Jeremy Roschelle, *SRI International, Menlo Park, CA*
Steve Leinwand, *AIR, Washington, DC*

Given the long history of technology in mathematics education and the many differences in approach and application, useful research must now go beyond making claims about technology in general or in isolation from its use; specific approaches must be described—with their entailments, assumptions, and goals—and evaluated on their merits.

—Roschelle, Rafanan, Bhanot, Estrella, Penuel, Nussbaum, Claro, 2010

Our Position

It is the position of NCSM that in order to develop all students' mathematics proficiency, leaders and teachers must take responsibility for the systematic classroom integration of effective technologies to enhance curriculum, pedagogy, assessments, and approaches to equity. Using research informed practices, NCSM members need to identify the technologies and make decisions about when and how to use these technologies in ways that strengthen students' mathematical thinking and learning. This position can be accomplished when leaders and teachers:

- Understand that technology is not an isolated element but a powerful tool that must be fully integrated into the teaching and learning process.
- Use research to guide decisions about what types of technology and how best to use them.
- Provide sustained professional learning opportunities prior to and during all phases of implementation.
- Recognize that learners—both adults and students—progress through varying levels of comfort as they begin to use technology before they can realize its full impact.

Today's world makes a burgeoning array of technologies available to classrooms, ranging from graphing calculators to computers to electronic whiteboards. Unfortunately, not all types and uses of technology lead to meaningful benefits in teaching and learning (Dynarski, Agodini, Heaviside, Carey, Campuzano, Means, 2007). Research can offer useful guidance about effectiveness of various options as well as best practice for technology use (Means, Roschelle, Penuel, Sabelli, & Haertel, 2003). Teachers and leaders who are aware of the research that shows how technology can enhance students' mathematical thinking will be better able to advocate for technology decisions based on the potential impact in the classroom.

Research That Supports Our Position

Growing and strong research literature highlights benefits to mathematics teaching and learning when technologies that are specific and appropriate to mathematics are systematically integrated into classroom practice (Heid & Blume, 2008a). Frequently cited benefits of integrating appropriate technology include increased conceptual understanding (including representing, generalizing, abstracting, modeling and working with symbols), better problem solving, broader participation and deeper student engagement (Heid & Blume, 2008b). Technology can also increase interactivity within the mathematics classroom, enabling students to explore mathematical concepts (Kaput, 1992) and providing immediate feedback to students and their teacher (Bransford, Brophy, & Williams, 2000). Research shows that using technology in this way not only helps students learn the same mathematics better (Roschelle, Pea, Hoadley, Gordin, & Means, 2000); it enables "democratic access" to more important and deeper mathematical thinking (Kaput, 1994).

Mathematics education leaders are likely to be asked to weigh in on the selection of particular products. This can be challenging because technology changes rapidly and specific products may come and go. Further, "gold standard" scientific evaluations of specific technologies are far and few between (National Mathematics Advisory Panel, 2008). The available research supports several important ways that technology can enhance mathematics teaching and learning through the following:

- strengthening the display and presentation of mathematical work;
- enabling dynamic representations of mathematical ideas;
- supporting formative assessment practices; and
- enhancing collaborative learning.

This research can help support good technology choices by identifying mechanisms that link technology to the enhancement of students' mathematical thinking and learning. Each is discussed below.

STRENGTHENING THE PRESENTATION AND DISPLAY OF MATHEMATICAL WORK

A first cluster of enduring and stable findings in research on learning addresses the presentation and focus of student mathematical activity. Display technology, such as projectors, document cameras and electronic whiteboards, can make it easier for teachers to focus on important ideas (Ruthven, 2009). Likewise, contrasting examples of concepts and misconceptions or different solution strategies is a very basic and important technique for advancing student learning. Display technology can make it easier to juxtapose examples and achieve the desired conceptual contrast (Bransford et al., 2000). Further, these technological tools can support rich mathematical tasks by increasing interactivity. Moreover, by handling some of the routine aspects of graphing, a graphing calculator can focus student thinking on the more important higher order concept – such as the relationship between a coefficient of an equation and the shape of the resulting function (Chandler & Sweller, 1991). Simultaneously, the increased interactivity of a graphing tool can engage students in exploration and inquiry and support conceptual learning.

ENABLING DYNAMIC REPRESENTATIONS OF MATHEMATICAL IDEAS

The Multimedia Principle of learning (Mayer, 2006) holds that students learn best from linked graphical and linguistic symbols such as graphs and algebraic expressions (National Mathematics Advisory Panel, 2008). In school mathematics, students need support to form meaningful connections between graphical and linguistic representations and to build conceptual connections. Technology supports these best practices of mathematics teaching (Hiebert & Grouws, 2007). Technology can also provide graphical images that accompany expressions, as in the case of graphing calculators, and also can use motion and animation to increase students' access to the mathematical meaning of the graphical notation. For example, in dynamic geometry, animation can help students to see how one particular construction of a circle inscribed in a triangle is an instance of the general case for all triangles, leading students from a single case towards generalization and proof (Laborde & Laborde, 1995).

Mathematics education leaders can ask, "How does this technology provide linked dynamic representations to deepen students understanding of mathematical connections among graphical and linguistic ways of expressing the same mathematics?" Further, once a technology is selected that features linked dynamic representations, leaders can also raise important questions about the teaching practices that leverage this technology. For example, learning with dynamic representations is strongest when teachers emphasize concepts and connections in their teaching and frame curriculum and activities to focus on big ideas (Heid & Blume, 2008a).

SUPPORTING FORMATIVE ASSESSMENT PRACTICES

One of the most firmly established learning principles is that students learn best with immediate feedback that identifies errors, prevents rehearsal of unproductive approaches, and reinforces success (Hattie & Timperley, 2007). A related best practice of teaching is adapting the pace, content, and supports provided to students to match developmental needs and capacity (Corno & Snow, 1986). It is important for this feedback and these adaptive teaching strategies to be informed by the ongoing use of formative assessments (Black & Wiliam, 1998).

Technology can support formative assessment in four complementary ways. First, technology can make it faster to collect and organize assessment information, as in the case with "clickers"—response devices that allow a teacher to poll students' answers (Littauer, 1972). Second, technology can increase the breadth of engagement in assessment by using classroom communication networks to collect work from all students simultaneously, creating pressure for all students to think, allowing responses to be collected anonymously, and providing a view of the variability of whole group to the teacher (Davis, 2003). Third, using handheld computer technology can deepen assessment to include conceptual reasoning by allowing teachers to create tasks that require students to do more than provide a factual answer. For instance, teachers can ask students to submit a graph, an algebraic expression, a sketch, or a snapshot of their work (Kaput, Hegedus, & Lesh, 2007). Fourth, technology can provide more actionable feedback to teachers. For example, technology can help teachers place student work on a learning progression from more basic to more advanced understanding of a concept and provide more targeted support (Anderson, Corbett, Koedinger, & Pelletier, 1995; Feng, Heffernan, & Koedinger, 2006).

A growing body of research has established the powerful role of technology in making it easier to achieve good classroom implementation of formative assessment processes. For example, an independent evaluation of the use of connected networks of graphing calculators for formative assessment found that teachers can deepen their understanding of student thinking and that students can make significant gains in learning algebra, especially when teachers use the technology to ask more meaningful mathematical questions (Owens, Pape, Irving, Sanalan, Boscardin, & Abrahamson, 2008). Likewise, a technology system that supported teachers' use of learning progressions to guide instruction dramatically enhanced student learning (Clements & Sarama, 2008). In general, formative assessment technology can provide teachers with assessment information that can be used to make wise instructional decisions that have a positive impact on learning (Means, Penuel, & Quellmalz, 2000).

Mathematics education leaders can ask, "How does this technology provide teachers and students with assessment information that is more timely, deeper, broader, and more directly useful in guiding further teaching and learning?" Once a technology is selected, leaders can emphasize the quality of tasks given to students, what they are likely to reveal about student thinking, and how teachers can use the assessment information to make instructional decisions that adapt to student needs and leverage the diversity of student ideas.

ENHANCING COLLABORATIVE LEARNING

Collaborative learning is another well-established practice that can accelerate and deepen student learning (Slavin, 1990). Especially with regard to mathematical communication and argumentation, students need to engage in discussing and explaining in order to learn. Explaining to peers and being helped by peers can produce strong learning gains (Webb & Palincsar, 1996). However, collaborative learning must be carefully structured to produce benefits; just asking students to "work together" is not enough (Cohen & Hill, 2001). Technology can help teachers to identify and manage the right structures, for example, by giving each student an individual aspect of a task so that each student has a unique and necessary role in the group (Slavin, 1980). Technology can also provide tools that help focus the collaboration on the important mathematics, for example, by providing tools for sharing diagrams and sketches that support students' explanation and argumentation. Further, the formative assessment technologies discussed above can support student collaboration when incorporated in a system such as Peer Instruction (Mazur, 1997). In such a system, after the classroom sees the diversity of student responses to a conceptual task, students work in pairs to convince each other of "the right way" to think about the problem. Collaborative learning and formative assessment have a strong natural synergy, and providing feedback on the collaborative learning process as well as what is being learned about the mathematics is one way to make it more meaningful and productive (Roschelle et al., 2010).

Mathematics education leaders can ask, "How does this technology organize productive structures for collaborative learning and increase student participation in mathematical explanation and argumentation?" Once collaboration technology is selected, leaders can productively focus on new assessment opportunities afforded by the technology, including assessment of mathematical communication as students work on collaborative tasks. Leaders can also focus on how to make sure all students are meaningfully engaged and accountable throughout their collaboration together (Slavin, 1990).

Integrating Technology into Classrooms

Technology use cannot be an isolated element of instruction. Rather, teachers must integrate technology with their approaches to curriculum, pedagogy, and assessment (Borko, Whitcomb, & Liston, 2009; Roschelle et al., 2000). Doing so successfully requires sustained teacher professional development (Zbiek & Hollebrands, 2008) and steady support from school and district leadership (Honey, Kulp, & Carrigg, 2000). Research can offer guidance to teachers and leaders in choosing goals and strategies to shape their efforts towards systematic integration of appropriate technology (Confrey, Sabelli, & Sheingold, 2002). Teachers and leaders who are aware of research will be better able to guide the formation of policies and plans that contextualize technology within a broader vision of improving mathematics classrooms.

When adopting new technology for use in the classroom, the challenges and changes a teacher experiences are numerous and perhaps daunting. Researchers have identified ways to understand the concerns teachers have about technology, the process teachers go through as they begin to incorporate technology into their instruction, the kinds of professional development support teachers need in order to be able to use technology successfully, and the importance of support from colleagues and school leadership during this ongoing process of learning to use technology in the classroom. Attending to all of these issues makes it more likely that teachers will be well prepared to use technology successfully to support mathematics teaching and learning.

Other Technologies in Mathematics Education

Social networking is also a prominent theme and fits with the recognition that learning is a socially-mediated process. When used effectively, Web 2.0 technologies can support mathematical communication and collaboration among students and with mentors (Renninger & Shumar, 2002; Stahl, 2009). In addition, online forums can be very helpful to teachers. As yet, the needed research to strongly guide practice in this area is still emergent.

Ready-at-hand informational resources are another emerging trend. For example, students can now type in a textbook page and problem and see a related video tutorial. Students can also access sites such as the MathForum, Wikipedia, and online calculators and tools. From the limited research available, it appears that "just in time" support can be very helpful to students (Renninger, Farra & Feldman-Riordan, 2000). Of course, it is also possible to spend a lot of time online with little educational benefit and some informational resources emphasize a very procedural approach to mathematics that does little to deepen mathematical understanding.

Other emerging areas of research on technology in mathematics education include the use of games, mobile phones, virtual realities, tangible computing and force-feedback (haptic) devices. Research is not yet sufficiently advanced to provide strong evidence on when and how these technologies are effective for learning.

Features of Technology in the Classroom and the Teaching and Learning Opportunities it Supports

Table 1 (see next page) summarizes the features of technology discussed thus far, the teaching opportunities they provide, the learning opportunities created, and examples of the technology.

The Process of Learning to Teach With Technology

To better understand technology adoption, researchers have sought to identify stages a teacher goes through during the implementation process. Zbiek and Hollebrands (2008) identify the PURIA model (Beaudin & Bowers, 1997) that characterizes how teachers learn to teach with technology. The PURIA model consists of five modes a teacher experiences as they begin to understand and use technology in their classrooms. The five modes are:

- Playing with the technology without a purpose;
- Using the technology for personal purposes, perhaps as a learner of mathematics;
- Recommending the technology to others, including a peer or a student, and beginning to explore informally together;
- Incorporating the technology into classroom instruction; and
- Assessing students' use of technology, including what are they doing and what are they are learning about the technology and about the mathematics.

The PURIA model reflects the needs of teachers as adult learners. One key insight of the process of implementing

Table 1

FEATURES OF TECHNOLOGY IN THE CLASSROOM AND THE TEACHING AND LEARNING OPPORTUNITIES IT SUPPORTS			
Technological Feature	**Teaching Opportunities**	**Learning Opportunities**	**For example**
Enhanced Display	Access to data, answers, problems, tasks, lessons More efficient use of time Customized presentations	Shared attention More effective time on task	Document cameras Shared access to websites Interactive white boards
Linked Dynamic Representations	Conceptual development Modeling Visualization	Meaningful connections Engagement in richer tasks More powerful access to multiple representations	Interactive software (e.g. Fathom, Geometer's Sketchpad, ConceptuaMath)
Classroom Connectivity	Explanation and justification	Collaboration and discussion	TI-navigator Clickers
Instantaneous, Non-judgmental Feedback	Formative assessments	Responsiveness to student thinking	Instructional courseware
Differentiation and adaptivity	Adaptive and customized assignments Multiple activities Responsive to student thinking Scaffolding	Individualization Linked to dominant learning style Hints	Learning Management Systems
Social Networking	Provides real-time learning support	Access to help/support	Class blogs Virtual coaching
Embedded Resources	Audio and video prompts Online calculators Dictionary and thesaurus Translator	Seamless access to supports	Embedded links Spellcheckers

technology that PURIA reveals is how teachers experience the first three modes—*play, use, and recommend*—and need to feel comfortable in each before they incorporate technology in their classrooms. When teachers aren't given a chance to feel comfortable and explore informally cases show it can be counterproductive for them to bring technology in their classroom (Zbiek & Hollebrands, 2008). In addition to feeling comfortable with the technology, many teachers need time to change their views about the role of technology, to understand what technology can bring to an understanding of important mathematics concepts, to feel ready to act as a collaborator with students during the process of using technology, and to be comfortable in new methods of instruction that incorporate technological tools.

One way to support teachers in gaining the skills, views, and dispositions necessary to successfully bring technology into their mathematics teaching is to create time for teachers to work with technology outside of formal workshops. It takes time to *play* with and *use* technology and then, as teachers are ready, it takes time and opportunities for teachers to work together to discuss, recommend, and explore technology with their colleagues The process teachers go through as outlined in the PURIA model often helps deepen their understanding of mathematics as well as the technology itself.

Indeed, researchers have articulated the concept of "Technological Pedagogical Content Knowledge" (TPCK) to highlight how expert teachers intertwine their knowledge of technology, pedagogy, and mathematical content (Mishra & Koehler, 2006). To develop TPCK, teachers need both formal professional development and opportunities to collaborate with other teachers while they are moving from early exploration *play* and *use* to *incorporating* and *assessing*. Time and opportunity for such interactions to occur is essential. Online communities, like Tapped In or the Math Forum can either provide a space for teachers from a school to work together online or connect teachers to technology innovators in other locations. The support a community provides, either on line or face-to-face or both, helps provide the support needed to solidify important understandings around new innovations (Schlager & Fusco, 2003).

Finally, it is worth noting that effectively using technology in the classroom often involves addressing a broad range of changes (Ruthven, 2009). Necessary changes can include re-arrangement of the physical space of the classroom, integration of technology with books and other curricular resources, adoption of new activity formats, and emphasis on new or different pedagogical skills.

How NCSM Members Can Support the Improvement of Student Achievement by Systematically Integrating Effective Technology

The opportunity to integrate technology into mathematics classrooms on a broader scale can be a catalyst for much-needed improvements in all aspects of mathematics teaching and learning. Alternatively, inappropriate, ill-conceived, or poorly-supported technology initiatives can be a wasteful distraction from the core practices of mathematics teaching and learning. NCSM members have the potential to make the difference by undertaking the following practices:

1. Advocating for the systematic integration of appropriate technology in all mathematics classrooms by:
    a. Using conversations about technology as opportunities to educate others about best practices in mathematics teaching and learning.
    b. Preventing technology fads from driving pedagogical decisions.
    c. Articulating the specific needs of mathematics teachers within technology polices and building relationships with IT leaders so that mathematics teachers garner the needed access and support.
    d. Securing buy-in, commitment, and necessary funding from administrative leaders for a systematic, integrated, long-term approach to incorporating technology in mathematics classrooms and providing necessary training in the effective use of this technology.

2. Promoting the unique value of technology in mathematics teaching and learning, including asking questions such as:
    a. How does this technology promote more effective classroom presentations and display of mathematical ideas?
    b. How does this technology provide dynamic representations to deepen students understanding of mathematical connections among graphical and linguistic ways of expressing the same mathematics?
    c. How does this technology enable formative assessment that is quicker, deeper, broader, and more directly useful in guiding further teaching and learning?
    d. How does this technology organize productive structures for collaborative learning and increase student participation in mathematical explanation and argumentation?

3. Developing and implementing detailed plans to support teachers in systematically integrating technologies as part of their permanent repertoire of improved classroom practices by:
    a. Providing high quality professional development, involving long-term engagement in professional communities and collaborative opportunities for teachers to informally explore new technologies.
    b. Promoting integration of technology within curriculum, pedagogy, and assessment and avoiding the temptation to see technology as an independent, isolated component.
    c. Seeking continuous learning about best practices in the use of technology in mathematics classrooms including keeping up with rapid evolution of applications of technology and the expanding research base for its use in improving teaching and learning.

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Mathematics Coaching Knowledge: Domains and Definitions

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A coach can be broadly defined as a person who works collaboratively with a teacher to improve that teacher's practice and content knowledge, with the ultimate goal of affecting student achievement. The National Mathematics Advisory Panel (2008) reports that school districts across the country are using mathematics specialists, including coaches, to improve instruction in elementary school systems. They also note that there is little research supporting the effectiveness of mathematics specialists.

Though limited, the evidence supporting the effectiveness of mathematics coaches is growing. There are a handful of studies showing indications of a connection between coaching and student mathematics achievement (Brosnan & Erchick, 2009; Campbell, 2010; Campbell & Malkus, 2009; Meyers & Harris, 2008; Wilkins, 1997), and if we broaden our focus to coaching in any content area, there is additional evidence that coaching is effective in supporting teacher change (Bowman & McCormick, 2000; Doughtery, 1993; Heberly, 1991; Kohler, Crilley, Shearer, & Good, 1997; Munro & Elliott, 1987; Showers & Joyce, 1996; Sparks & Bruder, 1987; Wineburg, 1995). A result of particular interest to professional developers is a finding by Knight (2004) that, within the first six weeks of the school year, instructional coaches reported that 85% of coached teachers had implemented at least one strategy learned in a summer workshop, compared to 10% of teachers who received no coaching support. Other studies have shown that coached teachers are more likely to engage in collaborative activities and that coached teachers believe their students learn more because their practice has been strengthened as a result of being coached (Sparks & Bruder, 1987; Smylie, 1989).

Not every study has found positive effects for coaching. Murray, Ma, and Mazur (2009) found no increases in student achievement due to peer coaching. Gutiérrez, Crosland, and Berlin (2001) found that coached teachers did not change their instruction in substantive ways. Olson and Barrett (2004) found that individual coaching sessions had limited success in supporting teachers' professional growth. These findings raise important questions about what it means to be an effective coach.

A closer look at one of the studies of mathematics coaching offers insights into a difficulty in studying its impact on instruction and student performance. While Campbell and Malkus (2009) found that the use of elementary school mathematics coaches had a significant positive impact on student achievement, the effect only emerged as a coach gained experience in the position. Moreover, the mathematics coaches in the study were highly trained, having completed five mathematics content courses and two leadership courses specifically designed to prepare them for their coaching assignments. According to Campbell and Malkus (p. 22), "simply allocating funds and then filling the position of elementary mathematics coach in a school will not yield increased student achievement. A coach's positive effect on student achievement develops as a *knowledgeable* coach and the instructional and administrative staff in the assigned school learn and work together" [emphasis added].

Exactly what knowledge is required to create a "knowledgeable coach"? Clearly, mathematics coaches should possess mathematics content knowledge, but what additional knowledge and skills are held by effective instructional coaches? While the literature is rich in providing details about what constitutes mathematical content knowledge

(Hill, Rowan, & Ball, 2005; Ball, Bass, & Hill, 2003; Shulman, 1986), mathematics coaching knowledge has largely been without formal definition.

Identifying what constitutes the knowledge for coaching is a dilemma that affects school leaders as well as researchers. With many schools turning to coaching as a school-based effort to increase teacher effectiveness and student achievement, a challenge presented to these institutions is determining what knowledge is held by effective instructional coaches. Currently, school leaders must wade through an impressive amount of literature to try to identify knowledge domains for effective coaching. (See Deussen, Coskie, Robinson, & Autio [2007] for an in-depth but not complete review of coaching literature.)

This article will describe efforts by the research project Examining Mathematics Coaching (EMC)[1] to define mathematics coaching knowledge. EMC is a five-year research and development project examining the effects of a coach's knowledge for coaching on a diverse population of K-8 teachers. Project leaders believe that knowledge of coaching significantly affects a coach's effectiveness as measured by impact on teacher practice, attitudes, and beliefs, and ultimately student achievement. EMC recognizes that researchers, professional development providers, and school leaders could benefit from more clearly defined knowledge domains for mathematics coaching. From the research point of view, exploring the impact of mathematics coaches using well-defined knowledge domains will lead to closer examination of the impact of mathematics coaches who seem knowledgeable and well-prepared for their coaching roles.

To establish coaching knowledge domains and develop an operational definition of coaching knowledge that capitalizes on the existing knowledge of experts in the field of mathematics coaching, EMC researchers convened a panel of experts. This work resulted in a set of domains of mathematics coaching knowledge and a definition for each domain.

Methodology for Creating Domains and Definitions

The project chose to use a modified Delphi method as its means to convene the expert panel. This method allowed us to bring experts in mathematics coaching to consensus around a particular topic and enhance decision-making. (For more information on the Delphi method and the variations it can take, consult Clayton, 1997; Garavalia & Gredler, 2004; and Chamberlin, 2008, and the references contained therein.) Through a series of online, text-based surveys, EMC engaged 12 panel members in three phases over 18 days. Of the 12 panelists, six are authors or co-authors of coaching or mathematics coaching books; four are directors of grant-funded professional development projects on mathematics coaching; one is a mathematics coaching practitioner; and one has studied coaching as a researcher in mathematics education. The panelists did not interact directly with each other, and EMC researchers did not know the specific authorship of panelist contributions.

The EMC Project provided expert panel members with this definition of mathematics coaching: *"A mathematics coach is an on-site professional developer who enhances teacher quality through collaboration focusing on research-based, reform-based, and standards-based instructional strategies and mathematics content that includes the why, what, and how of teaching mathematics."* Throughout the process, panelists were asked to reflect on models of coaching and report areas of coaching knowledge, unique from teaching knowledge, that contribute to effective mathematics coaching. The EMC researchers then identified domains of knowledge using qualitative analysis techniques.

At the conclusion of the panel's contributions, EMC researchers examined the panelists' responses to determine whether or not there was consensus among respondents regarding the definition of coaching knowledge provided. Based on the responses provided by panel participants, eight domains of coaching knowledge were initially identified by EMC researchers. (See Table 1 below.)

Table 1

DOMAINS OF MATHEMATICS COACHING KNOWLEDGE	
Assessment	Student Learning
Communication	Teacher Development
Leadership	Teacher Learning
Relationships	Teacher Practice

[1] Examining Mathematics Coaching is funded by The National Science Foundation through the NSF Discovery Research K-12 program, Award No. 0918326. Any opinions expressed herein are those of the authors and do not necessarily represent the views of The National Science Foundation.

The domains and draft definitions of each were then given back to the panel for critique and elaboration, although because the EMC researchers concluded that the domain and definition of communication were consistently and sufficiently identified and defined by panelists from the outset, "communication" was not included in the subsequent analysis by the panel. Once the collective thinking regarding these domains of mathematics coaching knowledge and definitions for each of the remaining seven domains were established, the expert panel provided individual levels of agreement and responded to open-ended questions on aspects of each of these definitions.

Using a five-point scale (1=strongly disagree, 2=disagree, 3=neither agree nor disagree, 4=agree, and 5=strongly agree), panelists rated their levels of agreement to definitions for each domain of mathematics coaching knowledge using two prompts: 1. This definition captures my thinking related to coaching knowledge of [each knowledge domain], and 2. This definition informs my work related to coaching knowledge of [each knowledge domain]. Based on analysis by EMC researchers, there was a high level of agreement and high level of consensus among panelists for the definitions of each of the seven domains.

Respondents were also asked to provide additional comments on definitions based on four open-ended questions that addressed words, phrases, or key features that respondents may have considered missing, unclear, or superfluous in each definition of each domain. The open-ended questions are presented in Table 2.

Based on responses from the expert panel, EMC revised the definitions of the seven domain areas, and these along with the initial definition of the domain of communication are reported in Table 3 (see pages 17-19). This generation of EMC Project-specific definitions, which reflected the EMC researchers' knowledge while considering the panelists' comments, allowed the definitions to be modified in ways that eliminated laundry lists, addressed any ambiguity in the articulation of skills, practices, and beliefs, and also framed the definitions in terms of knowledge. Some ideas were also moved from one domain to another by EMC researchers who applied a project-based filter that considered what a coach needed to know beyond what a teacher needed to know.

The EMC Project's primary purpose for conducting the study was to inform and guide instrument development to measure mathematics coaching knowledge. To help the reader understand how the panel definitions provided distinct ways of thinking about mathematics coaching knowledge, and how that allowed the project to enhance the definitions for the specific purpose of instrument development, both definitions are contained in Table 3, organized by domains.

Table 2

OPEN-ENDED QUESTIONS PERTAINING TO COACHING KNOWLEDGE DEFINITIONS
For each of seven coaching knowledge areas (100 words or less):
1. What words, phrases, or key features for the definition (if any) do you feel are missing and need to be considered for inclusion in the final definition?
2. What words, phrases, or key features (if any) do you feel are particularly unclear and need to be restated to minimize confusion or misunderstanding?
3. What (if anything) do you feel could or should be removed from the definition?
4. What other comments or suggestions do you have to enhance the overall quality and utility of the definition?

Use of Domains and Definitions

These domains and definitions present a starting point for further analysis of mathematics coaching knowledge. Until this effort by the EMC Project, identification of domains of knowledge and definitions for that knowledge were not compiled in a single resource. We believe that the present work has moved the field of coaching forward by identifying mathematics coaching knowledge domains and definitions with a high level of agreement and consensus among experts. We invite other projects and institutions to use these domains and definitions as a starting point for their own work on mathematics coaching.

For example, districts that employ mathematics coaches could use these coaching knowledge domains and definitions to identify teacher leaders who might be well-prepared to take on mathematics coaching roles. In addition, districts could use these domains to identify professional development courses that would be helpful for mathematics coaches. The EMC Project, in fact, has designed its own week-long professional development course for project coaches (to be given in 2011 and 2012) that addresses each of the mathematics coaching knowledge domains.

Other potential users of these domains and definitions are supervisors of mathematics coaches, who could use these domains and definitions to inform their support of mathematics coaches in the field. In our own project, we are able to identify which coaches appear to demonstrate the mathematics coaching knowledge domains and definitions identified in this study, and we are working to understand how these mathematics coaches make an impact on teacher practice and student achievement. This could lead to a better understanding of the degree to which specific knowledge domains contribute to desired impacts. To that end, the EMC Project has developed an instrument containing items based on the mathematics coaching knowledge definitions formed by the panel and is using this instrument to measure changes in coaching knowledge. (This EMC instrument is available for use by other educators and researchers; please contact the authors for more information.)

Of course, definitions also have value in providing a structure around which a community can reach a common understanding. We have presented these definitions at a number of national conferences and engaged participants in examining these definitions, and over time it has become clear that participants regard the definitions as very comprehensive. Some observers have expressed a concern about how realistic it is to expect a mathematics coach to know everything in every domain. It is our position that the definitions represent a starting point, so that as the community of mathematics coaches evolves, these definitions will be open to modification and discussion.

Indeed, as studies like the EMC Project continue to yield results, researchers may find that mathematics coaches who seem to have this coaching knowledge may still not have the impact one would expect because of constraints that emerge during the actual practice of mathematics coaching. This is why strong productive collaborations between mathematics coaches and the instructional and administrative staff in schools are also essential.

It may also be the case that even with the support of knowledgeable mathematics coaches, teachers may also need the support of ongoing school-based or district-based professional development that allows for in-depth explorations of mathematics content knowledge and pedagogical content knowledge as well as other aspects of mathematics teaching and learning.

Finally, even knowledgeable mathematics coaches are likely to continue to need their own professional development as they continue to work to reflect on and strengthen their mathematics coaching practice in a wide range of contexts that include a wide range of challenges.

Table 3

PROJECT DEFINITIONS OF KNOWLEDGE DOMAINS AND MODIFIED DEFINITIONS DERIVED FROM PANEL DEFINITIONS AND PHASE THREE PANELIST COMMENTS

	Panel Definitions	Project-Modified Definitions for Instrument Development
Assessment	A coach knows how to diagnose teachers' needs —personal, instructional, content, and management—and how data and assessment of student thinking inform instruction and work with teachers. The coach knows how to assess and use teacher content knowledge and pedagogical content knowledge to inform, grow, and support teachers. A coach deeply understands formative and summative classroom assessment and knows how to set goals for assessing effectiveness of lessons. A coach also knows how to select, adapt, and align curricula with assessments; knows how to use common learning trajectories; and knows when looking at student work is better than looking at numerical assessment results. The coach knows how to help teachers use assessment data to make informed decisions about instruction and student learning, and knows what teachers know about assessment, including different types, their uses, and limits.	A coach knows how to assess teachers' needs—personal, instructional, content, and management—and how to assess and use teacher content knowledge and pedagogical content knowledge to inform and support teachers. A coach knows how to determine what teachers know about assessment, including different types, their uses, and limits. A coach knows how to use data and assessment of student thinking to inform her or his work with teachers. A coach knows how to help the teacher learn how to set goals and assess lesson effectiveness. A coach also knows how to help the teacher learn when looking at student work is better than looking at numerical assessment results. The coach knows how to help teachers interpret and use assessment data to make informed decisions about instruction and student learning.
Communication	A coach knows how to communicate professionally with others about students, curriculum, and classroom practice. A coach knows how to mediate a conversation, by pausing, paraphrasing, probing, and inquiring. A coach knows how to ask reflective questions. A coach knows how to use nonverbal communication and knows how to listen actively in conversation. A coach knows how to communicate in problem-resolving conversations.	
Leadership	A coach knows leadership models and possesses the ability to identify, define, and communicate specific goals and objectives that relate to student success and align with the institution's vision for mathematics. The coach uses this vision and knowledge to inform work with other school leaders, to highlight the gap between teachers' espoused beliefs and actions, to develop trust with teachers and administrators, and to develop a deep understanding of the professional development process and impacts. A coach knows various ways to address challenges and how to communicate in ways that advocate for, negotiate with, and influence others.	A coach knows how to strategically identify, define, and communicate specific goals and objectives that relate to student success and teachers' professional growth, and align with the institution's vision for mathematics. The coach uses this vision and knowledge to inform her or his work with other school leaders, to bridge the gap that may exist between teachers' beliefs and their ability to implement instruction that reflects those beliefs, to earn trust with teachers and administrators, and to enhance teachers' content knowledge. The coach knows whether educational structures and policies impede or promote students' equitable access to quality instruction. The coach knows how to hold teachers, administrators, and schools accountable. The coach knows the coaching process and how to implement it. The coach knows how to address challenges and how to extend teacher cognitive processes regarding instruction – planning, doing, reflecting – and how to advocate for, negotiate with, and influence others.

Table 3 (continued)

	Panel Definitions	Project-Modified Definitions for Instrument Development
Relationships	A coach knows how to communicate professionally with a variety of audiences, and knows how to establish and maintain rapport and credibility with teachers based on trust, empathy, mutual understanding, and confidentiality. A coach knows about environments where positive relationships take place, including challenging and safe learning environments for teachers and students, collaborative working environments, and environments where people share common beliefs and goals with honest reflection. The coach knows how autonomy, issues of authority, and socio-cultural aspects of class, race, and gender for students and teachers influence relationships. A coach knows a range of concepts, theories, and frameworks (e.g., adult development, educational belief systems, cognitive styles, etc.) and how those relate to teachers, teachers' views of teaching and learning, and students.	The coach knows that the coaching relationship is grounded in content and how to use the relationship to support self-directedness in teachers. A coach knows how to communicate professionally with a variety of audiences, and knows how to establish and maintain rapport and credibility with teachers and other stakeholders based on trust, empathy, mutual understanding, and confidentiality. A coach knows about environments where positive relationships take place, including challenging and safe learning environments for teachers and students, collaborative working environments, and environments where people share common beliefs and goals with honest reflection. The coach knows how to work within the specific culture of the district and school. The coach knows how autonomy, issues of authority, and socio-cultural aspects of class, race, and gender for students and teachers influence relationships and influence perceptions and models of help and authority.
Student Learning	A coach knows how to create and manage mathematical learning environments that mediate factors in the K-8 spectrum including students' prior learning, age, race, gender, economic status, special needs, socio-cultural events, and school/district dynamics. A coach knows how to analyze student thinking and conduct mathematical error analysis, and has facility with a variety of instructional formats and strategies (mathematical discourse, mathematical exploration, meta-cognition, etc.) that help students engage in challenging and meaningful mathematics problems and tasks. A coach knows how to develop and how to provide teachers with learning opportunities aimed at improving student learning by analyzing student work.	A coach knows how to support teachers in analyzing student thinking and conducting mathematical error analysis, and knows how to support teachers in acquiring facility with mathematical processes (mathematical discourse, mathematical exploration, meta-cognition, etc.) that help students engage in challenging and meaningful mathematics problems and tasks. A coach knows how to develop and how to provide teachers with learning opportunities aimed at improving student learning by analyzing student work and student ideas as they are presented in the classroom. A coach knows how to help teachers recognize evidence of learning potential and deficits in student work. A coach knows how to help teachers become proficient at creating and managing mathematical learning environments in the K-8 spectrum. A coach knows how to support teachers in acquiring the ideas and the continuum of ideas in the K-8 mathematics classroom. A coach knows the research about student learning in mathematics.

Table 3 (continued)

	Panel Definitions	Project-Modified Definitions for Instrument Development
Teacher Development	A coach knows various models of teacher stages of development, adult change, and the continuum of learning that teachers often experience (e.g., from beginning to experienced to expert teacher). A coach knows how to diagnose where a teacher is, recognize potential learning trajectories, and differentiate strategies to support an individual teacher's growth. A coach knows the motivations for growth and barriers to growth and recognizes the role of reflection and feedback.	A coach knows various models of teacher stages of development, adult change, and the continuum of learning (e.g., from beginning to experienced to expert teacher; or from an unsophisticated view of teaching to a sophisticated one) that teachers often experience in exploring content knowledge, pedagogy, beliefs, and management. A coach knows how to ascertain a teacher's understanding of mathematics, teaching, and learning and is able to differentiate experiences to support an individual teacher's learning. A coach knows teachers' motivations for learning and barriers to learning and supports the development and use of reflection and feedback to enhance teaching and learning.
Teacher Learning	A coach knows about teacher motivations; the myriad ways that teachers know and understand mathematics content; and the teacher's pedagogical and pedagogical content needs, which may or may not be recognized by the teacher. A coach knows developmental continuums, potential learning trajectories, and teacher beliefs about learning. A coach knows how to develop tasks that support teacher learning through reflective practice and self-directed goal-setting, and knows when to consult and when to collaborate. A coach knows there can be a gap between a teacher's knowing a strategy and effectively using a strategy.	A coach knows about internal and external teacher motivations and about effectively engaging teachers in the coaching process. A coach knows the myriad ways teachers know and understand mathematics content and the teacher's pedagogical and pedagogical content needs, which may or may not be recognized by the teacher. A coach knows about how an individual teacher best learns, incorporating knowledge about developmental continuums and teacher beliefs about learning. A coach knows how to support teacher learning through reflective practice and self-directed goal-setting. A coach knows how to help teachers recognize that there may be a discrepancy between vision and practice and how to help the teacher address that discrepancy.
Teacher Practice	A coach knows teacher beliefs about teaching practice, along with a depth and breadth of knowledge of all types of practice for effective management and learning (e.g., lesson planning, school support structures, learning environments, models of instruction, mathematical tasks, assessment, and strategies that support students based on factors such as age, gender, culture and ELLs, etc.). A coach knows when to use these and how they translate into teacher actions in classrooms for effective teaching and learning. Additionally, a coach knows research on effective teaching and learning, instructional strategies, and cognitive development on how children and adolescents learn.	A coach knows how to discern teacher beliefs about mathematics teaching practice and holds a depth and breadth of knowledge of all types of practice and instructional resources for effective management and mathematics learning. A coach knows how these practices and resources translate into teacher actions in mathematics classrooms for effective teaching and learning.

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Standards for Computational Fluency: A Comparison of State and CCSS-M Expectations

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The adoption of *Common Core State Standards for Mathematics* (CCSS-M) by nearly all of the states (except Alaska, Minnesota, Texas, Virginia) represents an historic landmark in curriculum governance in the U.S. In just over two decades the U.S. has moved from a vision and standards for school mathematics developed by the National Council of Teachers of Mathematics (*Curriculum and Evaluation Standards for School Mathematics, 1989*) to consensus by nearly all of the states on grade-level mathematics learning goals for K-8 students.

While consensus on mathematics standards alone will not improve learning opportunities for students or provide necessary support for teachers, the standards do represent an important element in a standards-based reform system for nationwide improvement (Goertz, 2010). The CCSS-M aligned assessments under development by two state consortia (PARCC and SMARTER Balanced Assessment) have the potential to provide the other "bookend" of the standards system (Confrey & Krupa, 2010). What lies between these bookends (mathematics standards and aligned accountability assessments) is the important work of teachers, curriculum developers, and instructional leaders. In this regard CCSS-M provides an important opportunity for mathematics educators.

In 2006 a team from the Center for the Study of Mathematics Curriculum (CSMC) conducted a content analysis of a set of topics within the K-8 sections of 42 state standards documents. While not a comprehensive analysis of all the standards, a few topics were selected for in-depth analysis. One of these topics was computational fluency. Specifically, the team investigated when, according to the state standards, students are expected to begin study of computational methods and over what period of time (grades) they are expected to acquire fluency with whole number and fraction computation.

The study verified what many had believed—that state standards varied considerably with regard to the grade placement and language used to describe key learning goals K-8. The lack of consensus across the state standards is conjectured to be one of the primary causes for the repetitive nature of many of the textbooks available to schools and teachers over the past two decades. A full report of the CSMC analysis is available elsewhere (Reys, 2006).

In this article we compare learning expectations included in state standards, as described in 2006, regarding computational fluency with those outlined in CCSS-M. This summary is limited to four specific computational goals of school mathematics. Specifically, we summarize how the state and CCSS-M standards compare with regard to fluency with:

- Basic Number Combinations (Basic Facts)
- Multi-digit Whole Number Computation
- Fraction Computation
- Decimal Computation

In each case, we begin with findings from the earlier analysis of state standards. We then compare these findings to the set of parallel expectations (standards) articulated in CCSS-M.

Documentation of the changes in grade placement or nature of expectation will highlight areas where specific work is needed in both curriculum development and instructional planning.

Fluency with Basic Number Combinations (Basic Facts)

Basic number combinations are generally defined as the set of single-digit addition (or multiplication) combinations and their related subtraction (or division) combinations (e.g., 4+5=9; 9-4=5; 3x8=24; 24÷3=8). Fluency is the goal. That is, students are expected to recall quickly the sum, difference, product or quotient so that it can be used in various contexts, including problem solving, without undue delay.

Addition and Subtraction - Basic Number Combinations. In 2006, state standards articulated expectations regarding fluency in a variety of ways and sometimes at different grade levels. For example,

- Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory. (California, Grade 1)
- Demonstrate computational fluency for basic addition and subtraction facts with sums through 18 and differences with minuends through 18, using horizontal and vertical forms (Alabama Department of Education, 2003, Grade 2)
- State addition and subtraction facts. (Arizona Department of Education, 2003, Grade 2)

In addition to using particular language (e.g., state; commit to memory; fluency, horizontal and vertical forms) to specify the nature and parameters of the expectation, state standards also included guidance regarding when the work should start and often suggested initial parameters (number size or strategies) of development. For example, the Arkansas Department of Education Standards (2004) included the following progression spanning Kindergarten to Grade 2:

Kindergarten:

- Develop strategies for basic addition facts (counting all; counting on; one more, two more)
- Develop strategies for basic subtraction facts (counting back; one less, two less)

Grade 1:

- Develop strategies for basic addition facts (counting all; counting on; one more, two more; doubles; doubles plus one or minus one; make ten; using ten frames; Identity Property (add zero)
- Develop strategies for basic subtraction facts (relating to addition Ex. Think of 7 – 3 =____as "3 +___= 7"; one less, two less; all but one Ex.9 – 8, 6 – 5; using ten frames of the answers

Grade 2:

- Develop strategies for basic addition facts (counting all; counting on; one more, two more; doubles; doubles plus one or minus one; make ten; using ten frames; Identity Property (add zero) [standard repeated from Grade 1]
- Demonstrate computational fluency (accuracy, efficiency and flexibility) in addition facts with addends through 9 and corresponding subtractions (Ex. 9+9=18 and 18-9=9 add and subtract multiples of ten)

The 2006 analysis revealed that in most states, attention to developing fluency with basic number combinations began one or more years prior to when proficiency/fluency was expected. For example, 25 states introduced addition of basic number combinations in first grade. Introduction generally referred to exploration of strategies and/or focusing on a subset of number combinations. In 8 states fluency was expected by the end of grade 1. More states (27) specified this expectation by the end of Grade 2 and two states denoted it for Grade 3 (see Reys, 2006 for a detailed summary).

In CCSS-M the set of expectations and their progression regarding addition/subtraction of basic number combinations are as follows:

Kindergarten:

- Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. *(K.OA.2[1])*
- Fluently add and subtract within 5. *(K.OA.2)*

Grade 1:

- Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing,

[1] The labeling of standards used in this article is as they appear in the CCSS-M document.

with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (1.OA.1)

- Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). *(1.OA.6)*
- Develop strategies for basic subtraction facts (relating to addition Ex. Think of 7 – 3 =____as "3 +___= 7"; one less, two less; all but one Ex.9 – 8, 6 – 5; using ten frames of the answers

Grade 2:

- Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. *(2.OA.2)*

While the topic is generally introduced in Grade 1 in the state standards, it begins in Kindergarten in CCSS-M. The CCSS-M standards, like many state standards, clearly emphasize the importance of acquiring fluency by building onto and using physical and mental models and strategies based on conceptual understanding. The CCSS-M grade specification for fluency with addition and subtraction basic number combinations (Grade 2) is consistent with the expectation and grade placement of a majority of state standards in 2006. However, the terminology used within CCSS-M to denote particular number parameters (e.g., "add and subtract **within 10**" (emphasis added)) is not common in state standards. This expression is defined in the glossary of CCSS-M (p. 85) as:

> ***Addition and subtraction* within *5, 10, 20, 100, or 1000.*** Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Multiplication and Division - Basic Number Combinations. In 2006 most state standards denoted fluency with basic facts for multiplication in Grade 3 (13 states) or Grade 4 (22 states) and for division in Grade 4 (20 states). As with addition and subtraction, state standards specify introduction of this topic, generally with a subset of number combinations, one or more grades prior to the expectation of fluency.

In CCSS-M expectations related to fluency with multiplication and division combinations are concentrated in Grade 3. Two standards convey the expectation. The first emphasizes use of strategies based on applications of arithmetic properties:

- Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) *(3.OA.5)*

The second standard conveys the expectation of fluency (know from memory) by the end of Grade 3:

- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. *(3.OA.7)*

A comparison of the CCSS-M progression with state standards reveals a shift for most states. That is, in 2006 24 states expected students to acquire fluency with multiplication and division basic number combinations at a later grade, generally at Grade 4. Likewise, in the majority of state standards in 2006, acquisition of fluency with multiplication and division combinations is spread over at least two years (Grades 2 and 3 or Grades 3 and 4) while in CCSS-M this work is concentrated in Grade 3, with no mention in prior grades.[2]

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[2] There is preliminary work on relating addition and multiplication as operations in Grade 2.

Table 1

GRADE PLACEMENT OF LEARNING EXPECTATIONS RELATED TO FLUENCY WITH BASIC NUMBER COMBINATIONS FOR EACH OPERATION[3]					
Operation	**Grade**	**Number of States (N=39)**	**Operation**	**Grade**	**Number of States (N=39)**
Addition	1	8	Subtraction	1	7
	2	28 (CCSS-M)		2	27 (CCSS-M)
	3	2		3	3
	Not specified	1		Not specified	2
Multiplication	3	13 (CCSS-M)	Division	3	6 (CCSS-M)
	4	22		4	20
	5	1		5	3
	6	1		6	1
	Not specified	2		Not specified	9

Table 1 summarizes the grade placement of the expectation for fluency with basic number combinations for each operation across states in 2006 as well as within CCSS-M. As noted, the grade at which fluency with addition and subtraction number combinations was expected in 2006 is generally consistent with CCSS-M (Grade 2). However, CCSS-M specifies fluency with multiplication and division number combinations about one year earlier than most state standards and concentrates this work within one grade level (Grade 3).

Multi-Digit Whole Number Computation

Addition and Subtraction. Across the set of 42 state standards reviewed in 2006, the progression leading to fluency with multi-digit addition varied considerably. For example, in some states, students were expected to begin adding multi-digit numbers as early as Kindergarten or as late as Grade 3. The culminating standard (grade at which fluency was expected) ranged from Grade 1 to 6. The span within a given state between when the topic was introduced and when fluency was expected ranged from 1 to 4 grades, with two or three grades the most typical span.

In CCSS-M the progression towards fluency with multi-digit addition of whole numbers begins in Grade 1 with the following standard:

- Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. *(CCSS-M 1.NBT.4)*

The culminating standard for multi-digit whole number addition is in Grade 4:

- Fluently add and subtract multi-digit whole numbers using the standard algorithm. *(CCSS-M 4.NBT.4)*

The complete set of CCSS-M standards related to multi-digit whole number addition is shown in Table 2 (see next page).

Multiplication and Division. For multiplication of multi-digit whole numbers CCSS-M specifies a progression of standards (see Table 3, page 26), starting in Grade 3. Specifically, students are expected to first use strategies

[3] Thirty-nine state standards were included in the analysis of fluency with basic number combinations (three other states in the main analysis did not include standards for primary grades).

based on place value and arithmetic properties to multiply 1-digit whole numbers by multiples of 10 (less than 100). Strategy use continues in Grade 4 with more complex combinations (multiply 1-digit whole numbers by up to 4-digit whole numbers and 2-digit whole numbers by 2-digit whole numbers). The culminating standard is noted in Grade 5, specifying "fluency multiplying multi-digit whole numbers using the standard algorithm."

This progression is similar to what a few states outlined in 2006. For example, in most states, multiplication of multi-digit whole numbers began in Grade 3 (19 states) or

Table 2

PROGRESSION OF STANDARDS RELATED TO MULTI-DIGIT WHOLE NUMBER ADDITION/SUBTRACTION IN CCSS-M	
Grade 1	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. *(1.NBT.4)*
	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. *(1.NBT.5)*
	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. *(1.NBT.6)*
Grade 2	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *(2.OA.1)*
	Fluently add and subtract within 20 using mental strategies. *(2.OA.2)*
	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. *(2.NBT.5)*
	Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.6)
	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three- digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. *(2.NBT.7)*
	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (2.NBT.8)
	Explain why addition and subtraction strategies work, using place value and the properties of operations. *(2.NBT.9)*
	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g. by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. *(2.MD.2)*
Grade 3	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. *(3.NBT.2)*
Grade 4	Fluently add and subtract multi-digit whole numbers using the standard algorithm. *(4.NBT.4)*

Table 3

PROGRESSION OF STANDARDS RELATED TO MULTI-DIGIT WHOLE NUMBER MULTIPLICATION/DIVISION IN CCSS-M	
Grade 3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (3.NBT.3)
Grade 4	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.5)
Grade 5	Fluently multiply multi-digit whole numbers using the standard algorithm. (5.NBT.5)

Grade 4 (21 states). In about half the state standards, fluency is expected in Grade 4 (20 states). In 15 states, fluency is expected in Grade 5.

In general, the progression outlined in CCSS-M for multi-digit whole number multiplication is similar to that outlined in most state documents, although the work is generally condensed into two rather than 3 years. One difference between CCSS-M and most state standards is the reference to "the standard algorithm." The glossary of CCSS-M defines "computational algorithm" as, "A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly." However, no definition or description for "the standard algorithm" is provided.

The first CCSS-M standard regarding whole number division (multi-digit numbers) is at Grade 4:

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *(4.NBT.6)*

In Grade 5 the expectation is extended:

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *(5.NBT.6)*

The culminating expectation is found at Grade 6, again by demonstration of fluency with "the standard algorithm:"

- Fluently divide multi-digit numbers using the standard algorithm. *(6.NS.2)*

Table 4 (see next page) includes a summary of the grade at which the culminating standard (expectation for fluency) for each operation (addition, subtraction, multiplication and division) is denoted across state standards, based on the 2006 analysis. Also noted is the grade within CCSS-M where the expectation of fluency is noted. *In general, the grade at which CCSS-M specifies multi-digit whole number fluency with each operation is at or slightly later than specified in 2006 in most state standards documents.*

As noted earlier, the specific statement of the culminating standard for each operation in CCSS-M includes the expectation of use of "the standard algorithm." For example:

- Fluently add and subtract multi-digit whole numbers using the standard algorithm. *(4.NBT.4)*
- Fluently multiply multi-digit whole numbers using the standard algorithm. *(5.NBT.5)*
- Fluently divide multi-digit numbers using the standard algorithm. *(6.NS.2)*

However, a definition for "the standard algorithm" is not offered. If the authors of CCSS-M had a particular standard algorithm in mind, it was not made explicit nor is an argument offered for why a particular (standard) algorithm is expected.

Fraction Computation

Tables 5 and 6 (see pages 28-29) include all of the CCSS-M standards related to computation with fractions. As noted in Table 5, the CCSS-M standards related to addition and subtraction of fractions begin with addition of like denominators (Grade 4) followed by denominators of 10 and 100 (Grade 4) then focus on the more general

Table 4

GRADE PLACEMENT FOR CULMINATING LEARNING EXPECTATIONS RELATED TO FLUENCY WITH WHOLE NUMBER COMPUTATION FOR EACH OPERATION (N=42)					
Operation	**Grade**	**Number of States**	**Operation**	**Grade**	**Number of States**
Addition	1	8	Subtraction	1	1
	2	3		2	2
	3	14		3	15
	4	15 (CCSS-M)		4	15 (CCSS-M)
	5	5		5	5
	6	3		6	3
	N/S*	11		N/S	1
Multiplication	3	2	Division	3	0
	4	21		4	12
	5	15 (CCSS-M)		5	23
	6	3		6	6 (CCSS-M)
	N/S	1		N/S	1

**Not specified within state document.*

case, unlike denominators, by use of equivalent fractions in Grade 5. Unlike computation with whole numbers, CCSS-M standards related to addition and subtraction of fractions do not use the term "standard algorithm." However, a specific strategy, including its general form, is included in the statement of the standard:

> *Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* (5.NF.1)

A general finding in the analysis of state standards in 2006 was variability in the grade levels at which addition, subtraction, multiplication, and division of fractions was introduced (see Table 7, pg. 30) and when fluency was expected (see Table 8, pg. 30). As noted in Table 7, more than half of the states' standards reviewed introduced addition and subtraction of fractions in Grade 4 and this is also where the first standard in CCSS-M related to this topic is found:

> *Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.* (CCSS-M, 4.NF.3c)

However, CCSS-M deviates from all but one state by introducing multiplication of fractions in Grade 4. As Table 7 indicates, the majority of states (25 of 42) introduce multiplication of fractions two years later, in Grade 6. In fact, it is customary in most state standards to develop both multiplication and division of fractions in the same grade. CCSS-M's introduction of fraction division in Grade 5 represents acceleration of grade placement in all but 7 states.

In general, the timeline from introduction of the concept of fraction to beginning computation with fractions, then to expectation of fraction computation fluency is more condensed in CCSS-M than in state standards reviewed in 2006. For example, the first standard related to the concept of fraction in CCSS-M is at Grade 3; addition, subtraction, and multiplication of fractions is introduced in Grade 4;

and fluency is expected for these three operations on fractions in Grade 5. Introduction of fraction division appears in Grade 5 in CCSS-M, followed by expectation for fluency the following year, in Grade 6.

Decimal Computation

Relatively few CCSS-M standards address decimal computation; in fact, only two standards (one in Grade 5 and the other in Grade 6) specifically identify computation with decimals:

> *Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.* (CCSS-M 5.NBT.7)

> *Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.* (CCSS-M 6.NS.3)

Subsequent standards in Grade 7 use the more general set of "rational numbers" when specifying computation expectations. For example:

> *Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* (7.EE.3)

As noted, the second standard (CCSS-M 6.NS.3) refers to "the standard algorithm" without explicitly defining this strategy. CCSS-M also calls for work with fraction computation to precede that of decimal computation. Some state standards in 2006 mirrored this approach while others reversed the order, emphasizing computation with decimals prior to that of fractions, likely in order to build upon the similar strategies of whole number computation.

Table 5

PROGRESSION OF STANDARDS RELATED TO FRACTIONAL ADDITION AND SUBTRACTION IN CCSS-M	
Grade 4	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. *(4.NF.3c)*
	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. *(4.NF.3d)*
	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. *(4.NF.5)*
Grade 5	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.) *(5.NF.1)*
	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. *(5.NF.2)*
Grade 7	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. *(7.NS.1)*
	Solve real-world and mathematical problems involving the four operations with rational numbers. *(7.NS.3)*

Table 6

PROGRESSION OF STANDARDS RELATED TO FRACTIONAL MULTIPLICATION AND DIVISION IN CCSS-M.	
Grade 4	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? *(4.NF.4c)*
Grade 5	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? *(5.NF.3)*
	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. *(5.NF.4)*
	Interpret multiplication as scaling (resizing) by: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (nxa)/(nxb) to the effect of multiplying a/b by 1. *(5.NF.5b)*
	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *(5.NF.6)*
	Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. *(5.NF.7a)*
	Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. *(5.NF.7b)*
	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? *(5.NF.7c)*
Grade 6	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? *(6.NS.1)*
Grade 7	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. *(7.NS.2)*
	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. *(7.NS.2a)*
	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts. *(7.NS.2b)*
	Apply properties of operations as strategies to multiply and divide rational numbers. *(7.NS.2c)*
	Solve real-world and mathematical problems involving the four operations with rational numbers. *(7.NS.3)*

Table 7

NUMBER AND GRADE-LEVEL WHEN STATE STANDARDS INTRODUCE EXPECTATIONS RELATED TO COMPUTATION WITH FRACTIONS			
Grade	**Addition & Subtraction of Fractions**	**Multiplication of Fractions**	**Division of Fractions**
1st grade	2 states		
2nd grade			
3rd grade	7 states		
4th grade	22 states (CCSS-M)	1 state (CCSS-M)	1 state
5th grade	9 states	10 states	6 states (CCSS-M)
6th grade	1 state	25 states	27 states
7th grade	1 state	5 states	6 states

Table 8

NUMBER OF STATES AND GRADE-LEVEL WHEN STATE STANDARDS INDICATE EXPECTATION OF FLUENCY WITH ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF FRACTIONS[4]			
Grade	**Addition & Subtraction of Fractions**	**Multiplication of Fractions**	**Division of Fractions**
4th grade	1 state		
5th grade	15 states (CCSS-M)	2 states (CCSS-M)	1 state
6th grade	20 states	25 states	24 states (CCSS-M)
7th grade	6 states	13 states	14 states
8th grade		1 state	1 state

According to the 2006 state standards, computational fluency with decimals was to be developed in the late elementary and early middle school years, spanning from Grade 4 to Grade 7 depending upon the particular state. In CCSS-M, fluency with decimal operations is expected one year after students are first introduced to decimals. This represents another potentially condensed progression toward computational fluency.

Summary

The purpose of this discussion is to highlight within one area of the K-8 mathematics curriculum where CCSS-M represents a shift in current practice. The general findings of this analysis include:

- CCSS-M specifies computational fluency in the following areas in the SAME grade level as most state standards in use in 2006:
  - Addition and subtraction of basic number facts (Grade 2);
  - Addition and subtraction of multi-digit whole numbers (Grade 4);
  - Division of fractions (Grade 6).
- CCSS-M specifies computational fluency in the following areas at an EARLIER grade level compared to most state standards in use in 2006:
  - Multiplication and division of basic number facts (Grade 3);

[4] For this analysis, we used the culminating learning expectation that indicated students were working with common and uncommon denominators when adding and subtracting fractions.

Table 9

GRADE LEVEL AT WHICH CCSS-M INDICATES EXPECTATION OF COMPUTATIONAL FLUENCY						
	Gr1	**Gr2**	**Gr3**	**Gr4**	**Gr5**	**Gr6**
Basic Facts – Addition/Subtraction		x				
Basic Facts – Multiplication/Division			x			
Multi-digit numbers –Addition/Subtraction				x		
Multi-digit numbers – Multiplication					x	
Multi-digit numbers – Division						x
Fractions – Addition/Subtraction					x	
Fractions – Multiplication					x	
Fractions – Division						x
Decimals – Addition/Subtraction						x
Decimals – Multiplication/Division						x

– Addition, subtraction and multiplication of fractions (Grade 5).

- CCSS-M specifies computational fluency in the following areas at a LATER grade level compared to most state standards in use in 2006:
  – Multiplication of multi-digit numbers (Grade 5);
  – Division of multi-digit numbers (Grade 6).
- Attention to fluency with basic facts, whole number multiplication, and fractional operations is condensed over a shorter grade span in CCSS-M than in the majority of state standards in use in 2006.
- Decimal computation receives very little attention in CCSS-M (two standards) and follows the development of fraction computation.
- In several instances, CCSS-M specifies use of a 'standard' algorithm (e.g. for whole number computation) without specification of meaning of the specific algorithm intended. For fraction computation, a particular strategy is specified and associated with fluency.
- There is considerable attention in CCSS to applying computational procedures to "word problems."
- Prior to an expectation of fluency, CCSS-M standards specify use of models and individually derived strategies for computing.

This analysis confirms that there is a concentration of work on computation of whole numbers, fractions and decimals in Grades 3-6 (see Table 9). The majority of work with fractions is expected one year earlier than in most state standards published in 2006. Part of the rational for this shift is to "make room for" new emphasis on particular mathematics content (e.g., algebra) introduced and developed in the middle grades. However, given that this is a new roadmap for school mathematics and that there will be a significant transition period, it is not clear when or if middle grade teachers can count on students entering middle grades ready to undertake the advanced content.

As noted, this discussion has focused exclusively on computation. It is likely that in other topic areas there are similar or perhaps more dramatic shifts in learning goals at particular grades. These shifts will need to be carefully outlined and discussed by teachers and they will need new curriculum materials to help guide instruction designed to support student learning of the goals of CCSS-M.

CCSS-M provides a rather ambitious roadmap for changes in the K-8 school mathematics program - an important ingredient for systemic and widespread mathematics reform efforts. A strong formative and summative assessment system aligned with CCSS-M is also needed and under development by two state-led consortia. The next steps include formulating a system for closely monitoring and adjusting CCSS-M so that common standards represent a continually improving roadmap for intended learning goals. Equally important are curriculum resource development and large-scale efforts to improve teaching. As a leading educator has recently argued, unless there is a concerted effort to make improvements in teaching of mathematics, it is difficult to see how the goals of CCSS-M will be realized. (Ball, 2011).

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Organizing a Family Math Night

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Family involvement is something that all schools seek to cultivate, and a number of resources have been created to support these efforts. For example, *A Family's Guide: Fostering Your Child's Success in School Mathematics* (NCTM, 2004) is designed for families and summarizes what today's mathematics classroom is like, offers tips for family members on how to help children develop positive attitudes toward mathematics and presents practical suggestions for doing mathematics at home together. Another similar resource, *Helping Children Learn Mathematics* (NRC, 2002), outlines the important role of parents and other caregivers in supporting mathematics learning, including what it means to become mathematically proficient and what parents and caregivers can do to support these proficiencies.

We know that families are more likely to become involved in the school community when family involvement is encouraged by teachers and administrators (Drummond & Stipel, 2004). One strategy for encouraging family involvement is to host Family Math Nights at the school. While a number of books and articles have been written about Family Math Nights (e.g., Hall & Acri, Kyle, 1995; McIntyre, Miller, & Moore, 2001; Taylor-Cox, 2005), these primarily focus on the games and activities themselves. This paper describes how we planned and coordinated our Family Math Nights, the majority of which were held in high-poverty communities that serve minority populations, and where participation ranged from 200 and 400 family members. We believe our planning and coordination contributed to the success of these events.

Advertising, Recruitment, Incentives, and Funding

Each school designated a teacher or administrator as the organizer for the event. This particular individual did not necessarily need to be a mathematics specialist. The most important qualification is that they were dedicated to ensuring participation in the event by consistently following-up with students and families to encourage their attendance.

An important part of the process of ensuring participation was the design of a flyer to advertise the event. The flyer included logistical information as well as advertising that pizza and door prizes would be available. These flyers were distributed widely in the community as well as sent home with students. Family members were also encouraged to attend the event when dropping off or picking up their children or whenever they were present in the school.

Another important part of the process involved having families RSVP a week before the event in order to have an accurate count for refreshments and door prizes. Families were informed they would receive an additional number of game tickets for the event if they responded by the deadline. One school informed families that completed RSVP forms would be drawn daily from a box of completed forms for daily prizes. This particular strategy resulted in the greatest level of participation for any of the Family Math Night events we hosted.

The funding for an event like this may be of concern to some schools, but these events do not cost an extremely large amount of money to host, and community organizations and businesses are often willing to make contributions.

In addition, Title I funds were often available when these events were held specifically in low-income communities. For our Family Math Nights, our budget included approximately $300 for the supplies needed to construct the games and activities, approximately $400 was spent on pizza and beverages, and about $150 was spent on door prizes. Door prizes consisted of math games that families could play with one another (e.g., SET, 24 Game, Yahtzee). Classroom materials like individual whiteboards as well as other school supplies have been given out as well.

Finally, several of our schools had relationships with local colleges and universities, and as a result, a number of preservice teachers had been placed in these schools. They, too, were involved in the Family Math Nights. They played an important role in helping run the game tables, thus freeing up teachers to talk with family members about the mathematical goals of the game and how similar games could be played at home. This contributed to the engagement of family members while also providing preservice teachers with experience coordinating such events.

Layout and Floor Plan for Event

All of the Family Math Night events were held in the cafeteria of each school. Folding tables were set up along the perimeter of the cafeteria to set up game stations and each table was set up to accommodate folding chairs behind the folding tables for those assigned to run the games. A number of cafeteria-style tables were positioned in the center of the cafeteria where families could eat their pizza and drink their beverages. We found it to be essential that enough space was available around the perimeter of the room for lines to gather at each of the game stations. Figure 1 shows the typical layout of the cafeteria.

Coordination of Games and Raffle

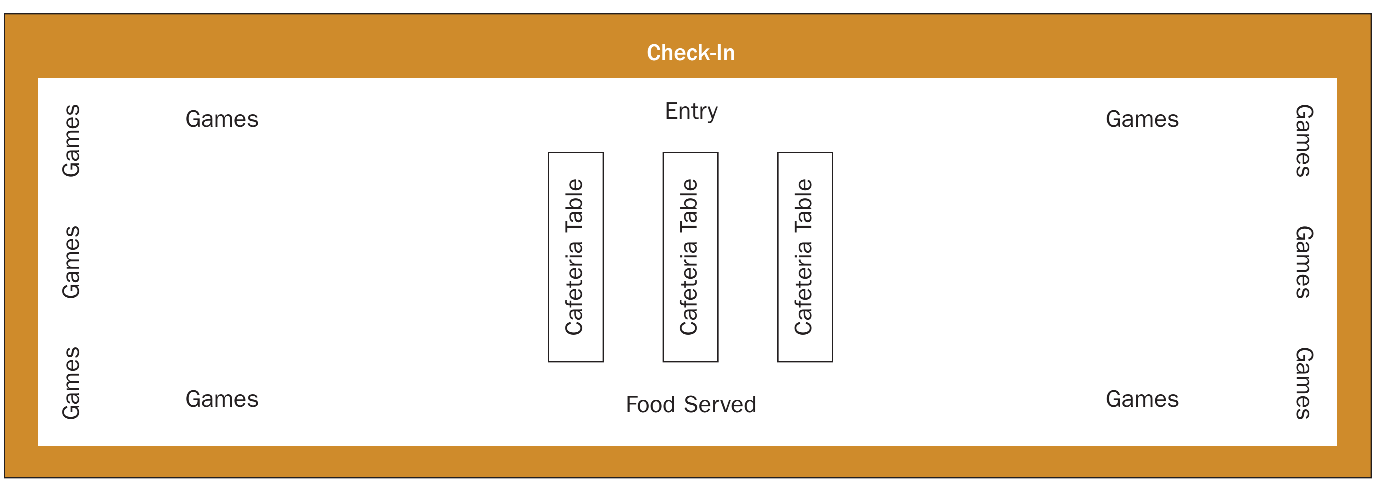
Family Math Nights typically were scheduled to start at 6 or 6:30 p.m. Games were played for about 1 hour and 15 minutes, with an additional 15 minutes spent wrapping things up and getting ready for door prizes. As students and their family members checked in just outside the cafeteria, each student received 10 single blue game tickets, with an additional 15 tickets provided to those who submitted their RSVP forms by the deadline. These tickets provided students with access to the game stations.

Double-portion orange tickets were created for use when students or family members participating in game activities won a game. The winning student or family member was given the "Keep This Coupon" portion of the orange ticket and the person running the game table kept the other half which was filled out with the family members name and contact information. At the end of the evening, tickets were collected from the game stations, and names were called out for prizes. Students and family members proceeded to pick up the door prize of their choice in the order in which they were called.

Description of Games and Handout

Games that were used varied depending upon the grade levels of the students and family members participating in the Family Math Night. Games were also selected in order to target areas of needed improvement based on state assessment data. Parents received a handout for each game that explained how to play the game, identified the mathematics that was addressed by the game, and discussed how the game could be played at home. A list of websites was also provided to help families become more aware of resources available to help support the mathematics learning

Figure 1 – Layout of Cafeteria



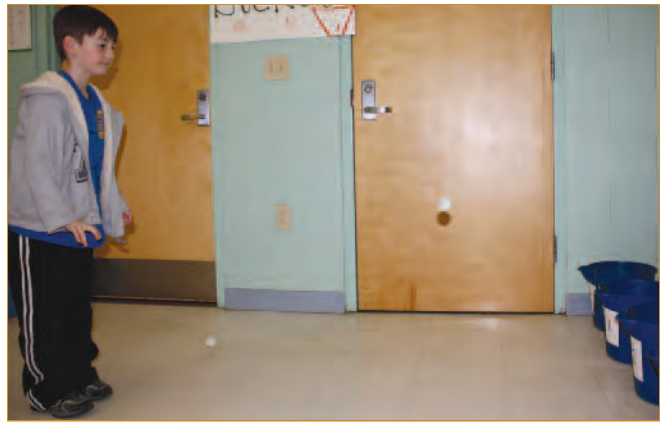
of their children. This included NCTM's Illuminations (http://illuminations.nctm.org/) and Figure This (http://www.figurethis.org/). The eight stations briefly described below provide some examples of the games most frequently used at our Family Math Nights.

STATION 1: ESTIMATION GAME

Players scoop out beans and put them in a cup. They estimate how many beans they think are in the cup and write down their estimate. Players then count the beans by placing them in groups of 10. For example if there are 48 beans, they make 4 piles with 10 beans in each pile and then a pile with 8 beans. Players compare their estimate to the actual count. Depending on their age, they can determine if the estimate is less than/greater than the actual count or find the difference between the two. The purpose of the game is to help strengthen students' number sense and their understanding of our base ten number system. At home, whenever possible, it can be helpful to provide children with the opportunity to count groups of objects by separating them into groups of tens and ones so they can understand that if there are 48 of something, then that means there are 4 tens and 8 ones.

STATION 2: MAKING TEN GAME

Players receive three ping-pong balls and try to get a combination of ping-pong balls to land in buckets (labeled with integers from 0 to 9). The object is to bounce the balls into the buckets to obtain a sum of 10. For example, if one of the ping-pong balls lands in the 8-bucket they can either get two more to land in the 1-bucket or one to land in the 2-bucket and another to land in the 0-bucket. The purpose of this game is to help students think about combinations of 10. For instance, if students get a ball in the "4" bucket, they will have to think about which bucket or buckets they will have to land in to get a total of 10. At home, children can be encouraged to think about number combinations that add to 10 by talking how many more of something you need in order to make ten.



STATION 3: TEN FRAME STATION

This game has a number of variations that can be played depending upon the level of the players. The youngest players are presented with a 10-frame that has a certain number of dots. The person running the game asks several questions, such as "How many dots are there?" or "How many more would I need to make 10?" Older players are presented with multiple 10-frames with various numbers and are asked to add or subtract the numbers. Families are provided with a handout of 10-frame cards so they can use them at home. Using these 10-frames can help children reason about addition and subtraction. For example, given the problem 8 + 5, children can realize that if you have 8, you need 2 more to make 10, and then you have 3 more, which makes 13 (e.g., 8+5 = 8+2+3 = 13). This kind of reasoning helps strengthen understanding without having to rely on memorized information. At home, when doing addition or subtraction, it is helpful try to connect numbers to 10 or groups of 10.

STATION 4 – 24 GAME

Players try to combine four numbers using the operations of addition, subtraction, multiplication, and division to make the number 24. For example, if the numbers were 2, 3, 4, and 6, players could say that (2x6) + (3x4) = 24. This game is most appropriate for older students and family members who understand all the operations (addition, subtraction, multiplication, and division). The purpose of this game is to developing fluency with addition, subtraction, multiplication, and division. Special 24 Game cards are not necessary to play this game at home. The game can be played by rolling four dice or selecting four cards from a deck of cards, with an ace being treated as 1 and 10 being treated as 0. It is important to note that not all combinations of four numbers can be combined to make 24.

STATION 5 – TANGRAMS

There are two variations to this station depending on the age of the player. Younger children get pre-drawn versions of puzzles where they will need to place the seven Tangram pieces in the appropriate place. Older children have to fill in a given shape without lines to show the pieces. The purpose of this game is to explore how shapes relate to one another and how they can be rotated to make certain designs. At home, these kinds of puzzles help strengthen spatial skills. Tangrams are relatively inexpensive and can be purchased for about $3. There are also online versions available at the following website: http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html?open=activities

Families can also make your own Tangrams. For instructions, visit this website: http://mathforum.org/trscavo/tangrams/construct.html

STATION 6 – PLACE VALUE GAME

Players randomly draw numbers from a deck of cards (labeled 0 to 9) or roll a 10-sided die. The goal is to make the largest or smallest number possible, but once you place a number in a location you cannot move it. For example, if you are trying to create the largest two-digit number possible and selected a 7, you could put it in the 10's place. However, if you selected a 9 as your next card, you could not replace the 7 in the 10's place, so the largest number you could make would be 79. This game is most engaging if two or more players compete against each other or compete against the person running the game. Another version of the game includes making the smallest number The purpose of this game is to emphasize place value understanding and to know that each digit in a number does not just represent that digit, but the place value associated with it. Using the language included in the game keeps the focus on place value understanding. Place value understanding can be strengthened at home by finding opportunities to break down numbers into their place value parts, particularly when working with money and making change.

STATION 7 – FRACTION BENCHMARKS

A piece of masking tape, labeled with 0, ¼, ½, ¾, and 1 is spread across the floor. Players flip over a card with a fraction that is not equal to any of the benchmarks and have to jump to the closest benchmark. For example, if a player flipped over a card with 13/24, he or she would try to jump as close to ½ as possible. The purpose of this game is to learn to compare fractions by relating them to various fraction benchmarks using their reasoning about the relative sizes of numerators and denominators. While cooking together at home, it can useful to discuss any fractions that are used in recipes in terms of nearby fraction benchmarks.

STATION 8 – INTERACTIVE WHITE BOARD STATION

Activities involving white boards are intentionally selected from websites that family members can access at home including the applets available at NCTM Illuminations (http://illuminations. nctm.org).

The purpose of this station is to help parents realize the ample resources that are available online that they can explore with their children at home.

Conclusion

Family Math Nights provide a useful opportunity for administrators and teachers to interact with students and their families in an informal manner. Events like these can provide families with math activities to do at home, they can help family members feel more comfortable asking teachers for suggestions about how to help their children with math at home, and they can also help family members appreciate the importance of the development of mathematical reasoning and sense making. Family Math Nights can also have an impact on teachers' attitudes toward family members, communicating their interest in being involved in the mathematics learning of their children. Events like these can change teachers' and parents' perspectives on new ways to interact with one another. Everyone benefits from participating.

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Reflections on Creating Strong Mathematics Coaching Programs

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American schools are striving to invent programs to increase student achievement in mathematics that will help American students compete in the 21st century economy. Many states are aligning their mathematics standards and benchmarks with countries that achieved high scores on international testing such as Japan, Korea, Germany, and Finland. The new Common Core Standards for Mathematics (CCSS, 2010), which 44 states adopted, reflect a goal to deepen students' understanding of mathematics. Teaching these new mathematics standards to students from pre-kindergarten through grade 8 requires an even stronger understanding of mathematics and the associated pedagogical content knowledge. Recently, to meet these needs, many large urban districts have turned to mathematics coaching programs as a component of their K-8 staff development plans (Russo, 2004).

Mathematics coaching programs that focus on strengthening the mathematical and pedagogical knowledge of teachers and include a focus on explicit instructional strategies that can be carried back into the classroom, we see gains in the mathematics achievement of students (Brosnan & Erchick, 2009; Herrelko, Jeffries, & Robertson, 2009; Ingvarson, Meiers, and Beavis, 2005; Russo, 2004; West & Staub, 2003). Resnick and Glennan (2005) suggest that on-site coaching for teachers in a range of content areas can improve the academic achievement of urban districts in those content areas.

For eight years, I have been a university facilitator for statewide mathematics coaching programs serving elementary schools in urban and rural settings in the State of Ohio. In this role, I provide professional development for mathematics coaches focused on mathematical content and pedagogical practices, keeping in mind that we want to prepare mathematics coaches who can help teachers use research-based instructional strategies and assessment methods as they teach mathematics. We also want to prepare mathematics coaches who can help teachers become reflective practitioners. Over the past 8 years, our data show that schools and districts participating in mathematics coaching programs show increases in the mathematics achievement of their students (Brosnan & Erchick, 2009; Herrelko, Jeffries & Robertson, 2009). In this article, I discuss some of the lessons we have learned about what helps make a mathematics coaching program successful.

Roles of School and District Administrators

School and district administrators play key roles in making a mathematics coaching program a success. This includes identifying the needs of the school or district to be addressed by the mathematics coaching program, establishing clear goals related to those needs and determining how progress toward those goals will be measured, committing the time and resources needed, selecting and hiring the mathematics coaches, and clearly articulating expectations regarding the roles mathematics coaches are to play as they provide support to schools. This can often include negotiating the roles of mathematics coaches with the teachers union to ensure that these mathematics coaches have the full support of their teacher colleagues. When mathematics coaching programs do not have the support needed from school and district administrators, problems can arise.

Some of these problems occur when mathematics coaches are used to address a variety of short-term needs that might arise in a school that are not connected to the established goals of the mathematics coaching program. For instance, when mathematics coaches are used to cover classrooms when teachers are absent, work directly with students who need more support, or copy and distribute material needed by teachers, it can mean that mathematics coaches have less time to provide the kind of support to teachers that strengthens mathematics teaching and learning over the long run.

Problems can also arise when administrators fail to establish scheduled time for coaches and teachers to work together separate from the time allocated to mathematics instruction, whether it is before or after school or during teachers' planning time, or fail to establish how teachers will be compensated for the time they spend with mathematics coaches outside of their contract day. It is also important to consider how time has been scheduled for other similar efforts, for instance, a literacy coaching program, and what precedents might have been set for teacher compensation.

Finally, it is important that mathematics coaching positions are negotiated in terms of where these new positions fit into the collective bargaining unit, contractual implications of these new positions, who is responsible for supervising mathematics coaches, and how salaries are determined. If these issues are not negotiated and resolved, tensions can arise between teachers and mathematics coaches that get in the way of productive working relationships.

Administrators play an important role in determining when a mathematics coaching program is to begin, timetables for posting mathematics coach positions and recruiting applicants, conducting mathematics coach interviews and selecting candidates, and then finally making decisions about where to place these mathematics coaches. Some districts make the decision to begin a coaching program just before the start of the school year. This places the district into a situation of hiring the first applicants who arrive at the district office as their mathematics coach rather than finding a teacher leader with the necessary skill set needed to be a successful mathematics coach. Using a timeline that starts sufficiently in advance of the launching of the mathematics coaching program increases the likelihood of hiring well qualified mathematics coaches who can be supported as they transition from their present positions to their coaching roles.

Defining the Role of a Mathematics Coach

A mathematics coach is a teacher leaders and a change agent working to strengthen the mathematical and pedagogical knowledge of teachers in order to strengthen student achievement in mathematics. This requires that the mathematics coach take the responsibility for being knowledgeable about research in mathematics education. It also requires that mathematics coaches have a compelling history of strong mathematics instruction in their own classrooms. Finally, mathematics coaches must understand what it means to support the mathematics teaching practice of their colleagues.

West and Staub (2003) defined the work of a mathematics coach as taking place in three stages: 1) a collaborator who helps a teacher in the planning process; 2) a facilitator who helps enact the plan; and 3) a facilitator who helps the teacher reflect upon what student learning happened during the lesson. The mathematics coach brings to the three-stage process an expertise in mathematics content and pedagogy including research-based practices, strategies, and methods that teachers can learn and implement in their own classrooms. The coach keeps the focus of the planning of the lesson, the enactment of the lesson, and reflection on what students seem to be learning during the lesson with implications for next steps. In addition, the mathematics coach supports the examination of teachers' assumptions about mathematics instruction in order to be able to challenge and support some of those assumptions and build a stronger mathematics teaching practice (McGonagill, 2002). These features of the role of a mathematics coach are consistent with how others describe this work as well (e.g., Becker, 2003; Mink, Owen, and Mink, 1993; Olson and Barrett, 2004). When these approaches are solidly put into place, mathematics coaching programs and any associated professional development are likely to continue even with high rates of administrator and teacher turnover (Balfanz, MacIver, and Byrnes, 2006).

Creating Schedules that Support Mathematics Coaching

Time needs to be identified during the school day for mathematics coach and teacher collaboration before and after mathematics instruction. Before the lesson, in a pre-conference meeting, the mathematics coach and the teacher identify the mathematical concept to be taught, review the teacher's mathematics lesson plan, and make any adaptations that might strengthen the lesson. During the enactment of the lesson, the mathematics coach might model particular instructional strategies, coteach the lesson with the classroom teacher as the lead instructor, or might play a role of assistant to the classroom teacher. After the lesson, in a postconference meeting, the mathematics coach and teacher debrief together by reviewing what happened during the lesson including a discussion of what students learned, the strengths and weaknesses of the lessons with regard to student learning, and next steps.

In my role as a university facilitator, I often observe mathematics coaches as they work with teachers, and I see big differences in what mathematics coaches are able to accomplish as a result of whether there is adequate time scheduled for mathematics coaches and teachers to collaborate together using the stages identified by West and Staub (2003). When time is allowed for the preconference, the teacher and mathematics coach work together smoothly in the classroom. When there was no time to confer prior to the teaching of the lesson, there is no coordination of efforts, no sharing of classroom responsibilities, no shared vision of where the lesson is going and how it will get there. Where there is no time for a postconference, reflection on what happened during the lesson and implications are not discussed, the opportunity for change and growth are lost. The strength of these collaborations between a mathematics coach and teachers rests on the available time that the teacher and coach have time during preconference and postconference collaboration and reflection. During my observations of mathematics coaching practice, these two meetings were typically missing because of union contracts that did not define or allow for these meeting times during the school day or because administrators usurped these times with other school duties or responsibilities. With little time to collaborate and reflect together, teacher growth and change is impeded.

The Importance of Building a Trusting Relationship

In order to earn the trust of the teachers, mathematics coaches must be allowed to keep the confidences of teachers. Developing trust takes time and effort. The mathematics coach must prove that he or she is trustworthy in the eyes of the teachers. The coach needs to be helpful, open to working in a manner that helps teachers feel comfortable, and not threatening. The mathematics coach needs time to build this kind of trust with teachers in order to learn about their strengths and limitations and be better able to support their learning. The mathematics coach should also be careful not to comment on the performance of specific teachers in an evaluative way, so teachers feel confident that discussions they have with mathematics coaches about areas of needed support do not end up being communicated to other teachers or administrators and do not appear on evaluation documents. When trust is established, teachers are open and willing to admit to the challenges they face in their mathematics instruction and request the help they need.

Holding Mathematics Coaches Accountable

Accountability for how mathematics coaches use their time in a school is important. Reports dealing with where, what, with whom the mathematics coach is working is a valuable source of information about how mathematics coaches are spending their time and who is receiving their support. The districts in which I worked each had a different means of coach accountability. One required a minute by minute accounting of the coach's time on a spreadsheet that was signed by the principal each week and submitted to the administration. Another provided a weekly template on one page where the coach noted the teacher and content that was worked on that week. These accountability tools can provide useful information for those who supervise mathematics coaches and can also provide important documentation for use in determining the effectiveness of a mathematics coaching program.

Who Could Be a Mathematics Coach?

Finding the right person to be a mathematics coach is fundamental to having a successful mathematics coaching program. Whether districts search outside their current teaching staff or move a classroom teacher into a coaching position, several professional qualifications are important to consider in both creating the position description and during the interview process. These include: (a) solid knowledge of mathematics and an enthusiasm for teaching mathematics; (b) well developed mathematics content knowledge for teaching; (b) well cultivated interpersonal skills and collaboration skills; (c) experience providing professional support to teachers; and (d) current teaching license or certification with a minimum of 5 years of teaching experience. Each of these is discussed further below.

(a) Solid mathematics content knowledge for teaching. Mathematics content knowledge for teaching is defined by the University of Michigan Learning Mathematics for Teaching Institute as having to do with the mathematical reasoning, insight, understanding, and skill needed to successfully teach mathematics to students (Ball, Hill, & Bass, 2005). The Ohio State University (OSU) Math Coaching Program (MCP) collected data that revealed that the level of mathematics content knowledge for teaching of those selected to be mathematics coaches was only marginally stronger than that of the teachers they would be coaching. As a result, weekly full-day professional development sessions were developed specifically for mathematics coaches that addressed important mathematical concepts, research about how students learn mathematics, and implications for instruction. By the end of one academic year, data showed there was significant growth in the mathematics content knowledge for teaching of these mathematics coaches as well as increases in the mathematics content knowledge for teaching of the teachers they were coaching, and these were both associated with gains in the mathematics performance of students (Brosnan & Erchick, 2009). These data suggest the importance of building a strong understanding of mathematics content knowledge for teaching among mathematics coaches through a substantial commitment to their ongoing professional development.

(b) Well-cultivated interpersonal skills and collaboration skills. Interpersonal skills are an essential ingredient for mathematics coaches. It is important for mathematics coaches to be able to actively listen to the teachers with whom they work, understand the dilemmas teachers face when they plan and enact mathematics lessons, and offer support without being judgmental. This supports the creation of a relationship that is collaborative and trusting (Feger, Woleck, & Hickman, 2004). The MCP model reflects the work of cognitive coaching as defined by Costa, Garmston, and Glickman (1994) where coaches intently listen to the teachers and ask reflective questions that elicit the thoughts and feelings about teaching and learning that can become the basis of their work together.

(c) Experience providing professional development support to teachers. There are important differences between working with adults and working with students. It is important for mathematics coaches to have had some prior experience successfully collaborating with colleagues and other adults, including experience supporting their thinking and practice related to mathematics teaching and learning, as these kinds of experiences are central to the development of a strong mathematics coaching practice. It is also important for mathematics coaches to show evidence of professional development practice that models the kind of practices that we want teachers to take on with their students.

(d) An active teaching license or certificate with minimum of 5 years of successful experience teaching mathematics. Earning a teaching license or certification means that mathematics coaches have successfully completed teacher preparation programs and passed state-required tests that are required of all certified teachers. At least five years of successful experience teaching mathematics, including the ability to be articulate about what it means to support the mathematics learning of students, means that mathematics coaches have something to offer teachers who are working to strengthen their mathematics teaching practice with their own students. This includes an appreciation for the complexities and challenges of teaching mathematics as well as what it means to strengthen one's mathematics teaching practice over time. This background knowledge and experience creates mutual respect and empathy between coach and teacher.

Conclusion

Having worked with three mathematics coaching programs as the university professional development provider, and having spent a number of years supporting mathematics coaches as they work in their districts and schools, I have had the opportunity to make some observations about what elements make a successful mathematics coaching program. It is my hope that this article has something to offer to others as they consider starting a mathematics coaching program that will have strengthened mathematics teaching and learning and help students be more successful in mathematics. Deciding to start a mathematics coaching program requires the creation and articulation of a shared understanding of program goals, a commitment of time and resources, and the selection of and support for mathematics coaches themselves. All of these are key to program success.

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Transformational Professional Development: Teacher Learning Through a Bifocal Lens

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In the summer of 2006, the Green River Regional Education Center (GRREC) in Bowling Green, Kentucky, began an initiative designed to strengthen student performance in mathematics through professional development designed to strengthen the mathematics content knowledge of K-8 teachers in the region. We framed this professional development effort as learning through a "bifocal lens." That is, we wanted participating teachers to learn more mathematics but we also wanted them to be able to think about how they were learning that mathematics and implications for their own teaching practice.

The project, called the Math Alliance Initiative, began as collaboration between GRREC and teams of teachers from 48 schools in 17 districts in the region. During the second year of the project, with additional funding through a 3-year Math and Science Partnership grant, the initiative was expanded to include teacher leaders from these participating schools as well. In total, over 220 teachers and teacher leaders from these 17 districts participated over the four years of the project.

A number of partners played key roles in the project including the Kentucky Department of Education, Western Kentucky University faculty, Global Education Resources, Measured Progress, and Carnegie Learning, with the GRREC overseeing and coordinating the effort. A number of master practitioners from the region who were experienced professional development providers also collaborated with these key partners to design and facilitate the professional development that was offered.

Each year, the project offered a 5-day Summer Math Academy during which participants explored mathematics content with a focus on reasoning and sense making. During the school year, the project offered four full days of professional development that focused on implications for instruction, including the use of instructional strategies that supported mathematical reasoning and sense making, as well as the use of formative assessment strategies that provided more information about what students understood and where they were struggling and implications of this formative assessment data for instruction. An important part of this work also involved becoming more articulate about what we want students to learn at each grade level. The Summer Math Academies and the school-day sessions are described in greater detail below.

Summer Math Academies in Years 1 - 3

Summer Math Academies were planned and facilitated by Carnegie Learning and our designated master practitioners, using pretest and survey data from participants to determine the particular focus of the mathematics that would be addressed. On page 46 is a table that lays out that mathematics content of the Summer Math Academies over the first three years of the project.

These summer academies were designed to strengthen and deepen participants' understanding of mathematics through the use of problem solving activities that built conceptual understanding and increased procedural fluency using tasks that addressed the specified content. This included exploration of cognitively demanding tasks created by Carnegie Learning that were intended to address the specified

Table 1: Content for Initial Three Years

	K – 1	2 – 3	4 – 5	6 - 8
Summer 2006	**Place Value** Modeling for Place Value and Conceptual Understanding of Base 10 System	**Place Value** Multiple Representations for Building Foundation of Place Value and Number Sense	**Fractions** Build Conceptual Understanding of Fractions and Operations	**Ratio** Build the Foundation for Algebra through Ratios and Proportional Reasoning
Summer 2007	**Fractions** Build Conceptual Understanding of Fractions and Multiple Representations	**Multiplication** Develop understanding of Multiplication as a Foundation for Algebra	**Fractions and Decimals** Conceptually Link Fractions to Decimals within Problem Solving	**Proportionality** Connecting Proportional Reasoning with Patterns, Relationships, and Linear Functions
Summer 2008	**Integers** Conceptual Understanding of Integers and Operations with Connections to Algebra	**Fractions** Conceptual Understanding of Fractions and Operations with Ratio and Probability Applications	**Linear Functions** Using Patterns, Relationships, Representations, and Functions to represent Algebra	**Functions** Quadratic and Exponential Function as mathematical tools for Understanding Change

content from an advanced perspective. This also included the exploration of cognitively demanding tasks addressing the selected standards for each grade level of participating teachers and teacher leaders. As a consequence, participants in these academies were able to experience the mathematics content as learners and then were able bring that experience to the question of how to engage their students in the mathematics content of their grade levels, thus creating the metaphor for learning through a "bifocal lens." Questions of pedagogy were also addressed through the modeling of best practices as staff from Carnegie Learning and master practitioners facilitated summer academy sessions. This attention to pedagogy included explicit discussions about instructional strategies designed to build conceptual understanding and procedural fluency using Teaching student-centered mathematics: Grades K-3 and Grades 3 -5 as resources (Van de Walle and Lovin, 2006).

Full Day School Year Sessions During Years 1 - 3

For the first two years of the project, participants met with Measured Progress for four full school days to address issues of formative assessment. Participants explored formative assessment strategies including how to identify clear learning targets by unpacking the standards for each grade level and developing "I can . . . " statements (Stiggins, Arter, Chappuis, & Chappuis, 2007) as well as exploring strategies for collecting information about what students were learning, exploring strategies for students to use to track their own learning, and considering how to use assessment data to strengthen instruction. This focus on formative assessment also created opportunities to revisit the use of the cognitively demanding tasks developed for each grade level during the summer academies by considering the particular formative assessment strategies participants might use as students worked on these particular tasks. Because participants were also expected to form and facilitate professional learning communities in their schools, school day sessions during the second and third years of the project provided them with tools to use in this school-based work, including an exploration of the use of protocols for examining student and the use of Japanese Lesson Study protocols. In both cases, the focus continued to be on the use of the cognitively demanding tasks developed during the summer academies as well as well as the use of other cognitively demanding tasks associated with existing instructional materials.

Year 4 of the Project

The fourth year of the project shifted its focus to geometry as a result of finding that many participants in the project appeared to have a limited conceptual understanding of important geometry content. This finding was based on classroom observations of participants as they taught geometry topics, conversations with participants about those geometry topics, and student performance on the geometry strand of state assessments. As a result, the goal of this fourth year of the project was to increase the geometry content knowledge of participants, particularly two-dimensional geometry, as well as helping participants understand what students needed to know about two-dimensional geometry at particular grade level based on the geometry standards and what cognitively demanding two-dimensional geometry tasks might support the learning of this content.

THE YEAR 4 SUMMER ACADEMY

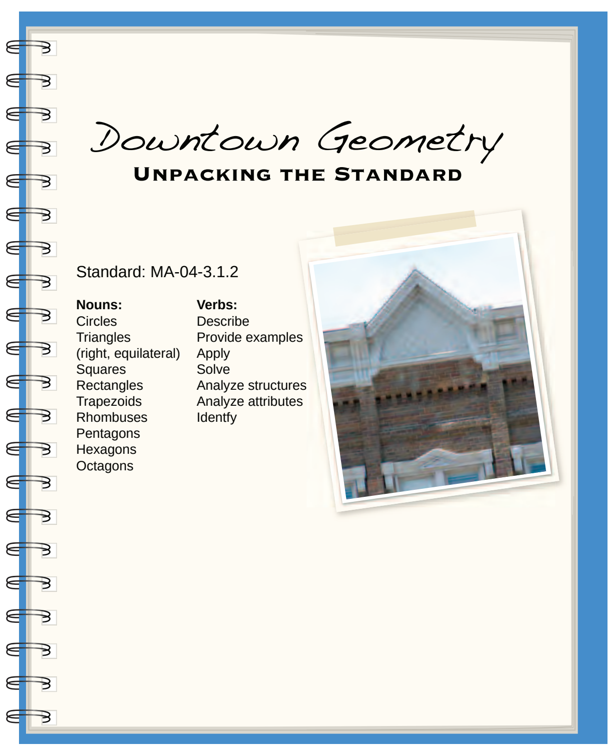
The Summer Academy for Year 4 was a three-day experience designed to be similar to those of the first three years of the project, creating a situation where participants were again learners of mathematics while also focusing on implications for their own instruction of students, thus continuing the metaphor of learning through a bifocal lens. Academy sessions were planned and facilitated by GRREC, math educators from Western Kentucky University, and master practitioners.

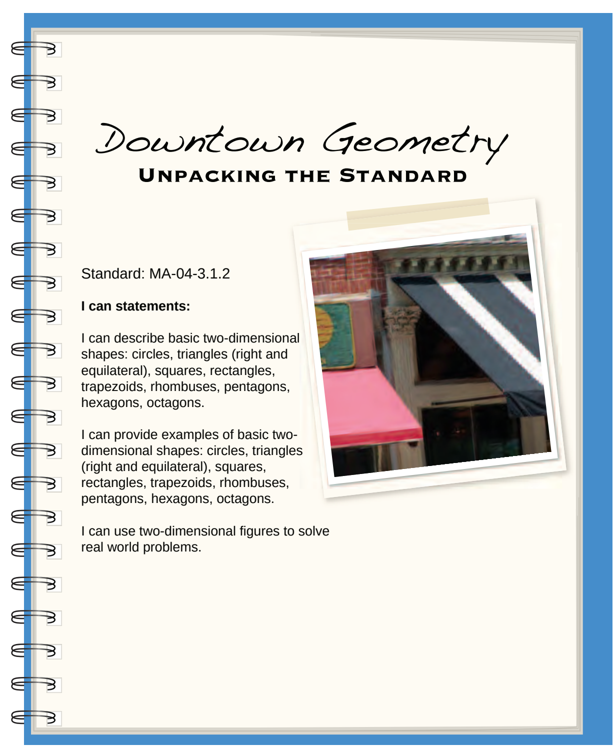
Day One of the Summer Academy focused on a deep examination of the geometry standards, experiencing and designing cognitively demanding tasks that addressed these standards, and considering the kinds of questions it would be important to pose to students as they worked on these tasks. Day One also included a field trip designed to address this geometry content and provide the context for additional geometry tasks. Day Two included learning how to use technology to expand upon the geometry field trip and the exploration of associated geometry tasks in ways that further developed an understanding of two-dimensional geometry while also making connections to the classroom. Day Three focused on strategies for engaging students in these geometry tasks, including the use of differentiated instruction to ensure that all students would be able to enter each geometry task and, though their engagement in these tasks, learn the geometry content specified by the standards. Each day of the Summer Academy is described in greater depth below.

Professional Development Day 1: Geometry Standards, Scaffolding of a Task, Classroom Discourse, Problem Solving, and Field Trip. On Day 1, we began with an introduction asking our participants to consider what they currently teach regarding geometry in their K-8 classrooms. Participants recorded the grade level content on chart paper and, through a "gallery walk" to examine these posters, began to have conversations about what they saw as gaps and repetitions in their current instruction.

Next, participants worked on a geometry task adapted from Carnegie Learning materials, *Bridge to Algebra* (2008) and *Geometry* (2008) entitled "Revitalizing Downtown"

FIGURES 1-2: *Sample from Unpacking the Standards Document Provided to Participants*





that centered around the theme of renovating different aspects of downtown with three Challenges: 1) "Safety Downtown" (*Bridge to Algebra*, p. 283-286), 2) "Downtown's New Skating Rink" (*Bridge to Algebra*, p. 305-310), and 3) “Downtown Condominiums, Nature, and Recreation" (*Geometry,* p. 49-52; *Bridge to Algebra*, p. 319-320). After working through the "challenges," there was an opportunity to discuss the grade level appropriateness of these tasks, with considerations of how to gear up or gear down the task to the appropriate cognitive complexity to assure access for all children, including scaffolding for those who might need more support and extensions for those who might be ready for greater challenge. There was also an opportunity to explore questioning strategies and "Talk Moves" (Sheffield, March 2006; Chapin, O'Connor, & Anderson, 2003) associated with this variety of tasks.

On the afternoon of the first day of the Summer Academy, participants departed on a field trip to the local downtown square in Bowling Green to explore the project's version of the "Downtown Math" adapted from the "The Math Connection Opening Your Eyes to Math: Experiencing a Math Trail Through Downtown Elkader" (Horstman, 2000). Participants were divided into six different groups and assigned to two unique architectural locations to complete a geometric scavenger hunt, using digital cameras provided by the project to record their findings. *(See Figures 1 & 2 on pg. 47)*

Participants completed the Scavenger Hunt and then went on to assigned locations to create two-dimensional tasks specific to the photographs they had taken, thus developing their own local version of “Downtown Math” that could be used with their own students at their own grade levels. *(See Figure 3)*

Professional Development Day 2: Use of Technology to Expand Upon the Geometry of the Field Trip. The second day of the session involved two breakout sessions. One session focused on the use of Google tools and strategies for incorporating the photographs from the scavenger hunt (*See Figure 4*) into movies that participants could use for geometry lessons in their own classrooms. Participants

FIGURE 3:
Sample from “Downtown Geometry: Fountain Square Park”

Downtown Geometry
FOUNTAIN SQUARE PARK

They call this Fountain Square Park. Is it really a square?

How can you prove it? (How do you know?)

How many children does it take to go around the fountain?

If we didn’t have enough children, what could we do?

FIGURE 4:
Sample of a Trapezoid found on 2D Scavenger Hunt

also learned how to use Picasa and Google Docs to embed their own "Downtown Math" tasks (*See Figures 5-10*) within these movies. The intended outcome was for participants to be able to help their own students develop their own movies using a similar "Downtown Scavenger Hunt."

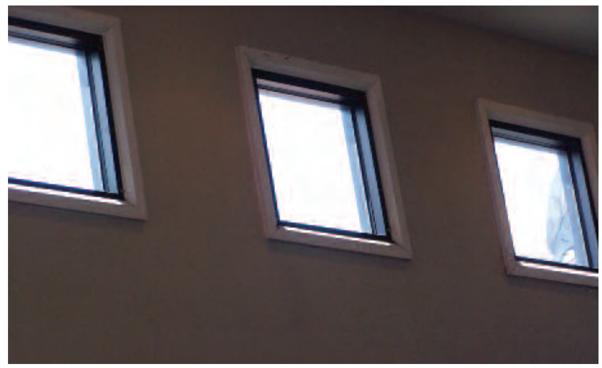
The remainder of Day Two focused on continuing to strengthen and deepen the geometry knowledge of participants, with a focus on the conceptual underpinnings of many of the formulas used in geometry and the development of the vocabulary of geometry, as well as exploring how the geometry taught at their grade level is a foundation for what comes next. With these goals in mind, participants were asked to work collaboratively on high-cognitive demand task adapted from *Geometry: Teacher's*

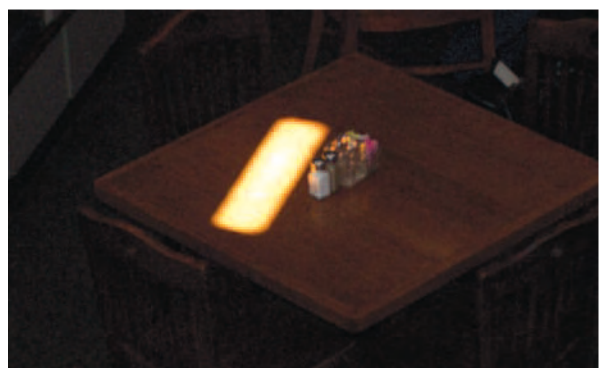
FIGURE 5:
Sample Participant Snapshots for Downtown Math

Teachers thought about asking students determine the mathematical relationships of the shapes in the tile. They also thought about asking students to cut out the shapes and compare them to the pattern block manipulatives.

FIGURES 6 & 7:
Sample Participant Snapshots for Downtown Math

Teachers noticed that the windows were square but the light filtered in to the restaurant in a rectangular shape. The teachers discussed developing questions surrounding this oddity.





FIGURES 8-10:
Sample Participant Snapshots for Downtown Math

Teachers noticed in the park that depending on the perspective and angle for taking the photograph of the concrete edging of the grass that the angle appeared to be obtuse, right, or acute. They would like to develop problems to help students explore this scenario.

Implementation Guide (Carnegie Learning, 2008; p 7-12) that focused on two-dimensional geometry involving the calculation of the number of gallons needed to seal an octagonal shaped deck without necessarily knowing the formula for calculating the area of any regular polygon. Participants also took part in two "shape sorts" involving a variety of two-dimension shapes that increased in difficulty. After sorting the shapes and displaying the sorted shapes on presentation paper, participants took part in a gallery walk, after which there was an in-depth discussion of the features of these shapes, the names of these shapes, and how these their sorting work related to the geometry content of their grade level.

Professional Development Day 3: Engaging Students in Geometry Tasks with a Focus on Differentiated Instruction. Day Three focused on the use of differentiation strategies for engaging students in the geometry tasks that had been identified for classroom use at particular grade levels, given the geometry standards and "learning targets" for each grade level. This included further discussion of formative assessment strategies, in order to be able to determine what students understood and where they were struggling, as well as how students might be strategically grouped as they worked on these tasks. Our hope was that participants might be better able to discern the extent to which they were actually differentiating their instruction and the extent to which they were appropriately and fluidly grouping students as they differentiated their instruction. They had the opportunity to do a reflective activity called "Where Am I Now?" (Stiggins, et al., 2007) through which participants reflected on their formative assessment practices and prioritized goals for the following school year. This also included worked in vertical teams to discuss how the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006) addressed two-dimensional geometry at each grade level. Discussions focused how the geometry topics they were currently teaching compared to the *Focal Points* recommendations.

At the end of summer institute, participants were given a bag of geometry resources that included the following:

- *Big Book of Math for Elementary K-6: Read, Write, Research* (Zike, 2003)
- *Big Book of Math for Middle School and High School: Read, Write, Research* (Zike, 2003)
- *Navigating through Geometry in Prekindergarten-Grade 2* (NCTM, 2006)
- *Navigating through Geometry in Grades 3-5* (NCTM, 2006)
- *Navigating through Geometry in Grades 6-8* (NCTM, 2006)

Children's literature related to geometry given to all participants for use in their school:

- *Three Pigs, One Wolf, and Seven Magic Shapes*(Maccarone, 1997)
- *Where We Play Sports: Measuring the Perimeters of Polygons* (Roza, 2004)
- *Spaghetti and Meatballs for All! A Mathematical Story* (Burns, 1999)
- *The Greedy Triangle* (Burns, 1997)
- *Sir Circumference and the Dragon of Pi* (Neuschwander, 1999)
- *Sir Circumference and the Round Table* (Neuschwander, 1997)

Our hope was that participants would now have the resources to strengthen the teaching and learning of two-dimensional geometry at each grade level so they could now facilitate conversations about this content with colleagues in their professional learning communities about the alignment of geometry topics at each grade level and across grade levels and how to use these resources to better differentiate their geometry instruction at each grade level.

FULL DAY SCHOOL YEAR SESSIONS DURING YEAR 4

Participants continued to meet for four full days during the school year during this last year of the project, now with a focus on the CCSS standards for geometry, the use of cognitively demanding geometry tasks, and strategies for differentiating instruction. As in prior years, participants were also expected to organize and facilitate professional learning communities in their schools where what was learned through the project could be shared with colleagues. And action plans for strengthening mathematics teaching and learning could be developed.

Conclusion

The purpose of the Math Alliance Project was to provide an opportunity for teachers to deepen their conceptual understanding of mathematics as learners while also creating opportunities for them to strengthen how they teach mathematics to their students. It is our hope that teachers who participated in this project not only now have a stronger mathematics teaching practice but are also better prepared to support colleagues as they endeavor to strengthen their mathematics teaching practice as well. (*See Figure 11*) Roger Lewing says, "Too often we give children answers to remember rather than problems to solve." What we want is an education that teaches us how to think rather than what to think. We hope the teachers who participated in our project are prepared to be teacher leaders who do just that!

FIGURE 11:
Teachers Pondering Problems for Own Version of "Downtown Math"

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We Need Elementary Mathematics Specialists Now, More Than Ever: A Historical Perspective and Call to Action

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In early June 2010, the Common Core State Standards (CCSS) were released. One year later, 45 of this country's states have agreed to adopt the Common Core and are transitioning to these new mathematics standards. One could argue that the adoption of the CCSS for Mathematics has, to a great extent, altered the daily responsibilities for many elementary mathematics specialists. In many schools and districts, elementary mathematics specialists have become the "go-to" people with regard to everything having to do with the CCSS for Mathematics, including what the Standards for Mathematical Practice are intended to look like in classrooms, what the Mathematical Content Standards mean and how they are woven together, what professional development might be needed for teachers and administrators, how parents might be informed and involved, as well as what the Partnership for Readiness for College and Careers (PARCC) or Smarter Balanced Assessment Consortium (SBAC) assessments are likely to contain. Elementary mathematics specialists have also become responsible for knowing about resources available to support the implementation of the CCSS for Mathematics, including the materials posted to the Tools for the Common Core Standards (http://commoncoretools.wordpress.com/) where the Progressions documents can be found, where a tool for analyzing curriculum materials can be accessed, and where the Illustrative Mathematics Project and other related efforts can be explored. In other words, elementary mathematics specialists are becoming the school or district level "transition agents" for the Common Core State Standards for Mathematics. Mathematics specialists at the elementary school level are becoming increasingly important as we acknowledge the complexities of elementary mathematics teaching and learning. But how did this all get started, anyway? Calls for mathematics specialists, mathematics coaches, or elementary mathematics instructional leaders are certainly not new to the mathematics education community.

A Brief History of Calls for Elementary Mathematics Specialists

The departmentalization of elementary schools was very popular in the 1960's and early 1970's but was suggested as early as the 1920's in an effort to ensure that content-focused teachers taught all of the mathematics (or science or social studies) at a particular grade, typically Grades 4 through 6 (Becker & Gleason, 1927). During the 1970's, projects like the Developing Elementary Mathematics Enthusiasts (DEME) Project were created to identify building-based mathematics "enthusiasts" who cared enough about the importance of mathematics to assist their colleagues by serving as afterschool and beforeschool mentors and generally providing the math support for the building (Fennell, 1978). In 1981, the National Council of Teachers of Mathematics (NCTM) recommended that state certification provide for a teaching credential endorsement for elementary mathematics specialists. Then, in 1984, NCTM President John Dossey called for elementary mathematics specialists in an article in the *Arithmetic Teacher* (Dossey, 1984). From these early beginnings, the importance of the mathematics specialist role began to emerge.

* The author thanks Maggie McGatha, University of Louisville for her suggestions and review of this article.

While mathematics educators have advocated for elementary mathematics specialists for over three decades (Fennell, 2006; Lott, 2003), the ExxonMobil Foundation deserves much credit for shaping the role of elementary mathematics specialists and supporting the teachers who took on these leadership roles. When the Foundation's mathematics education program began in 1987, one of its goals was to support the use of mathematics specialists in the primary grades (Miller, Moon, & Elko, 2000). Most of these ExxonMobil Elementary Mathematics Specialist Projects provided professional development support for elementary mathematics specialists by deepening their mathematical knowledge for teaching, exploring learning theory, providing opportunities to examine and discuss curriculum materials, and considering a range of issues related to mathematics instruction and assessment. This work included opportunities involving curriculum and curriculum materials, learning theory, assessment and instruction. Bob Witte and Jean Moon were largely responsible for these pioneering efforts, and even today, the Foundation continues to support initiatives in support of mathematics specialists in areas of the country where ExxonMobil has a significant corporation presence.

Recommendations about the need for elementary teachers with interest and expertise in mathematics continued to appear in a range of publications. For instance, the National Research Council's Everybody Counts (1989) noted the following:

> "*The United Sates is one of the few countries in the world that continues to pretend – despite substantial evidence to the contrary – that elementary school teachers are able to teach all subjects equally well. It is time that we identify a cadre of teachers with special interest in mathematics and science who would be well prepared to teach young children both mathematics and science in an integrated, discovery-based environment.*" *(p. 64)*

The *Principles and Standards for School Mathematics* (NCTM, 2000) suggested that specialist-related models, including mathematics teacher leaders and mathematics specialists, be considered as a way to ensure the mathematical expertise of those responsible for knowing and teaching the content and process standards contained within the *Principles and Standards* and thereby strengthen and deepen the mathematics learning of students. *Adding it Up* (NRC, 2001) also discussed mathematics specialists within a departmentalized setting (e.g., one teacher teaching all the Grade 4 mathematics) as well as describing school-based mathematics specialists who could be responsible for supporting mathematics teaching and learning in one or more buildings by coaching/mentoring teachers, providing professional development, co-teaching mathematics lessons, or providing intervention or enrichment through "pull out" programs. A Report from the Conference Board of the Mathematical Sciences entitled *The Mathematical Education of Teachers* (CBMS, 2001) called for efforts to strengthen the mathematics preparation of *all* elementary teachers but also recommended that all mathematics in Grades 5 - 8 be taught by mathematics specialists. It should also be noted that the *No Child Left Behind* legislation signed in 2001 and implemented in 2003, requiring the annual reporting of annual Adequate Yearly Progress (AYP) data for mathematics and reading, has prompted many schools and districts to identify elementary mathematics specialists, elementary mathematics coaches, and mathematics instructional leaders as part of an effort to increase the mathematics performance of their students on state assessments.

Unfortunately there is currently little quantitative data to support the use of elementary mathematics specialists, elementary mathematics instructional leaders, or elementary mathematics coaches to strengthen mathematics teaching and learning at the elementary grades (NCTM, 2009). However, in her research brief for the National Council of Teachers of Mathematics on elementary mathematics specialists and coaches, McGatha (NCTM, 2009) noted that while research on the teacher leader model of the mathematics specialist is, for the most part, nonexistent, Gerretson, Bosnick, and Schofield (2008) found that using elementary mathematics lead teachers to focus only on mathematics instruction allowed them to have more time for planning and allowed them to focus their professional development. In addition, descriptive data and anecdotal evidence suggest that such individuals can have a positive impact on the mathematics performance of a school or district. The National Mathematics Advisory Panel report (2008) recommended that "research be conducted on the use of full-time mathematics teachers in elementary schools" (page xxii). This recommendation was based on the Panel's findings relative to the importance of teacher content knowledge and their recognition that most preservice teacher education programs for elementary teachers do not address the teaching of mathematics in sufficient depth. It is well known that mathematics-related coursework at the preservice level is typically limited to two mathematics courses and one course related to the teaching of mathematics (Karp & Fennell, 2010). Despite the lack of research, many schools, districts, and even states continued to pursue the use of mathematics specialists as a way to address the need to strengthen student performance in mathematics.

State-Level Efforts to Support Elementary Mathematics Specialization

As interest in elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders grew, states began to take notice. Maryland's *Keys to Math Success – A Report from the Maryland Mathematics Commission* (MSDE, 2001) used suggestions from the *Principles and Standards for School Mathematics* (NCTM, 2000), *Adding it Up* (NRC, 2001) and the *Mathematical Education of Teachers* (CBMS, 2001) to help justify their recommendation that certification for elementary mathematics specialists be pursued. The Maryland report is not unique, as other state reports have also advocated for elementary mathematics specialist certification. That said, at a time when virtually every state offers certification for reading specialists, fewer than fifteen states have actually enacted certification for elementary school mathematics specialists. At present, the following states have developed certification guidelines for elementary mathematics specialists: Arizona, California, Georgia, Maryland, Michigan, North Carolina, Ohio, South Dakota, Texas, Utah, and Virginia with Kentucky, Louisiana, Wisconsin, and probably others to soon formally become "members" of this expanding network of states that acknowledge state certification for mathematics specialists. See the Elementary Mathematics Specialists and Teacher Leaders website at http://www.mathspecialists.org for a review of all state certifications for elementary mathematics specialists.

At present, Virginia is the only state that requires certification and a Master's Degree for elementary mathematics specialists. The Virginia program, established in 2007 and supported by the National Science Foundation, is directed and led by Virginia Commonwealth University. This state initiative includes a core of mathematics and mathematics education courses leading to elementary mathematics specialization that participating colleges and universities have all agreed to offer. In addition, the research component of the Virginia initiative is often referenced, as it tracks the role and responsibilities of elementary mathematics specialists, and the impact of their work on student achievement (Campbell, 2009; Campbell & Markus, 2009, 2011).

In recent years the Association of Mathematics Teacher Educators adopted guidelines (AMTE, 2009) for teacher credentialing and degree programs and the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of Mathematics Teacher Educators (AMTE) and the Association of State Supervisors of Mathematics (ASSM) have issued the following joint position statement (NCTM, 2010):

> *The AMTE, ASSM, NCSM, and NCTM recommend the use of Elementary Mathematics Specialists (EMS professionals) in pre-K–6 environments to enhance the teaching, learning, and assessing of mathematics to improve student achievement. We further advocate that every elementary school have access to an EMS. Districts, states or provinces, and institutions of higher education should work in collaboration to create (1) advanced certification for EMS professionals and (2) rigorous programs to prepare EMS professionals. EMS professionals need a deep and broad knowledge of mathematics content, expertise in using and helping others use effective instructional practices, and the ability to support efforts that help all pre-K–6 students learn important mathematics. Programs for EMS professionals should focus on mathematics content knowledge, pedagogical knowledge, and leadership knowledge and skills.*

Unfortunately it is far too often the case that many elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders are appointed to such positions without elementary mathematics specialist certification or without even the proper vetting related to their content, pedagogical, and leadership knowledge and skills. For instance, a survey of Maryland school districts (Ruehl & Wray, 2009) found that many districts employed literally hundreds of mathematics specialists, elementary mathematics coaches, and mathematics instructional leaders. This might have been due to increasing need for such specialists given the pressure to produce gains in mathematics performance in order to show Adequate Yearly Progress (AYP).

Now that many districts and states are transitioning to the Common Core Standards for Mathematics (CCSSO, 2010), the need for these elementary mathematics leaders has increased even further. As discussed earlier, elementary mathematics specialists in schools and districts have become the transition agents for the Common Core State

Standards for Mathematics, working with teachers to consider the Standards for Mathematical Practice and the shift in mathematics content standards—both of which require that teachers have a deeper understanding of what it means to do and learn mathematics—as well considering how these practices and content may be assessed through the Partnership for Readiness for College and Careers (PARCC) or the Smarter Balanced Assessment Consortium (SBAC). The work of these elementary mathematics specialists will continue to be important and leads to an

Table 1

MATHEMATICS SPECIALISTS MILESTONES	
1984	NCTM Recommends State Certification Endorsement for Elementary Mathematics Specialists
1987	Exxon Foundation's support of K-3 Mathematics Specialists
1989	Everybody Count's support for specialization in mathematics at the elementary school level
2000	NCTM's *Principles and Standards for School Mathematics* suggests exploration of models for elementary mathematics specialists and teacher leaders
2001	CBMS' *The Mathematical Education of Teachers* recommendation that mathematics in the middle grades should be taught by mathematics specialists, starting at least in the 5th grade.
2001	*Adding it Up*'s review of the need for mathematics specialists
2001	PL 107-110 the *No Child Left Behind Act*
2003	NCTM President's Message: *The Time Has Come for Pre-K-5 Mathematics Specialists* – Jonny Lott
2006	NCTM President's Message: *We Need Mathematics Specialists NOW!* – Francis (Skip) Fennell
2007	Virginia Commonwealth University, Virginia Science and Mathematic Coalition – Virginia Mathematics Specialist Project
2008	National Mathematics Advisory Panel Recommendation – Elementary Mathematics Specialists
2009	Elementary Mathematics Specialists and Teacher Leaders Project established at McDaniel College
2009	AMTE Standards for Elementary Mathematics Specialists
2010	NCTM, NCSM, AMTE, ASSM – Joint Position Statement – The Role of Elementary Mathematics Specialists in the Teaching and Learning of Mathematics
2010	Common Core State Standards released
2011	45 States Transition to the CCSS and PARCC and SMART consortial assessments

obvious question: "What is it that elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders actually do?"

Elementary Mathematics Specialists: What Do They Do?

A wide variety of position titles may be used in association with an elementary mathematics specialist role. These include: elementary mathematics coach, elementary mathematics instructional leader, mathematics support teacher, mathematics resource teacher, mentor teacher, and lead teacher. There may be other position titles as well. The roles and responsibilities associated with these position titles can also vary widely depending upon the particular context of the position (Miller, Moon, & Elko 2000). More important than the actual position titles, however, are the expectations for the position. An analysis of a variety of elementary mathematics specialist initiatives suggests the following responsibilities for those who serve in an elementary mathematics specialist role at either the school or district level.

- **Mentoring/Coaching.** Many elementary mathematics specialists spend much of their day working with other teachers in one or more schools. Such mentoring or coaching often focuses on mathematics content and pedagogy and includes the following: co-planning mathematics lessons with teachers; working with teachers to identify important instructional needs; co-teaching mathematics lessons with teachers; assisting with the monitoring of student progress; and debriefing with teachers to discuss the impact of lessons that were co-planned and co-taught.

- **Providing Professional Development.** Increasingly, many elementary mathematics specialists are planning and facilitating school-based or district-wide professional development in mathematics, especially given the transition to the Common Core State Standards for Mathematics and the limited funding for professional development in so many districts.

- **Assisting with Curriculum and Instruction.** Elementary mathematics specialists may work at the school and district level to align curriculum frameworks, link instructional materials to important standards, interpret the literature on "best practices" and the research on the teaching and learning of mathematics with colleagues, and address any school and district needs.

- **Coordinating Interventions.** Some elementary mathematics specialists are responsible for coordinating and implementing intervention programs in mathematics. These may be in-class interventions, where the elementary mathematics specialist assists the classroom teacher to address the needs of struggling students. Alternatively, this may be outside-of-class Tier 2 and Tier 3 interventions associated with Response to Intervention (RTI) initiatives.

- **Supporting Professional Learning Communities.** Perhaps one of the most important goals of any elementary mathematics specialist is to provide the kinds of supports that contribute to the establishment of professional learning communities within schools—communities that truly foster a self-reflective culture of learning among teachers (Moon, 2002). These communities can help extend the support provided by the elementary mathematics specialists as colleagues begin to take on responsibilities for their own learning and begin to share aspects of their own practice.

Elementary mathematics specialists are not likely to be expected to engage in the evaluations of teachers or complete any paperwork associated with teacher evaluations. However, many elementary mathematics specialists regularly work with their teacher colleagues as they prepare for aspects of the teacher evaluation process including any formal classroom observations associated with the process. They may also work closely with teacher colleagues to address needs identified by school administrators during the evaluation process.

Another consideration associated with the expectations for the elementary mathematics specialist position has to do with the funding source. Those supported by Title I funding or even special education funding are more likely to have teaching or assisting students as part of their specialist responsibilities. This may also include co-teaching, with a particular emphasis on assisting Title I students or those with an Individualized Educational Plan (IEP), or perhaps being responsible for organizing and implementing "pull out" programs for students with these needs. Those supported with other funding may be more likely to have responsibilities that involve more support for teachers. Sometimes positions are funded through a variety of funding programs, thus creating positions that involve a mix of direct services to students and support for teachers.

How Do We Know This is Working? A Challenge for the Field

As noted earlier, there is little quantitative data measuring the impact of elementary mathematics specialists on student performance in mathematics. This poses a significant challenge to schools and school districts that have funded and filled these kinds of positions. What kind of data might be important to collect? How long would it take for a school's achievement in mathematics to reflect the influence of a school-based mathematics leader?

While research related on the impact of mathematics specialists, instructional leaders, and mathematics coaches is extremely limited, data continues to emerge, and there is reason to believe that mathematics specialists can help teachers in making significant changes in their instructional practices and that these leaders have a positive impact on the instructional practice of teachers with whom they work. Several researchers (Rowan & Campbell, 1995; Campbell, 2007; Campbell and Malkus, 2009, 2011; Erchick et al., 2007) have been at the forefront of examining the impact of elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders on the mathematics achievement of students. While the causality issues here are complicated, there are some issues to consider. Just hiring an elementary mathematics specialist is not a "quick fix" with regard to improving a school's mathematics performance—no surprise there. Achievement gains may not be immediate and may vary across grade levels. Other intervening factors, like the time allotted to mathematics instruction, may also have an impact on student performance. In the meantime, it is hoped that research will consider a broad range of outcomes related to aspects of an elementary mathematics specialist's responsibilities.

A Resource: The Elementary Mathematics Specialists and Teacher Leaders Project (ems&tl)

The Elementary Mathematics Specialists and Teacher Leaders Project (ems&tl), established in 2009 at McDaniel College in Westminster, MD and supported by the The Brookhill Foundation, is dedicated to serving the needs of elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders. The project includes the following components:

a. A national clearinghouse that addresses the growth, development, and ongoing needs relative to elementary mathematics specialists;

b. Collaborative work with a core group of elementary mathematics specialists from central Maryland with a focus on the development and review of the project's case-based work.

c. Professional development for mathematics specialists —both locally and nationally, through the National Council of Supervisors of Mathematics' (NCSM) leadership conferences, prior to National Council of Teachers of Mathematics regional conferences, and during the NCSM Summer Leadership Academy.

d. Access to indicators of the impact of the work related to mathematics specialists at the regional and national level through the study of course offerings at the college/university level; review of state certification efforts; and analysis of school and school district programs that involve specialists, with particular attention to student achievement and teacher background.

The ems&tl Project regularly updates its national clearinghouse website. The website, located at http://mathspecialists.org, has received well over 200,000 hits and includes the following sources of information relative to the work of the elementary mathematics specialists: school district-based initiatives which involve elementary mathematics specialists; college and university graduate level programs for mathematics specialists; state certification guidelines for elementary mathematics specialists; publications ranging from texts to testimonies; and a discussion forum which includes "This Worked!" activities for elementary mathematics specialists. If you have not yet visited this site, we encourage you to do so— it's for you!

Moving Forward

For over three decades elementary mathematics specialists, elementary mathematics coaches, and elementary mathematics instructional leaders have been suggested as a possible solution (or step along the way) to ensuring strong mathematics teaching practice and higher student achievement in mathematics. There is enough history and acknowledgement to finally say, this 'seems like a good idea,' now lets move forward!

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