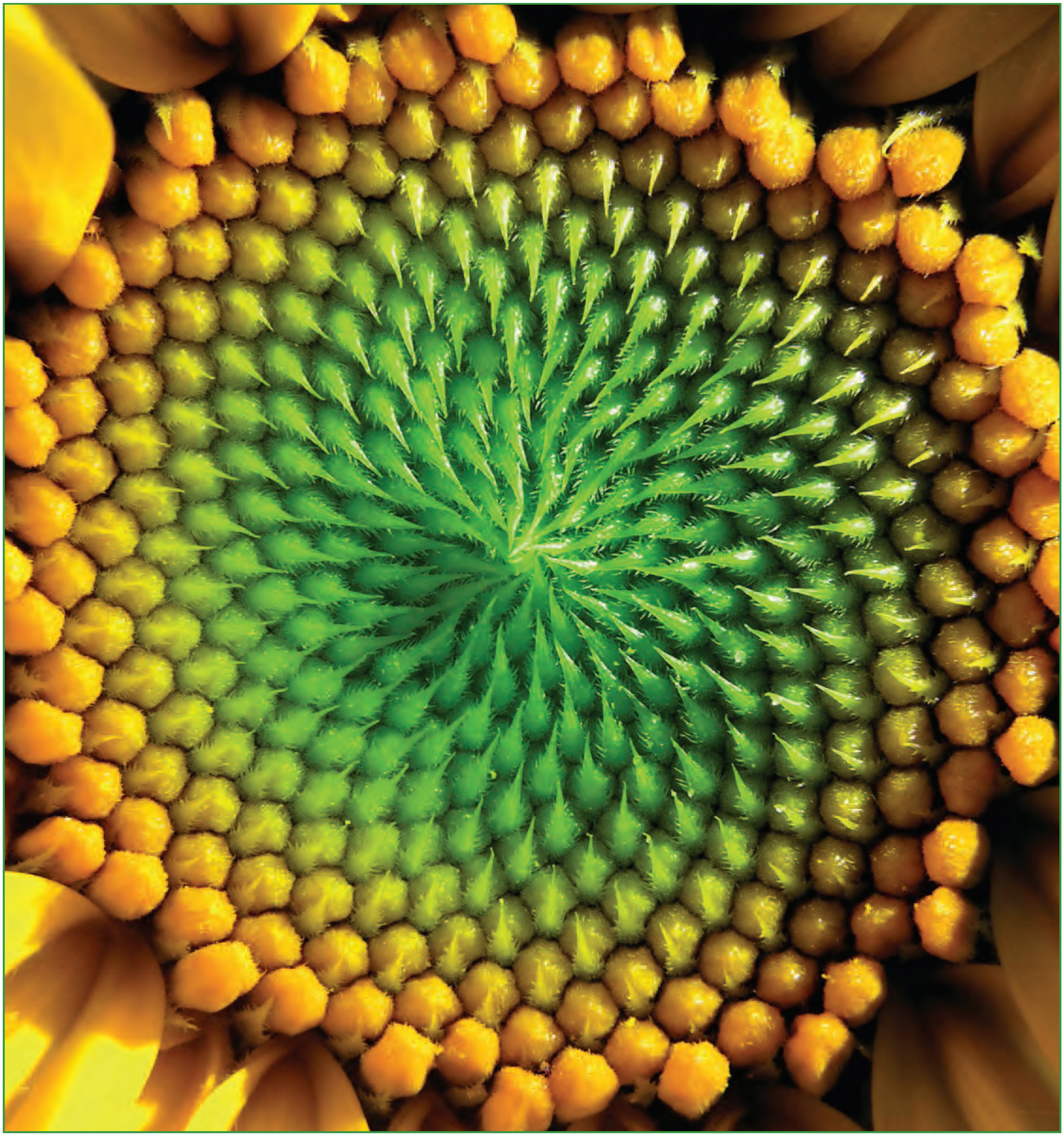
NCSM Journal

of Mathematics Education Leadership

SPRING 2012 VOL. 14, NO. 1



National Council of Supervisors of Mathematics www.mathedleadership.org

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Brief commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We want to hear about your reactions, questions, and connections you are finding to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Linda Ruiz Davenport, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

Inquiries about the ***NCSM Journal of Mathematics Education Leadership*** may be sent to:
Linda Ruiz Davenport
Boston Public Schools Elementary Mathematics Office
443 Warren Street
Dorchester, MA 02132
Email: ncsmJMEL@mathedleadership.org

Other NCSM inquiries may be addressed to:
National Council of Supervisors of Mathematics
6000 East Evans Avenue
Denver, CO 80222-5423
Email: office@ncsmonline.org
ncsm@mathforum.org

***Note:** Information for manuscript reviewers can be found on the inside back cover of this publication.

Table of Contents

Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education

- Fostering inquiry into key challenges of mathematics education leadership

- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice

- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Linda Ruiz Davenport, *Boston Public Schools, Boston, Massachusetts*
Angela T. Barlow, *Middle Tennessee State University Murfreesboro, Tennessee*

As this spring issue goes to press, the Annual NCSM Conference in Philadelphia is just a few weeks away. Those of you who plan to attend the conference may have begun looking through the conference program to find sessions of interest. Which of the "Major Speakers" do you want to hear?[1] Which conference strands are of particular interest, and which sessions within those strands?[2] Which Special Interest Group Sessions might you want to join on Wednesday afternoon for some work with others who share your interests? You may also have begun thinking about colleagues you will see and perhaps talk with over coffee (or other beverages!), exchanging stories of your own work in your own contexts. All in all, the conference is an opportunity to hear from other mathematics education leaders and think about lessons for your own leadership efforts.

In many ways, our *Journal for Mathematics Education Leadership* shares the same mission as these conference experiences. It creates an opportunity for mathematics education leaders to tell their stories to others in leadership roles. It also creates opportunities for those of us in leadership roles to learn from the important work that others are doing in our field. And while there are not the same kinds of opportunities to ask questions during a session or talk with presenters afterward, there are opportunities to converse with our authors through our *Letters to the Editors* where you can share your thoughts about particular articles, raise questions, and make connections to your own work or the work of others. We suspect that many authors would be happy to write back with a response!

It is with pleasure that we bring you this Spring 2012 issue full of lots of interesting articles for you to consider.

First, we hear from Lynn Breyfogle, Kay Wolhuter, and Amy Roth McDuffie about their efforts to support curricular reasoning through a set of interrelated projects that involved elementary teachers and middle school mathematics teachers. By "curricular reasoning" they mean the kind of thinking processes that teacher might engage in as they work with curriculum materials to plan, enact, and then reflect on instruction. Their work has important implications for the variety of roles teacher leaders might play as they work with colleagues to support the thoughtful and purposeful use of curriculum materials to support student learning.

We have another opportunity to think about how teachers use curriculum tools to plan, enact, reflect on, and then revise their instruction as we read through the article by

[1] Major speakers include Freeman A. Hrabowski, III; Deborah Loewenberg Ball; Virginia Bastable, Deborah Schifter, and Susan Jo Russell; Robert Q. Berry, III; Hugh Burkhardt; Marilyn Burns; Francis "Skip" Fennell; David Foster; William Haver, Vickie Inge, and Vandi Hodges; Timothy Kanold; William McCallum; Suzanne Mitchell; and Mary Kay Stein.

[2] Conference strands include Bridging the Knowing-Doing Gap: Implementing the Common Core State Standards; ALL Means ALL: Promoting Equity in Mathematics Education; Mathematics Leaders Translate Vision to Practice: Developing and Supporting Coaches, Specialists, and Teacher Leaders; Making Mathematics Learning Visible: Implementing Formative and Summative Assessments to Support Student Learning; and Leadership to Enhance Mathematics Teaching and Learning for All: Helping Administrators and Other Leaders Support High Quality Mathematics Instruction.

Randall Groth, Jennifer Bergner, and Harel Barzelai. This author team examines how high school mathematics teachers make use of algebra tiles to address important mathematics content in the context of a lesson study project. There are important lessons for how facilitated reflections on lessons, using lesson artifacts and video, can help teachers reflect on and strengthen how they work with curriculum tools to support student learning.

In the article from Michelle Stephan, Didem Akyuz, George McManus, and Jennifer Smith we have yet another opportunity to consider how teachers might collaborate to strengthen their mathematics instruction by coming together to create a mathematical community of teacher learners. They describe the work of one such community of middle school mathematics teachers that designed and then taught a set of lessons addressing the addition and subtraction of integers. They provide a useful list of the characteristics of communities of teacher learners and discuss how these characteristics were reflected in the work they did together. They also, importantly, identify the kinds of supports that are needed to create and sustain these kinds of learning communities.

From there we shift gears to consider an article from Sara Eisenhardt and Jonathan Thomas about an early numeracy intervention project designed to provide diagnostic and intervention services to kindergarten through third grade students. The project involved professional development for Mathematics Intervention Teachers through a newly created Kentucky Center for Mathematics. The article reports on the nature of this professional development and its impact on teacher beliefs and practices and well as its impact on student learning, with some discussions of lessons for how districts and states might undertake such efforts at scale.

In an article by Eric Hsu, Diane Resek, and Katherine Ramage we hear about a project that involved preservice and inservice mathematics teachers and focused on changing their conceptions of mathematics. Through this project, teachers engaged in doing mathematics, selecting mathematics tasks to do with others and getting feedback, and considering what it would mean to do this kind of mathematics with their own students. The authors provide us with lots of examples of rich problems from the project, details about challenges and successes as teachers engaged in these problems, suggestions for what to watch for as teachers pose these kinds of problems with colleagues and with students, and some resources that might be useful for other mathematics education leaders who might want to take on similar efforts.

Much of the work discussed so far in this issue of the journal involved some level of collaboration among schools, districts, and universities. Cathy Kinzer, Lisa Virag, Sara Morales, and Ken Korn, in their article about partnerships for learning, discuss what it means to collaborate successfully across these different kinds of institutions and organizations. Their Innovation Configuration Map contains guidelines that, if attended to in explicit ways, have the potential to strengthen such partnerships considerably.

In our closing article, we hear from Regina Mistretta about her efforts to prepare teachers to build strong collaborations with parents and families around mathematics teaching and learning. She describes a professional development effort that provided teachers with opportunities to build these collaborations over time, with a focus on mathematics content and pedagogy, what it meant to share one's mathematical thinking, and why this was an important part of the process of learning mathematics.

We invite you to read through these rich and interesting articles yourselves to find connections to your own mathematics education leadership work. We also invite you to let us know about these connections through a letter to the editor, or perhaps through an article of your own, where you recount some aspect of related work you might be doing. Such stories have so much to offer us all!

And perhaps we will see you in Philadelphia!

Supporting Teachers' Effective Use of Curricular Materials

M. Lynn Breyfogle, *Bucknell University*
Kay A. Wohlhuter, *University of Minnesota Duluth*
Amy Roth McDuffie, *Washington State University Tri-Cities*

Many of us working in mathematics education leadership roles have experienced the importance of and challenges in effectively supporting teachers as they learn to implement new curricular materials or materials new to them. Teachers need to be able to discern the important daily mathematical concepts embedded in the context of the unit, how these unfold over the school year, and how these relate to grade level expectations in terms of district or state standards (Roth McDuffie, Wohlhuter, & Breyfogle, 2011). In addition, teachers need to be able to consider students' prior knowledge and implication for how they might need to adapt, supplement, or omit portions of the materials to meet students' needs. For new teachers or teachers using new materials, this can truly be a daunting task. In this article, we draw from our professional development work in many classrooms (see Breyfogle & Spotts, 2011; Latterell & Wohlhuter, 2004; Roth McDuffie & Eve, 2009) to focus on how mathematics education teacher leaders, working directly with teachers in schools, can support teachers in this process. We view teachers leaders as persons who provide mathematics leadership within a building or a district and could have roles and titles such as mathematics specialists, mathematics coaches, principals, directors of curriculum and instruction, university mathematics educators, professional development leader, or other individuals who work to support mathematics teaching and learning in a school or district. We begin with a brief background and description of curricular reasoning—a type of reasoning that we have found is helpful for teachers to develop—and then discuss two settings in which teacher leaders can actively support teachers' development of this reasoning.

Importance of Curricular Reasoning

Two important shifts having to do with the effective use of curricular materials have occurred over the past two decades: (a) the publication of curricular materials aimed at problem solving, reasoning, and students' conceptual understanding of mathematics and (b) the development of curricular standards with increased accountability for learning measured by performance on state assessments. In response to these shifts, mathematics education leaders have realized the importance of helping teachers develop thinking processes to engage in as they work with curricular materials to plan, implement, and reflect on instruction, a process we refer to as *curricular reasoning* (Breyfogle, Roth Mc Duffie, & Wohlhuter, 2010; Roth McDuffie & Mather, 2009). Although curricular materials can strongly influence the nature of, and approaches to, mathematics teaching and learning, curricular materials alone do not ensure an effective lesson (Boaler, 2002). Teachers' decisions significantly influence this process. Below we focus on how teacher leaders can support the development of teachers' curricular reasoning while engaging in an observation-conferencing cycle and supporting teacher collaboration.

Observation-Conference Process Focusing on Use of Curricular Materials

The observation-conference process provides opportunities for dialogue between the teacher and teacher leader. In our work with different groups of teachers, this process was used to understand the classroom context and to determine the level of implementation of research-based effective teaching practices. In both situations, the observation-conference process was repeated on a regular basis (e.g., monthly) and typically occurred within the school day during the teachers' planning period, usually on the day of or following the observed class sessions. Some teachers agreed to participate in the projects, some of which included the videotaping of lessons, but most were encouraged by their building administrators and consented to participate as a result.

In the research that focused on understanding the classroom context, the observation-conference process occurred between individuals that already knew each other. For the cadre of teachers focused on research-based effective teaching practices, we held professional development sessions afterschool or on established professional development days for in order to establish a rapport with teachers. Additionally, in some of our work, on-going professional development sessions occurred throughout the year of working with the teachers. For example, in one project, one of the authors was asked by the building principal to serve as a mentor to his four mathematics teachers who were struggling with the implementation of NSF-funded middle school materials during a two-year period. She first established a rapport with the teachers prior to these interactions by providing a two-day professional development session focusing on research-based effective teaching practices during pre-established professional development days. She then conducted the observation-conference sessions during the school day and provided two-hour after-school monthly professional development sessions focused on an issue of teaching that emerged from one of the conference-observations.

As teacher leaders, we can support teachers' curricular reasoning by encouraging teachers to focus on students' needs during discussions about the planning of lessons and to engage in focused reflection in the post-observation conference. While enacting this dialogue involving curricular reasoning, teacher leaders can demonstrate respect for teacher knowledge by employing deep listening and suspending their personal assumptions about the classroom (Glover, 2007). In this section we elaborate on ways in which these practices support curricular reasoning and improved instruction.

Making decisions based on students' needs. Effective teaching is characterized by teachers understanding students' mathematical knowledge, what mathematics students need to learn, and how best to help learning occur (NCTM, 2000). Teachers develop these practices by applying curricular reasoning to: identify and understand the mathematics, anticipate potential approaches that learners might bring to a lesson, and consider students' backgrounds and experiences (Breyfogle, Roth McDuffie, & Wohlhuter, 2010). Teacher leaders can assist in this process by explicitly discussing the above practices and posing appropriate questions during the planning of lessons.

Our work with teachers includes research that focused on understanding the teaching and learning process in beginning mathematics teachers' classroom (Latterell & Wohlhuter, 2004). How teachers provided learning opportunities was one component of a two-year study. An eighth-grade teacher with the long-term goal of students understanding properties of linear relationships in tables, graphs, and equations knew it was important for students to develop foundational ideas about rate of change. *The Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) curricular materials used walking rates as one context for exploring rate of change. The teacher supplemented the curriculum by having students observe classmates' walking rates before they engaged in the textbook's task that asked students to determine how different walking rates (e.g., 1.5 m/s, 2m/s, 2.5m/s) affected the distance traveled and time needed for traveling. She observed students working in groups and listened to them as they shared their solutions. Based on her observations the teacher considered the extent to which students were ready for the next lesson that focused on making tables and graphs.

Effective teacher leaders facilitate this kind of lesson planning by raising questions that help teachers identify and understand important aspects of curricular reasoning, including what mathematical ideas are embedded in the lesson and how students' backgrounds and experiences may affect learning. Possible questions for the rate of change lesson include:

- How do these mathematical ideas fit together?
- What is the trajectory of learning for rate of change embedded in or underlying the design of the curricular materials?
- What evidence do you have about students' current understanding that helped you determine to focus on rate of change in this way?
- How did your students learn about the concept of rate of change in previous years and where is it leading in your curriculum?

- From your experience, what have students struggled with about this idea and how has your planned lesson addressed this?
- What specific gaps or misunderstandings about rate of change might be uncovered during the lesson?
- How will you know what each student understood about the concept rate of change at the lesson's conclusion?
- How will you use what you learned about students' knowledge to determine the content of the next lesson?

Raising questions like these models the types of questions teachers should be regularly asking themselves.

Providing opportunities for focused reflection that supports curricular reasoning. Post-observation lesson discussions provide an opportunity for facilitating teachers' curricular reasoning development by focusing reflection on teaching and learning. This means that teacher leaders serve as sounding boards and mirrors, allowing teachers to reflect on their lessons and consider ways they might both revise how they would teach this lesson again in the future and also adjust the next lesson to meet students' needs. We have found this type of reflection to be instrumental in their curricular reasoning development.

In the project in which one of the authors served as a mentor to the four middle school teachers, each teacher set individual pedagogical goals for improving their teaching at the start of each year. The teachers chose goals such as finding ways to differentiate instruction to challenge all of her students and selecting tasks to increase students' level of engagement. During observations, the mentor kept these in mind and focused her note-taking on this particular aspect of the lesson, including identifying specific times on the video that could be revisited and discussed with the teacher during the interview following the observations. These goals and the teachers' level of success with the goals were individually evaluated at each of the subsequent monthly observation conferences.

The teacher leader observed a 6th grade teacher who set the personal goal of challenging her students while she taught a lesson with this objective: to determine if triangles could be made given any set of side lengths while classifying triangles according to the lengths of their sides (e.g., scalene, acute, equilateral or not possible). The plan for the lesson was for students to intuitively develop the Triangle Inequality Theorem by trying to construct eight different triangles given sets of side lengths. For this activity students were provided pipe cleaners of varying lengths and a lab-sheet (See Figure 1 on next page) to record their findings. Concerned about the accessibility of this task with all of her students, the teacher asked the students to draw and label an additional column on the table that said, "sum of two shorter sides" and told the students that they were going to see a rule. While walking around observing the small groups, she provided explicit suggestions like "focus on the 'length of largest' to 'sum' columns" that funneled the students' thinking rather than allowing them to generate their own conjectures. An unintended consequence of these kinds of prompts was that the teacher took away the problematic aspects of the task such that the students were completing steps rather than engaging in mathematical reasoning. Sensitized to the goal of challenging all of the students, the teacher leader made notes of these instances so that during the observation interview she could raise questions with the teacher. Showing the videotaped excerpts and asking questions like, "Why did you choose to provide this suggestion to this group?" or even more focused prompts such as "How did your hints/prompts affect students' engagement in mathematical thinking and reasoning of this task?" were intended to help the teacher identify moves that contributed to decreasing the cognitive demand of the task, as well as identifying more productive moves in teaching. Questions such as these help the teacher realize how seemingly minor changes to the curricular materials (e.g., adding the column to the lab sheet) and prompting the students with hints have the adverse effect of reducing or eliminating opportunities for students to engage in the reasoning processes on which the lesson objectives were aimed (for further elaboration see Roth McDuffie, Wohlhuter & Breyfogle, 2011).

To promote and support teachers' engagement in curricular reasoning while reflecting on lessons, teacher leaders can raise other questions such as:

- Did the sequence of tasks build understandings appropriately?
- Did you anticipate students' needs in preparing them to engage in the tasks?
- During the lesson's summary portion, did you sequence and connect ideas in the materials to solidify learning and to prepare for future lessons?

FIGURE 1: *Labsheet for Exploration*

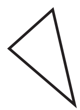
Name____Date____________________

MODULE 1 **LABSHEET 2C**

Sides of a Triangle (Use with Questions 15 and 16 on page 17.

Directions Try to form a triangle with each stick combination. For each triangle you form,
- make a sketch and classify it as *scalene, isosceles*, or *equilateral.*
- record the lengths of its sides in the appropriate columns.

If you were not able to form a triangle, write *not possible*.

Stick combination	Sketch of triangle	Type of triangle	Length of the longest side	Length of the two other sides
3 in. 4 in. 5 in.		scalene	5 in.	3 in., 4 in.
2 in. 2 in. 5 in.				
3 in. 5 in. 5 in.				
6 in. 6 in. 6 in.				
2 in. 3 in. 4 in.				
2 in. 3 in. 6 in.				
3 in. 5 in. 8 in.				
5 in. 5 in. 8 in.				
4 in. 4 in. 8 in.				

 Math Thematics, Book 1 **1-53**

(Billstein & Williamson, 1999)

Raising questions like these provides opportunities for focused reflection that supports curricular reasoning and models the types of questions teachers should be regularly asking themselves.

Supporting Collaborative Work Involving Curricular Reasoning

Teacher leaders need to establish and support teacher communities (Lattimer, 2007). Studies of collaborative teaching environments indicate that when teachers focus on students' learning, teaching practices and students' learning improves (e.g., McLauglin & Talbert, 2006). While various models for collaboration exist (e.g., professional learning communities, lesson study, video clubs), some common characteristics for effective collaborations emerged in our work with teachers' curricular reasoning.

Typically in our research, teachers collaborated in grade level teams, and in some cases the entire school staff participated (Roth McDuffie, 2009; Roth McDuffie & Eve, 2009). The team's work included: examining students' test data on state-wide or district assessments; designing and analyzing common classroom assessments; studying state curriculum documents for grade-level learning targets; analyzing curricular material's purpose, methods, scope and sequence across grades, and alignment with standards; and co-planning lessons, observing each others' teaching, and analyzing and reflecting on students' work. Teachers continually reflected that these collaborative activities affected their practice because activities were based in and relevant to their practice and students. In addition, these activities kept teachers focused on understanding and learning about students' thinking, teaching approaches, mathematics content knowledge, gaps and learning outcomes rather than limiting work only to "swapping new strategies and activities" that did not lead to real change. How teacher leaders structured and supported this collaborative work influenced the extent to which schools improved.

In a different two-year project, we collaborated with two elementary schools, with all teachers working in grade level teams as described above. In the second year, a new principal came to one of the schools. Although this principal supported and encouraged teachers' collaborative work, given that she was new to the school, she was reluctant to establish clear expectations or hold teachers accountable for collaborating with their teams to improve teaching and learning. This new principal's stance stood in contrast both to the other school and to the school's previous principal. Patterns for the relationship between a teacher leader's stance or actions and teachers' engagement in collaborative teams were evident. With these experiences and other researchers' findings in mind, we found that teacher leaders needed to both *support* and *expect* teachers' collaboration. Additionally, teachers needed to be held accountable for outcomes from their efforts. This accountability helped the whole school to prioritize collaborative school improvement. Below we discuss ideas for ways to support and expect collaborative work as teachers interact with their curricular materials and engage in curricular reasoning.

- Recognize that building this culture centered on improving students' learning requires time in the day and occurs over time in the year(s). Provide time and space (e.g., prioritize collaborative work in scheduling) for work to take place, and then ask for agenda and reports for team's activities and progress to maintain accountability.
- Help teams to develop goals for work centered on students' learning (different from collegial interactions which degenerate into "trading worksheets"). Depending on the model for collaboration, many resources are available to guide this process (e.g., Lewis, 2002; McLaughlin & Talbert, 2006).
- Ensure that teachers feel safe to try different approaches. Support responsible risk-taking by allowing them to keep collaborative planning, common assessments, and lesson observations separate from teachers' evaluations, and understand that innovations will need revisions and improvements. Expect teachers to justify their experimental approaches with research-based literature.
- Invest in the teams' efforts by attending meetings and serving as an active member. Teacher leaders' attendance provides opportunities to model deep listening and discourse that builds on participants' ideas. In addition, by listening to discussions, teacher leaders can identify ways to support teachers' work and address any early obstructive behaviors before they become a problem (e.g., a leader can hold a private discussion with a teacher who may be showing signs of obstructing the work to reflect back behaviors and explore how the teacher could better support the learning team).
- Allow teachers to drive and own the process, respect their knowledge and expertise (Lattimer, 2007), and value different ways each teacher can contribute.

Table 1. Roles for Participants in Collaborative Learning Settings

PARTICIPANT ROLE	DESCRIPTION OF CONTRIBUTIONS FROM PARTICIPANT IN THIS ROLE
Seasoned Practitioner	Anticipates trouble spots and strengths based on years of working with students.
Researcher	Reads professional literature, reviews and uses supplemental curricular materials, considers perspectives from educational research and theory, and/or attends outside workshops/speakers, and regularly shares new knowledge.
Organizer	Coordinates meeting scheduling, initiates agenda planning, keeps meeting notes and records, makes sure materials are prepared in advance, and reminds participants of responsibilities for follow through.
Encourager	Provides supportive comments, makes sure all voices and ideas are heard and valued, attends to emotional needs of participants, and keeps work moving in a positive direction.
Experimenter	Offers to try new approaches in his/her room as a test case, open to new strategies and/or using new materials, willing to pioneer new ideas (especially when others are reluctant to change and need to see it tested first).
Obstructer	Finds obstacles or reasons not to collaborate and improve. Participants should not permit others to take on this role, and teacher leaders may need to intervene to ensure that all participants are expected to avoid this role.

In considering the last recommendation, we identified a range of roles that teachers can take on in a collaborative environment. Participants' roles and the corresponding contributions that we have encountered are described in Table 1. These roles highlight a need for teacher leaders to value teachers' expertise, strengths, and voice in designing, planning, and implementing the collaborative learning, and if needed, to help teachers identify their roles. Teacher leaders and participants must expect and communicate that the *obstructer* is not an acceptable role. Note: critically examining and carefully considering new approaches/materials (a form of curricular reasoning) is an important part of the process to ensure that new approaches are not tried just because they are different, and this type of thinking should not be confused with obstructing the work.

Conclusion

In this article, we have conceived of "teacher leaders" quite broadly to include persons with various roles in a building and/or district and who are in a position to support teachers in their professional development and learning. A theme underlying the recommendations and approaches we discussed is that teacher leaders need to actively look for and provide opportunities to engage teachers in examining their practices and supporting their students' needs and learning. Specifically, we focused on developing curricular reasoning as an important part of becoming an effective mathematics teacher. Teacher leaders can help develop teachers' curricular reasoning by considering powerful questions to ask during teachers' design and lesson planning and when observing teachers, and also by encouraging and supporting their collaborative work.

References

Billstein, R., & Williamson, J. (1999). *Middle Grades Math Thematics.* Evanston, IL: McDougal Littell Inc.

Boaler, J. (2002). *Experiencing School Mathematics: Traditional and Reform Approaches to Teaching and Their Impact on Student Learning.* Mahwah, NJ: Lawrence Erlbaum Associates.

Breyfogle, M. L., Roth McDuffie, A., & Wohlhuter, K. A. (2010). Developing curricular reasoning for grades preK-12 mathematics instruction. In B. J. Reys and R. E. Reys (Eds.) *Mathematics Curriculum: Issues, Trends, and Future Directions.* Reston, VA: NCTM.

Breyfogle, M. L. & Spotts, B. (2011). Professional development delivered right to your door! *Teaching Children Mathematics*, 17(7), 420-426.

Glover, E. (2007). Real principals listen. *Educational Leadership*, 65(1), 60-63.

Lappan, G., Fey, J.T., Fitzgerald, W.M., Friel, S.N., & Phillips, E. *Connected Mathematics.* Menlo Park: Dale Seymour, 1998.

Lattimer, H. (2007). To help and not hinder. *Educational Leadership*, 65(1), 70-73.

Latterell, C.M. , & Wohlhuter, K.A. (2004). Beginning secondary mathematics teachers: A snapshot across our state. *Focus on Learning Problems in Mathematics*, 26(1), 23-33.

Lewis, C. (2002). *Lesson Study: A Handbook of Teacher-Led Instructional Change.* Philadelphia: Research for Better Schools.

McLaughlin, M., & Talbert, J. (2006). *Building school-based teacher learning communities: Professional strategies to improve student achievement.* New York: Teachers College Press.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

NGA & CCSSO. (2010). Common Core State Standards for Mathematics. Retrieved from http://www.corestandards.org.

Roth McDuffie, A. (2009). Mathematics Curriculum Implementation via Collaborative Inquiry: Focusing on Facilitating Mathematics Classroom Discourse. In D. Slavit, T. Holmlund Nelson, & A. Kennedy (Eds.), *Perspectives on Supported Collaborative Teacher Inquiry* (pp. 46 – 71). Oxford, UK: Routledge.

Roth McDuffie, A., & Eve, N. (2009). Breaking the Area Boundaries. *Teaching Children Mathematics*, 16(1), 18-27.

Roth McDuffie, A. & Mather, M. (2009). Middle School Teachers Use of Curricular Reasoning in a Collaborative Professional Development Project. In J. Remillard, G. Lloyd, & B. Herbel-Eisenmann (Eds.), Mathematics teachers at work: Connecting curriculum materials and classroom instruction (pp. 302-320). Oxford, UK: Routledge.

Roth McDuffie, A. , Wohlhuter, K. A. & Breyfogle, M. L. (2011). Tailoring Instructional Tasks to Meet Students' Needs. *Mathematics Teaching in the Middle School*, 16(9), 550-555.

Supporting Teachers' Understanding and Use of Algebra Tiles

Randall E. Groth, Jennifer A. Bergner, Harel Barzilai
Salisbury University, MD

One of the most formidable challenges in school mathematics is helping students learn algebra. The construction of algebraic habits of mind is an important milestone in students' mathematical development (Driscoll, 1999). To reach this milestone, students must learn algebra with understanding rather than simply learning by rote (Kieran, 2007). Although conceptually oriented teaching approaches show promise for building student understanding (Senk & Thompson, 2003), making the shift to a reform-oriented curriculum is a non-trivial matter. Teachers often must re-examine their personal paradigms of mathematics instruction while also learning about new content and instructional materials. Mathematics education leaders need to be cognizant of teachers' specific professional development needs with regard to algebra teaching and learning and determine how to address them.

Learning appropriate uses for manipulatives is a common need among teachers making the shift to conceptually oriented instruction (Chval & Reys, 2008). Manipulatives afford unique opportunities for students to explore the structure of mathematical ideas, but simply placing them in front of students will not, in and of itself, improve learning (Ball, 1992; Chappell & Strutchens, 2001; Moyer, 2001). Teachers play important mediating roles in how they introduce manipulatives and encourage students to use them. Those who are most successful are able to support students without taking over the thinking involved in the problem at hand (Stein & Bovalino, 2001). In order to do this, teachers themselves must have a deep understanding of the mathematical ideas in a lesson and the manner in which manipulatives can be used to help reveal the conceptual structures associated with these ideas.

Algebra Tile Model

Algebra tile manipulatives can be used as tools to foster conceptual understanding in beginning algebra courses. A diagram of the pieces in a typical set of tiles is shown in Figure 1. For an example of the use of algebra tiles, consider the multiplication $(x + 1)(x + 3)$. The product can be interpreted as the area of a rectangle whose dimensions are $(x + 1)$ and $(x + 3)$. A diagram for performing the multiplication is shown in Figure 2. The shaded area in Figure 2 indicates that the area of a rectangle with dimensions $(x + 1)$ and $(x + 3)$ is $x^2 + x + x + x + x + 1 + 1 + 1$, or $x^2 + 4x + 3$. As another example, if students were to factor $x^2 + 4x + 3$, they could begin by building the rectangle shown in the shaded area of Figure 2. The task of factoring could then be interpreted as reading off the dimensions of a rectangle with area $x^2 + 4x + 3$. Doing so

FIGURE 1. *Pieces in a typical set of algebra tiles*

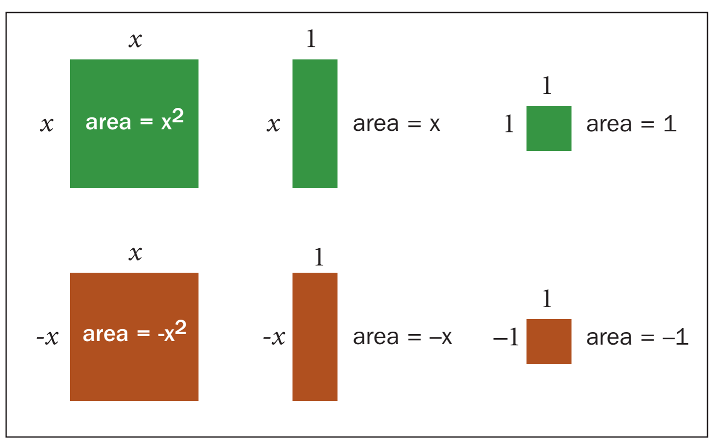
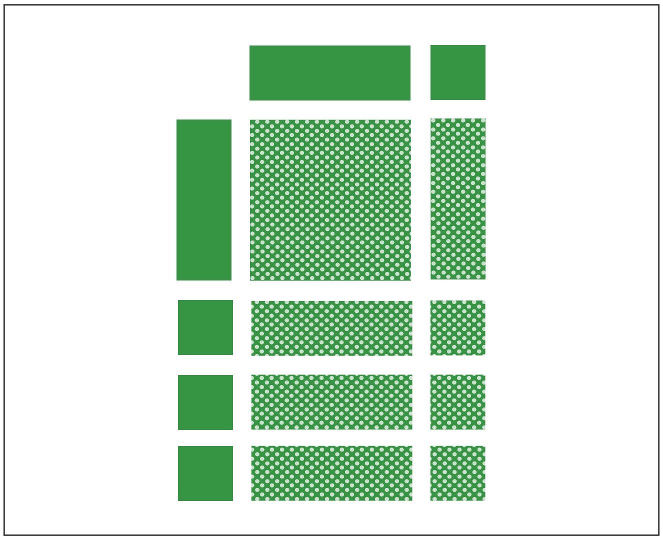


FIGURE 2. *Algebra tile diagram for polynomial multiplication and factorizations*



would give the factorization $(x + 1)(x + 3)$. Additionally, one could interpret the $x^2 + 4x + 3$ area as a dividend. The $x + 1$ length could then be thought of as a divisor, with the width, $x + 3$, being the quotient determined from constructing a rectangle with and area of $x^2 + 4x + 3$ and length of $x + 1$.

Lesson Study Professional Development

We had the opportunity to work with a group of four high school mathematics teachers as they used algebra tiles with their students as part of a university-sponsored lesson study project. The group was established as part of a larger funded project that included three school districts (Groth, 2011). The teachers were new to the lesson study process and to using algebra tiles with students. One teacher in the group, Janet (all teacher names are pseudonyms), had recently learned of algebra tiles at a professional conference. When presented the opportunity to engage in lesson study, Janet suggested to the group that they focus on becoming better acquainted with the tiles and using them to teach polynomial factoring. The other teachers in the group agreed that polynomial factoring was a difficult topic for their students and were willing to explore the potential of the algebra tiles to teach polynomial factoring during their lesson study work together. Because the algebra tile model was new to them, they expressed interest in the opportunity to learn how the algebra tile model might be used to address the mathematics content they were responsible for teaching by working with their colleagues and university faculty during lesson study.

The lesson study process for the project is depicted in Figure 3. Its structure allowed teachers to gradually polish and refine ideas for instruction as they worked together. The rectangles in Figure 3 represent the phases in a lesson study cycle. Arrows between the rectangles indicate the progression that occurred from one phase to the next. Teachers were given one semester to progress through each cycle and completed two cycles (see Table 1). As indicated in Figure 3 and in Table 1, the first phase consisted of constructing a lesson collaboratively. The lesson study goals were not dictated by university personnel. Instead, teachers chose learning goals in collaboration with one another (Lewis & Tsuchida, 1998). Once the goal of teaching

FIGURE 3. *A lesson study professional development model*

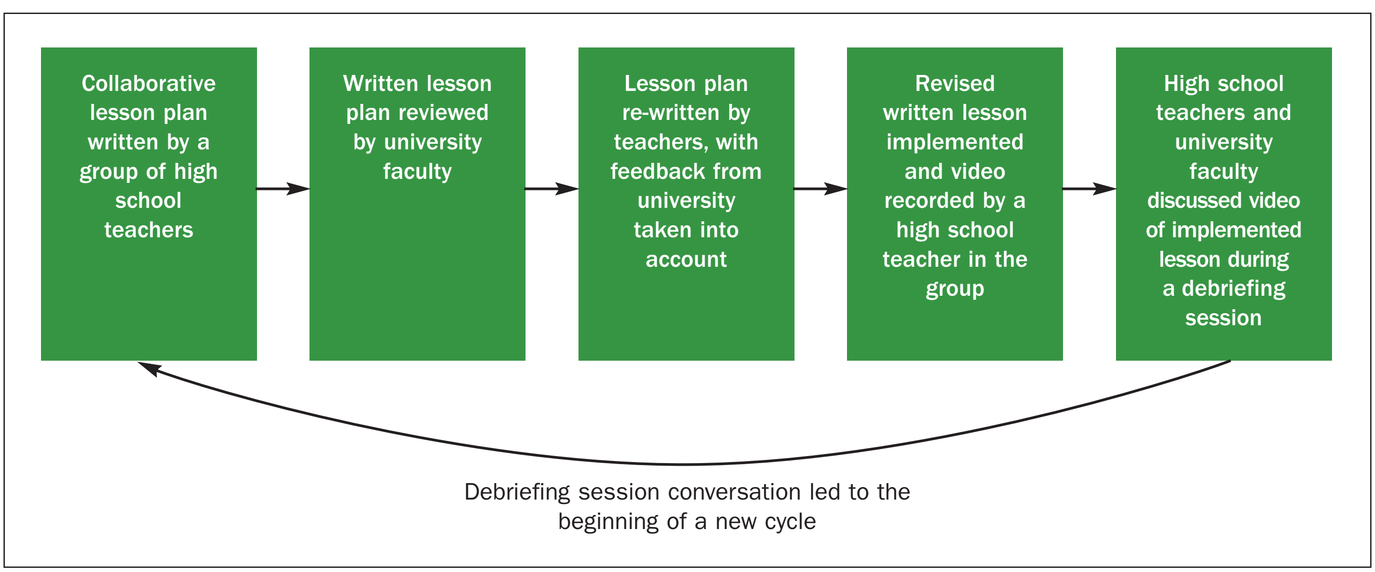


Table 1 — Lesson Study Timeline

Fall semester: Lesson study cycle 1 Lesson-implementing teacher: Janet	Spring semester: Lesson study cycle 2 Lesson-implementing teacher: Martha
1F. Algebra tile lesson designed collaboratively by teachers 2F. Algebra tile lesson reviewed by university faculty 3F. Teachers re-wrote lesson, taking university faculty feedback into account 4F. Janet implemented the lesson with her class and it was video-recorded 5F. Debriefing session occurred in which university faculty and teachers viewed and discussed video-recorded lesson. This motivated another cycle of lesson study in the spring dedicated to improving algebra tile usage during instruction.	1S. Algebra tile lesson from the fall cycle revised by teachers using feedback from cycle 1 2S. Revised written lesson reviewed by university faculty 3S. Teachers re-wrote lesson, taking university faculty feedback into account 4S. Martha implemented the lesson with her class and was video-recorded 5S. Debriefing session occurred in which university faculty and teachers viewed and discussed video-recorded lesson. Discussion focused on how the lesson could be further refined and improved.

polynomial factorization using algebra tiles was identified, the teachers collaboratively wrote a lesson to be implemented by one of the members of the group.

After the teachers wrote their lesson plan, it was sent to university faculty for review. Two university mathematics faculty members reviewed each lesson. The faculty reviews were solicited because previous research has shown that perspectives from outside a lesson study group can be valuable for identifying areas for improvement in lessons (Fernandez, 2005; Yoshida, 2008). Reviewers made written comments on potential strengths and weaknesses of the lessons. Within two weeks of the submission of each lesson, university faculty feedback was sent to the teachers. Teachers were then asked to use the feedback, to the extent they judged feasible, to revise the lesson before implementing it. Throughout the process, they had the freedom to accept or reject any feedback using their professional judgment.

After the initial written lesson was revised, one member of the lesson study group taught it while another member of the lesson study group video recorded it. The videos were later viewed by all of the teachers during a debriefing session, along with the second and third authors of this article. Although it is ideal to have all of the teachers present in the room when the lesson is implemented (Lewis, 2002), when schedules do not allow for this, a video of the lesson can be used as an alternative. During each debriefing session (one per cycle), the teacher who implemented the lesson and the videographer provided context to help explain the lesson video. After this information was shared, the video was played, and debriefing session participants took notes on perceived strengths and weaknesses of the lesson. When the video concluded, each individual participating in the meeting was prompted to share his or her perceptions. The arrow from phase 5 (debriefing) to phase 1 (planning) in Figure 3 shows that the debriefing session conversation from the first lesson study cycle sparked a second cycle.

As the lesson study process was carried out, several artifacts were retained to record its history and analyze teachers' learning:

- Written lessons produced by the teachers;
- Written feedback on the lessons given by university faculty;
- Video recordings and transcripts of the lessons teachers implemented during the first and second cycles of the lesson study; and
- Audio recordings and transcripts of the debriefing session conversations involving the university faculty and teachers during the first and second cycles of the lesson study.

The authors of this article collaboratively analyzed the artifacts to identify key learning experiences during the project. Our goal in doing so was to help other mathematics education leaders anticipate elements of teachers' knowledge that may need development and support as they begin to use algebra tiles and other manipulatives to address important mathematics content.

Key Learning Experiences for Teachers Beginning to use Algebra Tiles

In the remainder of this article, we describe what we consider to be the most important elements of conversations that occurred as teachers learned to use algebra tiles:

- Grounding students' work with algebra tiles in the concept of area;
- Helping students understand how the length of a tile is meant to represent a variable quantity;
- Using algebra tiles to establish the conceptual ground for factoring rather than just illustrating procedures;
- Choosing polynomials with the potential to encourage genuine problem-solving; and
- Encouraging problem-solving classroom discourse with algebra tiles.

We conjecture that several of these elements apply not only to the group of teachers described in this article, but will also apply to other teachers learning to use algebra tiles. Some of the elements also apply to the use of manipulative models in general, such as focusing on concepts, problem solving, and rich classroom discourse.

GROUNDING STUDENTS' WORK WITH ALGEBRA TILES IN THE CONCEPT OF AREA

Although the algebra tile model is based upon dimensions and areas of rectangles, it was challenging for teachers to think about how to make this connection in their classrooms. In the group's first collaboratively written lesson, the relationship between algebra tiles and area was not mentioned at all. When the teachers sent the written lesson to us for review, we recommended that they connect students' previous experiences with area outside of algebra classes to work with polynomials. One recommended activity was to have students make as many arrangements of 12 unit blocks as possible to find the factors of 12 before doing similar work with trinomials. This work would then lead to situations where length and width were variable quantities.

After the reviewers' comments were shared with the lesson study group, the teachers met with each other to decide how to use the feedback to edit the initial written lesson. Janet (a pseudonym, as are the rest of the teacher names in this report) then taught the revised lesson in her class. One striking feature of the lesson video was that the word "area" was not used at all in reference to the tiles. Janet gave a name to each piece in the algebra tile set, and then characterized the task of factoring a polynomial as arranging the appropriate pieces into a rectangle. This made each factorization into a jigsaw puzzle-like task to perform. When Janet wanted to prompt students to provide the factors of a given polynomial, she asked the question "What do these have in common?," being somewhat ambiguous about what "these" referred to. When students did not respond to the initial question, she would ask a more directive question such as "How many columns are there?" so a student would offer the correct response. If the idea of area had been used in connection with the tiles, it would have allowed her to focus students' attention instead on the question, "What are the dimensions of a rectangle with the area of the given polynomial?" During the debriefing session for this lesson, we again mentioned the connection to area and encouraged teachers to make this connection during the cycle 2 lesson.

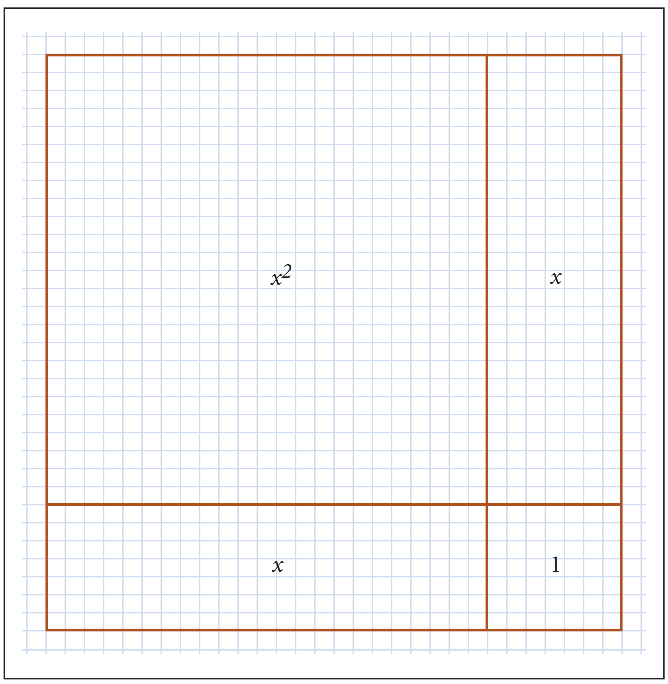
When implementing the cycle 2 lesson, the teachers *did* attempt to connect the concept of area to the algebra tile model. The implementing teacher for the cycle 2 lesson, Martha, asked students to think about what a 3 by 3 rectangle would look like with the unit tiles. She then asked students to think about an x by x rectangle, encouraging them to generalize the model to rectangles with variable lengths. Despite this progress from the cycle 1 lesson, it still proved difficult to connect the algebraic concepts represented by area to the lesson at points where this connection would be useful. For example, after Martha began discussing a rectangle with dimensions $(x + 2)$ and $(x – 3)$ with students, the word "area" was not used for the remainder of the lesson. Instead, when students were given factoring problems, Martha told them to decide what was "above" and "to the left" of the "box" rather than to find the dimensions of a rectangle with a given area. During the debriefing session for this lesson, we once again took the opportunity to suggest that teachers frame polynomial factorization tasks as determining the length and width of a rectangle whose area is represented by a given polynomial. After viewing the cycle 2 lesson video, teachers recognized this as being necessary for strengthening student's conceptual understanding.

HELPING STUDENTS UNDERSTAND HOW THE LENGTH OF A TILE IS MEANT TO REPRESENT A VARIABLE QUANTITY

Another key instructional decision discussed during debriefing sessions was that teachers required students to put their algebra tile diagrams for factorization problems on grid paper. This occurred during both the cycle 1 and cycle 2 lessons. Although this move produced neat-looking student papers, it also implied that the x-length in the model had a set integer value. (See Figure 4.) Algebra tile manipulatives are intentionally constructed so that unit tiles cannot precisely measure out the x-length, but when drawing tile pieces on graph paper, this structural characteristic of the model is lost. During debriefing sessions, teachers initially supported the decision to use grid paper for algebra tile diagrams because they felt the grid paper and colored pencils helped students organize their work. Janet, however, did express concern that having students work on grid paper caused some to think that the pieces representing variable length actually represented a fixed length of a certain number of grid squares. She acknowledged hearing students express this misinterpretation of the model as she circulated about the classroom during the cycle 1 lesson. Janet was satisfied that the problem was addressed, however, through her individual conversations with students as they worked.

Teacher leaders guiding teachers' first experiences teaching with algebra tiles would do well to keep in mind the problematic nature of representing variable lengths with fixed plastic pieces. Designers of algebra tiles try to address this dilemma by making unit squares that will not divide

FIGURE 4. *Grid paper sketch of an algebra tile diagram*



lengths that represent variable quantities. This technique, however, has the potential to cause almost as much confusion as having students sketch tiles on grid paper. While grid paper sketches imply that the variable quantities have fixed integer values, the tiles themselves can be interpreted to represent variable quantities as fixed non-integer values. To overcome this conceptual hurdle, it can be helpful to consider online virtual manipulatives. Some online versions of algebra tiles (e.g., http://nlvm.usu.edu/en/nav/vlibrary.html) (Cannon, Dorward, Heal, & Edwards, 2001) use built-in sliders that allow users to change the length of x to portray variable quantities. As the sliders are moved, the side lengths in algebra tiles and the rest of the diagram change accordingly. Once students understand the fundamental idea of variable side lengths, their work with plastic tiles or grid paper can be recognized as portraying just one possible value for x.

USING ALGEBRA TILES TO ESTABLISH THE CONCEPTUAL GROUND FOR FACTORING RATHER THAN JUST ILLUSTRATING PROCEDURES

The algebra tile model is meant to make polynomial multiplication and factorization accessible to students by building a bridge from students' understanding of area to the development of more formal algebraic techniques. In some cases during the lesson study project, this intended sequence was essentially carried out in reverse. That is, students were asked to use the tiles to check or illustrate the result of using previously learned procedures for polynomial factorization and multiplication. Part of this was due to the fact that students had studied conventional techniques for polynomial factorization and multiplication earlier in the year, and rather than attempting to ignore previous instruction on the topic, the group built it into their plans.

At the outset of the project, the lesson study group wrote in their lesson plan that they considered knowledge of polynomial multiplication to be a necessary prerequisite for working with algebra tiles. At some points in the first implemented lesson, Janet asked students to check their work by multiplying polynomials symbolically. This move encouraged students to appeal to a procedure to judge the correctness of their answers rather than using the structural characteristics of the tile model to understand and justify the formal procedure. The tiles then essentially became a way to illustrate a procedure rather than a means to provide conceptual grounding for the process of factoring polynomials. This difficulty lingered through the second implemented lesson. In that lesson, Martha at one point told students, "You will be using your knowledge of factoring to figure out what goes in the box." When a student presented an incorrect algebra tile diagram for factoring $x^2 - 2x + 1$ to the rest of the class, Martha explained why it was incorrect by demonstrating a conventional symbolic procedure for factoring she had previously taught. She *did*, however, seem to believe that there were limitations in appealing to previously-learned procedures to understand a concept. For instance, she told students at the outset of the lesson that it would have been ideal to use the tiles earlier in the year, when they were just beginning to learn polynomial multiplication and factoring.

Failure to ground students' work with algebra tiles strongly in the idea of area appears to be a cause of having to appeal to procedures to determine the correctness of student work. When the concept of area is missing from an algebra tile lesson, the primary means for determining the correctness of any given factorization is to check (or create) tile arrangements by going back to previously-learned procedures. Therefore, during the debriefing session for the second implemented lesson, university faculty again recommended making a stronger connection between the algebra tiles and the idea of area. It was suggested that teachers frame polynomial factorization tasks as determining the length and width of a rectangle whose area is represented by a given polynomial. Doing so allows students to check their work by examining the dimensions and area of rectangles instead of relying solely upon memorized procedures. The teachers took up discussion of this idea during the debriefing session for the second lesson they implemented, and began to consider using the algebra tiles at the outset of instruction instead of waiting until students had learned procedures for polynomial multiplication and factoring. Introducing the tiles before formal procedures gives students a chance to recognize formal symbolic procedures as convenient abbreviations of their concrete work with the tiles. Teachers did not have the opportunity to pursue this sort of re-sequencing of instruction during lesson study, but the lesson study process introduced the idea as a goal for future work.

CHOOSING POLYNOMIALS WITH THE POTENTIAL TO ENCOURAGE GENUINE PROBLEM-SOLVING

Teachers beginning to use algebra tiles also should develop the art of careful task selection. In the first written lesson teachers produced during the lesson study project, the tasks they selected largely resembled those that would be given using a conventional approach to teaching factorization. In some cases, these tasks happened to be suitable for tile-based approaches (e.g., factor $x^2 + 6x + 9$). In other cases, the tasks used time inefficiently because of the large number of tiles that would be necessary for producing the accompanying representations (e.g., factor $x^2 +15x + 36$). When we reviewed the first written lesson, we suggested changes in the included tasks students were to perform. Polynomials with negative coefficients were missing from the problem set, so their inclusion was recommended in place of polynomials that simply required a large number of tiles. Teachers incorporated this suggestion into the implemented lesson during the in-class tasks students were to perform and as part of the homework assigned for the day.

Despite teachers' acceptance of suggestions on task alteration during the first cycle of lesson study, their second written lesson indicated that it would be profitable for them to continue to delve more deeply into the issue of task selection. In the written lesson plan the teachers produced at the beginning of cycle 2, they noted that "some students will have difficulty as the numbers get bigger," showing that they still considered the absolute values of the coefficients to be the primary determinant of problem difficulty. In a review of the second lesson, it was noted that the constant term in each polynomial the teachers planned to present was positive, so including some with negative constant terms for students ready for such a challenge was suggested. Non-prime constant terms were suggested as another means of increasing the level of challenge, since the number of rectangular arrays that can be formed is greater than for primes. It was also suggested that non-factorable trinomials be included among the problem set in order to cause student discussion of the characteristics of such polynomials.

Some of the written suggestions from the lesson reviews were incorporated into the tasks teachers used in the implemented lesson although the changes were mostly to smaller features of individual tasks. For instance, the suggestion to include polynomials with negative constant terms was adopted by asking students to factor $x^2 - x - 6$. However, the idea of including some polynomials that would not factor was not implemented. Instead, at one point during the cycle 2 lesson, Martha stated, "You will see that it will work out, and if it doesn't work out, you mess around until you get one (a rectangle)." Some of her students may have interpreted this statement to mean that any quadratic could be factored with algebra tiles even though Martha may have only intended to convey that all of the polynomials given in class and in the text could be factored with the tiles. Hence, although teachers' attention was drawn to some of the subtle but important differences in tasks suitable for algebra tiles during lesson study, task selection persisted throughout the project as an important area for further attention.

When working with teachers on framing tasks, teacher leaders may find it useful to incorporate the levels of cognitive demand framework described by Smith and Stein (1998). It provides a means for explicitly discussing the types of student thinking required in tasks posed by teachers. The four levels in the framework can be summarized as follows:

- ***Memorization:*** tasks that simply require memorization of facts, rules, or definitions;
- ***Procedures without connections:*** tasks whose completion relies solely upon execution of previously learned procedures;
- ***Procedures with connections:*** tasks that require students to use procedures but also prompt them to explore the procedure's conceptual underpinnings; and
- ***Doing mathematics:*** tasks that have no prescribed solution method; students need to draw on conceptual knowledge to devise solution strategies.

Although we prompted the group to reach toward higher levels of demand by suggesting tasks that required novel student thinking (e.g., non-factorable polynomials and those with negative coefficients), we did not explicitly share the four-level framework. In subsequent professional development work, the first two authors have found the framework to be useful for fostering meaningful conversations about higher-level tasks. Similarly, Arbaugh and Brown (2005) found that using the framework helped improve teachers' choice of tasks. Such experiences suggest that it may be profitable for lesson study facilitators to explicitly introduce the notion of level of cognitive demand as a device to help teachers choose and design genuine problem-solving tasks.

ENCOURAGING PROBLEM-SOLVING CLASSROOM DISCOURSE WITH ALGEBRA TILES

A final observation addresses the contrast between the classroom discourse in the lessons implemented during cycles 1 and 2. In posing the first factoring problem in the lesson, Janet started with the premise that the tiles comprising the polynomial must form a rectangle. Rather than showing students precisely how to form the rectangle, she took suggestions from the class. As the class worked, she asked some students to demonstrate their strategies for forming rectangles for different polynomials. At times, this meant that trial-and-error strategies were demonstrated by students. On the other hand, during the cycle 2 lesson, Martha tended to funnel students toward the solution she was looking for by asking a series of questions that required only one-word or one-number responses.

Part of the reason for the difference between the two classrooms may have been that the scaffolding questions to be asked as the lesson moved from example to example were not specified in the group's written lesson. This allowed for a greater degree of individual interpretation about which types of questions would be most effective. Therefore, as teachers construct written lessons, it can be useful for supervisors to work with them to decide how questions will be posed before implementing the lesson. Although it would be nearly impossible to write all of the questions teachers are to ask during a lesson, it is helpful to have consensus that the types of questions posed will encourage students' intellectual engagement with problem solving rather than restricting their thinking with narrow questions and directives.

As teacher leaders work with teachers to help encourage problem-solving discourse, the NCTM (2000) communication process standard can be a useful reflective device. It emphasizes the importance of student-to-student communication, stating that students should "analyze and evaluate the mathematical thinking and strategies of others" (p. 60). It also connects classroom discourse to task selection, stating, "Students need to work with mathematical tasks that are worthwhile topics of discussion. Procedural tasks for which students are expected to have well-developed algorithmic approaches are usually not good candidates for such discourse" (p. 60). As teachers view lesson video during debriefing sessions, they can be encouraged to analyze the extent to which the classroom discourse models the recommendations of the process standard. In cases where alignment is lacking, the process standard can help in the diagnosis of root causes. For instance, in the lesson Martha implemented, opportunities for student-to-student communication were lacking. She also tended to lower the levels of cognitive demands of tasks by providing many directive questions while teaching. Tracing problematic aspects of classroom discourse back to their root causes provides information that can be used to re-structure and polish instructional plans during lesson study.

Conclusion

We hope this paper offers insights to mathematics instructional leaders about challenges they may encounter as they help teachers begin to use algebra tiles with their students. We have described a number of ideas teachers learned as they engaged in collaborative planning and conversation, but it should be noted that we gained just as much from the opportunity to interact with the group. As we reflected on the group's progress and challenges, we began to re-think the ways in which we introduce algebra tiles in our university level mathematics and mathematics education courses for pre-service and in-service teachers. In particular, we now focus more strongly on the connection to area as the underpinning conceptual ground for the algebra tile model. Once that connection is established, it becomes easier to discuss the ideas of tile length as a variable quantity, the distinction between illustrating procedures and teaching concepts, the importance of choosing appropriate polynomials for students' work, and optimal classroom discourse patterns for tile use. As teachers learn to construct lessons incorporating these elements, their students gain rich opportunities to learn algebra with understanding.

Acknowledgement

The work reported in this article was supported by Mathematics Partnership grant funds from the Delaware Department of Education.

References

Arbaugh, F., & Brown, C.A. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education,* 8, 499-536.

Ball, D.L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator*, 16 (2), 14-18, 46-47.

Cannon, L., Dorward, J., Heal, B., & Edwards, L. (2001). *National library of virtual manipulatives for interactive mathematics.* MATTI Associates LLC. Retrieved April 17, 2009 from http://nlvm.usu.edu/en/nav/vlibrary.html

Chappell, M.F. & Strutchens, M.E. (2001). Creating connections: Promoting algebraic thinking with concrete models. *Mathematics Teaching in the Middle School*, 7(1), 20-25.

Chval, K.B., & Reys, R. (2008). Effective use of manipulatives across the elementary grade levels: Moving beyond isolated pockets of excellence to school-wide implementation. *NCSM Journal*, 11, 3-8.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6-10*. Portsmouth, NH: Heinemann.

Fernandez, C. (2005). Exploring lesson study in teacher preparation. In Chick, H.L., & Vincent, J.L. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. 2,* pp. 305-312. Melbourne: PME.

Groth, R.E. (2011). Improving teaching through lesson study debriefing. *Mathematics Teacher*, 104, 446-451.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F.K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Reston, VA: NCTM and Information Age Publishing.

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change.* Philadelphia, PA: Research for Better Schools, Inc.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator*, (Winter), 14-17, 50-52.

Moyer, P.S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175-197.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Senk, S.L., & Thompson, D.R. (Eds.) (2003). *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah, NJ: Erlbaum.

Smith, M.S., & Stein, M.K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, 3, 344-350.

Stein, M.K., & Bovalino, J.W. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School*, 6, 356-359.

Yoshida, M. (2008). Exploring ideas for a mathematics teacher educator's contribution to lesson study. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education, volume 2: Tools and processes in mathematics teacher education* (pp. 85-106). Rotterdam: Sense Publishers.

Conditions that Support the Creation of Mathematical Communities of Teacher Learners

Michelle Stephan, *University of North Carolina, Charlotte*
Didem Akyuz, *Middle East Technical University*
George McManus and Jennifer Smith, *Lawton Chiles Middle School*

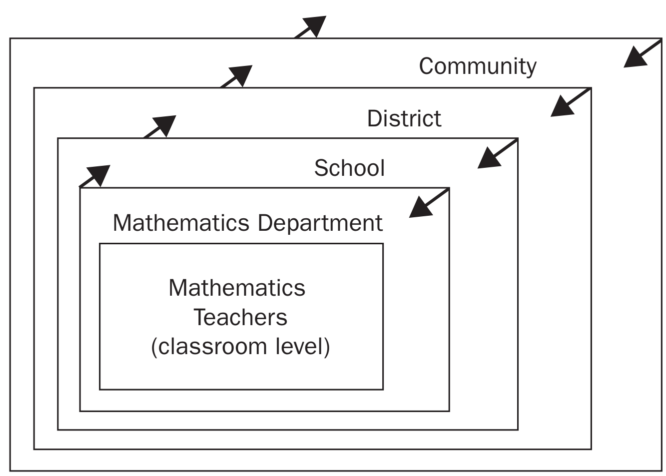
In his seminal work, Richard DuFour argues compellingly that if educational reforms are to be sustained long-term, schools must transform themselves into professional learning communities (DuFour, 1998). Professional learning communities (PLCs) can be distinguished from traditional school cultures in that all personnel from the principal to the classroom teacher are committed to collaboration using data about student learning to make decisions regarding school policy and practice. Supporting a school-wide culture that acts in this way is more difficult than it seems since teachers are accustomed to (and quite comfortable with) working in isolation. Encouraging teachers to make their students' learning public to other teachers is met with resistance because it exposes our personal teaching practices and beliefs to our peers.

In this paper we describe the conditions under which one group of mathematics teachers formed their own community of learners. We use the term communities of learner (COL) to refer to groups of two or more individuals (teachers, administrators or others) that collaborate about student learning, in contrast to an entire school body that we consider a professional learning community (DuFour, 1998, DuFour, 2004, DuFour 2007). Therefore, within one school that is operating as a PLC, there could be multiple smaller COLs at work. However, not all teams of teachers operate as COLs just because they meet on a regular basis. Our goals are to explicate 1) the characteristics of COLs and how they differ from traditional teacher teams and 2) the conditions that underlie the creation of strong COLs within schools with the hope that our work can be used to influence other cultures interested in doing the same.

Theoretical Perspective

The theoretical perspective that guides our discussion is rooted in situated learning theory (Wenger, 1998). In this view, teachers' practices and decision-making are situated within various other groups. As Cobb and McClain (2006) explain, teacher change is enabled and constrained as they form teacher networks that function within the confines of other groups within the educational system. For example, teachers must work within the context of their specific department, which is situated within the school, the district and the community at large, including parents, school boards, state legislative bodies and university officials. Teacher networks are viewed as nested within broader contexts and teacher networks can form across levels, e.g., collaborations across departments or across schools (see Figure 1).

FIGURE 1. *Nesting relationship among various education cultural groups*



In this figure, our COL can be located in the inner rectangle and is supported and constrained by other mathematics teachers within the school, which in turn operate within the larger setting of the school, within a district, and within the larger community. The participants at each level who operate within more than one level are referred to as *brokers* (Cobb and McClain, 2006). For example, one of the teachers in this paper served on various boards at the district and state level. She was, therefore, able to bring information about district and state policies regarding mathematics education to her COL.

Further, and more importantly, there is a bi-directionality of influence in that not only can people, policies. or activities in the outer levels enable and constrain the policies and practices at inner levels as is the traditional view, but also, communities at the inner levels can effect policy and practice outwardly at the department, school, district, or broader levels. As an example, our COL was asked to present our work on student interviewing to the monthly principals' meeting at the district level and also to a district-wide committee of teachers and assistant principals from each school. Our presentation contributed to the notion that assessing students via listening to them solve tasks before the unit is implemented can be a powerful tool for teaching, promoted the notion that data that is used in data-driven teaching can also be qualitative in the form of listening to students' preconceptions, and contributed to the creation of a district-wide definition of "mathematical big ideas." This example illustrates that the members of our COL served as brokers who contributed to the interchange of information between the teaching community and the district-wide committee of teachers, administrators, and policy makers. Additionally, our work had the effect of spreading from the classroom to the district (outward direction) which led to increased support for our efforts to use data-driven practices in our classrooms (inward direction) in the future through additional common planning sessions of 3 ½ hours, the creation of a leadership team, and days off from teaching to focus on collecting data to design effective instruction.

The purpose of our paper is to describe the policies and practices within multiple layers of Figure 1 that allowed for the emergence of a community of learners at the innermost level, a group of five mathematics educators. We do so by first describing the participants in the COL. We then elaborate on the three most important characteristics of PLCs that also serves as the foundation of a strong but smaller community of learners, using an example from our mathematics planning to illustrate the characteristics of COLs, and then use these three characteristics to define what makes COLs (and PLCs) different from traditional teams of teachers. We are not making any claims that our school was a PLC. Rather, we are claiming that five mathematics teachers were able to form and sustain a powerful COL situated within the context of a school that was attempting to instigate change towards a PLC. Next, we return to our theoretical framework to describe the policies and practices at multiple levels of our educational system in order to explain the conditions that led our COL to be successful and sustain its work into the subsequent years. Finally, we reflect on what we have learned in order to provide implications for other schools or teachers that might want to start their own communities.

The Mathematics Community of Teacher Learners

Our community of learners (COL) consisted of four seventh-grade mathematics teachers from a middle school serving approximately 1300 students from an upper-middle-class suburb of Orlando, Florida and one doctoral student from a local university. These four mathematics teachers included Stephan, Smith, MacManus, and Dickey, all of whom came together partway into the school year as the result of an opportunity to participate in a doctoral research project that involved using instructional material addressing integers developed the year before and tested by Stephan, McManus, and Smith (Stephan, 2009) rather than the instructional material addressing integers in the Connected Mathematics curriculum materials adopted by the district. The COL agreed to plan their instruction on integers together and, as part of this process, would also talk daily to reflect on their instruction. Since there was no teacher's manual for this instructional unit, frequent meetings were important for planning their instruction as well as tracking the mathematics learning of students and implications for their instruction.

Of the four teachers that formed the COL, only Smith could be considered a veteran with 10 years of teaching in special education. The other three teachers, Stephan, McManus, and Dickey, had taught 4, 3, and 1 years, respectively. One teacher, Stephan, had 5 ½ years experience teaching and doing research at the college level, and this year, she was teaching full time in the middle school, half of her time devoted to teaching and the other half as a mathematics coach for the school. Stephan and Smith had co-taught in an inclusion setting for three years and Stephan and McManus had co-planned daily the year prior to this paper. Akyuz, the doctoral student, was serving an internship in Stephan's classroom and attended class three times per week for the entire school year.

We began meeting one week before our instruction on integers began. In those initial meetings, Stephan and Akyuz shared readings on the historical development of integers as well as research articles examining students' understanding of integers. From there, the focus shifted toward the instructional materials themselves, and we envisioned how these materials might be used with students—what Schoenfeld (2000) refers to as developing a lesson image. All five participants met together at least once per week for a formal meeting while more informal meetings occurred on an almost daily basis. If only two or more of us could meet, one of the members took the responsibility to debrief the other team members.

FIGURE 2. *Big ideas for the integer unit*

Big Idea One: Interpreting Net Worth as a Positive/Negative Difference
- Net worth as a combination of a positive and negative value
- When a negative value is greater than a positive, the combination is negative

Big Idea Two: Using Zero as a Point of Reference for Calculations
- Referencing zero to determine net worth
- Referencing zero to compare two net worths
- Referencing zero to add or subtract integers
- Cancelling equal positive and negative quantities

Big Idea Three: Comparing Integers with a Vertical Number Line
- Higher negative numbers are further away from zero
- Structuring the gap between two integers to find the difference

Big Idea Four: Reasoning with a vertical number line to determine the results of addition and subtraction operations
- Determining the effect that operations have on a quantity
- Finding results of integer operations on the vertical number line
- Commutativity of subtraction with integers does not hold true

Big Idea Five: Determining the meaning of positive/negative signs
- Using flexibility with symbols to find unknown operations
- A minus sign is different than a negative sign

CHARACTERISTICS OF COMMUNITIES OF LEARNERS

In this section we consider three characteristics of our Community of Learners with some examples of those characteristics. We want to stress that just because teachers meet together to plan instruction, that does not make them a Community of Learners. COLs have very specific characteristics that set them apart from teacher planning teams.

Characteristic #1: ***Student-Centered Teaching: Determining learning goals, assessing students' conceptions, and supporting students who are struggling.*** Prior to instruction, all five members of the COL met to discuss the learning goals for the integers unit. The "big ideas" important for integer addition and subtraction are listed in Figure 2 below (see Stephan & Akyuz, in press, for more details). Instruction began in the context of net worth and used finance to build subsequent integer concepts and operations with a particular focus on addition and subtraction. Consequently, the big ideas listed below are cast in the context of finance and mathematics with more specific learning goals listed beneath each.

The broad goal of our instruction was to begin with a realistic context, giving students opportunities to think about integers as net worth, debts, assets, and transactions, and then move them toward more abstract reasoning with integers such as computing the difference between -1000 and -3000. At completion of the unit, we used these big ideas to write common assessment problems. In additional, we assessed students daily in order to find out more about their developing understanding, and if any students were struggling, they received individualized instructional attention from the teacher.

Characteristic #2: ***Focus on student learning, not just teaching.*** One of the hallmarks of DuFour's PLC notion is that organizations should become learning institutions rather than only teaching ones. On a smaller COL scale, we attempted to model this, with most of our conversations focusing on student learning. Generally, formal meetings were reserved for teachers to discuss the current goals of the instructional sequence, the next few goals in the sequence, students' current and anticipated thinking, and our means of supporting that thinking. To initiate discussions in these meetings, teachers might bring in examples of students' work, either in written form or from memory, and we would then use student thinking as the springboard for setting subsequent mathematical goals for instruction. If students' thinking did not match what we predicted in our lesson imaging, then we wrote new activities to strengthen and build on their current thinking. If students' thinking flowed as predicted, then we engaged in further lesson imaging by working out the next few pages of the instructional activities to remind ourselves of the intent of the instruction and to anticipate what strategies our students might create to solve the problems, both productive and not. We developed questions that we might ask to help students who are struggling as well as questions designed to further strengthen the thinking of students who were ready for a challenge.

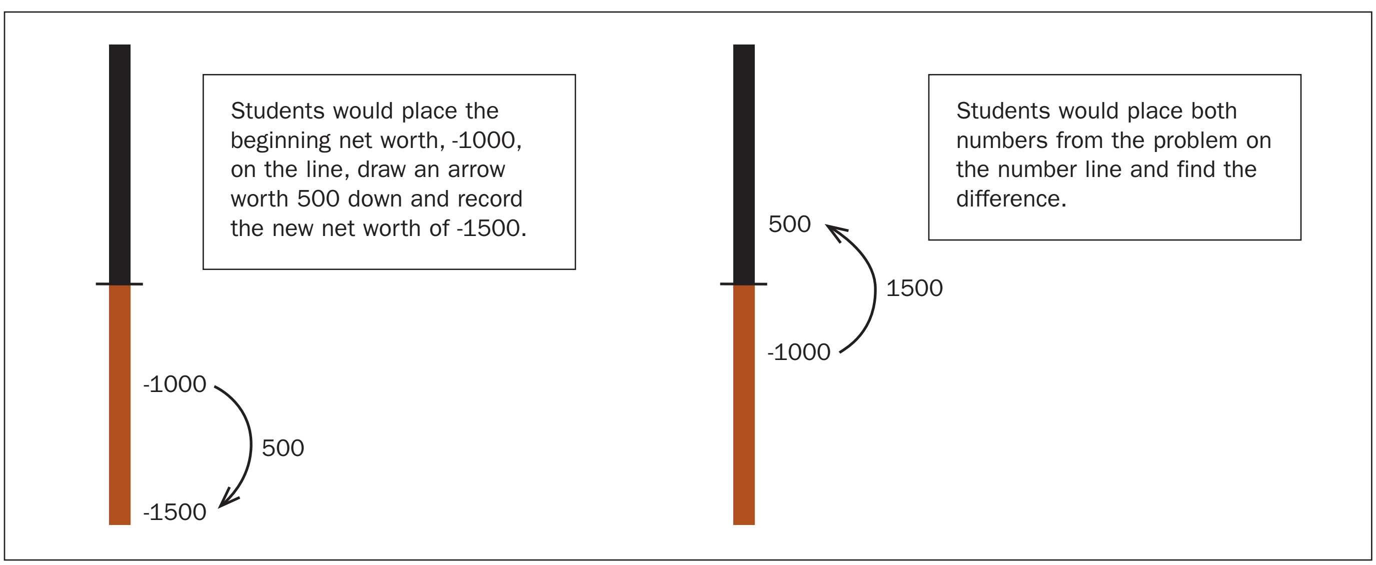
As a part of our lesson imaging, we used anticipated student thinking as the vehicle to generate possible questions for discussion in our classrooms. Each of us valued NCTM's process standard (2000) stressing the importance of creating opportunities for students to engage in meaningful, genuine mathematical discourse. This type of environment includes supporting students' conjecturing, proving, and revising conjectures based upon new ideas. Consequently, our instructional tasks were posed in ways that were intended to support conjecturing, and we discussed ways in which the teacher could highlight students' conjectures when they arose in class, so these could be used as a basis for our classroom discussions.

Characteristic #3: *Ongoing data-driven decision making/ assessment.* In describing our experiences above, we have shown that our decisions about instruction were influenced primarily by our analysis of student learning on a daily basis. The data we used to make these decisions were examples of students' work from that day, including both classwork and homework. In many instances, our analysis of these data led us to make adjustments to instruction. For instance, our unit used the context of finance to teach integers with students understanding that a person's net worth is the difference between his total assets and total debts, and we introduced problems that asked students to determine a person's new net worth when a transaction caused his original net worth to change. Based on what we saw happening with our students as they worked on these kinds of problems, we decided to introduce a vertical or "net worth" number line as a means for recording their operations with integers. This net worth number line was colored black on top (positive) and red on the bottom (negative).

At one point in our instruction, we asked students to use this net worth number line to determine someone's new net worth if their original net worth was -$1000 and they incurred a debt of $500. Students created at least two different ways of reasoning with the number line, as shown below (Figure 3).

At the time, the students' second strategy surprised us and became the major focus of our analysis. Students who were modeling the problem situation this way were having

FIGURE 3. *Two different ways students reasoned with the number line.*



difficulty interpreting their actions on the number line. The confusion occurs because the two numbers on their line represent different quantities: 1000 signifies a net worth and the 500 is a transaction. Our instructional intent was that student models would most easily fit the problem situation if the transactions were represented by the arrows. As a consequence of analyzing students' different ways of thinking about and modeling integer operations, we made adjustments to instruction the next day. We posed a similar task and asked students to symbolize their actions on the number line again. We then asked students to decide which inscription more accurately depicted what happened to the net worth, and students decided to place transactions on the arrow to show whether the net worth was changing for the better or worse.

In one of our last meetings, we focused on developing an assessment for the integer unit that all of us would all use with our students. We began by recalling what our mathematical goals had been at the beginning of the unit and any unanticipated goals that had arisen during implementation. Each teacher chose several of these goals and wrote possible questions. Stephan collected the questions and created the final unit assessment, sent it back to us for analysis and discussion, and then revised the assessment based on our feedback. Schedules did not allow for a formal debriefing meeting after we had given the assessment to our students, but we held several informal debriefs, and found that students had shown proficiency with almost all of the big ideas of the unit. A few students still had difficulty interpreting the meaning of a negative sign when problems only contained one, instead of two signs, such as -1000 – 2000 rather than -1000 – (+2000). Even though it was a minority of students, we agreed that our instruction next year should focus more heavily on problems that contain only one sign. In this way, summative assessments can show not only the success of an implementation but also lead to changes for the next year.

CONDITIONS FOR SUPPORTING COLS

In this section we revisit the theoretical model in Figure 1 and explore the policies and practices of the various communities at other levels of schooling that made it possible for our COL to thrive. We start by looking at the innermost level, the classroom teachers that comprised the COL, and move our way toward the outermost rectangle, the community.

Support #1 [Classroom Level]: We cannot stress enough that the emergence of our COL would never have been possible without the commitment of the teachers. This commitment involves teachers who believe that to teach, you must be a student. We believe that an *inquiry pedagogy*, which involves examining your classroom practice in collaboration with other teachers, is a rare phenomenon, yet one that is extremely rewarding. While it may be debatable whether or not you can mandate teacher participation in a COL of this type, we feel that voluntary participation was a key to creating and maintaining our shared goal.

While it can be frightening to share one's student data with other teachers, creating a safe environment for sharing data is essential for the community to thrive, and this requires teachers who believe that instructional decisions should be informed by student thinking and learning. One of the characteristics of our environment that helped to create safety was that our topics of discussion rarely focused on teacher actions but, rather, centered on student thinking. McManus and Smith recounted numerous times that they felt comfortable in our meetings because they never felt like their pedagogy was under scrutiny or attack from other teachers. In fact, when they would ask questions like, "How should I have taught this differently?" the lead teacher would always bring it back to students rather than the teacher by asking, "How were your students reasoning?" The focus on students' thinking made our conversations less personal and gave the teacher some basis for making their own decisions about changes to their pedagogy.

Support #2 [Classroom Level]: In our case, Akyuz's dissertation served as the catalyst for the emergence of our COL. However, we emphasize that Stephan and Akyuz could have conducted this study in the isolation of their classroom, but deliberately chose to let it serve as an opportunity to create a community of learners. This means that the establishment of COLs relies on at least one strong teacher leader that recognizes these opportunities when they arise and can take advantage of them. We contend that a strong teacher leader is one that is perceived by his or her peers as having expert knowledge of teaching in his or her field, knows how to create a safe environment for teacher collaboration, and focuses 90% of professional conversations on student learning and the implications for practice rather than administrative tasks. Good teacher leaders have what Collins (2001) refers to as a "hedgehog concept," a strong commitment to one particular aspect of teaching, and can therefore see (and is looking for) opportunities to further their agenda. The teacher leader's hedgehog concept in this COL has always been to be the best at student-centered mathematics instruction and found ways to encourage other teachers to join her in strengthening their practice in this manner. In fact, when this project was over and the next school year began, the teacher leader was transferred to another grade level and was not available to work with these teachers again, but this limitation did not stop these teachers from forming their own COL and sustaining the practices of our collaboration. Thus, when asked whether or not COLs working at this level can be sustained, the answer is a resounding "yes," as they are still going on after four years even after the original teacher leader has moved on.

Support #3 [Mathematics Department Level]: Our mathematics department chair continually highlighted her vision that teachers collaborate with one another during common planning time. During department meetings and in personal conversations, she encouraged teachers to use their planning time to work together as much as possible. She modeled this practice herself with one other teacher in her grade level. In addition, she often set aside department meeting time so teachers could collaborate on creating common assessments for the courses they all taught. Without this support from the mathematics department chair, it is possible that our COL would have had difficulty forming.

Support #4 [School Level]: The role that our principal and assistant principal played in supporting our COL was crucial. First, the principal's vision of our school involved working towards becoming a Professional Learning Community. At the beginning of the year, she distributed a short article describing PLCs to each member of the teaching staff so that those who had not heard of them would become more informed.

Our principal continued to share this vision with us at faculty meetings and in her one-on-one conversations with each of us. Knowing this was not enough to initiate a PLC environment, she held monthly meetings with teachers in key leadership roles (e.g., coaches, department heads, technology support). In these meetings, the teachers attempted to create a vision for the school that was consistent with PLCs. By virtue of being the mathematics coach of the school, Stephan participated in many of these meetings, and therefore, was exposed to the PLC vision.

While teacher leaders were creating the school vision with their faculty and each other, the principal made key structural decisions in order to better support the emergence of COLs. She implemented a common planning period for teachers, explaining that she expected us to use that time to plan together. She hired more teachers from within the staff to serve as coaches in mathematics, technology, language arts, reading and cooperative learning. By creating these positions she was attempting to convey her vision that teachers use each other and their coaches to make meaningful inquiries into their practices.

For her part, the assistant principal in charge of the mathematics program also provided a means of support for our COL by giving us approval to set aside our adopted curriculum materials for five weeks in order to explore our teaching of this new unit. This decision required us to focus on student reasoning *without* the support of a teacher's manual that often includes material that may address the student thinking that is likely to come up in an instructional unit. We are not suggesting that it is always useful to set aside the teacher's manual for curriculum materials, as these often provide important supports for instruction. But in *this* case, having to think together about the mathematical goals, the student thinking, and our strategies for engaging that thinking *without* the support of a teachers manual created a context in which these kinds of discussions were essential. We acknowledge that COLs can *also* productively work through and discuss supports provided by a teacher's manual to address the mathematical ideas, student thinking, and strategies for engaging that thinking as an ongoing part of their planning for instruction.

Support #5 [District Level]: Our district was in its third year of implementing Connected Mathematics Project 2. Three years prior to the creation of our COL, a team of teachers and administrators from all twelve middle schools in our district formed a group called CDDRE who, in conjunction with key district personnel, adopted not only the Connected Math Project 2 program but also made a strong commitment to a student-centered approach to teaching advocated by the NCTM Standards (2000). The purpose of the CDDRE meetings was to come together in collaboration with teachers from all over the district to discuss strategies for improving mathematics instruction and strive toward our shared goal of increasing the number of students who score at the proficient level on the state test.

CDDRE administrators and teachers were taught how to use data to drive decision-making at all levels and were charged with shaping their school mathematics programs in this manner. Data was often defined in these meetings as quantitative assessments on pre- and post-tests. Teachers from this CDDRE worked together to create common assessments to be voluntarily used by teachers at various schools. Stephan participated in the committee to create these common district assessments. Our department chair, assistant principal, and a lead teacher were regular members of this CDDRE and all felt that the district vision for mathematics aligned perfectly with our principal's vision for our school.

Support #6 [Community Level]: An important means of support came from the Florida Department of Education and the local University. Recently, the Florida Department of Education convened a group of experts to rewrite the state standards for mathematics instruction using the NCTM Curriculum Focal Points (2006) as their guide. These new standards advocate teaching for understanding using only a few big ideas per grade level, rather than teaching for mastery of a large number of concepts, as was previously the case. The vision of the new State Standards is that students should be encouraged to explore, conjecture, justify and represent mathematics in meaningful and sophisticated ways while becoming proficient in skills. This message is consistent with the vision of our district as we implement a reform-based curriculum and of our principals as they provide means of support for transforming traditional mathematics classrooms to inquiry environments. In addition, the adoption of these new reform-based standards provided a catalyst for teachers in our district and our COL in particular to start investigating approaches to teaching that were more student centered. We see the adoption of the Common Core State Standards as a key opportunity to create a number of COLs within our school that explore new strategies for teaching mathematics based on student thinking and learning.

Another important source of support for our COL came from the mathematics education research community. Stephan had previous research experience as a college professor. Her research focused on supporting students' development of mathematics and designing instruction that was inquiry based. As a consequence, she was able to bring information about student learning from the research community to the COL.

Constraints: Because of the strong alignment of vision across so many levels of our school community, our COL did not encounter many constraints. As we planned instruction on integers, we were of course constrained by the district's instructional plan, as well as the State's benchmarks for students. We therefore, had to create instruction that aligned with the district's plan and taught the required benchmarks for that concept. For us, that was a minor constraint and did not inhibit our inquiry.

We also experienced many of the same constraints that are reported across school district: lack of time, money and resources. However, we were able to use what resources we had to accomplish the goals of our COL and our administrators provided us not only with common planning time but also provided us with additional time together using coverage from paid substitutes when the need arose.

Implications

We believe that there are several components that supported our Community of Learners.*

- ***Voluntary Commitment to the COL.*** One key to the success of our COL was that the teachers came together around a common purpose that had everything to do with the practical daily life of teaching. Simply put, we wanted to improve our instruction on integers, and created a COL focused on student learning to accomplish this goal. Since the COL goals were created by teachers working together toward a common goal, we were more invested in the success of our community than if we had been mandated by the principal to create a COL. That is not to say that COLs are not possible when principals charge their teachers to participate in one. In our experience, when COLs are not optional, they begin as teams and take much more time and focus to become fully operating COLs.
- ***Safe Environment for Pedagogical Discussions.*** Creating an environment in which it is safe to share your practice was a crucial characteristic of our COL. Because our conversations always started with how our students were thinking and learning, teachers did not feel their practice was under attack. This is not to

* We are grateful to an anonymous reviewer who helped us articulate these implications from our work.

say that there were never tensions among participants. Disagreements are necessary for growth of a COL. However, the disagreements focused on student thinking, and ways the team might adjust instruction to better support the students, rather than judging the success or failure of a teacher's actions in her classroom.

- ***Strong Teacher Leader(s).*** We cannot overemphasize the importance of placing strong teachers who share the vision of PLCs in leadership roles within the school. These leaders need to possess the talent to recognize when other teachers share a common interest in student learning and the ability to encourage those teachers to form COLs to accomplish their goals. They also must possess knowledge of and commitment to serving as a broker across different levels of the school community. For example, to encourage other teachers and administrators within his or her school or throughout the district to put learning at the core of teaching, the broker must be willing to and know how to seize opportunities to engage others in this vision when they arise. District administrators as well as local ones must search for this type of broker and place them in formal leadership positions if their school is to become a PLC in the long run. Administrators must also provide the resources needed by their teacher leaders such as common planning time and extra planning time.

- ***Shared Vision of Teaching.*** Another crucial characteristic of schools that work as PLCs and include COLs is that a vision of learning and teaching is shared at the beginning of their work. Because the members of our COL shared the same vision, all we needed was a teacher leader to bring us all together to engage in work focused on student learning, along with the support of our administrators who helped facilitate our work instead of constraining it.

- **Goal Alignment.** One of the significant characteristics of our experience forming the COL was that key representatives and policies at various levels, from the teachers on up to the community, had goals that were aligned around student learning and standards-based teaching. The new state standards, the district's CDDRE group, administrators and the teacher leaders at our school knew the value of student-centered learning, data-driven decision making, and inquiry learning for both students and teachers. Additionally, many of them served as brokers themselves to convey the vision of teaching as learning and learning as inquiry among their constituents in both outward and inward directions (e.g., parents, school board members and teachers and other administrators whose vision may or may not have aligned).

Conclusion

In this paper, we have described the characteristics of a Community of Teacher Learners that align with the goals of DuFour's Professional Learning Communities. While PLCs focus on a school-wide vision for attending to student learning, our COL notion refers to smaller groups of teachers that come together and work in the same manner as the larger PLC that DuFour envisions. While our particular middle school was not operating as a PLC, our principal attempted to initiate the process early on, and supported our smaller efforts to engage in practices similar to the larger PLC. We have argued that without the support of multiple brokers (e.g., administrators, teacher leaders, state standards, and district policy makers), our chances of initiating and sustaining our work as a COL would have been minimal. Additionally, our work was not only supported by other leaders outside of our COL, but also we influenced practices outside of our COL, just as Cobb and McClain's (2006) theory suggests. Teachers in other disciplines have now formed teams at our school, although it is debatable whether they are operating as true COLs yet. Furthermore, our COL members have served as brokers to speak to other educational constituents both within and outside our district.

References

Cobb, P. & McClain, K. (2006). The collective mediation of a high-stakes accountability program: Communities of networks and practices. *Mind, Culture and Activity*, 13(2), 80-100.

Collins, J. (2001). *Good to great: Why some companies make the leap…and others don't.* New York: HarperCollins Publishers.

DuFour, R. (2004, May). What is a professional learning community? *Educational Leadership*, 61(8), 6–11.

DuFour, R. (2007). Professional learning communities: A bandwagon, an idea worth considering, or our best hope for high levels of learning? *Middle School Journal,* 39(1), 4-8.

DuFour, R. & Eaker, R. (1998). *Professional learning communities at work: Best practices for enhancing student achievement.* Bloomington, IN: National Educational Service.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (2006). *Curriculum Focal Points for PreKindergarten through Grade 8 Mathematics.* Reston, VA: NCTM.

Schoenfeld, A. (2000). Models of the teaching process. *Journal of Mathematical Behavior*, 18(3), 243-261.

Stephan, M. (2009). What are you worth? *Mathematics Teaching in the Middle School*, 15(1), 16-23.

Wenger, E. (1998). *Communities of practice.* New York: Cambridge University Press.

Early Numeracy Intervention: One State's Response to Improving Mathematics Achievement

Sara Eisenhardt, *Northern Kentucky University*
Jonathan Thomas, *Northern Kentucky University and The Kentucky Center for Mathematics*

A significant number of children struggle with quantitative ideas during the first few years of their academic careers and fail to construct a meaningful sense of number. These early struggles, if not addressed, can limit students' mathematics performance as they move through the grade levels. If these struggling students are fortunate enough to receive intervention services, most often they are provided too late and are provided by teachers with little knowledge of numeracy development. Duncan et al (2006) identified the predictive power of early mathematics knowledge and found that knowledge of numbers and ordinality were the most powerful predictor of later learning. The study recommends future research to identify "promising early math interventions" (p. 21).

Research on the factors contributing to student achievement discovers again and again that teacher expertise is one of the most important variables in determining student achievement (Darling-Hammond & Ball, 2004). Research also suggests that many elementary school teachers in the United States lack essential knowledge for teaching mathematics and this lack of knowledge directly impacts how well they teach mathematics (Ball, 1990; Ma, 1999). In fact, many elementary teachers report that they do not have sufficient conceptual understanding of mathematics and rely on rote computations and algorithms for instruction (Gerretson, Bosnick, & Schofield, 2008).

There seems to be agreement that mathematical performance is unlikely to improve without serious attention to the ongoing professional development of elementary teachers of mathematics. The research findings and student achievement data reflect the compelling need for high-quality professional development opportunities for elementary teachers of mathematics that focuses on developing knowledge of the conceptual foundations of elementary mathematics, the features of effective mathematics instruction, how to use curriculum materials to support instruction, and strategies for using assessment data to inform that instruction. There is a critical need to identify effective early intervention programs to enable more students to be successful mathematicians.

Review of Literature

Engaging teachers in identifying which concepts and skills they want students to learn, anticipating students' challenges, and understanding the nature of students' misconceptions improves teachers' instructional practices and results in more positive and significant student outcomes (Blank, de las Alas, & Smith, 2007; Carpenter, Feneman, Peterson, Chiang, & Loef, 1989; Cohen & Hill, 2001; Lieberman & Wood, 2001; Saxe, Gearhart & Nasir, 2001). Kennedy (1998) conducted a literature review to identify the features of effective professional development programs and found that "'programs whose content focused mainly on teachers' behaviors demonstrated smaller influences on student learning than did programs whose content focused on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject" (p. 18). Kennedy's literature review suggests an important role for content-emphasis in high-quality professional development. The most useful professional development directly relates to the teachers' work and involves a cycle of assessment, active teaching, observation and reflection (Darling-Hammond & McLaughlin, 1995).

Ongoing and sustained professional learning that provides teachers with opportunities to collaborate together allows teachers to learn together, apply their learning to their classroom context, and reflect on what is effective and why (Loucks-Horsley, Hewson, Love, & Stiles, 2009) and also promotes the creation of a shared understanding of what constitutes effective mathematics teaching and learning. The process of learning in small, supportive groups with colleagues promotes the likelihood of teachers changing their instructional practices (Dunne, Nave & Lewis, 2000). Situating collaborative conversations in dilemmas that teachers experience in their teaching creates meaningful and authentic opportunities for teachers to examine their instructional practices. Little (1990) found that collaboration focused on authentic work resulted in high-quality solutions to instructional challenges, increased teacher confidence and resulted in significant gains in student achievement. Findings from the National Commission on Teaching and America's Future (2010) indicate this kind of collegial interchange is a requirement of professional learning designed to strengthen instruction.

Context of the Study

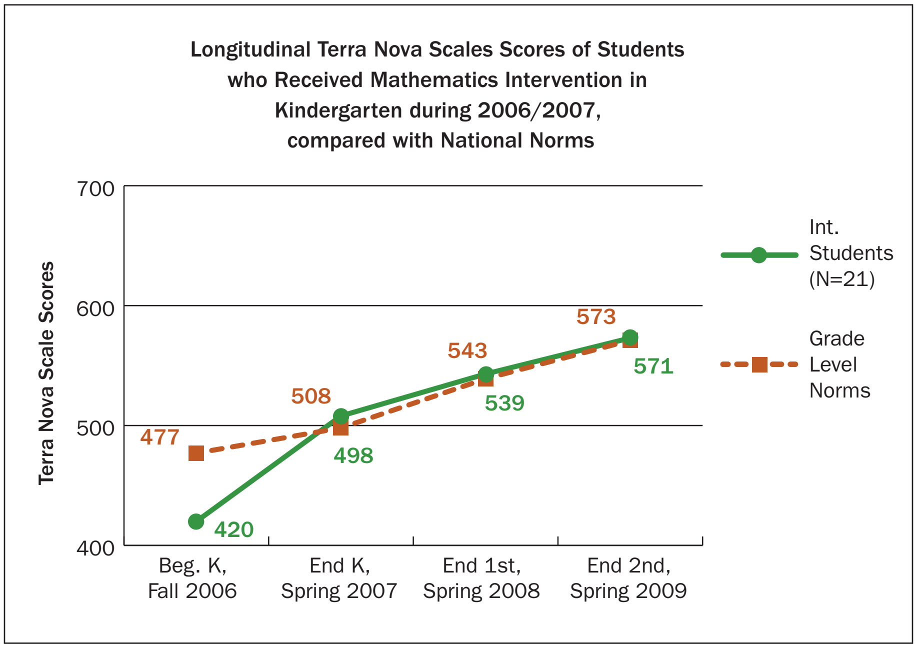
In spring 2005, the Kentucky Legislature passed House Bill 93 that called for the development of a multi-faceted strategic plan to improve K-12 student achievement in mathematics. One important focus of this bill was the need to provide developmentally appropriate and research-based diagnostic and interventions services to kindergarten through third grade students through a newly created Kentucky Center for Mathematics (KCM). The KCM chose as part of its mission to develop expertise among the Mathematics Intervention Teachers (MIT) community in order to affect significant positive changes in student learning of mathematics at the primary grades.

Typically, students participated in 30-60 minute intervention sessions daily (in addition to the regular mathematics instruction) and these sessions might involve either individual students or small groups of students. The KCM purposefully allowed some flexibility in this area so that schools might construct an individualized model for intervention to meet the needs of varying school contexts/structures. The only 'non-negotiables' were that intervention sessions did not conflict with the students' classroom mathematics instructional time and that MITs' time must be spent conducting mathematics intervention with children. Students who no longer needed additional support, based on assessments administered by the MITs, were released from the program.

Given the considerable evidence supporting the effectiveness of sustained and job-embedded professional development (Loucks-Horsley, Hewson, Love, & Stiles, 2009), the KCM required that MITs involved in this project participate in ongoing job-embedded professional development provided by the KCM. This consisted of a program of professional development that grew to include an intensive 5-day summer institutes, periodic 2-3 day within-year institutes, weekly online team meetings, and periodic face-to-face collegial team meetings.

Initially, the MIT professional development was associated with either Number Worlds (Griffin, 2004; Sarama & Clements, 2004; SRA, 2007) or Math Recovery (U.S. Math Recovery Council, 2006, 2008; Wright, Martland, & Stafford, 2000; Wright, Stanger, Stafford & Martland, 2006) intervention programs. The professional development focused on developing MITs' knowledge of the complexity of numeracy development. MITs learned about the stages of numeracy, characteristics of the various stages in the learning trajectory, and the instructional strategies appropriate for advancing student development along the learning continuum during the summer sessions. The weekly online meetings and the collegial team meetings provided a forum for MITs to discuss their individual students, professional challenges, and new professional insights with other MITs. Beginning in 2007, many MITs who were using Number Worlds also chose to participate in professional development associated with Math Recovery or Add+VantageMR (U.S. Math Recovery Council, 2008) and used a combination of the approaches in the intervention teaching. Add+VantageMR is, foundationally, very similar to Math Recovery in that both programs use similar professional development frameworks and progressions to map children's mathematical development. Indeed, Add+VantageMR and Math Recovery both rely upon learning frameworks developed by Wright et al. (2000; 2002; 2006); however, one key difference is that Add+VantageMR was designed for small group and whole class interventions while Math Recovery was designed for more intensive one-on-one interventions. This difference is articulated in professional development that emphasizes instructional experiences groups or individuals respectively.

FIGURE A



External evaluators conducted a randomized study comparing the pre- and post-achievement levels of participating students and students eligible for participation but unable to receive services due to the MITs capacity limit. The external evaluators also identified similar sites without an MIT and tracked the progress of students deemed eligible for intervention at those sites. The end of the first year and every year thereafter on the Terra Nova assessment noted significant gains in student achievement among students receiving support from these MITs for students in kindergarten through third grade. More specifically, pre- and post-test student achievement results on the Terra Nova at the end of the first year of implementation indicate that first grade intervention students supported by the Number Worlds intervention (1000+ students) achieved, on average, an increase of more than one year and the first grade intervention students supported by Math Recovery gained on average more than two years growth. On average, the intervention students made gains that exceeded their peers eligible for the intervention who were not provided with these services. Sustained impact was evidenced by longitudinal Terra Nova data demonstrating that intervention students were performing at or near grade level a year or more after exiting the program (Figure A). Average first grade achievement results over four years demonstrated results at or near grade level expectancy after one year of intervention despite beginning average scores well below those expected of entering kindergarten (Figure B).

The student achievement results suggest that the KCM was successful in its efforts to strengthen the mathematics achievement of low-performing students' in kindergarten through grade three for those students supported by the interventions offered by MITs participating in the professional development provided through this initiative. The purpose of this study is to identify the factors that contributed to the success of the professional development initiative.

Methodology

PARTICIPANTS

The primary participants in this study were the MITs who participated in the professional development initiative and provided instructional interventions based on individual student understanding of and fluency with number. The program started with 46 MITs in the summer of 2006, an additional 41 MITs in the summer of 2007, and an additional 27 MITs in the summer of 2008. Interviews were also conducted with building and district administrators and classroom teachers whose students were serviced by the MITs.

DATA SOURCES

The study used data from end of the year surveys, semi-structured interviews, and field notes of observations that allowed for a convergence triangulation of data across perspectives. Surveys were administered following each year of participation in professional development to assess MIT perceptions of the nature of mathematics, beliefs about teaching and learning mathematics, self-efficacy regarding their own proficiencies in mathematics, and the usefulness of the professional development in improving student learning. Semi-structured interviews and observations of MITs engaged in teaching, collaborative planning, and professional development sessions were conducted beginning in July 2007 and concluding in May 2010.

During the 2007-2009 school years, observations and informal interviews were conducted in professional development sessions during the summer and in collegial team meetings with regular education classroom during site visits to seven schools (three urban, two rural and two suburban schools) during the school year. During the fall of 2009, participant observations and interviews were also conducted during four days of Math Recovery training. Additional interviews were conducted and recorded via telephone.

A total of 112 semi-structured interviews were conducted with 47 MITs, 56 regular education teachers of participating students, and 9 administrators from July 2008 through May 2010. The purposes of second and third year semi-structured individual and focus group interviews and observations were to learn: 1) how participation in program activities contributed to teacher growth, 2) what learning was most transformative, 3) what changes in instructional practices resulted from participation in the program, and 4) how these instructional changes impacted their students.

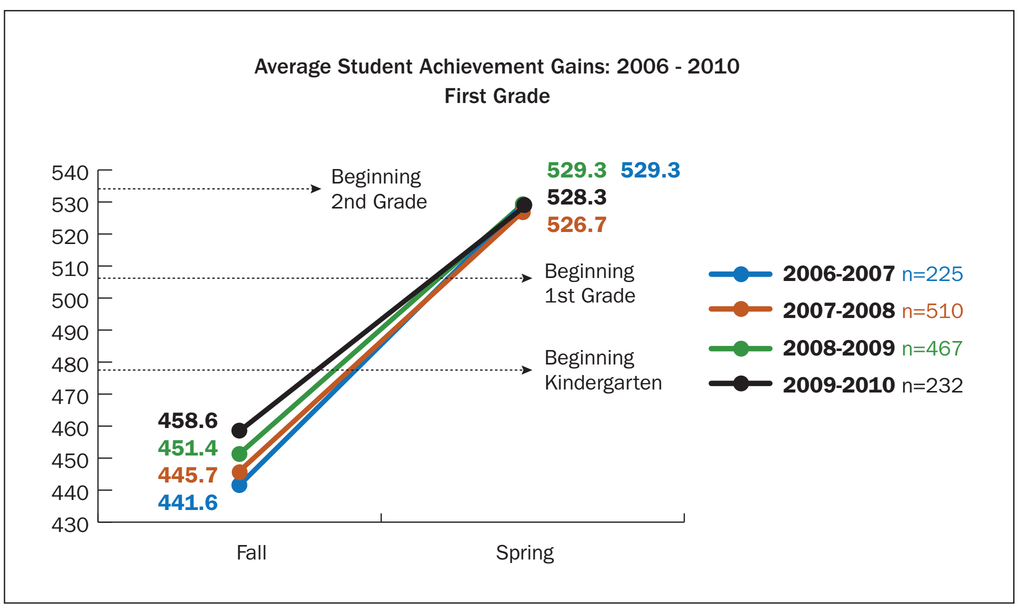
Qualitative data was analyzed using the constant comparative method of grounded theory (Glaser and Strauss, 2009) and involved the constant interplay between the researcher, the data, and the developing theory. All interviews and field observations were transcribed and analyzed using three cycles of analysis: open coding, axial coding and selective coding. On this basis, a theory was developed that enabled a rich description of the components that contributed to MITs' professional knowledge and aspects of their interventions that contributed to increased student achievement.

Results

MATHEMATICS BELIEFS SURVEY

The Mathematics Beliefs Survey results MITs were generally confident of their knowledge of mathematics and generally enjoyed mathematics. A high percentage of MITs across all three years of the program indicated they liked doing mathematics and were interested in mathematics. Changes in MITs' attitudes towards mathematics from the pre-participation survey to the post-participation survey demonstrated significant changes in the MITs' attitudes towards mathematics in a positive direction. As a result of participation in this professional development, an increased percentage of MITs also indicated that they looked at underlying reasoning, application, and use of hands-on activities and that anyone can learn mathematics and that they know they understand a concept when they successfully explained it to another person. For example, MITs were less likely to agree or strongly agree that, "To understand mathematics, students must solve many problems following examples provided." Significant declines were also present with the following questions: "Doing mathematics consists mainly of using rules." and "Knowing step-by-step procedures is necessary to solve mathematical problems" (University of Cincinnati Evaluation Services Center, 2009). Complete evaluation reports are posted on the Kentucky Center for Mathematics website at http://www.kentuckymathematics.org/research.asp.

FIGURE B: *Program Consistency*



SEMI-STRUCTURED INTERVIEWS AND OBSERVATION DATA

Three main themes emerged from the analysis of interviews and observations of MITs' responses during professional development sessions attended. These themes were:
1) conditions and culture
2) professional competencies, and 3) changes in practices and beliefs.
Conditions and culture describes the requirements of the MIT role and the level of collaboration.

Professional knowledge describes the professional knowledge of teaching including knowledge of mathematical content and pedagogy. Change describes the changes in MITs' instructional practices and beliefs about how children learn mathematics. While these themes had distinct qualities to them, there was much overlap and interplay between these. These themes will be presented separately for the purpose of reporting.

Conditions and culture. A majority of MITs were classroom teachers selected by their building or district administrator to serve in this this role. Participation in ongoing job-embedded professional development was a job requirement. During the professional development, MITs deepened their conceptual understanding of early numeracy, exploring mathematical tasks from many different perspectives, and exploring those different perspectives together. They spent many hours reviewing, analyzing, and discussing video clips of students responding to similar mathematical tasks using the Stages of Early Arithmetic Learning (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & Glasersfeld, 1988; Steffe, 1992) and other constructivist researchers to make sense of what they saw students doing and thinking. A culture of inquiry was modeled throughout the professional development so MITs could experience what it meant to participate in meaningful learning together. It is important to note the professional development focused on developing mathematical content knowledge and pedagogical knowledge, not just how to teach a particular intervention program.

MITs reported that they had never been involved with any professional learning that was as challenging and rigorous. As one MIT stated, "The training for this program is intense, but all of us have learned so much about the research behind good math education. . ." Another MIT expressed the importance of the affirmation she felt while also revisiting her beliefs and practices:

> And when I hear some of the things I'm already doing, it affirms me that I am doing the right thing. I need to know if I am doing the right thing and if I am not, I need to know what I should be doing. Without the support, I would get frustrated and teach the way I always have. I know I can call my regional coordinator and talk through my challenges. The collegial meetings give me the chance to solve issues and get reassurance that what I am doing is what I should be doing. In this way of teaching, you have to rely on what you know and what the student is doing. It's not like, read the manual and do the next lesson. It's like, really trying to understand what the child is thinking and what settings [instructional tasks] will move him forward.

The MITs also appreciated the opportunity to participate in the culture of inquiry created in the professional development and were able to connect it to their own instructional practices. As one MIT remarked, "The leader ran the meeting in the same manner that it was supposed to be implemented in the classroom. It was not simply knowledge that was passed along, it was modeled."

Professional competencies. The professional development activities engaged the MITs in developing knowledge of early numeracy progression, diagnostic and formative assessment strategies, and strategies for designing instruction based on assessment data. While many of the MITs were selected because of their teaching expertise, every participating MIT interviewed reported that they did not have the necessary knowledge of early numeracy development. The following is representative of many MITs' responses, "I guess I knew there was a numeracy foundation. I just did not know what made up that foundation—what they specifically needed to know." They described developing deep insights about the components of early numeracy development and clearly articulated their knowledge of early numeracy as a result of their participation in the professional development.

> It helped me to understand the development progression of early numeracy. It takes you from children that can't count by ones, those who have no number correspondence, and it teaches you how to help them develop a solid numeracy foundation with forward and backward number sequences, structuring numbers with five, ten and twenty.

A significant number of MITs also shared that they learned the value of observing how individual students solved mathematics problems. The following is one example of such sentiment, "I am thinking a lot about how the kids are getting the answer. I watch their thinking more than I ever have before. This training helped me think about the kids individually." Many MITs reported that the practice of sharing and discussing video clips of students solving mathematical tasks demonstrated the value of analyzing student thinking as a tool for focusing instruction on the individual needs of students.

Changes in practices and beliefs. MITs' approach to teaching mathematics changed as a result of their participation in the professional development activities. MITs reported they were engaging students in more discussions about how students approached and solved mathematical tasks and they were asking students many more probing questions about the students' thinking. As one MIT remarked, "'Did you understand that?' used to be the most probing question I asked. But now it's, 'Why did you think to do it that way, I wouldn't have thought about that?'" The asking of these kinds of questions provided MITs with a deeper understanding of how their students were making sense of the mathematics. "I have learned how important it is to ask probing questions. The questions can both guide the students to think deeper about the math and they help me to understand their thinking so I can better guide their learning."

Many MITs reported an increase in their use of manipulatives designed to support the development of early numeracy (e.g., five frames, ten frames, empty number lines, covered counters). Many MITs expressed the importance of students constructing mathematical understanding and how different this was from their prior understanding of the importance of procedures and memorization in learning mathematics. The following response represents a common theme expressed by every participating MIT interviewed.

> It is kind of sad to think the way we were taught to teach. No wonder my children had learning gaps. When I was ready to teach tens and ones, I now realize that half of them probably could not identify 12 and 20 or they would confuse these. I remember telling parents that their children just needed to memorize the facts. Well, no wonder my children didn't get it.

The changed perspective of children constructing knowledge effected changes in MITs planning for instruction and the pacing of instruction.

> I have taught math for 24 years. I used to just follow the manual. I really didn't know if the kids had the basics before I started teaching something new. You were so limited, I had so much to teach in such a short time. Now I use the assessments and that guides my instruction. This year I take as long as it takes to make sure they get it.

Many of the MITs recognized that their earlier limited knowledge of how children developed early numeracy limited their instructional practices to rote memorization and modeling. Overwhelming, MITs reported teaching with a great emphasis on developing understanding and less on the surface features of "doing as I show you."

Many MITs reported that their participation in the professional development increased their self-efficacy. "I feel like I did not know what I was doing with the math. And I thought I was a good math teacher. Now I feel I am more capable of working with the struggling students and I have more confidence now." It was a common occurrence for MITs with 20 or more years of teaching experience express how changes in their teaching were resulting in an increased sense of efficacy and renewed enthusiasm for teaching.

Discussion

The professional development associated with the KCM initiative went beyond "adding" knowledge and skill to transforming MITs' knowledge, beliefs, and instructional practices about mathematics teaching and learning with struggling students from kindergarten through third grade. This was achieved through the design of professional learning that drew on the research literature on effective professional development—a focus on mathematical knowledge for teaching related to the MITs job responsibilities that was ongoing and sustained and situated in a culture of collaboration and support. This resulted in increased MIT competencies, changes in instructional beliefs and practices, and increased student achievement.

MITs acquired knowledge of the stages of numeracy, characteristics of the various stages in the learning trajectory, and the instructional strategies appropriate for advancing student development along the learning continuum. The training and ongoing support activities provided an authentic lens for understanding their students' learning challenges and deepened their understandings of content and pedagogy. By situating analysis and planning in classroom practice, teachers were able to connect and implement ideas from current research in their instructional practices. The gap between professional development sessions provided the teachers an opportunity to reflect on the professional literature and the authentic cases presented in their teaching. In many ways, this gap served as a bridge to what they learned in their training sessions to what their students were actually doing. This resulted in MITs who were students of numeracy content and pedagogy and could structure learning experiences based on assessment of student thinking. The combination of deepened knowledge of how children develop early numeracy and an understanding of how formative assessment, discussions, and manipulatives support student learning resulted in significant changes in the MITs approach to teaching early numeracy. They moved from reliance on the textbook to reliance on professional knowledge and student thinking and became "teacher engineers."

The requirement of ongoing participation in professional learning as a part of the MITs' work enabled *all* MITs to deepen their knowledge and understanding over a period of three years while being supported as they implemented changes in teaching. These supports provided MITs the opportunity to discuss ongoing questions and challenges they had in changing their approach to teaching early numeracy.

MITs and facilitators of the professional development worked collaboratively and the lines between expert and novice were blurred. They created a culture wherein professional relationships were valued and promoted the principles of: 1) collegiality and collaboration; 2) everyone engaged as active learners; 3) learning is ongoing. As a result, MITs felt secure enough to share individual struggles and reflect on their learning with collegial support. This created a community who became ongoing learners of student thinking and numeracy.

Implications

This study suggests implications for implementing broad scale reform efforts designed to strengthen mathematics teaching and learning in primary grades through intervention strategies. It provides insights on how MITs can strengthen the mathematics achievement of struggling students through early and focused interventions and collaboration with the classroom teacher. It provides further and compelling evidence that job-embedded sustained learning, a culture of collaboration and exploration, and focus on deepening teachers' understanding of the specific mathematics content and pedagogy related to one's teaching are critical features of professional development designed to achieve these goals. It also provides insights that these interventionists need strong knowledge of how children develop early numeracy and the opportunity to develop a strong practice as interventionists in order to have an impact.

The study provides insights that may increase the efficacy of other school-wide and district-wide professional development initiatives. Careful consideration and planning are needed to identify the conditions and culture that provide the necessary structure and support for success. The content focus of the training sessions needs to ensure depth of knowledge growth and flexible application of the knowledge to meet the diverse needs of students. The results of the professional learning should result in transformative practices and beliefs and empower the teacher to be the architect and engineer of student learning.

References

Ball, D. L. (1990). The Mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal*. 90(4), 449-66.

Blank, R.K., de las Alas, N., & Smith, C. (2007). Analysis of the quality of professional development programs for mathematics and science teachers: Findings from a cross-state study. Washington DC: CCSSO.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematica l thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499-531.

Cohen, D. K., & Hill, H. C. (2001). Learning policy: When state education reform works. New Haven, CT: Yale University Press.

Darling-Hammond, L., & Ball, D. L. (2004). *Teaching for high standards: What policymakers need to know and be able to do.* Philadelphia, Consortium for Policy Research in Education and Washington, DC: The National Commission on Teaching and America's Future.

Darling-Hammond, L., & McLaughlin, M W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan,* 1995, 76(8) pp 597–604.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, C., Klebanov, P., Pagani, L., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2006). School readiness and later achievement. *Developmental Psychology*, 43 (6), 1428-1446.

Dunne, F., Nave, B., & Lewis, A. (2000). Critical friends: Teachers helping to improve student learning. *Phi Delta Kappa International Research Bulletin* (CEDR) (28), 9-12.

Gerretson, H., Bosnick, J., & Schofield, K. (2008) A case for content specialists as the elementary classroom teacher. *The Teacher Educator,* 43(4): 302-314.

Griffin, S. (2004). Building number sense with Number Worlds: a mathematics program for young children. *Early Childhood Research Quarterly*, 19, 173-180.

Glaser, B. G., & Strauss, A. L. (2009). The discovery of grounded theory: Strategies for qualitative research. Piscataway, NJ: Transaction Publishers.

Kennedy, M. M. (1998). Form and substance in in-service teacher education (Research Monograph No. 13). Arlington, VA: National Science Foundation.

Lieberman, A., & Wood, D. (2001). When teachers write: Of networks and learning. In A. Lieberman & L. Miller (Eds.), Teachers caught in the action: Professional development that matters (pp. 174-187). New York: Teachers College Press.

Little, J. (1990). The Persistence of privacy: Autonomy and initiative in teachers' professional relations. *Teachers College Record*, 91 (4), 509-536.

Loucks-Horsley, S., Hewson, P. W., Love, N., & Stiles, K. E. (2009). Designing professional development for teachers of science and mathematics. Thousand Oaks, CA: Corwin Press.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum Associates.

National Commission on Teaching and America's Future. (2010). Team up for 21st century teaching and learning: What research reveals about professional learning. Condensed Excerpts. Retrieved from http://www.nctaf.org/documents/TeamUp-CE-Web.pdf.

Sarama, J., & Clements, D. H. (2004). Building blocks for early childhood mathematics. Chicago: SRA/McGraw-Hill.

Saxe, G. B., Gearhart, M, & Nasir, N. S. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education*, 4, 55–79.

Scholastic Reading Association. (2007). Number worlds: A prevention/intervention math program. Chicago: SRA/McGraw-Hill.

Steffe, L. (1992). Learning stages in the construction of the number sequence. In J. Bideaud, C. Meljac, & J. Fischer (Eds.), *Pathways to number: Children's developing numerical abilities* (pp. 83–88). Hillsdale, NJ: Lawrence Erlbaum.

Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies.* New York, NY: Springer-Verlag.

Steffe, L.P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application.* New York: Praeger Scientific.

U.S. Math Recovery Council (2008). Add+VantageMR Professional Development Program. Brentwood, TN: US Math Recovery Council.

U.S. Math Recovery Council (2006). Math Recovery Intervention Specialist Program. Brentwood, TN: US Math Recovery Council.

University of Cincinnati Evaluation Service Center (2009). *Kentucky Center for Mathematics: 2008-2009 program evaluation data.* Retrieved from http://www.kentuckymathematics.org/research/docs/2010/08-09InterventionReportFinal.pdf.

Wright, R. J., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publications/Sage.

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). Teaching number: Advancing children's skills and strategies. London: Paul Chapman Publishing Ltd.

Changing Teachers' Conception of Mathematics

Eric Hsu, Judy Kysh, and Diane Resek, *San Francisco State University, CA*
Katherine Ramage, *Kara Associates, Berkeley, CA*

Introduction

This paper describes the efforts of the authors to work with a group of in-service and pre-service math teachers to change their conception of mathematics. We describe the teachers' starting conception and our desired re-conception. We then detail our efforts to craft moving experiences that would shift their thinking. We discuss why it is insufficient to "just tell them" and conclude with a cautionary note about the differing experiences of our teacher leaders who did or did not craft similar experiences for their fellow teachers.

WHAT IS THE ESSENTIAL WORK OF MATHEMATICS?

Mathematicians are often asked, "What do you do exactly? Haven't all the math problems been worked out?" Many people seem to think this is a very reasonable question, because what they know is from a traditional school mathematics experience, organized to guide them to memorize definitions and recall known routines to solve known problems.

To compare this common view of mathematics with the popular idea of what a scientist does, imagine a person in a lab coat tackling an unsolved challenge, performing experiments (some of which fail and explode, some of which finally succeed), getting stuck and frustrated until finding an alternate approach that causes a great "Eureka!" or perhaps making an unexpected connection to other knowledge, and then arguing with skeptical peers about results and plans and finally convincing people with logic and passion. This popular perception, perhaps glamorized, is essentially accurate, and the spirit of this description fits the work of mathematicians as well (perhaps without the lab coats and explosions).

Re-stated, the essential work of science and mathematics is to:

- Analyze non-routine situations;
- Explore a situation, fail productively, and persevere through frustration;
- Find new and multiple ways to approach a problem;
- Connect knowledge and relate ideas to form new understanding; and
- Communicate ideas and use logic to convince others of their value.

It is unnatural to educate students mathematically without their experiencing this essence. Yet the traditional school math experience emphasizes mental discipline and mathematical literacy, which are indeed important, but at the cost of losing the whole substance of mathematical thinking.

Furthermore, there are practical ramifications of this traditional approach. First, the more disconnected the routines are, the more difficult it is to remember them (NRC 2001, 2005). Second, the boredom of barren tasks drives interested minds out of science and math. Third, such a presentation filters strongly for students who have an existing faith that memorizing and reciting is important to their future success (NRC, 2001, 2004, 2005).

WORKING WITH TEACHERS LOOKING FOR CHANGE

Usually, math teachers have themselves been educated in a system where math is taught and valued in bits and pieces. They then perpetuate the vicious cycle when they organize their students' math experience in the same impoverished way. The authors had an opportunity to try to break the cycle by working with teachers who were motivated to change their teaching.

REvitalizing ALgebra (REAL) was an NSF Math Science Partnership aimed at improving the teaching and learning of algebra by developing teacher leaders and sparking lasting change in math departments at secondary schools and universities. The teachers in the project came from three different teaching settings: secondary mathematics teachers, graduate students in mathematics at San Francisco State University (SFSU), and undergraduate mathematics majors at SFSU.

For REAL, we selected teachers who said they believed changing their own teaching would enable their students to be more successful despite outside influences on the students' lives or constraints of their schools. The secondary teachers were teaching algebra, the undergraduates were paid to assist in the secondary classrooms, and the graduate students were teaching remedial algebra courses at SFSU. We worked with two cohorts of 27 members each consisting of approximately nine secondary teachers, nine graduate students, and nine undergraduates. The three co-directors worked with each cohort in a three-hour class each week through the first academic year and then everyday for three weeks during the summer. During the second year, supported by reduced teaching loads, secondary teachers led teacher teams in their home departments and math graduate students were paid a stipend to work with colleagues who were teaching in the mathematics department. (There was no follow-up for the undergraduates, who had been paid to support the secondary teachers, beyond encouraging those who were interested to enter teacher preparation programs.) We refer to participants from all three groups as 'teachers.'

During the semester prior to beginning work with the teachers, the three co-directors spent a half-day a week observing their classes. Their observations were not structured but they observed the lessons and the reactions of the students from the viewpoint of mathematicians and experienced mathematics teachers. At the same time the evaluator, Katherine Ramage, used a framework for observations developed jointly with the co-directors and attached to this article as Appendix B. Although we did not use a formally validated observation tool, the observers for the evaluation who used this tool regularly conducted inter-rater reliability checks, and these results showed good reliability.

From our extensive classroom observations and initial conversations with our participants, we found that, for the most part, teachers saw math as a batch of rules and facts, or at best, an ordered list of isolated definitions and procedures to be taught by them and remembered by their students. Textbooks and standardized tests determined the content and sequence of the math they taught. Teachers reported experiencing their own learning of mathematics as having problems presented that they would then solve by searching their memories for a statement or procedure that, when applied, would give them the answer. They also remembered respecting those who could show what they knew by solving problems quickly, often in a matter of minutes.

Creating A New Conceptualizing of Mathematics

Inspiring sympathetic teachers to enrich their conception of mathematics was the challenge. The heart of our approach was to create situations where they could:

1. Enjoy rich mathematics as students;
2. Practice identifying rich activities;
3. Practice facilitating such activities as teachers; and
4. Work towards believing their own students could learn in this way.

We also spent time in the workshops dissecting math problems and explicitly detailing connections among different parts of the algebra curriculum.

After we describe these approaches, we will reflect on why we took the trouble to set them up. Charisma and authority are not enough to change deeply ingrained beliefs. Logic was not enough to convince. We had learned these lessons before, but we re-learned them during this project, and probably will re-learn them in the next.

TEACHERS ENJOY RICH MATH AS LEARNERS

For teachers to change their ideas about the nature of mathematics, they first needed to experience 'doing mathematics' as we envisioned the subject. During a part of every REAL meeting, both during the academic year and the summer program, the teachers worked in groups on rich mathematical problems. We define a rich problem as having the following attributes: (Hsu, 2007)

- The "mysterious" part of the problem is mathematical.
- The problem has very little overt scaffolding.
- There are several ways to do the problem.

- Students of different skill levels can learn from this activity.
- The problem has natural extensions.

To solve these problems, teachers had to explore. They could not immediately see a procedure or a theorem to apply. There were usually several ways to begin exploring. In every problem some ways of exploration were accessible to the teachers with the least formal mathematical training, and those teachers often had insights that were of use to the most sophisticated members of the group. We randomly assigned the teachers to problem solving groups, so for the most part, there would be a mix of levels of mathematical sophistication in each group.

As a result of these tasks:

a. Teachers experimented and failed productively.
By working on these problems, the teachers saw that solving a mathematical problem involved "getting their hands dirty" through exploration and not sitting back to wait for a bright idea to come to mind. They began to see trial and error and learning from mistakes as legitimate and necessary mathematical problem solving tools that could be used as tools for learning.

For example, looking at a problem such as "Which numbers can be written as the difference of squares?" teachers would start looking at differences of squares such as $4^2 - 3^2 = 16 - 9 = 7$. After looking at some more examples, they might conjecture that all differences in squares are odd numbers. That they could not write 2 as a difference of squares was a verification, but then someone tried $4^2 - 2^2 = 16 - 4 = 12$, and they would see that their initial conjecture was wrong and needed modification. The teachers learned that being wrong was not detrimental; it helped them think toward a solution. In mathematics, genuine problem solving will proceed in a fitful manner. It will not normally proceed smoothly. Teachers needed to realize that when they protect their students from being wrong or thinking incorrect thoughts, they are keeping their students from solving problems on their own.

b. Teachers saw that people weren't just "better" or "worse" at mathematics.
On the occasions when our random process turned up homogeneous groups, the least sophisticated groups sometimes took the problem farther than those who had taken more direct mathematical approaches. Teachers learned that they could not assume in general that one person is 'better' than another mathematically as they saw different colleagues excel on different tasks and they began to see mathematical talent and learning as a mix of attributes.

Teachers commented on getting better at things where they were initially weak, such as visualizing. They learned by doing and by working with others who had different strengths and commented in discussion and in their daily written comments on using strategies they had not used before.

We wanted these experiences to keep them from pigeonholing their students as starkly as they had done initially when they talked about "strong and weak" students or "low students" and about their classes as "low level" or "slow algebra."

c. They worked together and learned from each other.
The problems were chosen so there were a number of non-routine insights needed. This was meant to make them explore as in (a), and to break down status differences and stereotypes as in (b), but also so that they would need each other's help! We wanted them to see the value of having students work together, and we wanted them to believe that their students could learn in this way, and that it would not be 'weak' students learning from 'strong' students with the 'strong' students being burdened by teaching and not learning anything new.

In particular, a non-routine problem makes people argue about the mathematics and about problem-solving strategies. An essential piece of the work was putting our teachers in situations where they had to communicate using mathematics. Many of our teachers were not used to communicating about math to investigate, to question, or to convince. They were used to being the classic sage on the stage transmitting well-honed signals to their students.

d. Teachers enjoyed doing mathematics.
Almost all teachers agreed that their favorite part of the REAL class was working on the problems. Many admitted they had not really enjoyed doing mathematics in the past. In the past, they may have been good at getting the answers and found joy in recognition that comes with success or in the approval their teachers, but in their anonymous evaluations they claimed the joy of actually solving a problem and knowing that they'd done it was greater. This piece should not be undervalued. It is amazing how many students

come into a math major loving to do mathematics and leave it, perhaps with respect for their hard work at mastering difficulties, but without actually having that same sheer enjoyment of the mathematical work. Others, who were successful in high school, never reach the point of enjoying mathematics in college.

TEACHERS DEVELOP TASTE IN SELECTING PROBLEMS

Immersing teachers in rich problems helped them develop skills for solving them and a taste for working on them. However, many teachers did not have the ability to recognize a rich problem when they saw one. We had experienced this lack of recognition in previous teacher education projects so we were prepared to work on this issue. The interesting thing is that the language is slippery enough that teachers would agree that the five aspects of a rich activity were desirable, but these teachers would then have trouble using that language to identify productive problems. Part of this uncertainty comes from little experience working on rich problems, and part of it comes from a lack of experience in facilitating student work on rich mathematics. There is an art to seeing the possibilities in a problem and to seeing how possible solution paths can lead to interesting mathematical discussions. In general “taste” is hard to describe but becomes shared through repeated experiences. It became a term we used internally to describe teachers’ evaluation of problems in terms of their richness.

We weren’t sure how to cultivate this taste, so we took the simple approach of finding a number of tasks that we ourselves thought were rich and ones that were not so rich and then gave the mixed list to groups of teachers to sort. We followed the sorting with a whole group discussion of each problem.

In one activity, we asked teachers to work with a partner to search for two problems on the web – one that was rich and “group worthy” and one was apparently rich but not really. We asked them not to label them and write each on an index card. We then selected six from those submitted and wrote them on the sheet shown in Figure 1.

Teachers worked in groups of four to sort the problems into rich and not rich. Then we had a whole class discussion about which problems were rich and how ones that were not might be made richer or used for another worthy purpose (Hsu, 2007).

We also engaged in some important complementary activities where we asked participants to try to enrich problems by removing some of the overt structure or specific directions for tasks. (This is described in detail in Hsu et al., 2009.)

As a result of these tasks:

a. Teachers practiced analyzing a problem in detail and its value for inspiring mathematical thinking.

That is, a problem isn’t about a subchapter in a textbook where it appears, or about the kind of problem it is and the recipe that solves it. A problem is a task that inspires student thinking and enables them to develop mathematical solution methods in a teacher facilitated group work setting. The teachers began discussing what mathematics could be brought out in approaches to a problem and what kinds of connections could be made by sharing multiple solutions. From this point of view, many of the mathematical tasks we give in classrooms are impoverished, often meant to inspire a single mathematical approach. Sometimes these kinds of problems are necessary, but it is important for us to know what we are sacrificing.

There are no set rules for when to direct students and when to let them explore. A certain amount of material needs to be covered, thus a certain amount of direction is needed. At the same time, a certain amount of richness needs to be present to make for an engaging class. Students need time to explore on their own, but teachers are constantly faced with demands to focus on what appears to be the content of the tests. How much of each kind of instruction to include is a judgment call and making that call is part of the art of teaching. It all depends on the teacher, the curriculum, and the students.

b. Teachers began adapting their tastes while productively saving face.

Some teachers would initially be satisfied with the level of richness of a rather limited problem. But through our pushing of the discussion and through listening to their peers, people began to raise their expectations for what was a rich problem. They were usually able to save face by noting that the problem would be rich for students who were sufficiently inexperienced. Indeed, this observation can be made of most tasks—even mundane mathematical recipes are intriguing to those who have never been taught the recipe. We thought this face saving was in itself a worthwhile lesson.

FIGURE 1

How Rich Are These Six Problems?

1. Within Eldoria, a little country far away, you can place a call with one of two companies.
   - *EZ phone charges $24/month for the first 3 hours and then 8¢/minute.*
   - *U-Call phone charges $30/month for the first 2 hours and then 5¢/minute.*

   a. After how many minutes of local calls will the two plans cost the same?

   b. Make a graph of each cell plan on the same set of axes. Make sure to label your axes.

2. There are many rules that fit the information in the In | Out table below:

In	Out
5	16

   a. Your task is to find at least 10 different rules that work. You can use multiplication, division, addition, subtraction and exponents and you can use more than one operation in a single rule.

   b. The table below has a bit more information than the one above, but that only makes things more interesting. Find as many rules as you can that fit both rows of this table.

In	Out
1	2
2	5

3. a. On the same set of axes, plot the graphs of 1/2 x^2, x^2, $2x^2$.

   b. On a second set of axes, plot the graphs of $-2/3\ x^2$, $-2x^2$ and $-x^2$.

   c. Write a paragraph explaining the 'a' in ax^2 affects the graph of x^2.

4. Two of the most commonly misused laws are called "the product of powers" and "the power of a power."
   Aka: $x^a \bullet x^b = x^{a+b}$ and $(x^a)^b = x^{ab}$, respectively.

   Task: Prepare an explanation of these laws, as if teaching someone who is learning this for the first time.

5. Take any three consecutive integers. Square the middle number and multiply the first and the third numbers. Compare your answers. Use algebra to find out why this will always happen.

6.

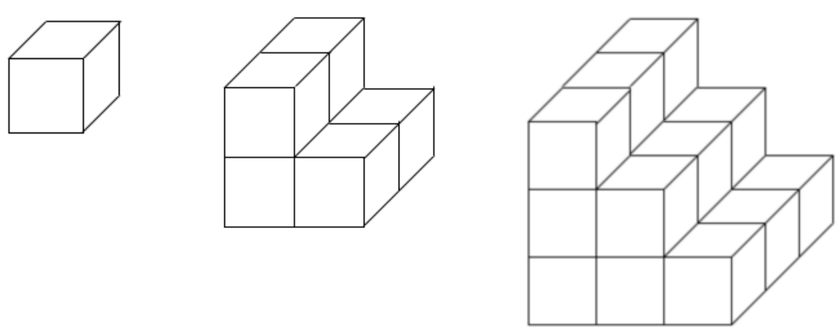


Figure 1 Figure 2 Figure 3

Each figure is constructed of cubes (1 cm x 1 cm x 1 cm).

   a. Find the surface area of figures 1, 2 and 3.

   b. Find the surface area of the 50th figure.

Though we do not have an objective measure of "taste," we do believe that most of our teachers improved their taste over time. Comparing their proposed rich tasks from early in the workshop to later ones, it did seem that the richness of the tasks increased, although we were impatient with the rate of change.

TEACHERS LED PROBLEM SOLVING IN SMALL GROUPS OF GOOD WILL

The tasks described so far were intended to give teachers the opportunity to enjoy active learning and develop the taste to select rich activities. But there was still a gap between these experiences and their confidence that they could create and sustain such learning situations in their own classrooms. It is a subtle and difficult task to support teachers in facilitating group work among their students and we probably did not budget enough time for this in the workshops.

In REAL, our strategy was to first deal with the difficulties inherent in facilitating group work even in an ideal situation of student goodwill and then introduce other tasks to handle teacher difficulties in non-ideal situations.

We wanted to create a situation where people could concentrate on facilitating group learning away from the stress of difficult students, unrealistic academic calendars, and unhelpful curriculum materials. The problems were chosen so as not to require any mathematics beyond algebra, partly to emphasize the richness of the mathematics in algebra, and partly to avoid giving advantages to those teachers who were concurrently studying advanced mathematics.

First we facilitated their work in groups to solve selected problems so they would experience the problem they would be teaching as a student. Then they worked together to prepare to teach the problem to a subset of the class. Nine teachers co-taught in trios. Each trio taught two of the other problem groups, so they had a class of six students, which typically would be run as either three pairs or two trios of students. Learners gave anonymous feedback on index cards to the teachers, and afterwards, the problem teams reconvened to compare notes on their experiences either as teachers or learners. There were no directions given to the learners regarding their comments; only that they should make comments they thought would be useful to the teachers.

In the first session we allowed freedom in choice, which resulted in a wide range of teaching approaches, some more engaging than others. In two subsequent sessions, we gave more explicit guidance to focus on certain aspects of teaching, such as asking good questions, handling unequal participation within a group, or how to move groups to key checkpoints in the activity without ruining their creative thinking. We began the guidance through a discussion with all the teachers, getting them to specify problem areas for their practice teaching. Then we asked them to prepare in their working trios strategies for addressing these problem areas. One of the co-directors met with each trio to discuss their strategies. Finally, after the teaching episode, the trio reflected on the success of their plans.

As a result:

a. Teachers found it was really fun to facilitate a rich problem with interested students.

It's a great feeling to manage an experience for people who get excited and engaged. This is a feeling that our teachers did not typically get in their classrooms. Even if the situation we set up was artificial, it was a real reminder of the potential joy that comes from helping interested learners.

b. Many teachers were surprised to see people could struggle and solve problems.

Because we did not initially force the facilitators to let people struggle, teachers used a wide range of approaches, from letting people struggle with encouragement and well-placed questions and hints to telling people which path to take and then explaining the answer at the end. Initially leaving the choice of facilitation methods to the teachers provided the basis for insightful discussion when their 'students' shared their reactions.

Teachers saw that intervening lightly by giving learners more time to struggle, offering fewer directive hints, and asking them to describe or explain their thinking often gave people a chance to persevere and to come up with marvelous insights on their own without being told.

c. Many teachers were surprised when their helpful interventions and explanations were not welcomed.

Some teachers were surprised to see people struggle on problems that they, as the facilitators, perceived as simple, but most remembered how they themselves had not immediately found productive paths on their first

FIGURE 2

HW 17 (2/8/05)

1. (ALL) Please discuss one class comment from the Flag Hoist discussion that you agreed with and one you disagreed with.

2. (Teachers & Grads) Think about an algebra problem that could provoke an interesting class conversation with your students, as in the 'Flag Hoist' video. Plan to give your class the problem during class or for homework and have a 'good' class discussion about the problem before Feb 22 (in two weeks). At the end of the discussion, give your students an anonymous quiz on the content of the discussion. Bring them to class on Feb 22. (It should be a VERY short, one question quiz.)

3. (ALL) Reading. Read (or reread) the "Messy Monk" article. Imagine yourself in the author's role. What do you think would be the most difficult point in the lesson for you?

Copes, L. (2000). Messy Monk Mathematics: An NCTM Standards-Inspired Class. *Mathematics Teacher*, 93(4), 292–298.

HW 18, Due 2-22-05.

1. Remember that at the end of the rich discussion you gave your students an anonymous quiz on the content of the discussion. Bring the quizzes to class on Feb 22. (It should be a VERY short, one question quiz.)

2. Read *The Nature of Classroom Teaching*, Ch. 2. On pages 2 and 3 from the article the author gives an example of a situation that is mathematically problematical for students and one that is not mathematically problematic. Come up with one example of each from algebra.

3. Read *Engaging Schools*, Chapter 1.

4. Respond to the following questions in relation to the *Engaging Schools* reading.
   - How can you tell whether a student is engaged in your class?
   - What strategy did a teacher use in a class you took that got you engaged?

encounter with the problem. In fact, a substantial proportion expected people not to be able to solve the problems without their hints to push them along to the teacher's solution.

Heavy-handed intervention and explication by facilitators was often met with resentment for "stealing the thunder." Facilitators were sometimes surprised when their 'students' liked their own solutions and representations better. Needless to say, it was unusual for teachers to get such honest and constructive feedback in their normal practice.

d. Co-teaching made visible the many choices and mathematical observations a teacher makes.

Each co-teaching trio found themselves discussing where the class was and how to choose the next move at each step. Occasionally one of the trio would act as a silent observer, but in general, the trios naturally discussed key teaching moves, such as whether groups were working quickly enough, which groups needed help with math or with internal status differences, which groups ought to present and in which order. Even though we did not require them to consult with each other, it naturally occurred in all cases.

TEACHERS STROVE TO BELIEVE IN THEIR STUDENTS

In the previous tasks, teachers grappled with the subtleties of handling group work and rich problems with very cooperative students. But for teachers to incorporate this new sense of mathematics in their own classrooms, they needed to become convinced that their own students were capable of doing mathematics in this way, and that their students could learn important content in this way.

Many said their advanced students would work on problems in ways similar to those of their colleagues but they were not convinced that the students who had trouble with algebra were capable or would be willing to work in this way.

To convince teachers that all of their students could benefit from working on rich problems, we gave them assignments that involved teaching their own classes in new ways and then reflecting on what happened. See Figure 2 for two consecutive assignments. In our observations of the secondary math teachers, we saw very little change during that first year. Based on our records of classroom observations, it was not until the following year that change was observable. It is unclear whether teachers needed time to integrate the new approaches or if they needed to start fresh with new students to make such big adjustments. These secondary math teachers also faced directives and mandates from their districts around curriculum materials, especially for students failing algebra, which may have contributed to their pace of change. The graduate instructors did incorporate changes in their initial year, but their classes were only one semester long, so they started with new students in the middle of the year. The graduate students were also relatively new to teaching algebra so may have found it easier to try something different.

In addition to asking the teachers to try new lessons, we showed them tapes of some lessons, which are described in Appendix C. Some of the classrooms in the tapes were approaching mathematics in the new way and others were of more traditional classrooms. The teachers were not given specific aspects to watch for, even though we always had a list of topics to discuss. The facilitator would ask for comments on positive aspects of the instruction and then open the discussion for general comments. At first teachers were reluctant to criticize other teachers, but later in the program, some became overly critical. In either case, starting with positive comments served to balance and enhance the discussions. The differences in the degree of engagement of the students in the tapes where instruction was student-centered compared to those that were teacher-centered were remarkable, and some of the student-centered tapes were of classrooms with students from lower socio-economic levels. It was, however, very difficult to find good examples of student-centered classrooms in urban schools. This lack of evidence made it difficult to convince some teachers that their students would be capable of learning in such a classroom.

We assigned readings about lack of success in mathematics learning for African-American and Latino students. See Appendix A for the list of readings we used. These articles were discussed during the REAL class, and during the follow-up year, the graduate students and many of the secondary teachers, who had participated in the first year, chose to read and discuss these and other articles in their meetings with their department colleagues. But some still remained unconvinced that their students would have the ability to learn mathematics in a student-centered environment.

Our first task in attempting to convince teachers that the important content could be learned through problem-based lessons was to give them the vision of what such curriculum would look like. The mathematics problems the teachers worked on in the REAL class, were usually not designed to teach particular content since we wanted the teachers to work together as equals on the problems; although, their backgrounds with respect to mathematics content varied widely from very few college math courses to graduate students working toward a masters degree in mathematics. Together we examined problem-based lessons from reform curricula, which approached algebra in a student-centered way.

The teachers needed to realize that all the mathematics they had been teaching as separate procedures could be looked on in a different way. The lessons from the reform programs provided an opportunity to see their curriculum organized around big ideas. In Hsu et al. (2007) we reported on our experience working with teachers to identify and build activities around big ideas. However, pressure for students to do well on state tests was a major impediment to using a problem-based approach. Through readings and discussions, we succeeded in convincing teachers that tests composed of many problems that required rapid application of procedures did not assess whether students could reason and solve problems nor their understanding of concepts. However, most believed that their mathematics programs would be in jeopardy if their students did not perform well on those tests. We needed to convince them that their students could do as well or nearly as well if they did not teach to the test. This turned out to be an extremely difficult task, and most decided to compromise and teach in a student-centered way part of the time, but to teach to the test some of the time also. Unfortunately, there is some evidence that mixing these approaches may not work (Pesek, 2000).

Once teachers had a vision and believed they should change their teaching, they were still not confident that they could conduct class in this manner. They feared behavior problems, that in groups only the 'fast' students would do any work, that many groups would give up, and that they, as facilitators, would not be able to get them restarted. There is much to learn about group facilitation and good facilitation is vital to running a student-centered classroom. Through our work in REAL we realize that this third concern requires continuing, follow-up support, far more than our project was able to offer. We discuss some of the issues of helping teachers to acquire these skills in an article on differentiating instruction (Hsu et al., 2007).

Why Bother With These Moving Experiences?

Some of the participants were frustrated by our refusal to "just tell them clearly what we wanted them to do in their classrooms." In fact, much of this frustration was demonstrated by teachers who eventually came to strongly respect our judgment through some mix of their beliefs, our authority, and their experiences. The transformation seemed to occur over the summer when they were away from us and had more time to think. They had fought the new ideas we were supporting during our classes, but in the fall, they began to practice them with fervor in their own classrooms. When we visited their schools to observe, they had rearranged their classes to accommodate groups, they were assigning fewer but meatier problems for class and for homework, and they were structuring their teaching around big ideas as opposed to isolated procedures.

Some of the frustrated participants came to believe we used our methods out of our own ideological commitment to constructivism, and that we wanted them to invent the answers themselves. But this was only a small piece of the puzzle. If we could just tell them what to do and have them go forth to be great teachers, we would. But we believe they really needed the moving experiences and new images of how students learn to understand to the level of turning words into practice.

WHY NOT JUST TELL THEM WITH CHARISMA AND AUTHORITY?

Perhaps the first idea that comes to mind is that if we could explain our conception clearly, repeatedly, and with charisma and authority, the participants would be able to internalize it. Indeed, we did occasionally share our opinions and educational values in the course of our project, especially when a co-leader was in the audience as a fellow teacher. It would have been a very short project if this were sufficient. But cognitive science and research on how people learn (NRC, 1999) make it very clear that relying on charisma and authority alone has serious limitations.

Everyone has been inspired by hearing a speaker. But most people have also had a follow-up experience of not remembering the details and logic, or even worse, losing interest upon reflecting soberly, and perhaps feeling fooled. This is like the math students following a wonderful speaker and nodding along and then afterwards realizing they didn't really understand. Similarly, for instance, we had a healthy, enjoyable discussion in the first weeks of our workshop where most people agreed that it was important for us to have student activities that engaged the students and had them think about the concepts underlying rules and definitions. Then we sent them away to put together a sample conceptual lesson. When we saw the results, we were sobered. Only two of the twenty-seven activities they produced met our criteria for a conceptual activity, and one of those was by an undergraduate who wasn't yet teaching her own class! This experience was to be expected, because these ideas are subtle and hard, but even though we expected that it would take time the results were an unpleasant surprise.

Even if we delivered our message with such charisma that everyone would be inspired in a lasting way, we could not guarantee that they would do the same as lead teachers. In our project, we wanted change to spread from the lead teachers we worked with directly to their department colleagues in secondary school and the university through the meetings in the second year. Even if they accepted our authority, as many of them did, none of them held the same authority with their peers.

WHY NOT JUST LOGICALLY CONVINCE THEM?

A second idea might be to not rely on authority and charisma, but to simply state the change we wanted to see and make a strong logical argument. Then we could allow teachers to debate the issues and the logic would convince.

1. These words are easy to misunderstand.

Some words are not easy to misunderstand, like "abelian" or "polynomial," because they can be defined rigorously. However, in educational work we can only use approximate words with as many different meanings as there are people. For instance, take merely one brief phrase used in the opening, "analyze non-routine situations." What does it mean to analyze? Some teachers thought it was enough to provide a numerical answer with some related computations. Some wanted an argument, but only a rigorous one. Some welcomed partial answers and creative approaches even if they were not well articulated.

Then, what is a "non-routine" situation? In the course of our work, we found some teachers would count a standard problem type whose numerical values were hard fractions as non-routine. Others took a routine problem and appended a fun non-mathematical task, such as using the solution to uncover a secret phrase or answer a riddle. Others posed a question that did not have a known recipe for solving it. Even the word "situation" is trouble. Are "situations" always realistic world settings? Can they be fanciful? Can they be abstract situations?

If we could precisely define what we mean by "analyze non-routine situations," we would have done so. But there was no way to do this in words. The best we could do was to provide shared experiences and to ask the participants to discuss them. While this practice led to some frustration among the participants, there does not appear to be a simple way of getting the notions across.

Multiply this ambiguity across the whole range of vocabulary, and it is almost impossible to have genuine discussions about these issues without actually experiencing the exploration, analysis, and communication together. These common experiences are necessary to create meaningful common vocabulary.

2. These words are loaded.

Some words sound so good that everyone aspires to those labels. Everyone wants to believe that their students are "engaged" and understand things "conceptually" and that they are teaching "the big ideas" of the course. But we believed many of our teachers had never experienced the depth of engagement, conceptual thinking, and course conceptualizing that we pushed for, so simply agreeing to value those would be pointless.

Also, in the wake of the math wars, many terms were loaded, and people who considered themselves on one side or the other of reform or tradition had prejudices towards "group work" or "basic skills" and other hot-button phrases. These prejudices interfered with real conversation about what effective group work or practice looks like. More subtly, there was a small but important core of teachers who insisted they were above the whole reform-traditional debates and took the best of everything in moderation. These teachers seemed the least willing to change their practice, as if any suggestions to them would perturb their equilibrium, and later they turned out to be the most upset that we were not directly prescribing a position. They suggested that our putting them in rich learning situations was a way of tricking them into agreeing with reform beliefs.

3. We can't give them recipes for the whole wide range of future challenges.

Mathematics is more than a collection of problem recipes. Teaching mathematics is more than a collection of teaching recipes. A number of our teachers admitted that they had hoped we would set out for them which excellent teaching to do and which great problems to use and then pinpoint when to use them.

A Test: Teacher Leaders Try To Move Department Colleagues

We conclude with a striking example of the importance of colleagues working together on rich mathematics problems, namely the experiences of project secondary teachers working with their departments. Our secondary teachers spent the second year of their project participation as lead teachers for teams of their math teacher colleagues (whom we will refer to as "department teachers"). The secondary school teachers, both the teacher-leaders and their department colleagues, were given a released period for a full year to participate and the graduate instructors were paid stipends for their time. All the groups met multiple times each week.

We had decided to let the lead teachers determine the form and the pacing of the teacher meetings. We visited their team meetings on a weekly basis along with many of their classes, and we gave feedback as to what we saw happening. We made suggestions about activities they could do as a department. We continually reminded our lead teachers of the ultimate goals of having authentic conversations about improving practice, getting department-wide commitment to looking honestly at their teaching, and making this part of their department culture. But while we pushed the big ideas and concepts of the professional development, we left the details up to them out of respect for local autonomy.

Response was different at different sites. The eight school teams exhibited different levels of engagement. The first level, which every teacher team reached, was to work on curriculum materials together and to collaborate to insert some isolated special rich activities. Only four secondary math department teams reached a next level, where the department's culture changed so that the teachers became part of a community that worked together on mathematics, on teaching and learning, and on sensitive issues of race, ethnicity, and expectations. In addition to working on curriculum materials they spent their meeting time reflecting on and discussing tough issues, and made decisions with effects that were apparent in their day-to-day work in the classroom. These four departments were the ones that started their meetings together with work on rich mathematics problems and then worked through a sequence of activities similar to the ones they had experienced. One other team had already developed a department culture that supported their work on improving methods of instruction but they did not really progress beyond where they had started.

In retrospect, we believe the lack of stronger guidance during the second year was a mistake. Most of the lead teachers did not have the intellectual leadership skills or enough pre-existing status in their departments to facilitate activities that involved risk taking (Hsu, 2009). The schools that did have strong lead teachers used a number of the approaches, which we have described, and they did move their department cultures to change during the project. The other lead teachers did not push their teacher teams through the initially uncomfortable engagement in solving mathematics problems together that appears to have been a necessary step toward change. Because as project leaders, we potentially did have the authority and capital to push the teams to that uncomfortable place, we are left with a big question. Would mandating the use of the approaches described in this paper and working with department leaders to co-facilitate the school-site meetings have caused deeper and more lasting change? Maybe all the teachers needed to participate directly in the whole REAL program.

Conclusion

Most of the teachers in the REAL project expanded their views of mathematics and of what mathematics is important for their students to learn. Those teacher leaders, who took the next step, used what they had learned to facilitate problem solving with their colleagues and lead them into both deeper mathematical discussions of curricular issues and discussions of more sensitive issues of teaching and learning expectations.

It is worth noting that only those departments that started by solving problems together moved on to the other activities: identifying rich activities and working towards believing their own students could learn in this way. And only those teachers made lasting progress toward changing the culture of their departments to include ongoing discussion of mathematics and of improving their teaching in order to support success for students of all racial and ethnic groups. We conclude, as we did in an earlier article on mentoring, that working together on mathematics problems allowed teachers to relax their defenses and start to build the trust needed to participate in frank discussion of more sensitive issues (Hsu, 2009).

References

Hsu, E., Kysh, J., Ramage, K., & Resek, D. (2009). Helping Teachers Un-structure: A Promising Approach. *The Montana Math Enthusiast*, 6 (3).

Hsu, E., Iwasaki, K., Kysh, J., Ramage, K., and Resek, D. (2009). Three REAL Lessons About Mentoring. In G. Zimmermann, L. Fulmore, P. Guinee and E. Murray (Eds.), *Empowering Mentors of Experienced Mathematics Teachers.* Reston, VA: National Council of Teachers of Mathematics.

Hsu, E., Kysh, J., Resek, D. (2007). "Differentiated Instruction Through Rich Problems." *New England Mathematics Journal.* 39, 6-13.

National Research Council. (2005). *How Students Learn: Mathematics in the Classroom.* Committee on How People Learn, A Targeted Report for Teachers, M.S. Donovan and J.D. Bransford, Editors. Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

National Research Council. (2004). *Engaging Schools: Fostering High School Students' Motivation to Learn.* Committee on increasing high school students' engagement and motivation to learn. Board on Children, Youth, and Families, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council (1999). *How people learn: Brain, mind, experience, and school.* Bransford, J.D., Brown, A.L., Cocking, R. (Eds.), Committee on developments in the science of learning. Washington D.C.: National Academy Press.

Pesek, D., Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal of Research in Mathematics Education.* 31, 525-540.

Wenglinsky, H. (2004, November 23) Closing the racial achievement gap: The role of reforming instructional practices. *Education Policy Analysis Archives*, 12(64). Retrieved from http://epaa.asu.edu/epaa/v12n64/.

APPENDIX A

REvitalizing ALgebra | ***Readings on Cultural Differences***

Boaler, J. (2003). When Learning No Longer Matters: Standardized testing and the creation of inequality. *Phi Delta Kappan*, 84(7), 502-506.
Also available at http://ed.stanford.edu/suse/newsbureau/railside_pdk1.html

Boaler, J. “Sugar and Spice” http://www.stanford.edu/~joboaler/MTL_2002.doc

Delpit, L., (1988) “The Silenced Dialogue: Power and Pedagogy in Educating Other People’s Children”, *Harvard Education Review*, August.

Delpit, L.(1995) “Cross-cultural Confusions” in Other People’s Children. The New Press, New York.

Koh, J. (Writer) (1995). My brown eyes [1 videocassette (18 min.) VHS]. San Francisco: NAATA/CrossCurrent Media.

Massey, G. C., Scott, M. V., & Dornbusch, S. M. (1975, November). Racism Without Racists: Institutional Racism in Urban Schools. The Black Scholar.

Moses, R. “Radical Equations: Math Literacy and Civil Rights”, Chapters 1, 5 and 8.

Ogbu: <http://www.eastbayexpress.com/issues/2003-05-21/feature.html/print.html>

Rothstein, R. *Class and Schools: Using Social, Economic, and Educational Reform to Close the Black-White Achievement Gap*, College Columbia University, May 2004 (Introduction)

Singham, M. (1998). Canary in the Mine. *Phi Delta Kappan*, September

Singham, M. (2003). The Achievement Gap: Myths and Reality. *Phi Delta Kappan*, April.

Steele, C. on Stereotype Threat: <http://www.theatlantic.com/issues/99aug/9908stereotype.htm>

Treisman, U. (1992). Studying Students Studying Calculus: A Look at the Lives of Minority Mathematics Students in College. *College Mathematics Journal*, 23(5), 362-372.

APPENDIX B

REAL CLASSROOM OBSERVATION FRAMEWORK (2005-06)

Creating a classroom culture which/where

- Requires students to respect each other and the class. This might manifest in students being forced to explain and listen to each other.
- Empowers students to speak about personal topics and math. (Look for evidence that it is valuable use of class time to have students speak about personal topics.)
- Students are working on improving cooperation in groups, e.g. silent activity of having to find out the rules without talking.
- Teachers are thinking about groups. Students are used to working in groups and the room is set up for groups.
- Teachers are extending wait time for students to think and then talk.
- Teachers are displaying compassion for students, valuing personal connection as part of classroom atmosphere, e.g., longer wait time, interest in the wrong way to do a problem, making personal contact with student after the student didn't answer.
- Teachers are recognizing incorrect solutions and spending time examining students' thinking.
- Teachers are interested in why students are misbehaving rather than assuming they are bad. Look for evidence that teachers are thinking about what math will make them behave better.
- Teachers are getting around to all students. Classes are not dominated by a small group of students. (Look for evidence of inclusion)

We want to see evidence of teachers

- Giving consideration to quality of activity over coverage, e.g., taking more time for activities that need it, going outside for activities such as "To the Moon."
- Focusing on the big idea and creating a context.
- Focusing less on procedures.
- Moving away from primary focus on correct answers
- Including ELLs, not making it easier for students to read but making them have to cope with the language. Having students read aloud is good as is working in groups. Not depriving ELLs of good math.
- Teacher's thinking moving towards honors and remedial classes looking the same, with both taught conceptually.

When using reform curriculum materials, we do want to see teachers

- Pushing students to explain
- Offering good alternate explanations.
- Improving the quality of math problems making them more challenging.

(cont. on next page)

APPENDIX B *(cont.)*

We don't want to see teachers

- Discouraging students to use concrete supports like manipulatives.
- Skipping the challenging problems and problem solving strategies.
- Making students do all problems.
- Going directly to teaching FOIL rather than letting students discover.

When using traditional texts, we want to see teachers

- Doing something different such as putting the math into a context
- Letting students discover and evolve their own understandings, e.g., laws of polynomials, figuring out proportions on their own rather than the teacher giving cross multiplication.
- Improving the quality of math problems, making them more challenging.

With regard to teacher knowledge, we want to see teachers

- Having a rich understanding of math evidenced in group work supported by rich discussions.
- Thinking about the question "Why are we doing this?" and lingering more on problems and big ideas like slope.
- Realizing that a really "bad day when students struggle and don't reach conclusions" is a good day. They have to come back to it and resolve it later. (Eric thinks that he came up with a bad problem if the students get it right away.)

With regard to assessment, we want to see

- Richer, more embedded assessment.
- Teachers not as frightened of or driven by standardized tests.
- Teachers asking students to solve problems in more than one way and reducing the number of problems.
- More authentic questions and fewer procedural ones.
- Multiple measures, anything that differs from tests and quizzes.
- The ultimate— stopped using class time for test prep.

We don't want to see

- A small number of high stakes assessments.
- All individual quizzes and test.

APPENDIX C

SOME VIDEO MEDIA USED IN REAL

TIMSS 1995: US and Japan. These were once available on videotape, but now can be downloaded at http://timssvideo.com/97 and http://timssvideo.com/67. Used to contrast questioning of US classroom (short wait time, fill-in-the-blank, quick arithmetic) with the Japanese classroom (problem solving, sense-making, student presentations, computer graphics).

Flag Hoist and Plugged Funnel from Miriam Sherin's VAST Project. Excellent example of two tasks which provoke rich classroom discussions around productive, authentic mathematical disagreement. The tasks also support kinesthetic thinking.

Where is the 10? from Jo Boaler. An example of a rich problem with many ways into it. The video shows some intense group work where the group dynamic and teacher facilitation keeps everyone persevering through a difficult task.

Candle Questions from Driscoll's Fostering Algebraic Thinking. Show parts 1 and 3 to give two contrasting examples of problematic questioning strategies (highly non-directive versus not pushing for explanation) along with challenges of unequal status in groups.

Getting Around to Groups:
- TIMSS 1999, US http://timssvideo.com/58 and
- Sandie Gilliam Group Work Highlights
http://gallery.carnegiefoundation.org/collections/quest/collections/sites/gilliam_sandie/archive.htm *(Constraints videos and Reflections)*
Pair these two videos as an introduction to group work. The TIMSS 99 video shows a teacher inefficiently trying to use direct instruction for each separate group. Sandie has a far more restrained approach. Her class is an IMP 3 class, so they've had three years to socialize into passable group work. She walks around and monitors and intervenes only to push groups along. A highlight is the video "Whole group discussion, Eliminating constraints 2" which has the fascinating piece where students keep working on the problem together over break, including one boy hitting another for not letting a girl participate.

TIMSS 1999 Exponents
http://timssvideo.com/69
An interesting class where a teacher gives students exponent laws and asks them to prove the 0 case and negative numbers case. Students aren't given enough time to work through the laws themselves, so they all are convinced that 2^0 is 0.

How Many Seats? Lesson Study by Catherine Lewis. http://www.lessonresearch.net/howmanyseats.html
A wonderful lesson study cycle with lots of honest reflection by the teachers. The teachers shift to observing student thinking as opposed to teacher moves and grapple with the pitfalls of using tables to represent functions.

My Brown Eyes, by Jay Koh. http://www.master-comm.com/mbevideo.htm.
Film about a resourceful, independent Korean child having a horrible introduction to an American school that is not prepared for cultural difference. An entryway into discussing cultural assumptions.

Partnerships for Learning: Using an Innovation Configuration Map to Guide School, District, and University Partnerships

Cathy Kinzer, Lisa Virag, Sara Morales, and Ken Korn, *New Mexico State University*

There is a pressing need for effective partnerships between government, business, communities, schools, universities, and other stakeholders in education. This need is magnified by the current involvement of foundations, business, and government in educational endeavors such as the Common Core State Standards for Mathematics and research grants funded through the National Science Foundation. These endeavors often require focused collaborative interactions between all stakeholders that ultimately support a learning system and students' achievement. The purpose of this article is twofold: (a) to report key findings that emerged from our research partnerships, and (b) to offer an instrument that assesses the readiness and measures the effectiveness of educational research partnerships between schools and universities.

Background

As mathematics educators, researchers, and mathematicians, we have engaged in university and school- or district-based projects for over eight years. The central goals of our collaborative work have been to improve mathematics achievement for students and develop cultures for continued learning through two projects funded by the National Science Foundation (NSF): the *Scaling Up Mathematics Achievement* project (Kinzer, 2007a) and the *Gadsden Mathematics Initiative* (Kinzer, 2007b). These NSF projects have required complex district and university organizations to work together effectively in order to attain project goals. This article builds from a prior publication of the initial partnership work (Kinzer, C. Wiburg, K. Virag, L. 2010). Based on our learning from these projects, the Innovation Configuration (IC) map was developed *(see pgs. 59-62)*. It provides a tool for assessing the key elements necessary for initiating and maintaining successful research partnerships. The IC map provides a softer approach than using a checklist or evaluative instrument and provides a range of descriptors that are helpful in developing a robust research partnership.

The Need

Partnerships between universities and schools and districts are usually very complex and vulnerable. Building professional relationships requires thoughtful collaboration focused on explicit, shared project goals. These associations are influenced greatly by the personalities of the key stakeholders, their abilities to develop a mutual working culture within a changing political landscape, and the establishment of structures and processes for implementing and monitoring project goals over time (Darling-Hammond, 2010; Elmore, 2005; Fullan, 2001, 2005; Kinzer & Bradley, 2010). A rubric or IC map (Hall & Hord, 2001) helps partners see what is needed for professional working relationships based on project goals. The IC map also serves as a formative assessment tool for monitoring the development of the partnership over time and can serve to support thoughtful examinations of what it means to establish and maintain healthy, productive research partnerships.

Domains

The map is organized into four key domains: (a) Culture, (b) Structures, (c) Processes and Practices, and (d) Research. Each domain is divided into levels with key characteristics for each level.

A. CULTURE

It is important for partners to design a collaborative culture in which to develop and share knowledge and meet measurable goals over time to accomplish the intended work (Barth, 2002; Fullan, 2001). Building a collaborative culture to support the project requires considerable commitment and the ability to consider one's individual interest and the collective needs of the project. In particular, this means becoming familiar with the working culture of each partner, especially in terms of their beliefs about learning, teaching, change, use of resources, and how internal systems and policies impact (and sometimes impede) upon the shared work. Developing an understanding of each organization provides a basis for codeveloping cultural values, goals, and principles and/or a theory of action in the shared project space.

Assumptions can become barriers, and therefore, preliminary discussions are essential to determine the viability of a research partnership. Norms, beliefs, and strategies for collaboration should be explicitly discussed "up front" before investing in a shared project. This is because partnerships are greatly affected by how key stakeholders leverage their beliefs about how project goals are implemented and monitored, how problems are solved, and how the "reculturing" that occurs in working together takes place (Fullan, 2002). Preliminary conversations are likely to reveal previously unstated assumptions so they can then be discussed and addressed.

Agreements about the work requirements and the implementation process are critical for success for partnerships. For example, one NSF research grant was written with different district administrators, and when the project was funded there was new leadership in the school district. This required intensive work with the new administration to develop an understanding of the project design, commitments, costs, affordances, and research plans. A central administrator played an integral role in bridging the leadership changes to ensure a continued commitment to the project partnership and agreed upon scope of work.

As the partnership develops, ways of working and interacting should be co-constructed and clearly defined, with norms for the intended collaboration clearly established. This norming process often requires a shift in perspective, with a focus on creating partnerships that function as learning systems, with shared components and perspectives, rather than partnerships that consist of unique separate entities (Fullan, 2005; Senge, 2006). This involves developing a shared culture over time, building the collective capacity of the partners to learn together, both within each partnership and across the partnership, with a focus on the shared work of the project. Understanding the stages of group development provides insights into the process of building interdependence and ways of working together (Tuckman & Jensen, 1977). We began with the nuts and bolts of when, where, and how to manage the work and, over time, began to understand how to collaborate and communicate as partners to achieve project goals. This includes important details like clarifying roles and responsibilities to best ensure the project will benefit the school district as well as the university. It must be a reciprocal relationship without hierarchal dynamics that marginalize or minimize the partners.

Critical to successful partnerships is knowing the formal and informal, procedural policies and chains of command for effective communication and collaboration within the unique systems of each partnering school, district, and university. For example, one of the district-based co-principal investigators presented the organization chart of the school district to help the university researchers understand the structure of the district. This provided ways to think about how to effectively facilitate the logistics of the work. Understanding the leadership and management structures ensures that project conversations include the essential partners. Knowing the organizational structures of each partnership and who is responsible for procedures and policies can build relational trust.

Partnering requires flexibility regarding some responsibilities, such as the facilitation of the weekly meetings, but also requires firm commitments to other responsibilities, such as agreed commitments to the data feedback process, ways to address challenges, and monitoring of the work plan. Developing "partnership competence" requires honoring key stakeholders as individuals, while at the same time rising above a focus on individuals to the creation of a synergistic team with collective responsibility for achieving shared goals (Ravid & Handler, 2001).

A working partnership is grounded in an action/work plan with specific measurable benchmarks and mechanisms for monitoring progress toward achieving those benchmarks, with a management team of leaders who agree on project roles and responsibilities. These include a full commitment to regular meeting times, the use of equitable communication practices, and frequent data feedback to support decision-making and to inform stakeholders of progress towards achieving project goals.

In particular, it is important that expectations regarding data collection, analysis, and the process for public presentation are made explicit. Researchers should honor agreed upon processes for sharing data with school district administration or appropriate stakeholders. These data conversations across the partnerships are crucial for guiding the project, developing knowledge, and determining future directions. In our project, we found that processes for sharing data—including classroom observations, case studies, research articles, and stories from practice—provided opportunities to build a knowledgeable and viable partnership with a common vision and shared language to discuss that vision. This can only happen if there are protocols about how data are shared and discussed both within the project and with the broader community.

Opportunities for partners to learn together are essential for building trust, creating shared knowledge, and engaging in unified decision making. Such opportunities necessitate a culture of sustainable learning in the partnership in relation to the project goals and purposes. Developing common professional knowledge is important in order for the partnership to clarify purposes, strategies, and language for their own learning. The partnership itself can become a professional learning community, developing common knowledge and skills needed for the work at hand.

B. STRUCTURES

To be sustainable, it is important for partnerships to build their capacity to support and sustain distributed or shared leadership, both within and across the partnerships and with key stakeholders in the school district. District leaders should serve as co-principal investigators on projects. School board members and teacher-researchers can function as leaders across both the school and university domains. These opportunities to develop shared leadership help build collective capacity to achieve project goals and are paramount to the sustainability of projects.

In many cases, structures need to be created to support effective and collaborative communication across partnerships. Some of these communication structures help support the management of the project. For instance, it can be important to provide ready access to school district and university calendars so meeting dates and times can be easily set and posted.

In other cases, it is a matter of leveraging existing organizational structures within each member of the partnership, including understanding the broader contexts of these structures as well as the barriers and opportunities they present. For example, in schools where professional learning communities of teachers meet regularly to discuss project data and consider implications, it can be useful for researchers to become members as well—participating in the discussion alongside teachers.

The school, district, and university partners should focus on developing a systemic and inclusive approach through the partnership. This requires not simply developing and strengthening communicative structures between collaborators across the partnership, but among the broader community as well. Sustainable capacity for the project is strongly influenced by the development of both internal and external stakeholders' understanding and commitment to the project over time. Support from the wider school community and those who influence policy, such as school board members, politicians, or external stakeholders, is vital. Shared knowledge helps to build a comprehensive base for understanding the research project, especially when leadership changes. Many worthwhile partnerships have ended because of a single leadership change.

At times, existing leadership structures may serve as barriers for true collaborative practices. Partner projects should utilize readiness instruments and tools to identify where the partnership is, what the concerns are, and understand how change occurs within organizations (Banathy, 1996; Fullan, 2005; Hall & Hord, 2001). Learning brings change and the partners will need to understand how to assess the levels of implementation, collaboration, and determine the impact of their efforts.

One strategy to build collective capacity for the project is through supportive team structures. The *Scaling up Mathematics Achievement (SUMA)* research project utilized a district mathematics leadership team as a structure to think interdependently about mathematics teaching and learning in the district (Kinzer & Bradley, 2009, 2010). This leadership team included stakeholders from all levels of the system—teachers, administrators, parents, university researchers, mathematicians, professional development providers, and project staff. The leadership team had the opportunity to engage in classroom observations, analyze data, develop a shared vision of effective teaching, and provide feedback to the research project (Kinzer & Bradley 2009, 2010). Additionally, another project team considered management details, such as scheduling classroom observations, meeting with evaluators or statisticians, and when to arrange data sharing with key stakeholders. The teams must have access to relevant project information through effective communication structures.

C. PROCESSES AND PRACTICES FOR LEARNING

It is necessary to determine whether the schools or district even desire change or whether the organization is satisfied with the current state. If a partner district is not actively interested in change, or does not see a need for change, it may be difficult to form a collaborative partnership. Project data can often be a useful tool for addressing the need for change as well as documenting how that change can be accomplished.

In our partnerships, gathering data at both the classroom and district levels regarding changes in mathematics teaching and learning and sharing that data across the partnership was important. While the research team gathered much of the data, it was necessary to develop feedback loops to inform university and district partners, so all could meet together to discuss the meanings and implications of the data.

The careful use of data can be helpful in mediating project decisions, allowing decisions to be based not just on opinions, but on what is actually happening in terms of teacher and student growth. For that reason, a process for scheduled data and knowledge sharing is essential. This process, a learning cycle, uses data purposefully to support continued improvement.

In the SUMA project, the school district used its school-based professional learning communities to discuss project data and its implications for teaching. Data based decision-making increased in the schools. Because of the focus on data to support learning, there was growth in teacher's use of formative assessment data in their math classrooms.

As a result of these collaborative discussions about project data, both the schools and the university partners are asking better questions about the data and the implications for both the research project and student learning.

D. RESEARCH

School, district, and university partnerships require a commitment to share goals, provide appropriate resources, measure progress toward those goals, and utilize a recursive process to collaboratively study and learn through the research.

It is important for everything associated with the research agenda of the project to be made explicit, including project timelines, resources, research plans, data analysis strategies, and data reporting protocols. Any managerial details associated with the unfolding of these collaborative efforts also need to be clarified. Unexamined assumptions about this aspect of the project work and the proposed work plan can create obstacles.

The research focus of a project can provide opportunities for professional learning in the district, particularly when district-based teacher-researchers are involved in the research effort. These individuals are important connectors between the cultures of the school district and the university. While they are learning the skills and knowledge needed as researchers, they observe in classrooms and work with school district administrators and practitioners. These district-based partners support effective communication, as they have both the district and university contexts in mind, and are integral interpreters during the implementation of the project that can help bring coherence to the partnership.

The research effort can also provide opportunities for university partners to gain a better understanding of the challenges district leaders face as a project unfolds, especially with regard to district, state, and federal guidelines and expectations of compliance. As our research partnership progressed, we were often reminded of the fact that the school district has many masters, subjects, guidelines, and emergencies that need to be addressed. There are also the ongoing working realities and challenges related to changing policies, budgets constraints, and mandates. With a shared commitment to project goals and a viable process for collective decision-making, these types of challenges are minimized when engaging in school based research.

As a partnership, we developed a shared understanding of the essential components in effective mathematics classrooms and refined classroom observation instruments based on both research and the shared vision. Through this purposeful collaboration the research process has contributed to improvement of mathematics teaching and learning in the school district.

Summary

Partnerships between schools, districts, and universities require a collective responsibility for collaborative structures, processes, and resources for achieving shared project goals. It is essential that crucial conversations take place at the onset to address possible preconceived notions, assumptions, or conflicting agendas.

Conversations that are supported by norms, protocols, and explicit structures that reinforce collaboration and provide time to build trusting relationships can be instrumental in bringing partners together to engage in collective work. Partners learn from each other and create a shared culture for collaborative knowledge building and continued learning beyond the research project. Sometimes it is best to start with small projects so that confidence and competence in the partnership are built slowly and provide a foundation for growing efforts.

Both the district and the university will initially come to the table with very different lenses and ideas; the IC map can stimulate critical conversations about the shared work. The IC map is useful to collectively assess readiness; design and monitor progress, and strategically consider the roles and responsibilities of the school district and university within the project plans to implement a successful research partnership.

References

Barth, R. (2002). The culture builder. *Educational Leadership*, 59(8), 6-11.

Banathy, B. H. (1996). *Designing social systems in a changing world: Contemporary systems thinking.* New York, NY: Springer.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (2000). How people learn: Brain, mind, experience, and school (expanded ed.). Washington, DC: National Academy Press.

Darling-Hammond, L. (2010). The flat world and education: How America's commitment to equity will determine our future. In J. Banks (Series Ed.), *Multicultural education series.* New York, NY: Teachers College Press.

Elmore, R. (2005). Accountable leadership. *Educational Forum*, 69(2), 134-142.

Fullan, M. (2001). *Leading in a culture of change workbook.* Washington, DC: Jossey-Bass.

Fullan, M. (2002). The change leader. *Educational Leadership*, 59(8), 16-20.

Fullan, M. (2005). *Leadership and sustainability: System thinkers in action.* Thousand Oaks, CA: Corwin Press.

Hall, G., & Hord, S. (2001). *Implementing change: Patterns, principles, and potholes.* Boston, MA: Allyn & Bacon.

Kinzer, C. (2007a). Pointing in the right direction: The Gadsden Mathematics Initiative; National Science Foundation Award Number 0096674. *Noticias de Todos*, 3(2), 8-10.

Kinzer, C. (2007b). *Scaling up mathematics achievement* (National Science Foundation. Award Number 0733690). Washington, DC: National Science Foundation.

Kinzer, C., & Bradley, J. (2009). Developing a shared vision. *Journal of the National Council of Supervisors of Mathematic*s, 11(1), 39-41.

Kinzer, C., & Bradley, J. (2010). A district mathematics leadership team; Deepening the collective focus. *Journal of National Council of Supervisors of Mathematics*, 10, 33-46.

Kinzer, C., Wiburg, K., & Virag, L. (2010). University/public school research partnerships in mathematics. *Border Walking Journal*, 61-69.

Ravid, T., & Handler, M. G. (Eds.). *The many faces of school-university collaboration: Characteristics of successful partnerships.* Englewood, CO: Teachers Ideas Press.

Senge, P. M. (2006). *The fifth discipline: The art and practice of the learning organization.* New York, NY: Currency.

Tuckman, B. W., & Jensen, M. (1977). Stages of small group development revisited. *Group and Organizational Studies*, 2, 419- 427.

School, District, and University Learning Partnership Participants Innovation Configuration Map

The goal is Level 4. What level currently characterizes your school, district, and university partnership?

CULTURE: School, District, and University Learning Partnerships develop a **culture** for professional learning and improvement that:

	Level 4	Level 3	Level 2	Level 1
Norms for Collaboration	Enacts norms for interacting and ways of working that exemplify relational strengths and professional structures in the district and the university; encourages openness to modifying goals and objectives through deliberate reflection and analysis in relation to shared vision and articulated goals.	Takes explicit actions that have potential for building relationships, trust, and ways of working between the university and the district but agreed-upon norms or ways of working collaboratively are inconsistently implemented.	Uses norms or agreed-upon ways of working inconsistently, contributing to ineffective communication, mistrust, and inadequate implementation.	Assumes that norms or agreed-upon ways of working are in place without discussion, ownership, or shared responsibility.
Data Feedback Systems	Includes key stakeholders (internal and lateral) in inquiry and reflective dialog for analysis and decision making focused on project goals and measureable outcomes; makes strategic adjustments based on data from feedback systems.	Uses inquiry and reflective dialog periodically for analysis and decision making about project goals and outcomes; but communication and data/feedback loops are often unidirectional or do not include or inform appropriate stakeholders.	Rarely uses systemic feedback to engage in inquiry and reflective dialog for analysis and decision-making about project goals and outcomes with partners.	Lacks inquiry and reflective dialog for analysis and decision making with partners; no data feedback system in place.
Professional Knowledge	Supports a culture of sustainable learning in the partnership and in the partners' organizational structures (e.g., PLCs) related to project purposes and measureable goals to build a collective professional knowledge base for improving and guiding the project.	Engages in work together but there is a need for a systemic approach to building a professional knowledge base through developing structures like PLC's or ongoing processes for developing shared knowledge relevant to project.	Focuses on management issues with limited opportunities for professional learning and knowledge building that supports project goals.	Lacks explicit systems or processes for building professional knowledge within the project.

School, District, and University Learning Partnership Participants Innovation Configuration Map (cont.)

STRUCTURES: School, District, and University Learning Partnerships develop and employ **structures** and processes that support sustained purposeful learning at all levels of the system that:

	Level 4	Level 3	Level 2	Level 1
District as Learning Partner	Utilizes structures and processes for systemic improvement, including (a) necessary resources, (b) effective communication and decision-making processes, (c) ongoing relevant professional development (e.g., PLCs), (d) specific measureable prioritized goals that are monitored at several levels of the system, and (e) timelines for assessment, data collection and analysis, and feedback mechanisms for adjustments to the learning system.	Moves toward systemic implementation, with needed resources, communication structures, professional development monitoring, timelines, and data feedback systems that are used periodically by partners for improvement of teaching and learning.	Provides sporadic opportunities to build a learning system focused on the goals and objectives; lacks ongoing professional development and collaborative communication structures, contributing to district fragmentation and uneven support for implementing the project.	Lacks explicit structures or agreements for purposeful collaboration based on shared goals; each school or classroom works in isolation; inconsistent messages, a lack of resources, and little professional support for realizing project goals.
University as Learning Partner	Engages honestly in the partnership through shared commitment to the project, expertise, respect, and credibility; builds viable learning structures and processes to improve individual and collective practices and project interdependence.	Develops partnerships through intermittent efforts to provide modest support but lacks consistency in commitment to the collective project and partnership.	Provides periodic support to the project and due to the unreliable nature of the efforts there is a lack of relational and professional trust in the partnership.	Lacks responsibility, commitment, expertise, or infrastructure to support the project; not a collaborative functioning partnership.
Shared Leadership	Engages in ongoing shared leadership and decision making that includes all members of the partnership; provides opportunities for equitable voice and shared negotiated responsibility and mutual accountability based on collective measureable goals.	Engages in leadership that is distributed unevenly throughout the partnership; some but not all partnership voices are used for input and decision making; not everyone has agreed to or operates from delineated project goals and action plan.	Engages in leadership that is primarily autocratic and does not utilize partnership voices or input for decision making; only some agree to project goals.	Uses hierarchical leadership that is top down or bottom up, with no ongoing relationships between the levels of the system to support a collaborative partnership.
Purposeful Collaboration	Clarifies the goals and actions, utilizes strengths, abilities, and motivation of individual, social, and structural elements in the defined partnership based on shared values collective capacity, and a culture for purposeful collaborative learning.	Provides frequent opportunities for collaboration by partners focused on project goals; provides support for common but limited participation, communication, and collaboration in sustaining the vision, goals, and commitments of the project.	Accepts inconsistency in shared values and commitment to the project goals; conducts periodic meetings that are not clearly connected to the larger shared project goals and implementation.	Lacks shared values and purposeful commitment to the partnership.

School, District, and University Learning Partnership Participants Innovation Configuration Map (cont.)

PROCESSES & PRACTICE: School, District, and University Learning Partnerships develop **processes and practice** that:

	Level 4	Level 3	Level 2	Level 1
Readiness for Change	Utilizes belief surveys, readiness for change and/or needs assessments to determine how to engage in the project strategically and build a competent partnership based on shared goals.	Makes use of readiness for change or needs assessments to determine the current "state" of the partners and decide on long and short term goals regarding the project and partnership.	Determines a need to do things differently but lacks data or commitment for collaborative change by both partners in the project.	Lacks understanding of the initial state of the partnership; stake-holders are not ready to collaborate.
Learning Cycle	Employs explicit processes and practices to develop data literacy by building skills and knowledge to use data to support continuous learning (i.e., a learning cycle that is understood by partners and is useful to support learning at all lev-els of the system).	Utilizes processes to build data literacy and develops the expert-ise to use data in a learning cycle but the process is inconsis-tent or has limited impact on learning at the appropriate level.	Provides data as singular events not connected to a learning cycle.	Lacks data to support ongoing learning.
Assessment	Uses an explicit formative assessment process with specific benchmark checkpoints to assess progress on partnership, project goals and expected outcomes.	Analyzes progress through periodic formative assessment checkpoints to assess partner-ship and project goals.	Uses formative assessments infrequently to measure progress toward partnership and project goals; lacks full participation by all members of the partnership.	Lacks ongoing processes or focused efforts to measure growth toward shared goals.

School, District, and University Learning Partnership Participants Innovation Configuration Map (cont.)

RESEARCH/TOOLS: School, District, and University Learning Partnerships have **Research/Tools** that:

	Level 4	Level 3	Level 2	Level 1
Research Practices	Builds capacity of school district to understand the role of research in a learning system and the university's role to make research practical and useful for educators by bridging research and the applications to practice to improve teaching and learning.	Embeds research for professional learning to connect research and practice (action research, formal research projects, study groups, etc.); research is valued as part of a learning system.	Conducts research activities in isolation that are limited in usefulness or relevance for improving learning.	Lacks connections between research and practice.
Collective Responsibility for the Partners	Demonstrates commit-ment to university/district project and research process through a clearly defined written agreement with shared accountability and collective responsibility for shared goals.	Uses clearly defined written agreements but lacks consistent commitment or accountability to the research partnership by all key participants.	Uses vague agreements with no strong commitment or shared accountability to the research partnership by key participants.	Lacks clearly defined written agreement or commitment to the research partnership.
Resources for Implementation	Includes dedicated resources and budget from both partners to support the needs of the partnership/project goals, including time, money, space, and personnel for the duration of the project.	Provides some resources from both partners that, over time, can support project implementation.	Includes minimal resources from one or more of the partners, which impacts implementation and project outcomes.	Lacks dedicated resources from either partner for enacting the project.

Preparing Teachers To Cultivate Parent-Child Collaboration In Mathematics

Regina M. Mistretta
St. John's University, Staten Island, New York

Introduction

The National Science Foundation, through its funding of curriculum projects such as *Everyday Mathematics* that include a family component, demonstrates its recognition of the significance of families in mathematics education. This is just one example of several curriculum projects and school initiatives that value collaboration with families. Such efforts respond to the National Council of Teachers of Mathematics' call to build family understanding of current school mathematics goals and instructional practices so that home and school may support each other (National Council of Teachers of Mathematics, 2000).

However, in-service professional development efforts for teachers and administrators focus primarily on understanding the curriculum's teaching and learning objectives (Ball, 1996; Nelson & Sassi, 2000). Less emphasis is placed on efforts to understand how families view these objectives and to develop ways to involve families meaningfully in their child's learning of mathematics (Remillard & Jackson, 2006).

This is true for pre-service teacher education as well. The formal preparation of educators to partner with families in any form is under-emphasized in teacher education programs (Shartrand et al., 1994; Hiatt-Michael, 2001; Witmer, 2005). This is despite research findings suggesting that productive collaboration with families has a positive impact on attitudes towards mathematics and mathematics achievement (Bezuk, Whitehurst-Pane, & Aydelotte, 2000l; Kliman, 1999).

Examinations of the nature of teacher collaboration with families have documented that many teachers do not have adequate knowledge and skills necessary for promoting family partnerships that support students' academic achievement (Ratcliff & Hunt, 2009). For example, some researchers found teacher collaboration with families consisted primarily of a "laundry list that good parents do" (Calabrese Barton et al., 2004; p.3). Other researchers found that teacher collaboration with families focused only on "how to" strategies for dealing with situations such as "difficult parents" or parents of children with learning disabilities Ferrara and Ferrara (2005).

To strengthen how teachers might productively collaborate with families, we designed a university-sponsored mathematics professional development program that would provide opportunities for teachers to investigate "parent-child collaboration" while working with family members in their own mathematics classrooms, with an eye to how what they learned from these investigations might inform their efforts to build strong collaborations with families around mathematics teaching and learning. The term "parent-child collaboration" in mathematics refers to the manner in which a parent and child work together on mathematical tasks such as daily homework and projects.

Assessment of this professional development program was conducted to determine its impact on teacher understanding of how and why parents and children work together the way they do in mathematics and the role of the teacher in nurturing productive parent-child collaboration.

Literature Review

The most basic premise of Vygotsky's theory (1978) is that a child's intellectual development is a produce of their social environment. Vygotsky points out that this social environment contributes to the cultivation of a child's higher order thinking skills when adults provide guidance within a child's zone of proximal development—a cognitive state in which the child cannot yet quite solve a problem by themselves and is responsive to social guidance. This social guidance is often referred to as "scaffolding."

A link between Vygotsky's view and family involvement in mathematics education exists. Researchers find that families, as a unit of the social environment, act as positive influences for attaining success in mathematics when they provide assistance that reflects a scaffolding approach (Connor & Cross, 2003; Vygotsky, 1978; Wood & Middleton, 1975). Family members using such an approach are attuned to the needs of the learner, guiding the learner within his or her zone of proximal development, and readjusting their assistance as the learner progresses to a new ability level. Guidance of this nature reflects what Hyde et al. (2006) term as "quality" assistance that is just as important, if not more, as the quantity of assistance.

However, many family members face the challenge of their lack of familiarity with the reform mathematics curriculum materials (Burns, 1998). These family members may struggle when trying to assist their child in a manner reflective of the scaffolding approach. They may also feel uncomfortable abandoning a drill and practice approach that worked well for them when they were in school (Epstein & Jansorn, 2004). Researchers warn that unfamiliarity and resistance can challenge reform efforts when family members choose to assist their child in ways that only mirror their past learning environment as opposed to that of their child's (Remillard & Jackson, 2006).

Given the difference family members can make in a child's mathematics performance, it is important for teachers to support and encourage collaborations in ways that address the challenges family members may feel when they seek to support their child's mathematics learning. This is particularly true for family members that come from different learning environments, have low levels of formal education, or are from low-income communities. Civil and Bernier (2006) highlight the need to move teachers away from a "deficit model" where family members are underutilized and devalued, to a mindset where family members are valued as "intellectual resources" regardless of their economic, cultural, and educational backgrounds.

Calabrese Barton et al. (2004) reflect this focus on engaging family members, regardless of their backgrounds, using their Ecologies of Engagement Framework where they define parental engagement as "a dynamic, interactive process in which parents draw on multiple experiences and resources to define their interactions with schools and among school actors" (p. 3). This framework represents a shift from focusing primarily on what family members do to engage in their children's education, to also learning about the "hows and whys" behind their actions. This shift enhances Epstein's (1987) theory of overlapping spheres of influence that identifies students as the main actors in their education, supported by others at home, at schools, and in their communities. When attention is given to Epstein's concept of multiple forms of support, with a lens reflective of the deep understanding advocated by Calabrese Barton et al. (2004), it is likely that productive collaborations that benefit students, strengthen families, and improve schools can be designed.

To determine how best to structure a learning environment for teachers that addressed these productive collaborations while also providing parents with insight into their child's learning of mathematics, an investigation of best practices for teacher education and family involvement initiatives was conducted. During that investigation, it was noted that Situated Cognitive Theory (Choi & Hannifin, 1995; Jonassen & Rohere-Murphy, 1999) suggests to teacher educators that new knowledge comes from implementing and observing actual school-based teaching. Darling-Hammond & McLaughlin (1995), Lee (2005), and Sawchuck (2009), as a result of their evaluations of teacher education and professional development programs, found that continued support from teacher educators, coupled with opportunities for teachers to share feedback with their colleagues, cultivates professional growth in a community of practice.

When reviewing the research on how best to support family engagement, findings favored efforts that focus on building parents' understanding of the changes in mathematics teaching (Sheldon & Epstein, 2001), especially the use of manipulatives as tools for learning (Mistretta, 2004; Orman, 1993; Dauber & Epstein, 1993; Epstein, 1986). In addition, parents were found much more knowledgeable about their children's learning of mathematics at the close of a series of activities where both parents and children engaged in mathematics tasks together (Tregaskis, 1991; Lachance, 2007; Fagan, 2008). These established learning conditions for parents as well as those described previously for teachers provided the foundation for the professional development program that was crafted and is discussed in this paper.

Methods and Procedures

PARTICIPANTS

An inner-city nonpublic school population of 147 pre-kindergarten through 8th grade students and their parents, along with their seven mathematics teachers agreed to participate in the professional development program. There was one teacher for both pre-kindergarten and kindergarten, one teacher for each of grades 1 through 5, and one teacher for grades 6 through 8. The 2nd grade teacher had 18 years of teaching experience, while the 1st grade teacher had three years, the pre-kindergarten/kindergarten teacher two years, and the others were first year teachers. Five teachers were state certified and two were working towards it. Four teachers were Caucasian, two were Hispanic, and one was Pacific Islander. In addition to receiving professional development credit, the teachers also received a stipend for their participation in the program.

The students' ethnic backgrounds consisted of 82% Hispanic, 14% Afro-American, 3% Caucasian, and 1% Asian. There were 75 male and 72 female students, and there was one class per grade level except for pre-kindergarten and kindergarten, which were merged due to the small number of students in each.

All families in the school participated in the professional development program and received incentives for their involvement; these incentives included home instructional materials, student dress-out-of-uniform passes, and free raffle tickets for prizes consisting of school supply store and supermarket gift cards. Dinner was also served prior to each of the family sessions. All families were fluent in English and were classified as low socioeconomic status, with approximately 81-90% of the children qualifying for free lunch.

PROFESSIONAL DEVELOPMENT PROGRAM

The professional development program consisted of four 2-hour teacher workshops and three 2-hour family sessions that took place over eight weeks during the first half of the school year. In these sessions, participating teachers engaged their own students and parents in mathematics tasks, gathered and analyzed data, and shared findings with their colleagues.

During the four 2-hour teacher workshops, teachers prepared to facilitate the family sessions, using the same mathematics tasks they would later use with family members during the family sessions. Teachers also learned how to collect data (surveys, field notes, work samples, and written reflections) during the family sessions and analyze these data. Finally, teachers discussed their findings at workshop sessions scheduled a week after each family session in order to create opportunities to share and discuss data on an ongoing basis with the other teachers participating in the project. Because the grades 6 through 8 group of family members was large, several additional teachers joined these teacher workshop sessions in order to be able to provide support to the grade 6 through 8 mathematics teachers facilitating the family sessions for those grade levels.

The three 2-hour family sessions were facilitated by these participating teachers in the evening with support from the project staff and the school principal. The sessions were designed to inform and engage family members, promote reflection, and build collaboration between parents and children with regard to mathematical learning. Because tangrams were being used by teachers during their mathematics instruction as a result of prior professional development at the school, and were familiar to both teachers and students, these materials were also a focus of the family sessions.* All teacher workshop and family session guidelines as well as related hand-out materials used throughout the professional development program can be found in Teachers Engaging Parents and Children in Mathematical Learning: An Approach for Nurturing Productive Collaboration (Mistretta, 2008a).

The family sessions were announced with an invitation to parents that included a request for information about times that would best suit their schedules. To personalize

* Tangrams consist of seven geometric shapes including two large triangles, one medium triangle, two small triangles, one square, and one parallelogram.

the invitation, students designed their own covers, and teachers then stapled the invitation inside each student's cover and sent them home. The sessions were then scheduled, taking parent time constraints into account, to the extent possible.

At the beginning of the first family session, teachers asked parents to complete a survey designed to help them better understand how and why parents and children work together at home. After administering the survey, the teachers outlined the agenda for each of the family sessions. Teachers then led a discussion on constructivist teaching practices that addressed how these instructional practices use a developmental approach, with individual learners actively building new knowledge as they interact with people and things in their environment (Cathcart et.al, 2006). The discussion then turned to the topic of using manipulatives, specifically tangrams, as a tool to support mathematical learning. Teachers presented the tangram set and talked with parents about how they would be participating in tangram activities with their children in ways that were similar to how their children were exploring tangrams in their classrooms. A question and answer session followed so that parents could ask questions and comment on the content of the session.

The second family session provided a concrete, active learning environment for participating families. Teachers distributed tangram sets and let parents and children know they were about to engage in activities involving spatial reasoning, computational skills, and problem solving. Time was provided for free exploration to foster parents' familiarity with the pieces. After eliciting information about the size and shape of the pieces, teachers posed the following questions concerning the relationships among the pieces:

- How does the small triangle compare with the medium triangle?
- How does the small triangle compare with the large triangle?
- How does the medium triangle compare with the large triangle?
- What tangram pieces can be joined together to form other tangram pieces?
- How many ways can you cover the large triangle with other tangram pieces?

Additional small triangles were distributed in case families wished to use them when exploring the relationship between the small and large triangle. Teachers circulated among their families to give assistance and observe interactions between the children and their parents.

Families then discussed as a whole group what they had discovered about their tangram sets. For example, the small triangle is half the size of the medium triangle, the small triangle is one-fourth the size of the large triangle, and the medium triangle is half the size of the large triangle. Other discoveries were that the two small triangles can form both a square and a parallelogram shedding light on the fact that both shapes have the same area because they both contain the same amount of space (the two same sized small triangles) but just in different representations. By covering the large triangle in different ways, families discovered how the large triangle can consist of: two medium triangles, two small triangles and the medium triangle, two small triangles and the square, or two small triangles and the parallelogram.

To initiate an activity that involved spatial reasoning and connected their discoveries with a computational task (Fuys & Tishler, 1979; ETA/Cuisenaire, 2007), teachers asked their families to arrange the seven tangram pieces into an outlined cat that was distributed to them. A monetary value was assigned to the smallest triangle of the tangram set and teachers posed the following questions according to grade level:

- Grades Pre-K to 2: How much does the cat cost if the smallest triangle costs 1¢?
- Grades 3 to 5: How much does the cat cost if the smallest triangle costs 20¢?
- Grades 6 to 8: How much does the cat cost if the smallest triangle costs $3.25?

Parents and children were instructed to use what they discovered about the relationships among the tangram pieces to arrive at their solutions. For the Pre-K to Grade 2 families, an outlined cat with the tangram shapes drawn inside was used, and an additional 14 triangles were distributed so the cat could be covered with 16 triangles, thus providing these children with the option of finding the cost of the cat by counting by ones rather than adding larger numbers or multiplying. Teachers advised parents not to do all of the telling, but rather, explore their children's mathematical thinking by asking prompting

FIGURE 1: *Grade 1 Work Sample*

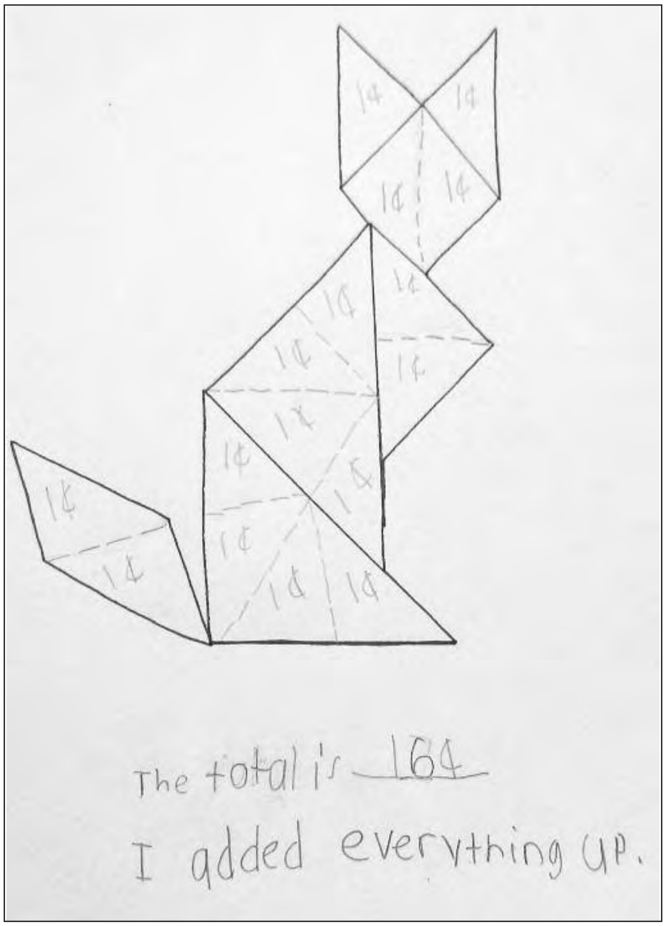
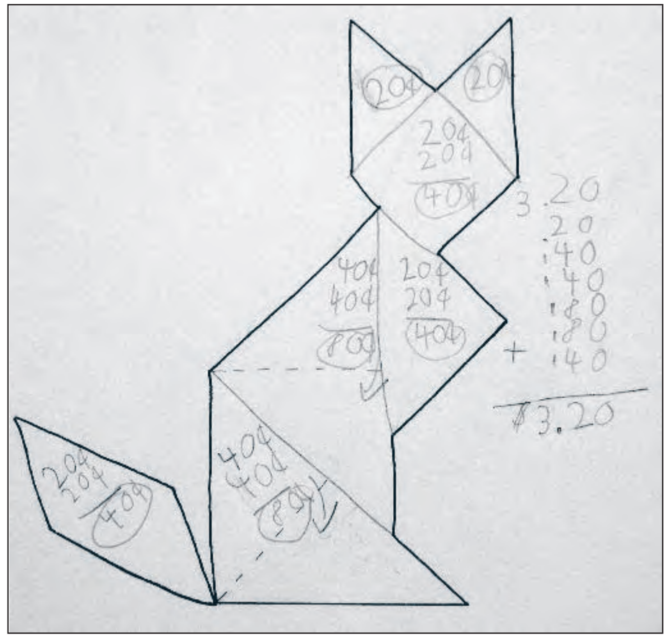


FIGURE 2: *Grade 3 Work Sample*



questions such “Where shall we begin?” “What do we know that can help us?” and probing questions such as “Can we approach this another way?” “Why?” and “How?”

The next step involved small and whole group reflection on this work (see Figures 1 & 2) and sharing ideas about how they obtained their solutions. Questions posed by the teachers included:

- What was your answer and how did you get it?
- Did you solve the problem in one or many ways?
- Did you and your child approach the task in the same way? If not, whose method did you use? Why? How did your methods compare?
- How did you help each other?
- What questions did you ask?

Teachers concluded this second family session by distributing paper tangrams and explaining a mathematics task for the families to do at home that extended the session’s experience. This mathematics task involved having families create their own tangram design and find its cost, given another assigned monetary value for the smallest triangle piece of the tangram set. Material designed to guide the parent explorations with their children was distributed and parents were asked to bring all completed work to the third family session.

The main goal of that third session was to share the family work on the tangram problem given at the end of the last session (see Figure 3) and reflect together as a community of learners. To initiate reflection, teachers invited families to talk in small groups about their work on the tangram problem, using the same questions posed during the second family session about the nature of their collaboration.

At the end of the session, parents and their children were asked to write a reflection about their experience collaborating, with parents or a teacher scripting the thoughts of any younger children whose writing skills were not yet developed. These reflections, along with the written materials that families brought to the session, were collected at the end of the session.

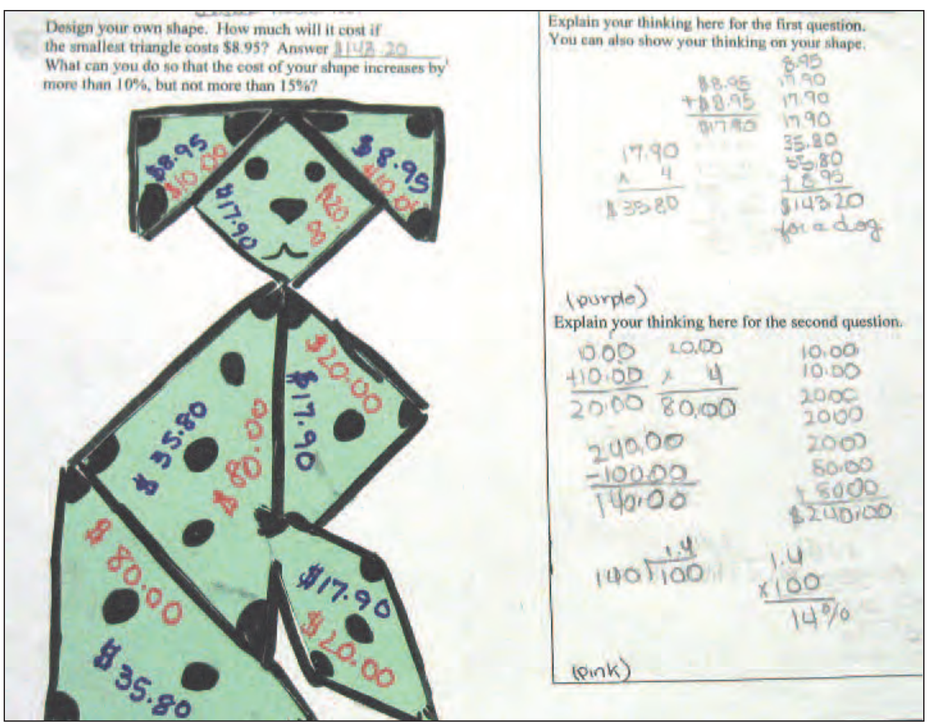
MEASURES

The ***parent survey*** (Mistretta, 2008b) consisting of 14 statements requiring 5-point Likert scale responses and one narrative response question served to investigate how the parents collaborated at home with their children in mathematics and the challenges they faced. ***Work samples*** of assigned tasks done at home were graded with a 4-point rubric to assess the quality of work completed by the families. ***Written reflections*** served to identify student and parent feedback concerning the most enjoyable and challenging aspects of their collaboration.

To keep a written record of their observations of parental assistance, the questions posed to children by parents, and the verbal communication among the families, teachers took ***field notes*** during small and whole group discussions and while observing the parents and children working on their tasks.

A ***journal*** consisting of four entries was kept by each teacher throughout the professional development program. This was used to assess and monitor initial perspectives of teachers about parental involvement, reactions to the findings from collected data, and their developing perspectives about parental involvement and their role in cultivating it. More specifically, the first entry required the teachers to write about their perspectives concerning parents' interest in and commitment to collaborating with their child, their willingness as a teacher to include parents in their children's mathematical learning, and any practices they currently implemented to include parents in their child's learning of mathematics. For the second journal entry, the teachers used their findings from the parent survey to describe what they had learned about what parents do most and least concerning their child's mathematical learning and any challenges parents face helping their child with mathematics. The third journal entry focused on using the field notes collected during the second family session to focus on the following: Describe the interaction you observed between the parents and children as they worked on tasks together; Describe the responses you heard during the discussions as families worked on tasks together; and Have these observation or responses informed your understanding of parent-child collaboration and your related instructional practices? If so, how? The fourth journal entry centered on the third family session and required the teachers to use their collected work samples, field notes collected during this session, and written reflections at the end of the session to respond to the following questions: What scores did most families achieve on the home mathematics task? Have the work samples (solutions and solution strategies) informed your understanding of parent-child collaboration and related instructional practices? If so, how? Describe the responses you heard during the discussions. Describe the enjoyable and challenging aspects stated in the written reflections. Has any particular solution, method of solution, or responses during discussions informed your understanding of parent-child collaboration and related instructional practices? If so, how?

FIGURE 3: Grade 7 *Home Activity*



Four ***group interviews*** with teachers were conducted and audio-recorded during each teacher workshop using questions that reflected those of the journal entries. Notes were transcribed afterwards and compared with each teacher's corresponding journal entries to assess consistency between their journal entries and interview responses as well as clarify any unclear responses in either the journal or interview.

Data Analysis

Each teacher analyzed the data concerning their classroom families. They tallied the parent survey Likert-scale responses and scores from their family work samples. They conducted content analyses on their parent survey narrative response question, field notes, and written reflections. Survey narrative responses were coded and tallied to determine emerging themes. Field notes and written reflections were coded and tallied to note trends in both the observed parent-child interaction and the written reflections from parent and their children. These data were also analyzed by project staff to ensure consistency of findings. In addition, teachers' journal entries and transcribed notes from small group interviews with teachers were coded and tallied by project staff to determine and compare emerging themes.

Discussion of Findings

After analyzing the teachers' first and second journal entries and related interview responses, it was clear that each teacher noted that at the onset of this project there seemed to be limited discussion between parents and children about how answers to mathematics problems are obtained. Teachers each indicated that most parents involved themselves in only checking that homework was done and reviewing for upcoming tests. Such limited parental involvement may have been a consequence of the teachers themselves unintentionally limiting parent involvement.

For example, all of the teachers initially acknowledged the value of involving parents, yet expressed a lack of confidence in the mathematics content knowledge of their parents. As a result, they each indicated their decision to give parents tasks they felt they could do—checking homework, reviewing for tests, and drilling multiplication tables. In addition, communication with parents about the mathematical learning going on in their classrooms consisted only of written letters focusing on classroom procedures such as when homework is given and how grades are calculated.

On a more positive note, the teachers' desire to learn how to more effectively involve parents in their child's learning of mathematics was evident. They all admitted they underutilized parents because they viewed their parents as not having the educational background to help their child, and didn't know how to alleviate the situation, but wanted to know how to involve parents more productively. This admission of and willingness to move away from the "deficit model" of parents previously described by Civil & Bernier (2006) provided an opportunity to develop new understandings of what it might mean to engage parents in the mathematics learning of their children.

When analyzing the narrative responses to the parent survey question, the teachers noted that parents referenced their lack of content knowledge and differing prior learning environments as reasons for their limited mathematical discussion with their children at home. The teachers each noted that the majority of their parents made comments such as "Mathematics today is taught differently than in my time. I don't want to confuse my child."

The teachers, in their third journal entry and related interview responses, each noted the benefits of engaging families in mathematics tasks in their own classrooms. They each viewed this setting as a means for building parents' content knowledge and understanding of "why we teach the way we do." They each acknowledged, as well, the usefulness of their recent opportunities to observe the forms of interaction between parents and their children and, at times, offer appropriate guidance. For example, one fourth grade teacher stated the following:

> "I see the need for me to help my parents realize it is essential to talk about math problems with their child even though they themselves may have struggled as a math learner. I have to encourage my parents to try and understand how their child arrives at their answers even though they themselves may approach the problem differently. I need to guide them to better understand how their child thinks so that they can productively help them."

These words merit recognition since they surface a realization of the need for teachers to support parents' efforts to better communicate with their children. A more focused lens on the specific words "I have to encourage my parents to try and understand how their child arrives at their answers even though they themselves may approach the problem differently," suggests the need for teacher educators to better prepare teachers to encourage connections between a variety of methods of solution. For example, when teachers facilitate communication within families about how differing methods compare and contrast, the approaches of both child and parent are recognized and valued, as opposed to one approach being viewed as inferior to the other. This type of communication among parents and children not only builds appreciation for diversity in methods of solution, but also supports the scaffolding approach (Vygotsky, 1978; Wood & Middleton, 1975) by linking multiple approaches to build deeper meaning for the child.

An encouraging moment for the teachers was when they each noted their parents transitioning from a role of telling to one of listening and guiding. Most of their parents initially took control of conversations in an explanatory manner, using only one method of solution (theirs), and posing short answer questions requiring a yes/no or a number response. As the family sessions progressed, each teacher witnessed parents' receptiveness to their guidance and suggestions, and detected more meaningful collaboration starting to occur. For instance, parents began to question more and tell less by posing the prompting and probing questions offered by the teachers.

The teachers also noted how much their parents' enjoyed talking with their children about their thinking and conversing with other families about their successes and challenges. The children were also observed by each teacher as very willing to explain their thinking to their parents. For example, one first grade teacher stated the following:

> "The parents found the activities fun and wished they used manipulatives for math when they were in school. The parents had the opportunity to see how their child can reason and think. In the beginning, the parents were just giving the children the answers. When I told them the children had to explain how they arrived at their answer, they really listened and started to ask guiding questions instead of telling. This was a valuable experience for me because I saw how the families genuinely want to help their children with math and can if I properly guide them."

Experiencing this receptiveness, on the part of both parents and children, allowed the teachers to see the value in playing the role of catalyst for productive family collaboration.

The teachers' fourth journal entry and related group interview indicated that each teacher found most of their family scores on the home mathematics tasks to be 3 or 4, indicating accurate solutions and methods of solution. The following reflection from a second grade teacher indicates a realization that productive family collaboration on mathematics tasks can indeed be a productive undertaking:

> "The parents really wanted to be involved in the learning process and did great work at home with their child. I learned through conversations with them that they work late and by the time they arrive home they don't have enough time to go over the homework in a way they would like to. I therefore need to involve them in a project like we just gave them to do at home with enough time and guidance to do it."

Through the analyses of these home mathematics tasks, teachers realized that quality collaboration on mathematics tasks *can* happen at home when the proper support is given.

Each teacher, after analyzing the parent and child written reflections about the collaborative experiences, noted that most children appreciated the opportunity to share their own methods of solution that were often different from their parents' way of answering. For example, a fourth grade student made the following comment on the collaboration with her mother: "This was great; now she listens to what I think." The teachers also noted that most of their children said they felt challenged when their parents asked them how or why they arrived at their answers, but this was helpful to them. For example, a fifth grader reported, "It was hard to say what I was thinking. It kind of hurt my head. But it did help me sort things out."

The teachers noted, as well, that most parents expressed an appreciation for being able to witness their child thinking, and viewed the time collaborating as an opportunity to build better understandings of each other. At the same time, most parents viewed listening and guiding their child's mathematical thinking as a challenge because they were used to telling their children the answers and how to obtain them. A parent's comment reflecting this point was "I'm starting to catch myself. I listen more now before jumping in. It's not easy though, but I'm getting there."

Conclusions

This professional development program, designed to support family engagement in mathematics, provided an opportunity for teachers to build parents' knowledge of mathematics content and pedagogy. But even more, it provided tasks and venues for the teachers to note the ways families collaborated and the reasons behind their actions. Teachers were able to witness the willingness of parents to collaborate with their children on mathematics tasks and activities. They were also able to witness the extent to which children enjoyed sharing their mathematical thinking with their parents. As a result, the participating teachers gained a deeper sense of value for the role of parents in the mathematics learning of their children, and recognized the role they might play in supporting this parent-child collaboration. Teacher preparation as described in this paper warrants consideration. Civil & Bernier (2006) state that teachers can influence the success or failure of efforts that seek to change the ways parents participate in their child's education. If parents need to productively involve themselves in their child's learning, teacher educators need to focus their attention on preparing teachers with the knowledge and skills necessary to cultivate such involvement.

References

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan*, 77, 500–508.

Bezuk, N.S., Whitehurst-Payne, S., & Aydelotte, J. (2000). Successful collaborations with parents to promote equity in mathematics. In W.G. Secada (Ed.), *Changing the faces of mathematics.* Reston, VA: NCTM, 143-148.

Burns, M. (1998). *Math Facing an American Phobia.* Sausalito, CA: Math Solutions Publications.

Calabrese Barton, A., Drake, C., Perez, J. G., St. Louis, K.; & George, M. (2004). Ecologies of parental engagement in urban education. *Educational Researcher*, 33(4), 3-12.

Cathcart, W., Pothier, Y., Vance, J., & Bezuk, N. (2006). *Learning mathematics in elementary and middle schools.* Upper Saddle River, NJ: Pearson Education, Inc.

Choi, J. I. & Hannifin, M. (1995). Situated cognition and learning environments: Roles, structures, and implications for design. *Educational Technology, Research and Development*, 43(2), 53-69.

Civil, M., & Bernier, E. (2006). Exploring images of parental participation in mathematics education: Challenges and possibilities. *Mathematical Thinking and Learning*, 8(3), 309-330.

Connor, D. B., & Cross, D. R. (2003). Longitudinal analysis of the presence, efficacy, and stability of maternal scaffolding during informal problem-solving interactions. *British Journal of Developmental Psychology*, 21, 315-334.

Darling-Hammond, L. & McLaughlin, M. W. (1995). *Transforming professional development for teachers: A guide for state policymakers* (Washington, DC, National Governors' Association) (ERIC Document Reproduction Service No. ED 384 600).

Dauber, S.L., & Epstein, J. L. (1993). Parents' attitudes and practices of involvement in inner-city elementary and middle schools. In N. Chavkin (Ed.), Families and schools in a pluralistic society (pp. 53-71). Albany, NY: SUNY Press.

Epstein, J. L. (1987). Toward a theory of family-school connections: Teacher practices and parental involvement. In Klaus Hurrelmann, Frederick Kaufmann, & Frederick Losel, eds., *Social Intervention: Potential and Constraints.* New York: DeGruyter.

Epstein, J. L. (1986). Parents' reactions to teacher practices of parent involvement. *The Elementary School Journal*, 86(3), 277-293.

Epstein, J.L. & Jansorn, N. (2004). School, family, and community partnerships link the plan. *The Education Digest*, 69(6), 19-23.

ETA/Cuisenaire (2007). Tangrams make cents. In *The super source*. Vernon Hills, IL: ETA/Cuisenaire.

Fagan, N. (2008). Identifying opportunities to connect parents, students, and mathematics. *Teaching Children Mathematics*, 15(1), 6-9.

Ferrara, M. & Ferrara, P. (2005). Parents as partners: Raising awareness as a teacher preparation program. *The Clearing House*, 79(2), 77-82.

Fuys, D. & Tishler, R. *Teaching mathematics in the elementary school.* New York: Harper Collins, 1979.

Hiatt-Michael, D. (2001). *Preparing teachers to work with parents.* (ERIC Digest. ED 460123).

Hyde, J. S., Else-Quest, N. M., Alibali, M. W., Knuth, E., & Romberg, T. (2006). Mathematics in the home: Homework practices and mother-child interactions doing mathematics. *Journal of Mathematical Behavior*, 25, 136-152.

Jonassen, D.H. & Rohrer-Murphy, L. (1999). Activity theory as a framework for designing constructivist learning environments. *Educational Technology Research and Development*, 47(1), 61-79.

Kliman, M. (1999). Beyond helping with homework: Parents and children doing mathematics at home. *Teaching Children Mathematics*, 6(3), 140-46.

Lachance, A. (2007). Family math nights: Collaborative celebrations of mathematical learning. *Teaching Children Mathematics*, 13(8), 404-408.

Lee, H. (2005). Developing a professional development program model based on teachers' needs. *The Professional Educator*, 27(1&2), 39-49.

Mistretta, R.M. (2008b). Cultivating parent-child collaboration concerning mathematical learning: A necessary objective for teacher preparation programs. In C.J. Craig & L.F. Deretchin (Eds.), *Teacher Education Yearbook XVI: Imagining a Renaissance in Teacher Education* (pp. 348-362). Lanham, MD: Rowman & Littlefield Education.

Mistretta, R.M. (2008a). *Teachers engaging parents and children in mathematical learning: An approach for nurturing productive collaboration.* Lanham, Maryland: Rowman & Littlefield Education.

Mistretta, R.M. (2004). Parental issues and perspectives concerning mathematics education at elementary and middle school settings. *Action in Teacher Education*, 26(2), 69-76.

National Council of Teachers of Mathematics. *Principles and standards for school mathematics.* Reston, VA: NCTM, 2000.

Nelson, B. S.,& Sassi, A. (2000). Shifting approaches to supervision: The case of mathematics supervision. *Educational Administration Quarterly*, 36, 553–584.

Orman, S. (1993). Mathematics backpacks: Making the home-school connection. *Arithmetic Teacher*, 40(6), 306-309.

Ratcliff, N., & Hunt, G. (2009). Building teacher-family partnerships: The role of teacher preparation programs. *Education*, 129(3), 495-506.

Remillard, J. T., & Jackson, K. (2006). Old math, new math: Parents' experiences with standards-based reform. *Mathematical Thinking and Learning*, 8, 231-259.

Sawchuck, S. (2009). Teachers' staff training deemed fragmented. Education Week, 28(21). Retrieved June 9, 2009, from http://www.edweek.org//ew/articles/2009/02/04/21development.h28.html?tkn=MXSFFr5J5

Shartrand, A., Kreider, H., & Erickson-Warfield, M. (1994). *Preparing teachers to involve parents: A national survey of teacher education programs.* Cambridge, MA: Harvard Family Research Project.

Sheldon, S. & Epstein, J.L. (2001). Focus on math achievement: Effects of family and community involvement. Paper presented at the 2001 annual meeting of the American Sociological Association, Anaheim, CA.

Tregaskis, O. (1991). Parents and mathematical games. *Arithmetic Teacher*, 38(7), 14-17.

University of Chicago School Mathematics Project. (2001). *Everyday mathematics* (2nd ed.). Chicago: SRA/McGraw-Hill.

Vygotsky, L. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Witmer, M. (2005). The fourth *r* in education-relationships. *The Clearing House*, 78(5), 224-8.

Wood, D., & Middleton, D. (1975). A study of assisted problem-solving. *British Journal of Psychology*, 66, 181–191.

JOURNAL OF MATHEMATICS EDUCATION LEADERSHIP

Information for Reviewers*

1. Manuscripts should be consistent with NCSM mission.

   The National Council of Supervisors of Mathematics (NCSM) is a mathematics leadership organization for educational leaders that provides professional learning opportunities necessary to support and sustain improved student achievement.

2. Manuscripts should be consistent with the purpose of the journal.

   The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

   - Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education;
   - Fostering inquiry into key challenges of mathematics education leadership;
   - Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice; and
   - Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

3. Manuscripts should fit the categories defining the design of the journal.

   - Key topics in leadership and leadership development
   - Case studies of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
   - Reflections on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
   - Research reports with implications for mathematics education leaders
   - Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
   - Brief commentaries on critical issues in mathematics education
   - Brief reviews of books that would be of interest to mathematics education leaders

4. Manuscripts should be consistent with the NCTM Principles and Standards and should be relevant to NCSM members. In particular, manuscripts should make clear to mathematics leaders the implications of its content for their leadership practice.

5. Manuscripts are reviewed by at least two volunteer reviewers and a member of the editorial panel. Reviewers are chosen on the basis of the expertise related to the content of the manuscript and are asked to evaluate the merits of the manuscripts according to the guidelines listed above in order to make one of the following recommendations:

   a. Ready to publish with either no changes or minor editing changes.

   b. Consider publishing with recommended revisions.

   c. Do not consider publishing.

6. Reviewers are expected to prepare a written analysis and commentary regarding the specific strengths and limitations of the manuscript and its content. The review should be aligned with the recommendation made to the editor with regard to publication and should be written with the understanding that it will be used to provide the author(s) of the manuscript with feedback. The more explicit, detailed, and constructive a reviewer's comments, the more helpful the review will be to both the editor and the author(s).

---

* Please contact the journal editor if you are interested in becoming a reviewer for the *Journal*.

LEADERSHIP IN MATHEMATICS EDUCATION
NCSM NETWORK COMMUNICATE SUPPORT MOTIVATE

Membership Application/Order Form

Use this form to renew a membership, join NCSM, update information, or order items. Complete this form and return with payment. The information you provide will be used by the NCSM office for member communication, mailing lists, and the NCSM Membership Directory. Membership Dues are currently $85.

NCSM sometimes provides its mailing list to outside companies. These companies have been approved by NCSM to send catalogs, publications, announcements, ads, gifts, etc. Check here to remove your name from mailing lists. In addition, by checking this box, only your name without contact information will be included in the NCSM Directory. ☐

PLEASE PRINT LEGIBLY OR TYPE

First Name_______________________________Middle ________

Last Name ____

This is my complete address: ☐ Home ☐ Work

Address____

____

City__________________State_____Zip_______Country_______

Employer ____

Title ____

Telephone (______________)__________________________

Fax (______________)_______________________________

Email ____

Please check all that apply. I currently work as: (Optional)

☐ State/Provincial Department of Education Employee
☐ Government Agency (NSF, DOE, etc.)
☐ Member of Local Board of Education
☐ Superintendent
☐ District Mathematics Supervisor/Leader
☐ Principal
☐ Department Chair
☐ Grade-Level Leader
☐ Teacher Leader
☐ Author
☐ Coach/Mentor
☐ Consultant
☐ Curriculum Leader/Specialist
☐ Education Technology Provider
☐ Pre-Service Educator
☐ Professional Developer
☐ Publisher
☐ Teacher
☐ Other__________________________

Please check all that apply. I am a leader in mathematics education at the following levels:

☐ National
☐ Regional (more than one state/province)
☐ State/Province
☐ District/County/City
☐ Building
☐ University/College
☐ Senior High School
☐ Junior High/Middle School
☐ Elementary School
☐ Pre-Kindergarten
☐ Other__________________________

Since designations vary over time, check the one you feel best describes you:

☐ African American/Black
☐ Native American
☐ Asian American
☐ Pacific Islander
☐ European American/White
☐ Bi-Racial/Multi-Racial
☐ Mexican American/Hispanic/Latino
☐ Other__________________

Check the area you serve:
☐ Rural ☐ Suburban ☐ Urban

Do you influence purchasing decisions?
☐ Yes ☐ No

Age: ☐ under 25 ☐ 25 - 34 ☐ 35 - 44 ☐ 45 - 54 ☐ 55-64 ☐ over 64

Work Experience:
☐ First year in position
☐ 2-5 years in position
☐ 8-10 years in position
☐ 11-20 years in position
☐ 21-30 years in position
☐ over 30 years in position
☐ retired

Please check all that apply. Which of the following characterize the community you serve?
☐ High percent poverty
☐ High percent of English language learners
☐ Racial and ethnic diversity
☐ None of the above

Qty.	Item*	Member	Non-Member	P&H**	Sub-Total
____	**Monograph: Future Basics: Developing Numerical Power**	$15	N/A	N/A	_______
	PRIME Leadership Framework				
____	1-4 copies (each)	$16	$18	$4.95	_______
____	5-9 copies (each)	$15	$17	$10.70	_______
____	10-15 copies (each)	$14	$16	$14.50	_______
____	16-99 copies (each)	$13	$15	**	_______
____	100 or more (each)	$12	$14	**	_______
____	NCSM Member Pin	$2			$_______
			Merchandise Total:		$_______
	Membership Dues	**$85**			$_______
			Total order:		$_______

*Availability of products and prices are subject to change without notice.
Postage/Handling: Books are sent by USPS. For orders of 16 or more copies, contact NCSM Member & Conference Services for a postage and handling price. Outside the U.S. or for expedited orders, call for shipping prices.

Please return this form to:
NCSM Member and Conference Services
6000 E. Evans Avenue 3-205, Denver, CO 80222
Phone: 303.758.9611; Fax: 303.758.9616
Email: office@ncsmonline.org Web: mathedleadership.org

Payment Method: ☐ Visa ☐ MasterCard ☐ Discover Card ☐ Check/M.O. *(U.S Funds only)* ☐ P.O.**

Purchase Order # ______________________________________

Credit Card #____

Cardholder Name______________________________ Exp ____/____

Cardholder Signature ___________________________________

Purchase orders will be accepted for PRIME orders only. A purchase order number must be included. ***Please note:* an invoice will NOT be sent. Should you need an invoice, please use this order form.

Emeritus Membership: Please check the NCSM website (mathedleadership.org) for eligibility requirements.

National Council of Supervisors of Mathematics
6000 E. Evans Avenue
Denver, CO 80222-5423

Presorted Standard
U.S. Postage
PAID
Brockton, MA
Permit No. 301

LEADERSHIP IN MATHEMATICS EDUCATION

NCSM

NETWORK
COMMUNICATE
SUPPORT
MOTIVATE