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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Brief commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We want to hear about your reactions, questions, and connections you are finding to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Linda Ruiz Davenport, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM)

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***Note:** Information for manuscript reviewers can be found on the inside back cover of this publication.

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editor

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In reflecting on the contents of this issue, I found myself drawn to NCSM's recently released book: *It's TIME: Themes and Imperatives for Mathematics Education* (2014). In this document, NCSM "provides clear, research-based guidance on how to raise achievement in mathematics for every student and effectively implement the CCSSM in every classroom" (p. 1). Within their framework, the writers of this document describe three imperatives for systemic change: professional learning, collaborative structures, and coaching. It is in light of these imperatives that one should view the contents of this issue.

Professional Learning

> Raising achievement in mathematics for *every* student and effectively implementing the CCSSM in *every* classroom requires *extensive and ongoing opportunities for teachers to enhance their own professional learning and to build their capacity to reach all students.* (NCSM, 2014, p. 44)

In supporting professional learning, *It's TIME* outlines several imperatives to be met by mathematics education leaders. Within these imperatives, one finds references to assessing teachers' knowledge and skills so as to provide professional learning to meet individual needs. This issue of the journal contains two articles that provide assessment strategies for this purpose, both aimed specifically at identifying teachers' understandings of the Standards for Mathematical Practice (SMP).

In the first article, Bostic and Matney describe a performance assessment used during professional development and focused on the Common Core. Referred to as Unpacking the SMPs, this performance task requires teachers to role-play selected SMPs. Bostic and Matney describe their use of the assessment along with the insights gained regarding their teachers' understanding of the SMPs. In the second article, Olson, Olson, and Capen surveyed teachers across two professional development projects. By asking teachers to describe what *caught their eye* within a standard and how they might be intentional about a standard, the analysis revealed teachers' views regarding how the SMPs might influence their instructional practices. The authors recognized the teacher-oriented perspective that resonated within the teachers' responses and provide implications for professional learning regarding the SMPs. Common to both articles is the practice of identifying teachers' current understandings as a means for identifying their professional learning needs.

In addition to assessing teachers' knowledge and skills, the professional learning imperative includes discussion of the need to evaluate professional development in order to determine "if and in what ways the professional learning was successful" (NCSM, 2014, p. 45). To aid us in thinking about this, Boston and Steele describe the use of student work along with a set of rubrics for monitoring the success of professional development initiatives. In their article, they share examples of their work taken from two professional development projects and describe how their rubrics were helpful in monitoring and informing the process of instructional change.

Collaborative Structures

> Raising achievement in mathematics for *every* student and effectively implementing the CCSSM in *every* classroom requires *robust, well-functioning collaborative structures, including administrative teams, academic leader teams, and grade-level or course-specific teams.* (NCSM, 2014, p. 47)

The *It's TIME* document outlines the need to establish a professional culture in which teams of teachers work together with a goal of raising student achievement. Although achieving this goal requires a variety of foci, cultivating a professional culture and developing a shared vision of teaching and learning are clearly two key aspects of the work. To aid us in thinking about how to begin this process, Albert, Terrell, and Macadino describe how they supported a group of mathematics teachers in developing a professional learning community (PLC). Included in the description is a sample of the PLC's work with monitoring student progress on assessments. In addition, the authors highlight their efforts to develop a shared vision of mathematics instruction among teachers and administrators.

Coaching

> Raising achievement in mathematics for *every* student and effectively implementing the CCSSM in *every* classroom requires that *knowledgeable and trained coaches support instruction improvement and professional collaboration in every school.* (NCSM, 2014, p. 52)

To meet the goals set in this imperative, *It's TIME* (NCSM, 2014) describes the need for mathematics education leaders to provide mathematics coaches with professional development that, among other things, supports the coaches in "apply[ing] the tenets of skilled coaching" (p. 53). To this end, Yopp, Barlow, Sutton, and Burroughs provide insights into these tenets of skilled coaching. Specifically, they asked practicing coaches and coaching experts to assess the work of a novice coach depicted in video of a coaching session. Through their analysis of the responses, the authors provide guidance regarding professional development for coaches.

Although these imperatives for systemic change represent one small portion of the leadership framework in *It's TIME* (NCSM, 2014), the work embedded within them is significant. I hope that the articles in this issue support you in thinking about how to address these imperatives within your district – it's time. ✪

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Role-Playing the Standards for Mathematical Practice: A Professional Development Tool

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Abstract

This article describes a performance assessment to use during Common Core-focused professional development and shares insights from research using this assessment regarding about teachers' comprehension of the Standards for Mathematical Practice (SMPs). We asked 46 teachers from grades K-10 to read and make sense of the SMPs and then role-play a classroom scenario indicative of one SMP. This performance task is called Unpacking the SMPs. *Teachers' interactions during the role-play activity were intended to help them interpret the SMPs. From this role-play activity, PD providers were more aware of teachers' initial comprehension of the SMPs.*

Mathematics instruction in the era of Common Core State Standards for Mathematics (CCSSM) will require educators to reevaluate their current instruction (National Council of Teachers of Mathematics [NCTM], 2010). A critical element of the CCSSM is the overarching emphasis given to the Standards for Mathematical Practice (SMPs). The SMPs offer descriptions of mathematical habits and behaviors that students should demonstrate while learning mathematics (Common Core State Standards Initiative [CCSSI], 2010), which are listed in Table 1. The habits and behaviors in the SMPs are just as important as the Standards for Mathematical Content, and are central to developing students as mathematically proficient learners.

Prior research has explored whether and to what degree mathematical habits and behaviors like those found in the SMPs were happening in the classroom. For instance, video analysis of mathematics instruction indicated that generally speaking, teachers were not promoting habits and behaviors like those described in the SMPs (Hiebert

Table 1: Standards for Mathematical Practice

Standard for Mathematical Practice #	Title
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for regularity in repeated reasoning.

Note: Discussion about a specific SMP is denoted as SMP # within the manuscript.

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et al., 2005). An implication of this finding is that teachers need opportunities to learn about the SMPs so that they might enact instruction that supports them. Professional Development (PD) providers should design PD that assists mathematics teachers' understandings of the habits and behaviors found in the SMPs and how they can be promoted through their instruction. Fortunately, teachers want SMP-focused PD during this transitional era from state-level standards to the CCSSM (Bostic & Matney, 2013). Before doing any SMP-focused PD, however, it is prudent to assess teachers' prior knowledge about the new standards so that the PD best suits their needs. Hence we designed a performance assessment to make sense of teachers' ideas of the SMPs in order to better focus the design of future PD sessions to meet the needs of those teachers.

Performance assessments are just like any other measure: an assessment of and for learning that provides an indicator of an individual's knowledge (Wiliam, 2007). One type of performance assessment is role-play. Role-play offers a window into an individual or group's beliefs, thoughts, and actions (Van Ments, 1999; Yardley-Matwiejczuk, 1997). It is a performance assessment "in which participants 'take on' or 'act out' specific 'roles' often within a predefined social framework or situational blueprint" (Crookwell, Oxford, & Sanders, 1987, p. 155). If utilized correctly during PD, role-play is a focused and creative enactment of teaching and learning experiences. It places teachers in situations that have the same constraints and pressures that exist in their classrooms (Van Ments, 1999). Role-play has been used previously as a teaching and assessment tool regarding social and affective issues (Jones, 2007; Van Ments, 1999). Our dual purposes in this article are (a) to describe a performance assessment (i.e., role-play) that allows PD providers an opportunity to make sense of teachers' comprehension of the SMPs and (b) to share what we learned about K-10 mathematics teachers' comprehension of the SMPs as a result of the assessment.

Overview of the PD

Broadly speaking, the aims for the PD included: making sense of the SMPs; exploring inquiry through worthwhile tasks, mathematical discourse, and appropriate learning environments; and implementing classroom-based tasks that aligned with the SMPs and the Standards for Mathematical Content. Each author was a project director for one PD program and co-director on the other. One program supported elementary (i.e., K-5) teachers and the other focused on middle and high school teachers (i.e., grades 5-10). Prior to the PD, none of the teachers indicated that they had read or reflected on the implications of the SMP for their classroom instruction.

There were 46 teachers from a Midwest state who participated in the PD. One project served 23 grades K-5 mathematics teachers while the other supported 23 grades 5-10 mathematics teachers. The K-5 and 5-10 participants met separately due to geographic constraints. Demographic data for the participants are shown in Table 2.

Table 1: Standards for Mathematical Practice

Demographic information	K-5 cohort	Grades 5-10 cohort
Mean (SD) years of teaching experience	11.89 (6.27)	12.97 (5.08)
Number of female teachers	20	12
Number of male teachers	2	10

Note: Demographic information was not available for two participants.

Participants came from urban, suburban, and rural school districts in the Midwest and ranged in classroom experience from one to 26 years. On average, participants across both cohorts had approximately 12 years of teaching experience. Both projects included participants from at least one district with more than 20% of students from families below the poverty line.

Participants met four times for four-and-a-half hour sessions during spring 2012. Our focus in this paper is on the role-play used during the spring sessions of the respective PD meetings. The role-playing activity was named *Unpacking the SMPs.*

Unpacking the SMPs

Groups consisting of two to four participants were assembled. Elementary participants were arranged into groups of grades K-2 (i.e., primary) and 3-5 (i.e., intermediate) elementary teachers. Middle level and secondary participants were organized into groups of grades 5-7 (i.e., middle school) and grades 8-10 (i.e., high school) teachers. The grades 8-10 formation was made because many eighth-grade teachers also taught Algebra and/or Geometry. Each group created three role-plays, one for each of three assigned

SMPs: (a) SMP 1 or 6; (b) SMP 2, 3, or 4; (c) SMP 5, 7, or 8. We were careful to assign each SMP to one group within each grade band.

For each assigned SMP, participants carefully and closely read the paragraph descriptions of the SMPs. Following this, each group described the SMP in a manner that the following people might understand: a child in their respective grade levels, a parent or administrator, and a fellow teacher of mathematics. These descriptions were helpful in formatively assessing how participants made sense of their assigned SMPs and intended to share their ideas to various audiences.

After creating the descriptions, participants were expected to role-play three classroom scenarios, each role-play depicting an assigned SMP. Those not participating in the role-play (onlookers) were expected to look for evidence related to that specific SMP. Onlookers did not usually participate in the role-play except on one occasion. Groups were encouraged to behave as the teacher and students or role-play a scenario with only students. Participants were encouraged to choose their mathematical focus, problem, and context, drawing on their typical mathematics instructional experiences to portray their assigned SMP. For instance, participants could retrieve tasks from websites, computers, or textbooks, and structure themselves to show whole-class, small-group, independent work, or some combination of these formats. There was a ten-minute limit placed on role-plays; however, participants could provide interludes between sequences (e.g., role-play an introductory task, provide some narration, and then segue to a focal problem). Initially, groups were given 40 minutes to prepare their first two role-plays. As participants gained experience creating role-plays, they decided that less time was sufficient. Designing and preparing role-plays took approximately two hours.

Finally, a time of discussion, feedback, and questions over the particular SMP involving all participants occurred after both grade-bands presented their role-plays. The rest of the participants were tasked to share whether and/or to what degree the SMP was evident in the role-play as well as any additional thoughts. After each grade-band shared their role-plays and the group's discussion waned, the PD leaders synthesized teachers' ideas. Approximately five hours of the Spring PD were directly devoted to the *Unpacking the SMPs* role-play task.

Our role as PD providers was to help participants make sense of the objective and to facilitate ways to demonstrate evidence of the SMP. This was accomplished in various ways. A central reason for us to engage participants in this role-play activity was to formatively assess our participants' understanding of the SMPs. For this initial PD activity, we did not re-direct participants but instead encouraged them to re-read the SMPs and to unpack them according to their own predilection. Our role was to help participants make sense of the language in the SMPs. We encouraged reading strategies and using available resources (e.g., dictionaries) with unusual terms (e.g., decontextualize). Also, we supported participants' role-play ideas by discussing recent tasks they did in their classrooms that they believed addressed their SMPs. Finally, we initiated and facilitated discussions with questions such as "What did you notice in the role-play?" and "What evidence from the description shared by this group did you see in the role-play?" We also welcomed and encouraged onlookers to ask questions to those presenting the role-play. These questions and the ensuing discussions deepened our understanding of participants' impressions of the SMPs and provided a rich context for everyone to make sense of the SMPs for classroom contexts.

What Did We Learn from the Role-Play?

We videotaped the activity and examined the visual and audio evidence of the interactions, cues, writing, technology, and expressions used during the role-play and ensuing conversations using narrative analysis (Hatch, 2002). First, videos of the unpacking activity were transcribed. A table was created to organize ideas during the coding process. Each SMP was ascribed a column and each group of teachers was assigned a row. Second, we watched the videotapes and read transcripts simultaneously to familiarize ourselves with the data. Videotapes were paused after each role-play to allow the coders to discuss the activity. Initial ideas about each group's role-play were recorded as memos to reflect on during iterative and subsequent analyses. Next, we reviewed the memos within the matrix for overarching impressions that transcended across groups, grade levels, and/or SMPs. Later, impressions were reexamined for substantial evidence and a paucity of evidence. Impressions were retained when there was substantial evidence from the videotapes and/or transcripts. The final stage in the process was to rewrite the impressions as complete thoughts. The following findings are impressions of our participants' initial comprehension, which might be shared by teachers in your state, county/district, or city. We noted the teacher in the provided excerpts from the role-plays; all others in the role-play behaved as students. All names are pseudonyms of participants who enacted role-plays.

Impression #1: SMPs Are For Students

The first impression was that participants struggled with the notion that the SMPs are written for students to demonstrate. This is clearly evident in the language of the SMPs because every standard begins with "mathematically proficient students" (CCSSI, 2010, pp. 6-8). Ideally, the teacher should act as a facilitator, creating a context for students to exhibit these mathematical behaviors. The videos of the role-play consistently showed that participants struggled with determining what it is students should exhibit as evidence of the behaviors in the SMPs. For example, the teacher in the grades 5-7 role-play for SMP #7 did not allow students to wrestle with a mathematics question. The teacher (Angela) in the 90-second role-play showed that she was able to demonstrate evidence of this SMP but her students (Ryan and Benjamin) did not have any such opportunity.

> *Angela (Teacher):* Okay class, I am going to give you a story problem and I want you to figure it out in your head without using your calculator. We have 15 students in our classroom, and I want to give each student 9 M & M's. And I need to know how many M & M's I need to bring to school? 15 students are going to get 9 M & M's each.
>
> *Ryan:* I don't know what 15 times 9 is.
>
> *Angela:* Well how can you figure this out? . . . Is there something in there that you do know? Look at your numbers. You should break 15 apart, maybe.
>
> *Ryan*: 15 is 10 plus 5, it is. And I know that 10 times 9 is 90.
>
> *Benjamin:* And 10 times 5 is 45. Oh sorry, I mean 5 times 9 is 45.
>
> *Angela:* OK. So what could you do with 90 and 45?
>
> *Ryan:* Well, we could add those together, and then we get a 135.

In this role-play, the teacher led the instruction using an initiate-respond-evaluate (IRE) format and directed students' thinking with guiding questions. IRE is a teacher-led three-turn sequence that involves a teacher question, a students' response, and the teacher's evaluation of the student's response (Durkin, 1978-1979). Angela could have used wait time or posed an easier but similar question when students were struggling with large two-digit numbers. The only one providing evidence of looking to make use of structure to solve the problem is the teacher, who quickly offered the idea that 15 could be decomposed into 10 and 5 and then a few seconds later that something should be done with the two partial products (i.e., 90 and 45). The students used the provided hint of structure, but did not look for it themselves.

Students in the role-play were not provided with an appropriately rich problematic task, much less time to wrestle with it, and were not expected to demonstrate the behaviors indicated in SMP #7. Keep in mind that these role-plays were developed and practiced by the entire group, not just the teacher (Angela). The voice of who provides evidence for enactment of the SMP becomes clear through the role-play: the teacher. This episode was consistent with the other role-plays as those who played the role of the teacher demonstrated mathematical habits and/or behaviors described by the SMPs and perceived their role as model for students. Those playing the role of the students tended not to demonstrate habits and/or behaviors. Thus, participants felt that the teacher's role was to demonstrate the behaviors and habits described by the SMPs and encouraged students to notice how the teacher behaved mathematically.

Impression #2: Classroom Norms Impact Students' Outcomes

The second impression was that the norms of classroom environments impact the depth and quality of the SMP that may be exhibited, and participants seemed unaware of their influence. Expectations for learning, doing, and justifying mathematics are called sociomathematical norms (Yackel & Cobb, 1996). All but one role-play demonstrated the same two sociomathematical norms: students should respond only to teacher questions; and students should not engage in collaborative mathematical thinking. Teachers in these role-plays used an IRE discourse pattern. Alternatively, we noticed that one group's role-play demonstrated participants' awareness of effective classroom norms and expectations that fostered SMP-like behaviors.

This group of intermediate elementary (i.e., grades 3-5) participants was asked to role-play SMP #4: Model with Mathematics, although they also showed evidence of other SMPs. The task within their eight-minute role-play was for students to design a field trip that kept students in the local community, maintained low fuel costs, and took students to interesting places. The teacher, Sandra, provided clear directions and then asked the students, Bart and Sarah, to begin working. Sandra started the role-play with the following task:

> You guys are going to get to plan our field trip, but we do have some guidelines that you have to follow. The first is that each group will come up with a plan and an idea, an itinerary of our day, of possible places that we can go in our community. . . each group will have to pitch their idea to the class and then we'll vote on it and whichever one wins that's the field trip we'll get to go on. . . here are our limitations: . . .we don't have a lot of money and gas is expensive; so we only get to take the bus 25 miles. . . . The other thing is we are going to leave from school, but we have to be sure that we get back here. . . .You've got to explain to us where we are going to go and about how many miles it is going to take because we got to make sure we get back to the school.

The two students in this role-play, Bart and Sarah, created diagrams (i.e., models) characterizing their proposed field trip and shared them. Sandra asked Bart and Sarah to share interesting mathematical elements within their models (e.g., the order in which places on the field trip route are attended did not affect the overall total number of miles traveled). Bart and Sarah later critiqued each other's models and responded to questions from Sandra (i.e., SMP #3). Finally, Bart and Sarah showed that they were able to decontextualize the mathematics from a local area map, apply mathematics procedures to develop their models, and contextualize their findings within the field trip problem (i.e., SMP #2). Specific to this role-play, the teachers enacted norms such as students are expected to (a) discuss the effectiveness of the model and its representation, (b) discuss the mathematics within the model, and (c) reason quantitatively as described in the second SMP. This example characterized how a rich task, as well as mathematical and sociomathematical norms, influence students' engagement in the SMPs.

Impression #3: Misunderstanding SMP #1

The third impression was that there is a lack of evidence that our K-10 participants sufficiently understood the language within SMP #1. Participants' role-plays provided little evidence of any behavior described in this standard. For example, the high school participants role-played a scenario in which students worked with a system of equations using a graphing calculator. Language within SMP #1 stated that "older students might, depending on the context of the problem…change the viewing window on their graphing calculator to get the information they need" (NGAC, CCSSO, 2010, p.6). This role-play lasted approximately two minutes.

> *Harper (Teacher):* Class are you ready? We talked about how a system of equations with only one solution has a single ordered pair that works in both equations and only those two numbers work. I'd like to give you a new method of finding that solution by graphing the equations. Do you have your graphing calculators? Are they turned on?
>
> *Quinn:* My batteries are dead.
>
> *Harper:* Your problem is going to be to graph a system of equations. I would like you to graph y=-1/3x+22 and y=2x-20 on your graphing calculator then try to find the solution graphically to that system.
>
> *Quinn:* Only got one line on here.
>
> *Harper:* Did you graph the other equation?
>
> *Quinn:* Yeah I typed them both in here. I got one line.
>
> *Harper:* Why do you think that is?
>
> *Quinn:* You use bad parameters?
>
> *Harper:* Why don't you think about why you can't see that solution? Do you think there's another line in there?
>
> *Quinn:* I can't see anything.
>
> *Harper:* I want you to work as partners and try to solve that problem and reach a solution. (Pause) … You see two lines but can you give me the solution for it? … Can you give me the numbers from that picture you're looking at on your graphing calculator?
>
> *Quinn:* 0.5 and 8.
>
> *Harper:* That's very good.

These participants interpreted expanding the graphing window to examine a system of equations as evidence of making sense of a problem and persevering in solving it (i.e., SMP #1). A critical component to demonstrating SMP #1 is providing students with a worthwhile task that is problematic. No role-play for SMP #1 provided evidence of a problem or rich task that might engage students in perseverance or sensemaking about mathematical concepts or procedures. Furthermore, the intermediate elementary group discussed earlier in impression two was the only group to employ a problem. Note that here we define a problem as having three characteristics: a solution is not obvious; it is uncertain whether a solution exists; and a solution strategy is not readily apparent (Schoenfeld, 2011).

Summary

Participants attending these voluntary PD were motivated to improve their teaching and the role-play provided an assessment of their initial comprehension as well as a tool for fostering their learning about the SMPs. These impressions provided insight into teachers' comprehension of the SMPs and also pointed to features that we considered when enacting our PD that focused on the SMPs.

Implications for CCSSM-focused PD

An important implication stems from our first impression. Participants' role-plays suggested that participants thought they should demonstrate mathematical habits and behaviors described in the SMPs so that students might take them up. It was rare when these participants created a role-play where students engaged in the SMPs. After the role-plays, we discussed that the CCSSM were written for teachers and students. Participants commented that they were aware of this but did not demonstrate their awareness through the role-plays. There was a consensus among participants in both PD programs during these initial meetings that they felt their role as the mathematics teacher was to demonstrate habits and behaviors of mathematically proficient citizens; yet they tended to focus on how to carry out a set of procedures to solve a problem.

The teacher's role in helping students enact the SMP in their own learning must move beyond simply being a model and hoping that students will pick up on it. We would not have known the pervasive difficulty of this idea among our participants had we not assessed our participants' comprehension in a way that connected their ideas with classroom practices. Teachers may need support thinking about ways to gather evidence of students' engaged in the SMPs. Thus, we advise that mathematics education leaders develop PD tasks that remind teachers of the target audience for the CCSSM.

Another key implication of this study comes from the second impression. The role-play assessment was a useful tool for garnering shared experiences among PD participants to highlight important components of good teaching. We were able to assess that most participants paid relatively little attention to the way sociomathematical norms influence student's enactment of the SMPs through the role-play activity. Although that revelation was important for us as PD providers to assess, even more important to the PD was that there was one group's role-play that was an exception to the others. From this exception emerged an opportunity of shared experience among our participants to discuss the importance of sociomathematical norms through the lens of role-play done by Sandra, Bart, and Sarah. The role-play was not only a performance assessment by which we could come to understand the sense our participants were making of the SMP but also a task through which overcoming difficulties in teaching and learning could be explored.

From the third impression we learned that it is important for leaders in mathematics education to be careful about our assumptions regarding the sense teachers are making of the SMPs. We have been providing CCSSM-focused PD for teachers since shortly after the document was launched. Most teachers who attend our PDs explain that they have read the titles of the SMPs but never the paragraph below each title. Impression three highlights the difficulty some teachers have in making sense of a particular SMP even after a careful and close reading of the paragraph and discussion of that SMP with other teachers. The role-plays gave all of us, participants and PD leaders, a space through which we could discuss the meanings of the SMPs individually and as a group in more depth and explore habits of mind and behaviors of mathematically proficient students.

As we look to the future, we plan to refine *Unpacking the SMPs*. First, we plan to restructure the description in such a way that teachers might paraphrase the SMPs' descriptions. While the three unique descriptions were useful, participants struggled to sense the difference between an explanation for a principal/administrator and parent/guardian. Second, we will ask teachers to construct the paraphrased description and share it during one PD meeting, gather potential materials for the role-play afterwards, and then plan and execute the role-plays at the following meetings.

Conclusion

The dual aims of this manuscript were to discuss a performance assessment (i.e., *Unpacking the SMPs*) that allows PD providers an opportunity to make sense of teachers' comprehension of the SMPs and share what we learned about our participants' comprehension of the SMPs. The role-play activity was a useful performance assessment because it allowed us an opportunity to formatively assess participants' prior knowledge and initial comprehension of the SMPs. Our results from the role-play provided information regarding participants' ideas about the SMPs. An important benefit of the role-playing task was the rich data it provided about how the participants were making sense of the SMPs. The *Unpacking the SMPs* task offers clear benefit for any mathematics education leader aiming to support teachers' sensemaking of the SMPs. ✪

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The Common Core Standards for Mathematical Practice: Teachers' Initial Perceptions and Implementation Considerations

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Abstract

Teachers' responses to surveys involving two prompts after their first in-depth reading of the Standards for Mathematical Practice (SMP) in professional development settings are reported. Specifically addressing calls for research on how teachers are viewing their role in the implementation of the Common Core State Standards for Mathematics, and in particular the SMP, these data highlight what terminology teachers potentially focus on in reading the descriptions of the SMP. Additionally, the data highlight the roles that teachers envision themselves taking as they plan for and implement the SMP in their classrooms. We provide analysis of the teachers' responses, as well as discussion and suggestions for mathematics education leaders as they engage classroom teachers and other leaders in considering the implications for implementing the SMP with respect to student and teacher classroom roles.

The Common Core State Standards of Mathematics (CCSSM) have been established as a guide for mathematics education in the United States. This curriculum framework defines "what students should understand and be able to do in their study of mathematics" (Common Core State Standards Initiative [CCSSI], 2010, p. 4). As of this writing, "forty-five states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the Common Core State Standards" (CCSSI, 2014). Largely influenced by both the National Council of Teachers of Mathematics' process standards (NCTM, 2000) and the National Research Council's report *Adding It Up* (NRC, 2001), the CCSSM articulates eight Standards for Mathematical Practice (SMP) that "describe varieties of expertise that mathematics educators at all levels should develop in their students" (p.6). In describing this expertise, the beginning three words of each of the eight SMP are, "Mathematically proficient students." This phrasing is supported by a paragraph for each standard explicating what students are to do in their mathematical experiences to develop the necessary proficiency related to each SMP.

The SMP are listed in Table 1 (next page), and for brevity, only the title of each standard is given. Although the SMP describe proficiencies students should develop, little is said regarding how teachers should facilitate and develop these proficiencies with their students. However, standards documents addressing the teaching of mathematics to develop similar proficiencies have been published within the past quarter century (NCTM, 1989, 1991, 2000). In particular, researchers have investigated the degree of teacher awareness of the various previous NCTM standards documents, and the alignment between standards and teachers' beliefs (LaBerge, Sons, & Zollman, 1999; Markward, 1996; Mudge 1993; Perrin, 2012; Zollman & Mason, 1992). These studies indicated that there was a broad range with respect to teachers' awareness and familiarity of the NCTM standard

Table 1: Standards for Mathematical Practice

Standard for Mathematical Practice #	Title
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.

documents. Furthermore, in examining the alignment between the philosophies implied by the standards and teachers' beliefs, the studies found varying degrees of alignment with the NCTM standards documents. However, with the recent publication of the CCSSM, studies of teachers' understandings and perceptions of the CCSSM in general, and SMP in particular, with regard to their influence on teachers' professional practice are only emerging or nonexistent (see, Heck et al., 2011).

Consequently, we utilized professional development opportunities to conduct research specific to the CCSSM and SMP to inform our own professional development practices. In particular, we endeavored to ascertain what teachers of mathematics glean from initial readings of the SMP. In order to engage teachers in professional development related to the CCSSM and the SMP, we initially must know what the teachers identified in their initial reading of the SMP, and how they believe they can implement, or are implementing, the ideas outlined therein. In this paper, we present our findings and discussions based on our research related to the following two questions:

1) When teachers initially read the SMP, what do they report as noteworthy?

2) When teachers initially read the SMP, what aspects of each standard do teachers identify as influencing their intentions to address the SMP in their instruction?

The analysis of teachers' responses to the prompts given to the teachers provides a measure of what was viewed as noteworthy and what aspects they see as influencing their intentions to implement SMP. The wording of these research questions is mirrored by the questions for which the teachers were asked to self-report.

Recent Recommendations for CCSSM and Standards Research

Our research questions and analysis of data were informed through examining policy documents that offered timely perspectives related to the release of the CCSSM. Research on the CCSSM, implementation thereof, and effects on teachers' practice and student outcomes are identified as key areas in reports, and by various national organizations. In mid-2010, NCTM, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) released a joint public statement on supporting the implementation of the CCSSM. In this public statement, these organizations "strongly encourage and support both research about the standards themselves (e.g., research on specific learning trajectories and grade placement of specific content) and their implementation" (NCTM, NCSM, ASSM, & AMTE, 2010, para. 5).

In identifying areas for such research, Heck et al. (2011) pointedly noted that the NRC framework (2002), developed for investigating the influence of standards documents, foundationally acknowledges that standards documents "are unlikely to have a direct impact on student learning, but come to influence teaching and learning by first influencing key components of the education system, including curriculum, assessment, and teachers" (p. 2). Building on the NRC framework, Heck and colleagues outlined a priority research agenda specifically for understanding the influence and implementation of the CCSSM. In their report, Heck et al. described four areas of research study: case studies, investigations of relationships, status studies, and studies to improve the standards. Within each of these areas, they provided a variety of study types and foci. For example, they identified five priority areas for case studies, four for investigations of relationships, and four for status studies, while no specific priority is given for studies to improve the standards. Our work is situated in Priority Case Study Focus #5: Teacher responses to the

CCSSM, the only priority area dealing directly with teachers. This focus area outlined the following.

Since teachers' knowledge, interpretations, self-efficacy, beliefs, dispositions, and skill, as well as their specific intentions and plans, affect what transpires in classrooms, it is critical to understand how teachers respond to the CCSSM, and what kinds of classroom learning opportunities for their students result. (Heck et al., 2011, p. 13)

Within this priority case study focus, the research and discussions presented in this paper largely align with the following areas that Heck et al. outlined to focus studies undertaken to investigate teacher responses to the CCSSM: "What implications do teachers see for their mathematics instruction? What aspects of their mathematics instruction do they see as validated by the CCSSM, and what aspects do they consider in need of change based on the CCSSM?" (p.13). Our work represents teachers' initial full reading of the SMP. In particular, our first research question aligns with identifying potential aspects of their instruction that the teachers feel are either validated by the SMP or needing of change (vis-à-vis the language of "eye catching"). Our second research question aligns with teachers self-identifying the implications the SMP have for their instruction (vis-à-vis the language of "influencing intentions"). Although not a word-for-word reproduction of Heck et al.'s language, the research and discussion presented here provide baseline data of how teachers perceived (and potentially continue to perceive) the SMP as affecting their mathematics instruction.

Methods

A total of 23 teachers participated in this study. Each teacher participated in one of two different in-service professional development (PD) settings. There was variation in the grade levels self-identified as the teachers' primary teaching responsibility, the range spanned Early Elementary (K-2) through College or University. However, 17 of the 23 teachers reported primary teaching responsibilities at either the middle school (5) or high school level (12). All teachers had at least one year of prior experience teaching in the same large urban district[1]; however, demographic information with regard to specific school building assignments in the district was not gathered.

In each PD setting, the PD facilitator surveyed the participants with the sole purpose of gathering formative assessment data to inform the PD activities specific to the CCSSM. Before providing the participants with the portion of the CCSSM document containing descriptions of the SMP, the sentiment expressed by all of the participants in each setting indicated that *not one participant* had more than briefly skimmed the SMP descriptions. As such, each participant's "familiarity" with the SMP was considered as "not read" (as defined by Perrin, 2012). The participants in each setting first read the descriptions of the eight SMP, the titles of which are listed in Table 1. The full descriptions of the SMP that the participants read can be found on pages 6 through 8 of the CCSSM document (CCSSI, 2010). The participants were instructed to read, and were observed reading, each SMP description in its entirety. Sufficient time was provided for participants to read the three-page document and to formulate appropriate responses to two prompts: Prompt 1 – *Name one or two things that caught your eye as you read the standard*; Prompt 2 – *What is one way you are, or plan on being, more intentional about this standard in your teaching?* Participants responded anonymously (by way of a Google Form) to these prompts. Given the context as described here, we believe the responses presented in this paper reflect the perspectives of in-service teachers' initial complete reading of each SMP.

During the PD experiences, the facilitator immediately used the data he had gathered in real-time through the Google Form to engage the participants in discussions centered on the anonymous responses. The discussions generated by the facilitator's formative use of the data were informative for both the participants and facilitator. These discussions led the facilitator to engage in subsequent discussions with colleagues, and upon further examination of the data collected, led to a deeper investigation of the literature.

Using our research questions to guide our data analysis, we compiled and qualitatively examined each participant's response. Analysis of participants' responses to the two prompts was conducted using Grounded Theory principles (Strauss & Corbin, 1998) in which primary analysis and coding focused on identifying emerging and cross-cutting themes that were later reorganized and further classified.

[1] 2012-2013 Ethnic distribution for Grades PreK-12 for the district has been reported as follows: Hispanic (43.4%), Caucasian (30.2%), African American (12.0%), Asian (6.6%), and Other (7.8%).

Table 2: Classifications and Counts of Responses for Prompt 1

Standard for Mathematical Practice	Classification	Number of Times Identified
1. Make sense of problems and persevere in solving them	Making Sense; Checking Answers; Persevere; Explaining (Ability to)	8 7 7 4
2. Reason abstractly and quantitatively	(Coherent) Representations; Meaning of Quantities; Abstract Thinking/Reasoning; Contextualize/Decontextualize	7 7 5 4
3. Construct viable arguments and critique the reasoning of others	Listen/Read/Ask; Distinguish Correct and Flawed logic; Justify Answers/Conclusions; Construct Arguments; Critique	7 5 4 3 3
4. Model with mathematics	Solve Problems in Everyday Life; Assumption, Approximation, Revision	10 5
5. Use appropriate tools strategically	Consider tools; Tools to Deepen Understanding; Use Tools Strategically	10 5 5
6. Attend to precision	Definitions and Symbols; Precision; Carefulness	10 9 5
7. Look for and make use of structure	Patterns, Structures, Connections; Auxiliary Line; Respondent Provided Specific Example	13 3 3
8. Look for and express regularity in repeated reasoning	Repeated/Repetition; Shortcuts; Maintain Oversight of Process	13 6 5

For reliability purposes, one member of our team conducted initial analyses, and the two other team members conducted secondary analyses of the emerging themes, codes, and classifications defined in the initial analysis. Any discrepancies among the three analyses were discussed and reconciled through face-to-face and electronic communications. Reconciliation efforts were specifically focused on further defining and refining classifications of themes that emerged from the teachers' responses to the two prompts.

Due to the degree to which the SMP descriptions vary, emerging themes and codes for participants' responses for Prompt 1 were classified for each individual SMP. Conversely, although the standards differ, participants' responses to Prompt 2 related to how they intended to implement the SMP were such that emerging themes were categorized by one overarching classification scheme for all eight SMP.

Results

Classifying Responses to Prompt 1

In examining participants' responses to Prompt 1 – *Name one or two things that caught your eye as you read the standard* – we determined that if a participant's response was categorized under two or more classifications, each was counted. In other words, in examining the data presented in Table 2, if a participant's response to SMP 1 mentioned ideas related to perseverance *and* making sense, then that one participant's response was counted under the number of times each of those was identified. The words used as themes for the classifications in Table 2 are directly related to wording found in the description of each SMP. Counts were not recorded as to whether or not a response was unrelated to the standard, or if no response was made. Consequently, the total of the number of times themes were identified per standard is not always 23 (the total number of participants) in Table 2. For example, 24 themes emerged and were cross cut, linked, and categorized into the three classifications for SMP 8. For SMP 4, 5, and 7, we were only able to classify emerging themes totaling 15, 20, and 19, respectively. This perceived lack of response was most often indicative of responses that simply did not address the standard. However, overall, many of the phrases and verbiage found in the SMP descriptions appeared to strike teachers as noteworthy.

Classifying Responses to Prompt 2

Two overarching constructs emerged as themes in examining participants' responses to Prompt 2 – *What is one way you are, or plan on being, more intentional about this standard in your teaching?* – student oriented versus teacher oriented perspectives of teaching. In other words, participants' plans for implementing the SMP in their teaching practices were classified as either an action a participant was personally going to take to modify a practice in teaching mathematics (teacher oriented), or an action a participant was going to take to modify practices of students in learning mathematics (student oriented).

The student-oriented responses were further classified into two categories. A "student allowance" action (SOA) is a

Table 3: Classification Categories for Prompt 2

Classification Category	Example Participant Responses per Classification
0. No response or response did not address Prompt 2	Quantitatively is the easy part, thinking abstractly is the harder part. This seems like an oxymoron.
1. SOA – Student Oriented, Allowances	Allow students to develop reasoning and concepts through problem solving and exploring a variety of contexts. Giving my students more time to struggle with and interpret the meaning of problems themselves. Allow students to develop their own thoughts despite the scary paths they may travel.
2. SON – Student Oriented, Need, Self-Action, Student Responsibility	Making sure that students understand symbols and equations in order to be able to read problems and translate into mathematical equations. Students don't often realize the importance of details. They need their eyes opened to the repercussions. Students must know all aspects of a problem and not just a few cases.
3. TOA – Teacher Oriented, Assessment	I will award and/or acknowledge students for partial success rather than all or nothing. Visualizing a concept is very important to understanding a concept and being able to visually diagram a concept is a step that must be completed and evaluated to ensure students are picking up the intended concept in the lesson.
4. TOP – Teacher Oriented, Pedagogical/Instructional	I will do all steps to the problems out loud and explain why I did the steps and what I was thinking. I plan on making sure that I find ways to connect what I am teaching to real world application. I will teach students to give "constructive" criticism. I need to stop giving students my answer so fast. Initially, I should model the problem solving steps I use to approach a problem.

teacher action oriented towards something the student would be allowed to do. A "student need, self-action, responsibility" action (SON) is a teacher intention to promote student action identified by the teacher as necessary to achieve a particular SMP.

The teacher-oriented responses were further classified into two categories. A "teacher assessment" action (TOA) is a teacher action that the participant would take to purposefully assess student progress towards an SMP, whether in a formative or summative manner. A "teacher pedagogical/ instructional" action (TOP) is an action the participant intended to take specific to his or her instructional methods as related to the SMP.

Table 3 presents the four classification categories that emerged along with actual responses that were classified within each category.

In Table 4, we provide the counts for participant responses in each classification category. In some of the more lengthy responses, multiple themes emerged that allowed the response to be classified into two or more categories. Conversely, in a few instances a response did not address the standard or was left blank. Consequently, the total number of responses per standard in Table 4 is not always 23. Furthermore, as indicated by the Totals row in Table 4, a total of 205 separate themes within responses were classified into these categories.

Discussion

What Participants Identified as Noteworthy

When initially reading the descriptions of the eight SMP, the participants identified different noteworthy items. In fact, although the authors of the standards included key elements in each standard, the participants identified certain wording at the expense of other parts of the standard. For example, SMP 1 states that, "Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends" (p. 6). Interestingly, 4 of the 26 (15.3%) responses were categorized as Explaining (ability to). However, in our reading of the standard, the ability to "explain" in this standard specifically pertains to, and directly follows language related to "proficiency." In other words, nearly 85% of the responses did not identify this explicit proficiency oriented language as being particularly noteworthy for SMP 1.

Table 4: Counts Per Classification Category for Prompt 2

Standard for Mathematical Practice	Counts Per Classification Category					
	0. No	1. SOA	2. SON	3. TOA	4. TOP	Totals
1. Make sense of problems and persevere in solving them	0	8	6	2	15	31
2. Reason abstractly and quantitatively	1	2	6	3	15	27
3. Construct viable arguments and critique the reasoning of others	1	1	10	1	12	25
4. Model with mathematics	0	0	4	1	18	23
5. Use appropriate tools strategically	1	5	3	1	15	25
6. Attend to precision	1	1	3	4	17	26
7. Look for and make use of structure	3	1	4	0	17	25
8. Look for and express regularity in repeated reasoning	1	1	3	2	16	23
Totals	8	19	39	14	125	205
Percent Category of Overall Total	4%	9%	19%	7%	61%	

Interestingly with regard to SMP 2, responses mostly identified with either representations or the meaning of quantities. However, the constructs of contextualizing and decontextualizing were identified the least by the teachers in SMP 2. The use of representations to understand the meaning of quantities through the constructs of contextualizing and decontextualizing was recently identified as a key component to understanding SMP 2 (Olson & Olson, 2013).

Although many of the responses of noteworthy aspects of the SMP were interesting by the very nature of the variety of what participants identified in their initial reading of the descriptions, one term stood out in our analysis, shortcuts. Of the 24 responses to SMP 8, 6 identified shortcuts as a noteworthy aspect of this SMP. That is, one-quarter of the responses identifying noteworthy aspects of *look for and express regularity in repeated reasoning* focused on the notion of shortcuts. This is especially interesting in that for almost all of the discussion in SMP 8, the examples focus on the regularity in repetitive reasoning and how this may lead to a generalization of a mathematical idea. Perhaps this emphasis by some participants on shortcuts in SMP 8 could be a focus to better understand how participants' beliefs about the nature of mathematics obscure or reaffirm their mathematical interpretations of standards documents, and particularly more process-oriented standards such as the SMP.

Participants' Intentions in Implementing the SMP

Examining the data in Table 4, the number of responses coded as *Teacher Oriented – Pedagogical/Instructional* (TOP) is consistently larger than the number of responses for any of the other classifications. This focus on teacher-oriented pedagogical and instructional moves is perhaps not entirely unexpected. When implementing the standards, participants likely perceived the way in which they can bring SMP into the classroom is through controlling their instructional and pedagogical choices. However, these instructional choices are qualitatively different than the instructional choices that involve student-oriented actions, which are arguably more consistent with student-centered instructional choices. In fact, such teacher-oriented instructional actions were largely consistent with the first example statement for TOP in Table 3: *I will do all steps to the problem out loud and explain why I did the steps and what I was thinking.* That is, in general, the TOP category encompassed teacher actions that we identified as being analogous to a teacher stating, "I will do the mathematics I know for my students to illustrate how the SMP are important in mathematics learning."

Of the eight SMP, only two standards (SMP 1 and 3) involved responses coded for TOP that were less than 50% of the total responses for that standard. For the other six SMP, the percentage of total responses for the specific standard coded TOP ranged from 56% (SMP 2) to 78% (SMP 4). Overall, of the total number of responses, 61% were coded as TOP. That is, for the 205 distinct responses of how these teachers envisioned implementing the SMP, 61% of those responses (125 out of 205) involved teacher actions driven by what the participant intended to do in the classroom setting to show how mathematics learning involves the eight SMP.

Potential reasons for such identification with a TOP perspective might be best analyzed through the perspective of SMP 4. As noted, 78% (18 out of 23) of the responses for SMP 4 were coded for teacher-oriented pedagogical actions. SMP 4 is the standard that most discusses the importance of mathematical modeling as a process of doing and learning mathematics. Our interpretation of the data through our collective anecdotal experiences is that perhaps in their initial reading, participants envisioned that they are the ones responsible for modeling how to do mathematics in classroom settings. Similarly, if interpreted more as developing mathematical models to explain and predict phenomena in real-world settings, perhaps the participants still felt an initial compulsion to show students how such modeling is done through completing models and activities for the students as a way of exemplifying such processes. We interpret such compulsions as a likely by-product of the *apprenticeship of observation* (Lortie, 1975) that all teachers have experienced in their own lives as students of mathematics.

Similar feelings of needing to provide students with a teacher-oriented instructional perspective on learning mathematics through the SMP likely underpin responses to SMP 7 and SMP 8 that were coded as TOP. In particular, responses to each of these standards comprised 68% and 70% of the total responses for each of the standards, respectively. In other words, participants likely felt the need to show students the structure of the mathematics for which they should be looking (SMP 7), or to show them how regularity in repeated reasoning can lead to generalizations (SMP 8), and eventually "shortcuts" – the term identified by some of the participants as noteworthy.

Many participants' responses were categorized as student-oriented actions. Specifically, for SMP 1, 26% of the responses were coded for student-oriented allowances, and 19% were categorized as student-oriented need, self-action, and responsibility. That is, 45% of the responses to SMP 1 involved participants envisioning implementing the standard through a student-centered perspective. Perhaps SMP 1 allows for more student-centered implementation of the standards, as it is difficult to imagine how students will persevere and make sense of problems unless they are actively engaged in the learning and solving processes.

Lastly, 40% of the responses to SMP 3 (construct viable arguments and critique the reasoning of others) were coded as student-oriented need, self-action, and responsibility; 20% of the responses to SMP 5 (using tools strategically) were categorized as student-oriented allowances. In other words, in their initial reading, participants identified SMP 5 as a way in which they can allow students to take more ownership of their learning through the appropriate choice of tools; participants identified SMP 3 as a way in which they can promote student actions, and self-action, to take responsibility for constructing (for themselves) viable arguments and engaging classmates in critiques of mathematical ideas.

Through analyzing the participants' responses, we maintain that in their initial reading of the SMP, participants viewed some of the standards as more easily incorporated into student-centered learning environments. Furthermore, we also argue that participants' viewed other standards as perhaps more difficult to implement beyond direct instructional actions likely due to a myriad of reasons, not least of which is the way in which they experienced *modeling with mathematics, seeing structure in mathematical concepts, and were shown regularity in mathematical reasoning* throughout their own learning of mathematics content (i.e., the SMP viewed through the lens of their own apprenticeship of observation).

It is important to note that we are not arguing against or for the importance of teachers employing direct instructional actions versus student-centered actions in their classrooms to connect and provide meaningful exposition to introduce, augment, or summarize mathematics discussions. In fact, much literature exists on the importance of a variety of instructional approaches in mathematics classrooms, such as the Knowledge of How People Learn framework presented by the NRC (2000, p. 22). However, the preponderance of responses categorized as TOP indicated to us that when engaging teachers in discussions related to implementing the SMP, thoughtful challenges must be posed to teachers so they have the opportunities to re-consider what classroom actions are available in order for students to engage in mathematical study via the SMP.

Summary and Implications for Mathematics Education Leaders

Importantly, although little is currently known about how teachers have interpreted their future actions through reading the eight SMP, our work is well aligned with other efforts to engage teachers in such thinking and discussion. In particular, an NCSM resource provided on the organization's website is Illustrating the Standards for Mathematical Practice (NCSM, 2014). In one resource on the website, *6-8 Comparing Linear Functions – Presentation*, professional development participants are prompted to:

1. Individually review the Standards for Mathematical Practice.

2. Choose a partner at your table and discuss a new insight you had into the Standards for Mathematical Practice.

3. Then discuss the following question: What implications might the Standards for Mathematical Practice have on your classroom? (NCSM, 2014, slide 8)

In other words, the framework we provided here for how we utilized prompts for facilitating PD experiences is not necessarily novel. However, in collecting the varied participants' responses, and investigating the data through focused qualitative analyses, we believe important beliefs underlying teachers' instructional practices have been identified through their initial reading of the SMP. Such frameworks, we feel, are useful vehicles to use when engaging in PD experiences with the SMP so that those providing the PD have an opportunity to gather formative assessment data relative to understanding the beliefs of teachers related to their reading of the SMP.

Based on our work with these teachers, and examining the data, we argue it is critical for mathematics education leaders, teacher educators, and professional development facilitators to be sensitized to the potential that the likely prevailing approach to implementing the SMP will be from a teacher-oriented perspective. Such a perspective is important for PD leaders to continually be cognizant of, and thoughtfully engage and challenge teachers in alternative pedagogical approaches when facilitating discussion around the SMP. Not only are the teachers we surveyed viewing the SMP through the lens of what they must do instructionally for students to be proficient with the SMP, they are in large part also *not* viewing the SMP through the lens of what they will allow their students to do to so that the students can fully engage in the range of mathematical experiences delineated by the SMP.

The data serve as a reminder that even though PD may be provided from student-centered perspectives, teachers likely engage with PD from various TOP perspectives. That is, even when descriptions of mathematical practice are explicated, as is the case with the SMP, there will likely be a disconnect between the written word and teacher practice that must be acknowledged and bridged by mathematics education leaders and facilitators in an effort to make clear the perspectives that both the teachers and leaders bring to standards implementation. While the SMP are discussed before the Standards for Mathematical Content in the CCSSM, the simple fact of having three pages of SMP in the CCSSM will not, alone, produce the paradigm shift needed in teachers' instructional practices to move them from "what I will do to *show* my students a SMP" to "what I will do to *allow* my students to experience, for themselves, the interconnectedness of all of the SMP." Significant dialog regarding implementation efforts must be facilitated for teachers' understandings of each SMP to be explicated, challenged, and critiqued in thoughtful, respectful, and meaningfully beneficial ways so that a vision for instruction can emerge in which students are constantly and consistently engaged in mathematical study through the SMP. ✪

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Analyzing Students' Work to Reflect on Instruction: The Instructional Quality Assessment as a Tool for Instructional Leaders

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Abstract

In this article, we discuss how instructional leaders can use collections of students' work and the Instructional Quality Assessment (IQA) Mathematics rubrics to initiate conversations with groups of mathematics teachers and to monitor the success of professional development initiatives and curricular implementation efforts. In our work, collections of students' work are used to reflect on instructional practice, by considering the nature of instruction that supported students to produce the mathematical work and thinking. We ground the discussion in specific examples from two studies in which collections of students' work and the IQA rubrics were used to diagnose the effectiveness of professional development and curriculum implementation efforts, engage teachers in reflecting on practice, and inform next steps in the instructional change process.

In the current era of the Common Core State Standards in Mathematics (CCSSM; Common Core State Standards Initiative [CCSSI], 2010) and increased accountability demands on teachers to support strong learning outcomes for all students, teachers and administrators are focused more closely than ever on the nature of teachers' classroom practice in mathematics (Cobb & Smith, 2008; Spillane, Halverson, & Diamond, 2004). Supporting students in meeting the Standards for Mathematical Practice (CCSSI, 2010) identified in the CCSSM will require a sharp departure from traditional procedurally driven mathematics curricula and teaching practices, and successful implementation of CCSSM will require "significant changes in the practice of most US mathematics teachers" (Cobb & Jackson, 2011, p. 185). To address these new demands, school districts across the country will need to engage teachers in professional learning experiences and adopt or revise mathematics curricula to promote the ambitious vision of mathematics teaching and learning advocated by the Standards for Mathematical Practice (Cobb & Jackson, 2011).

Instructional leaders play a critical role in the success or failure of teachers' efforts to grow and develop their classroom practice (Boyd et al., 2011; Tickle, Chang, & Kim, 2011). Studies of successful systemic change in secondary mathematics, for example, have identified strong instructional leadership as an integral component of changing classroom practice (e.g., Stein & Nelson, 2003; Stein, Silver, & Smith, 1998). Supporting meaningful change means that instructional leaders must engage with the substance of a reform initiative rather than simply the broad-stroke forms (November, Alexander, & van Wyk, 2010; Spillane, 2000; Stevens, 2004). While short walkthroughs and teacher observations are important tools that an instructional leader might use to support teacher professional development (Fink & Resnick, 2001), strong instructional leadership also includes engaging in conversations with teachers (individually and in professional learning communities) about instruction outside the context of an observation (Rossi, 2007).

Given the constraints on leaders' time, however, frequent full-class observations and debriefing sessions with teachers around classroom practice are often not feasible. Similarly, having groups of teachers observe each other may not be logistically possible to implement on a regular basis. As such, instructional leaders need tools that support meaningful discussions about teaching and learning with their mathematics department outside of regular class time. Conversations in professional leaning communities about the nature of mathematical tasks (e.g., Arbaugh & Brown, 2005) and the analysis of students' work (e.g., Kazemi & Franke, 2004) have been shown to be effective in supporting reflection on practice and teacher change. Interventions with principals have demonstrated that instructional leaders with diverse backgrounds can engage meaningfully in conversations about mathematics tasks, episodes of teaching, and students' work (Boston, Gibbons, & Henrick, 2011; Steele, Johnson, Otten, Herbel-Eisenmann, & Carver, under review). Each of these studies made use of research-based tools to structure conversations among teachers and administrators.

In this article, we present one such research-based tool – the Instructional Quality Assessment (IQA) – that can be used with student-work artifacts to analyze and interrogate the nature of classroom practice, as a proxy for classroom observations. We describe two projects in which students' work was collected as a measure of and reflection on instruction. In these settings, the IQA rubrics were used to analyze the effectiveness of a professional development initiative and the implementation of a standards-based, algebra curriculum. We suggest ways that instructional leaders could use the rubrics internally to serve diagnostic purposes and, most importantly, as a learning tool for fostering rich conversations with teams of mathematics teachers about mathematics instructional practice. While students' work has been used successfully to engage teachers in assessing students' thinking and understanding of mathematics, we propose that analyzing sets of students' work can also be used to initiate conversations about the nature of instruction that supported students to produce the mathematical work and thinking. In this way, students' work provides a reflection on instruction that can promote teachers' self-reflection, self–discovery, and transformative growth (Steele & Boston, 2012).

The Instructional Quality Assessment Mathematics Rubrics

The Instructional Quality Assessment (IQA) Mathematics Toolkit was developed to provide a direct assessment of instructional quality based on live classroom observations or collections of students' work. Though initially created as a research instrument, the IQA can also serve as a tool to support rich conversations about instructional practices in mathematics. The IQA rubrics for classroom observations and students' work assess the rigor of instructional tasks, task implementation (i.e., how the demands of a task are enacted by teachers and students during instruction), classroom discourse (observation rubrics only), and teachers' expectations (students' work rubrics only). Research has consistently identified these four aspects of classroom instruction as impacting student achievement (Cobb, Boufi, McClain, & Whitenack, 1997; Hufferd-Ackles, Fuson, & Sherin, 2004; Staples, 2007; Stein & Lane, 1996). Figure 1 provides the *Teacher's Expectations* rubric and samples of teacher's expectations at each level. Excerpts from the rubrics for *Potential of the Task* and *Task Implementation* are provided in Figure 2 along with sample tasks and students' work indicative of the score levels on each rubric.[1]

The IQA rubrics are grounded in two bodies of research. First, the Mathematical Tasks Framework (Stein, Smith, Henningsen, and Silver, 2009) informed the IQA's assessment of instructional tasks separately from task implementation, and score levels within each rubric reflect the Levels of Cognitive Demand: *doing mathematics* and *procedures with connections* (i.e., high-level cognitive demands) and *procedures without connections* and *memorization* (i.e., low-level cognitive demands). Second, the collection and analysis of students' work as a valid reflection of instructional practice utilizes the research of Matsumura, Garnier, Pascal, and Valdes (2002). Design and generalizability studies determined that four sets of students' work, containing at least 4-6 samples per set and scored by two trained raters, provided a stable indication of a teachers' classroom practice highly correlated with observed instruction (Matsumura, Garnier, Slater, & Boston, 2008). As such, the analysis of samples of students'

[1] See Boston (2012) for a comprehensive description of the protocol for using the IQA rubrics and collecting and analyzing student work in research.

FIGURE 1: *IQA Mathematics Assignments rubric for* Teachers' Expectations *(Boston, 2012) and samples of* Teacher's Expectations

	***Teacher's Expectations* rubric**	**Samples of *Teacher's Expectations***
4	The majority of the teacher's expectations are for students to engage with the high-level demands of the task, such as using complex thinking and/or exploring and understanding mathematical concepts, procedures, and/or relationships.	Sample 1 (for the Level 4 task in Figure 2): "I wanted to see students really thinking creatively about the problem, using what they know about benchmark fractions and percents, and using the diagram in their explanations. I wanted clear explanations that make sense to the reader."
3	At least some of the teacher's expectations are for students to engage in complex thinking or in understanding important mathematics. However, the teacher's expectations do not warrant a "4" because: • the expectations are appropriate for a task that lacks the complexity to be a "4"; • the expectations do not reflect the potential of the task to elicit complex thinking (e.g., identifying patterns but not forming generalizations; using multiple strategies or representations without developing connections between them; providing shallow evidence or explanations to support conclusions). • the teacher expects complex thinking, but the expectations do not reflect the mathematical potential of the task.	Sample 1 (for the Level 3 task in Figure 2): "To write a story problem that could be answered by solving the equation." Sample 2: "I wanted students to be able to estimate perimeter and area and explain why they chose that particular estimate." [Teachers' expectations did not capture the main mathematical ideas of the task: developing students' understanding of perimeter and area by comparing the perimeter and area of irregular shapes.]
2	The teacher's expectations focus on skills that are germane to student learning, but these are not complex thinking skills (e.g., expecting use of a specific problem solving strategy, expecting short answers based on memorized facts, rules or formulas; expecting accuracy or correct application of procedures rather than on understanding mathematical concepts).	Sample 1 (for the Level 2 task in Figure 2): "My students always understand that quality work involves neatness, accuracy, and checks for accuracy. I continue to stress completeness, neatness, and accuracy… all problems attempted with minimal (1-2) mistakes." Sample 2: "High performers were students who had math facts memorized and breezed through the assignment."
1	The teacher's expectations do not focus on substantive mathematical content (e.g., activities or classroom procedures such as following directions, effort, producing neat work, or following rules for cooperative learning).	Sample 1 (for the Level 1 task in Figure 2): "This work was checked for effort rather than performance. Students must label their papers (name, date, class) and use pencil (NOT pen)."

work that conform to these requirements serves as a valid proxy for live classroom observation.

Using Student Work to Assess a Professional Development Initiative: The Summer Workshop in Mathematics Project

During Summer 2010, the Summer Workshop in Mathematics (SWIM) project[2] engaged 39 elementary and middle school teachers (grades 3-8) in two, one-week professional development workshops focused on the teaching of fractions and algebraic thinking. The goal of the project was to develop teachers' understanding of fractions and/or algebraic ideas and to support teachers in analyzing the cognitive demands of mathematical tasks in any mathematical content area. As the primary professional learning activity in each workshop, teachers engaged in solving cognitively challenging tasks with the potential to engage them as learners in the Standards for Mathematical Practice and to support the development of their conceptual understanding of fractions and algebraic ideas. Each workshop also provided opportunities for teachers: to

[2] The first author served as Principal Investigator on the SWIM Project, funded by a grant from the Heinz Endowments.

FIGURE 2: *Excerpts of the* Potential of the Task and Task Implementation *rubrics from the IQA Mathematics Assignments Manual (Boston, 2012) and corresponding samples of students' work.*

<table>
<tr><th></th><th>Potential of the Task rubric</th><th>Task Implementation rubric</th><th>Sample of Student's Work</th></tr>
<tr><td>4</td><td>The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:
• Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); or
• Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.

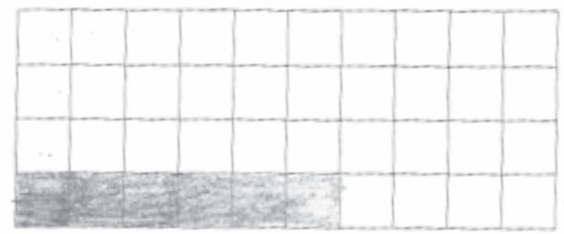
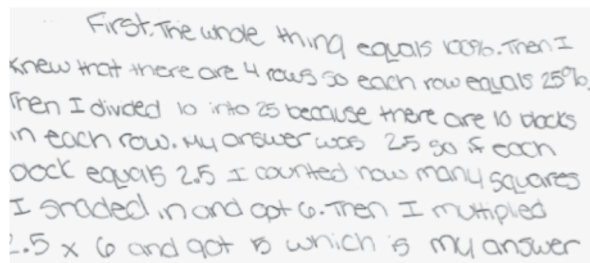
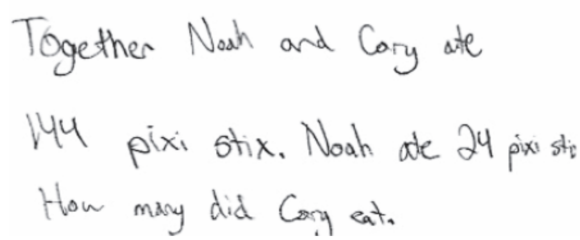
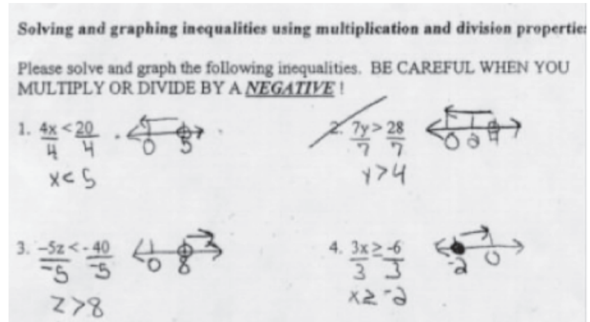
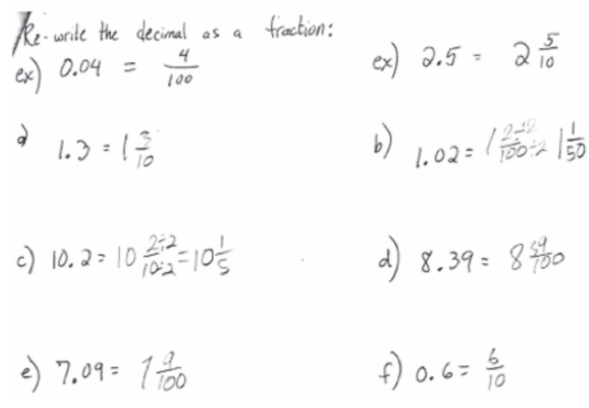
The task must explicitly prompt for evidence of students' reasoning and understanding. For example, the task MAY require students to:
• solve a genuine, challenging problem for which students' reasoning is evident in their work on the task;
• develop an explanation for why formulas or procedures work;
• identify patterns; form and justify generalizations based on these patterns;…</td><td>Student work indicates the use of complex and non-algorithmic thinking, problem solving, or exploring and understanding the nature of mathematical concepts, procedures, and/or relationships (i.e., there is evidence of at least one of the descriptors of a "4" in the Potential of the Task rubric.)</td><td>Shade 6 of the small squares in the rectangle below:

Using the diagram, explain how to determine:
a) the percent of area that is shaded

</td></tr>
<tr><td>3</td><td>The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a "4" because:
• the task does not explicitly prompt for evidence of students' reasoning and understanding.
• students may need to identify patterns but are not pressed to form or justify generalizations;
• students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;…</td><td>Student work indicates that students engaged in problem-solving or in creating meaning for mathematical procedures and concepts BUT student work lacks explicit evidence of complex thinking required for "4" (i.e., the Potential of the Task was rated as a 3 or 4…and there is a lack of evidence of the appropriate descriptors for a 4, but there is evidence of at least one descriptor of a 3).</td><td>Write a story problem for each of the following equations:
B) 144 = 24 +x

</td></tr>
</table>

FIGURE 2: *Excerpts of the* Potential of the Task *and* Task Implementation *rubrics from the IQA Mathematics Assignments Manual (Boston, 2012) and corresponding samples of students' work.*

	***Potential of the Task* rubric**	***Task Implementation* rubric**	**Sample of Student's Work** The potential of the task is limited to engaging stu-
2	dents in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, orplacement of the task. There is little ambiguity about what needs to be done and how to do it. The task does not require students to make connections to the concepts or meaning underlying the procedure being used… (e.g., practicing a computational algorithm).	Students engage with the task at a procedural level. Students apply a demonstrated or prescribed procedure. Students show or state the steps of their procedure, but do not explain or support their ideas. ..	
1	The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions…	Students engage with the task at a memorization level… (e.g., students provide answers only), OR even though a procedure is required or implied by the task, only answers are provided in students' work; there is no evidence of the procedure used by the students.	

compare tasks with different levels of cognitive demand (e.g., tasks that engaged students in reproducing procedures or memorized knowledge versus tasks that promoted reasoning, problem-solving and sense making); to analyze the cognitive demands of the tasks they engaged in solving; and to reflect on their experiences as learners and how the facilitator supported their learning. Beyond these reflections, however, teachers did not discuss the implementation of cognitively challenging tasks or how to enact the practices of the facilitator.

The questions guiding the study were: *Following the workshop, could teachers implement a high-level task in ways that maintained the cognitive demands, as evident in the sets of students' work? What were teachers' successes and challenges in maintaining high-level cognitive demands, as evident in the sets of students' work and expressed by teachers during the follow-up sessions?*

Participants and Data

Thirteen teachers from Project SWIM elected to attend follow-up meetings during Fall 2010, to incorporate ideas from the workshops into their classrooms.[3] The teachers were from two urban school districts and two suburban school districts in a mid-sized Northeastern city. Teachers taught in 11 different elementary and middle schools, and all teachers had responsibility for teaching mathematics the majority of the school day. Demographic data for the teachers is provided in Table 1.

In the follow-up sessions, teachers used samples of students' work to describe their experiences in implementing high-level tasks, including successes and challenges. As samples of students' work were shared with the group, teachers could comment on what they noticed and wondered about students' mathematical understandings evident in the samples of work and the nature of the

3 Other teachers declined participation in the follow-up sessions due to personal commitments, health issues, or teaching assignments in the new school year that did not include mathematics.

Table 1: Demographic Data for Teachers in the Project SWIM Follow-Up Sessions

School Setting:	Urban 11	Suburban 2
Age Level of Classroom (at time of project):	Elementary (K-5) 7	Middle School (6-8) 6
Teaching Certification:	Elementary (K-6) 7	Mathematics (7-12) 6
Gender:	Female 10	Male 3

lesson in which the work was produced. Data from the follow-up sessions included teachers' written reflections (e.g., on instructional cases and on their own lessons and students' work), written artifacts produced during the follow-up sessions (i.e., chart paper listing successes and challenges in implementing high-level tasks), and the facilitator's notes from the discussions.

Ten of the 13 teachers agreed to provide their sets of students' work as data, resulting in 39 sets of student work for the analysis (four per teacher, with one teacher submitting only three). While the small sample size limits generalizations to the entire population of teachers in Project SWIM, the group of 10 teachers is important because 8 of them identified this type of mathematics instruction as atypical of their everyday practice. The sets of students' work captured their genuine, initial efforts to implement cognitively challenging instructional tasks, as might be the case in many districts embarking on instructional change in light of CCSSM.

Analysis

Written artifacts, discussions, and teachers' reflections from the follow-up sessions were used to identify common successes and challenges in implementing cognitively challenging tasks, as reported by the teachers. Sets of students' work provided evidence of teachers' ability to maintain the cognitive demands of high-level tasks. Student-work sets were scored independently by two trained raters (the first author and a graduate research assistant not associated with the SWIM workshop), using the IQA Mathematics Assignments rubrics for *Potential of the Task* and *Task Implementation* (featured in Figure 2). The raters achieved 89% initial exact-point agreement, with all disagreements resolved through discussion. Consensus scores were used to produce descriptive statistics on the overall collection of students' work.

Results

Within the group if 10 teachers, 8 (80%) teachers implemented at least 3 of 4 high-level tasks in ways that maintained students' opportunities for thinking and reasoning. In other words, the student-work samples provided evidence (as rated on the *Task Implementation* rubric) that students had actually engaged with the cognitively challenging aspects of the tasks (as rated on the *Potential of the Task* rubric). Overall, in 39 sets of students' work, 33 sets (85%) featured high-level tasks (i.e., a score of 3 or 4 on the *Potential of the Task* rubric) and 26 sets (67%) featured high-level implementations (i.e., a score of 3 or 4 on the *Task Implementation* rubric); hence 26 of 33 high-level tasks (79%) were maintained at a high-level during implementation. These data provide evidence that the majority of teachers were able to implement cognitively challenging instructional tasks.

Successes and challenges arose as teachers shared their experiences in implementing the tasks. Discussions among teachers regarding students' work samples served as a vehicle for identifying aspects of ambitious mathematics instruction that were present or absent from the student-work samples. Teachers often noticed successes as they examined other teachers' sets of students' work. For example, teachers commented that *students solved the task in more than one way even though the task directions did not specifically ask for multiple strategies, and students consistently used "because" in their written explanations.* These insights arose as teachers *noticed* aspects of other teachers' sets of students' work, and typically generated discussion as they *wondered* how students had been 'trained' to solve the task in more than one way or include a conceptual explanation even when not explicitly prompted by the task (i.e., how these norms had been developed in the classroom).

Challenges arose in teachers' reflections and noticings on their own and other teachers' sets of students' work. Common challenges included: resources for high-level tasks (noticing that the task was not high-level); evidence of students' lack of a conceptual understanding (noticing that students could not solve the cognitively challenging aspects of the task); and the quality of written explanations (identified by a teacher regarding his/her own students' work, and relating to the low-quality of verbal explanations during the lesson). Challenges regarding teachers' own student-work sets were sometimes noted by the teacher initially, and sometimes arose in comparison to other teachers' student-work. For example, the challenge of improving students' verbal and written explanations was identified by several teachers in the first follow-up meeting. Teachers noted students' difficulty in providing verbal explanations during class, and how this was evident in students' written explanations on the student-work samples. While reviewing a set of students' work from another teacher, one teacher reflected back to her own students' explanations: "Even though students were writing 'explanations,' the explanations only involved procedural steps." A discussion ensued regarding the difference between procedural and conceptual explanations, and how to develop students' ability to create conceptual explanations, especially in classrooms where cognitively challenging mathematical work and thinking were new experiences for teachers and students. Teachers collectively took this issue on as a group, brainstormed ideas, and returned to the second follow-up session eager to share new instructional practices (e.g., having a student provide a verbal explanation as another student writes what is being said, then both students revise the explanation to clearly communicate the mathematical thinking; prompting students to explain their thinking as if they were talking or writing to someone in a younger grade or a classmate who was absent from the lesson).

Implications for Instructional Leaders

In addition to identifying successes and challenges in implementing tasks, the student-work collection also served diagnostic purposes, identifying successes of the professional development initiative and pathways for improvement for the teachers as a group (analogous to instructional leaders using student-work diagnostically within a department, school, or district). First, after participating in the professional learning experiences of solving cognitively challenging tasks, participants appeared to be successful in selecting high-level tasks (85% of tasks were high-level) and in supporting students' exploration of cognitively challenging instructional tasks (79% of tasks that began as high-level were maintained during instruction). These sets of students' work provided evidence that students solved tasks in a variety of ways, and used manipulatives, diagrams, and representations to support their thinking. Samples of student-work had unique strategies and ways of thinking, and did not look uniform (i.e., as though students had been directed on how to solve the tasks). If the teachers were within the same school or district, the instructional leader would want to capitalize on the fact that most teachers could identify and implement a high-level task successfully, and base future professional development initiatives on this foundation.

Second, high-level task demands that declined during implementation, even from a *Potential of the Task* score of 4 (i.e., the task explicitly required explanations of students' high-level work and thinking) to a *Task Implementation* score of 3 (i.e., implicit evidence of students' high-level thinking), could often be attributed to non-existent or low-quality written explanations. This indicates that, while teachers' experiences solving cognitively challenging tasks as learners enabled them to implement high-level tasks in ways that encouraged multiple strategies and representations, teachers did not appear to gain ways of developing students' mathematical explanations. As a next step, instructional leaders would want to provide opportunities for professional learning experiences specifically focused on supporting students to clearly explain their thinking, verbally and in writing.

Third, teachers with curricula lacking in high-level tasks often used open-ended assessment items or tasks directly from the workshop for their student-work collections. As teachers identify the need for curricular materials containing high-level instructional tasks, an instructional leader would want to provide teachers with increased access to curriculum and resources containing such tasks. However, research cautions that simply providing teachers with new or revised curricular materials does not guarantee that the materials will be implemented as intended (e.g., Remillard & Bryans 2004). In the next section, we discuss how instructional leaders can use collections of students' work to diagnose and support teachers' implementation of a cognitively challenging mathematics curriculum.

Using Student Work to Assess Curriculum Implementation: The Mathematical Practices Implementation Study

Another approach to supporting instructional change involves implementing new or revised mathematics curricula. The success of such an implementation presents a number of challenges for administrators, teachers, and students. At the high school level in particular, curricula that feature an abundance of high cognitively demanding tasks and support discourse-based pedagogies require substantial systemic support, and even with that support such curricula often lose traction when key personnel leave the district (Senk & Thompson, 2003; St. John et al., 2005). The Mathematical Practices Implementation (MPI) study is analyzing the implementation of one such curriculum, the Education Development Center's "Center for Mathematics Education" (CME) Project.[4] The goals of the study are to measure the extent to which implementation of the CME Project materials reflected the high cognitive demands of the curriculum, and to identify key factors that support or inhibit the principled implementation of the curriculum. By principled implementation, we mean teaching that is faithful to the overarching principles and mathematical habits of mind upon which a curriculum is built (Cuoco, Goldenberg, & Mark, 1996), moving beyond simpler measures of textbook use to capture the ways in which the curricular tools are used in teaching. To understand the extent to which teaching represents principled implementation, the MPI study seeks to measure a number of aspects of teaching practice, including teachers' mathematical knowledge for teaching, their understanding of the mathematical habits of mind, the influence of teacher professional development on implementation, and the ways in which classroom norms and practices support student engagement and learning. This analysis considered specifically the relationships between the potential of the tasks teachers select for students to work, the implementation of those tasks as measured by the student work, and the expectations teachers have for the work students will produce on those tasks.

Participants and Data

We identified two large metropolitan districts that were adopting CME Algebra I at the start of the study, and recruited 50 teachers at 12 school sites to participate in the study. These twelve school sites were housed in ten districts across five states, in or adjacent to urban centers serving a diverse student population. Teachers ranged in experience from 0 to more than 20 years of experience (see Table 2); 98% held a secondary mathematics certification, with the remaining 2% holding a certification in a secondary field other than mathematics. Teacher-participants at each site committed to submitting four days' worth of assignments completed during class time twice a year (fall and spring) across the first two years of implementation.

Analysis

Project personnel scored the student-work samples using the IQA rubrics for *Potential of the Task, Task Implementation*, and *Teacher's Expectations* (Figures 1 and 2). Raters that demonstrated 85% agreement or better on test items rated the project data samples. The project also assessed the academic rigor of the curriculum materials, rating each section of the Algebra I text using the IQA *Potential of the Task* rubric. These ratings were used to compare the potential of the specific tasks teachers selected for students to the section's potential in general (i.e., were teachers selecting the high-level tasks available in each section of the curriculum?). The study is presently at the end of its first year of data collection, with the first two sets of students' work rated for participating teachers. At present, the first year data set contains 85 discrete student work sets, which were analyzed for this study.

Results

Two important trends emerged from the student work ratings thus far that have implications for the support of a new curriculum implementation. The first trend relates to the potential of the tasks that teachers implemented with their students. Across the data set, the tasks teachers used with students almost universally reflected a lower *Potential of the Task* rating as compared to the text sections to which the assignments corresponded. Of the 85 student work samples rated, 67% were rated as a *Potential* of 2, indicating that students executed a clear mathematical procedure without providing implicit or explicit connections to meaning. This indicated that teachers in their first year implementing the new curriculum overwhelmingly selected

Table 2: Years of Experience for MPI Study teachers, Year 1

0 years (first year teaching)	8%
1 year	10%
2-5 years	28%
6-10 years	28%
11-15 years	18%
16-20 years	2%
More than 20 years	6%

[4] The second author serves as a Co-Principal Investigator on the MPI Study, funded by the National Science Foundation.

procedural tasks, choosing not to implement higher cognitively demanding tasks that asked students to make sense of the underlying mathematical ideas. While this finding may be disappointing, it is not necessarily unexpected given prior research regarding teachers' selection of high and low cognitively demanding tasks (e.g., Stein & Lane, 1996). We also noticed that the bulk of high cognitively demanding tasks that teachers selected declined in rigor with respect to implementation and sought to identify reasons that might explain these declines.

This investigation led to our second finding, which was particularly illuminating with respect to principled implementation. For the 22 tasks that began at a high level (3 or 4 on the IQA scale for *Potential of the Task*) and declined in implementation (1 or 2 on the *Implementation* scale), we also looked at the scores for the teacher's expectations for the assignment. Scores of 1 and 2 on the *Teacher's Expectations* rubric represent expectations that are either non-mathematical in nature, such as neatness or clarity, or that are not complex thinking skills, such as short answers or accurate application of procedural steps. Although the rigor of these expectations is appropriate for tasks of a low potential, they are not a good fit for tasks of higher potential. We defined tasks with a *Potential* score of 3 or greater and an accompanying set of *Teacher's Expectations* scores of 2 or less as *Potential-Expectation* mismatches. If the *Potential* and *Teacher's Expectations* were both low (2 or less) or both high (3 or greater), we identified this as a *Potential-Expectation* match.

Across the first year data set, 86% of tasks that declined from high to low cognitive demand also featured a *Potential-Expectation* mismatch. Only in 3 of 22 cases did the implementation of a task represent complex thinking despite a lack of explicit expectations for complex thinking as measured by the rubric. This suggested an important relationship between the rigor of the teacher's expectations and students' engagement with the task: a low rigor of expectation engenders student work that systematically does not attend to the high cognitive demand aspects of the task.

There were also some promising signs of change in the first year of the study. Between the fall and spring data collections, the average score on the *Teachers' Expectations* rubric rose from 1.65 to 1.96. This suggested a trend in which the rigor of teachers' expectations increased over the course of the first year of implementation. This finding also indicated that explicit conversations about the ways in which the expectations can support students' engagement in high cognitively demanding tasks (perhaps through emphasis on the mathematical habits of mind) might further support teachers in moving towards a principled implementation of the CME Project Algebra I curriculum. Supporting teachers in being able to describe and set expectations for rich mathematical thinking is also likely to support students in being more successful in engaging in the Standards for Mathematical Practice, a key aspect of the new multi-state assessment systems.

Implications for Instructional Leaders

Translating curricular resources into instruction that results in deep student learning can be a challenging task for teachers, particularly with curricula like the CME Project that support ambitious visions of teaching. This preliminary analysis of student work from teachers in their first year of implementing CME Algebra I suggest some specific ways in which instructional leaders might support such a curriculum implementation. The first area of support is the selection of tasks from a section of the text in which to engage students. To support teachers in selecting tasks that better represent the cognitive demand of a given text sections, instructional leaders and teachers might work on the mathematical tasks in the section together and discuss ways in which to support students in thinking through high cognitively demanding tasks. Particularly for districts that are moving from curricula with a heavier skills emphasis, teachers may be more disposed to select the familiar procedural tasks from a text section. Discussing the task selection process with teachers and understanding their decision-making process could help instructional leaders support a long-term systemic implementation of ambitious curricula.

Second, instructional leaders and teachers might find a benefit in co-designing expectations for students that support high cognitively demanding work. This work could be done either with respect to specific mathematics content, or in the form of general rubrics that teachers might apply across a broad range of student work. Working with teachers to set and communicate these expectations can help to send important messages to students that thinking and reasoning is a valued part of their mathematical work, rather than simply correct answers or properly executed procedures.

In the next section, we generalize across the two specific studies presented herein to discuss how instructional leaders might use collections of students' work and the IQA rubrics to support their work more broadly.

Using Student Work in Instructional Leadership: Initiating Conversations and Diagnosing Next Steps

The specific studies discussed herein represent two situations of instructional change that will be common to many districts embarking on successful implementation of the CCSSM: professional development to support teachers' initial attempts in using cognitively challenging instructional tasks, and the implementation of new or revised curricula consisting primarily of cognitively challenging instructional tasks. In this article, we have presented how the analysis of sets of students' work can be used to diagnose the success of each of these efforts and provide instructional leaders with data to inform instructional improvements. As applicable beyond our specific studies, mathematics teachers participating in instructional change efforts (i.e., sustained professional development experiences or curriculum implementation) consistent with CCSSM can be asked to collect samples of students' work to tell the story of their successes and challenges in implementing cognitively challenging instructional tasks. Across a school or district, collections of students' work can then serve as artifacts for instructional leaders to initiate conversations about instructional practice amongst teachers and to identify pathways for instructional improvements.

Initiating Conversations

In our work, we have utilized a variety of formats in leveraging students' work to initiate conversations with teachers. One method is to use an open case story format (Hughes, Smith, Boston, & Hogel 2008), where teachers share their experiences implementing high-level tasks, use the student-work samples as evidence of their successes and challenges, and allow other teachers to share their noticing and wonderings. Another method is to explicitly use the IQA rubrics to guide the conversation. Teachers can use the *Potential of the Task* rubric to identify the level of cognitive demand of the task and to identify specific aspects of the task that make it high-level. They are then asked to consider, "What is the evidence that students engaged with the high-level demands of the task?" and to use this evidence to score the set of students' work (holistically) based on the IQA *Task Implementation* rubric. Similarly, teachers can be asked to compare the score for *Potential* of the tasks used for instruction with either a) the *Potential* of the tasks featured in the corresponding section of the curriculum, to determine whether they are capitalizing on the cognitively challenging tasks featured in the curriculum or b) the rigor of their *Expectations* for the task, to identify *Potential-Expectation* matches or mismatches. Since teachers are looking across a set of responses rather than at individual student's work and thinking, commonalities often arise that can be attributed to the nature of instruction rather than to the students' mathematical thinking or ability.

In this way, reflecting on students' work (from their students and from other teachers' students) serves a self-diagnostic purpose, where teachers identify aspects of instructional practice that support or inhibit students' opportunities to engage in high-level thinking and reasoning. Instructional leaders can use collections of students' work to initiate similar conversations with teachers, where insights for instructional improvement are identified by the teachers themselves.

Identifying Pathways for Instructional Improvement

Analyzing collections of student work can also serve diagnostic purposes for instructional leaders, by considering, "What does the collection of students' work indicate about the quality of instruction and students' learning opportunities in my department, school, or district?" Using the IQA rubrics specifically, instructional leaders can use collections of students' work to address questions about:

- **Instructional tasks:** Are teachers using high-level tasks for instruction? Are teachers choosing the high-level instructional tasks featured in their curriculum?
- **Task implementation:** Are teachers implementing instructional tasks in ways that maintain the cognitive demands and mathematical purposes of the tasks? What are teachers' specific successes and challenges in implementing high-level tasks?
- **Classroom norms and practices:** What opportunities do students have to demonstrate and explain their mathematical work and thinking in writing? What representations are students provided opportunities to use? What counts as an explanation?
- **Teachers' expectations:** What is the level of rigor of teachers' expectations for students' mathematical work

and thinking? Are these expectations aligned with the tasks embedded in the curriculum?

In this way, analyzing collections of student work supports instructional leaders in identifying pathways for instructional improvement.

Of course, there are important questions regarding the quality of classroom instruction that collections of students' work cannot answer. For example, what types of questions are being asked? What is the quality of mathematical discourse? How do the teacher-student and student-student interactions support students' mathematical learning? These questions might best be addressed through focused classroom observations and could complement discussions around students' work.

Conclusion

Many school districts will need to implement professional development initiatives and new or revised curricula to enable teachers and students to meet the expectations of the Standards for Mathematical Practice (Cobb & Jackson, 2011). As supporting instructional change in the mathematics classroom is particularly challenging, especially with secondary teachers, school leaders will need to take an active role to guarantee the success of these efforts (Stein & Nelson, 2003). The analysis and discussion of collections of students' work using the IQA rubrics can provide principals, curriculum supervisors, and department chairs with reliable, valid, and mathematically rigorous tools to engage teachers in discussions and collect diagnostic data to monitor change. The IQA students' work rubrics provide valuable tools for instructional leaders by: identifying aspects of instructional practice that matter in terms of students learning; identifying areas of instructional improvement at the scale of a school or district; aligning well with the Standards for Mathematical Practice; and being well-suited for assessing professional development and curriculum implementation efforts. Moreover, the collection of students' work represents a way of discussing instructional practice that does not hinge on the availability for observations, provides a permanent artifact of practice that can easily be shared for analysis and discussion within a professional learning community, and is easier to collect and less intrusive to teaching than group observations or videotaping. Students' work also backgrounds the teacher's actions, making it a safer space (than video or observation) to discuss the successes and challenges encountered in one's own classroom. Within group of teachers, collections of students' work can serve as evidence of practice that is not subjective or reliant on teachers' recapping of events in a lesson, which can help to avoid judgment or debates over what happened. Most critically, discussing students' work from teachers' own classrooms positions teachers as decision makers in the process of instructional change, providing opportunities for collective- and self-reflection on teaching practice. Since the work was generated by students within their own school, district, or region, teachers often come to realize the mathematical capabilities of their own students by analyzing students' work from other teachers' classrooms. Collectively, teachers identify common issues and challenges in enacting ambitious instruction and construct pathways for improving their practice in ways that better support students' learning. ✪

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The Mathematics Excellence Partnership: Developing Professional Learning Communities

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Abstract

The Mathematics Excellence Partnership project was a professional development project aimed at supporting the development of 22 high school teachers of mathematics, including special education and bilingual teachers. In this paper, we share our school-based, bottom-up, collaborative design that supported the development of professional learning communities.

Achieving fundamental changes in teachers' content knowledge and instructional practices that influence student learning and performance requires new approaches to professional development (Bay-Williams, Scott, & Hancock, 2007; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Desimone, 2009; McLaughlin & Talbert, 2006). These new approaches entail more complex strategies that go beyond one-day professional development in which an expert in the field provides a workshop on a particular topic. Instead, these professional development approaches should be teacher-driven and shared collectively by all stakeholders. Key characteristics of effective professional development include, but are not limited to: a commitment to content and standards, such as the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* (2000) and more recently the *Common Core State Standards for Mathematics* (Common Core State Standards Initiative [CCSSI], 2010); the use of assessment data to ascertain relevant learning and pedagogical actions; professional activities that span over time; and adequate time for teachers to engage in professional activities. Regardless of the design of the professional development, the goal is assurance that *all* students learn mathematics.

Toward meeting this goal, Loucks-Horsley, Stiles, Mundry, Love, and Hewson (2010) argued, "When a school community has a shared commitment to high standards for all students, it is better prepared to take an honest look at student learning data and is more likely to experience dissatisfaction with results that fall short of its commitments, rather than complacency, resignation or defensiveness" (p. 34). Therefore, to foster among teachers a level of shared commitment to high standards, a professional learning community is needed to provide learning opportunities that benefit, support, and sustain teacher development and student learning overtime. According to Fullan (2005), sustaining teacher development in such a collaborative culture requires building a collective competence that "is the daily habit of *working together*, and you can't learn this from a workshop or course. You need to learn it by doing it and getting better at it on purpose" (p. 69). In the mathematics education community, *doing it and getting better* involves a major focus on advancing teachers' pedagogical practices by targeting particular mathematical content knowledge (Cohen & Hill, 2001; Hill & Ball, 2004). Consequently, the context for this aspect of development necessitates positioning "teachers' knowledge, build[ing] on their questions, and help[ing] and support[ing] them

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in evaluating their beliefs, and sometimes changing deeply embedded behaviors" (Weinbaum et al., 2004, p. 17).

The purpose of this paper is to report on a university-school partnership that provided professional development opportunities and implemented activities for enhancement of mathematics teaching while establishing a culture of collaboration. Specifically, we will give attention to our actions that supported the development of professional learning communities. We begin by presenting related literature reflective of the perspective that to foster positive collaborative work "flexible and alternative methods for continuing education and self-improvement [should be] instituted to support ongoing learning of mathematics and mathematics education" (NCTM, 1991, p. 184). Next, we describe the professional development project, including the school context. This discussion provides some sense of how the professional development partnership involved a bottom-up collaborative approach in which the participating teachers took the lead in constructing their own professional learning activities.

Related Literature

A Vision for Professional Learning

In 1991, NCTM asserted that teachers needed to demonstrate "the value of mathematics as a way of thinking and its application in other disciplines and in society" (p. 104). Recent research in mathematics education reform suggests that embedded in this assertion is the mathematical idea that learning "is not merely accumulating facts and information but also a way of shaping our beliefs, ideas and lives" (Boaler, 2010, p. 1). It is a way of helping students and teachers think about mathematical sense making and reasoning that moves them beyond the historical stance in which students ingest considerable amounts of mathematics facts, and yet experience difficulty applying this information to new and more practical situations (Beswick & Dole, 2001; Boaler, 2008; Mansilla & Gardner, 2008).

Despite these reform messages, a succession of research studies suggests that pedagogical practices continue to follow a traditional path: the teacher checks homework, demonstrates problems for new skills, and assigns students a series of similar problems from the mathematics textbook. In this familiar scenario, the teacher seldom focuses on developing the underlying conceptual features of problems solved by students (Hiebert et al., 2003; Rowan, Harrison, & Haynes, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003) "Today, the information revolution and the ubiquity of search engines have rendered *having* information much less valuable than *knowing how to think* with information in novel situations" (Mansilla & Gardner, 2008, p. 19, italics added). To support students' thinking in novel situations, teachers must provide meaningful contexts in which students may utilize their previous knowledge and acquire new knowledge. In this context, the mathematics is experienced in a dynamic way, a more fluid body of knowledge that allows for reaching conclusions and solving problems using a variety of methods and approaches (Mansilla & Gardner, 2008; NCTM, 2000).

Research has demonstrated that these classrooms, which embody problem solving and collaborative grouping, tend to have positive effects on students' mathematical disposition and learning (Boaler, 2008, 2010; Steffero, 2010). Fundamental to this finding is the notion that problem solving and collaborative work need to engage teachers and their students in a rigorous intellectual process in which making sense of mathematics content is pertinent to their lives. "Teaching mathematics requires an appreciation of mathematical reasoning, understanding the meaning of mathematical ideas and procedures, and knowing how ideas and procedures connect" (Hill & Ball, 2004, p. 331). An essential way to influence the teaching of mathematics in classrooms is through quality professional development activities, which focus on mathematical knowledge for teaching (Hill & Ball, 2004). Furthermore, in order to develop the pedagogical skills necessary to convey mathematics in this way, teachers need professional development experiences that will provide them exposure to learning mathematics in this manner.

The Role of Professional Development in Developing Mathematical Knowledge

The most compelling argument for providing teachers with professional development experiences in which the focal point is mathematical knowledge for teaching is highlighted in research by Silver (2003), Sowder (2007), and Supovitz and Turner (2000). A valuable presumption from this research is that effective professional development may influence teachers' understanding of content and subsequent pedagogical practices. Sowder (2007) stated, "Professional development provides an opportunity for teachers to learn more mathematics, even when the focus is on student thinking or curriculum or classroom events" (p. 163). Further, professional development should involve reform-oriented activities and standards (Garet, Porter, Desimore, Birman, & Yoon, 2001). In a report on the status of professional development in the U.S. and abroad, Darling-Hammond and her colleagues (2009) summarized the available research, revealing two key findings that were relevant to the project presented in this paper. First, "sustained and intensive professional development for teachers is related to student achievement gains" (p. 5). Second, "effective professional development is intensive, ongoing, and connected to practice; [focusing] on the teaching and learning of specific academic content" (p. 5). Such is the premise of professional learning communities, which may be the best way to attain truly momentous, broad range progress in teaching and learning (DuFour, Eaker, & DuFour, 2005).

The Professional Learning Community

Emerging from the literature are two relevant claims regarding professional learning communities. First, the hallmark of a professional learning community is the focus on learning, collaboration, and accountability (Banilower, Boyd, Pasley, & Weiss 2006; Darling-Hammond et al., 2009; DuFour, 2004; DuFour et al., 2005; Sparks, 2005; Sowder, 2007). With the support of their school leaders, teachers learn to work collaboratively through professional learning communities to advance pedagogical practices, improve student learning and performance, and hold themselves responsible for learning outcomes. These features require members of the learning community to organize their learning around three essential elements: what students need to learn, what indicators suggest that students have learned, and how to address the needs of students who are struggling to learn (DuFour, 2004). This argument is consistent with NCTM's Teaching Principle (2000) that states, "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 16). Moreover, as teachers embark upon attending to these features, they need to work as a group, developing an understanding of the importance of sharing and researching ideas, activities, and materials. Senge et al. (2000) offered this abridgment, suggesting, "A strong professional community encourages collective endeavor rather than isolate efforts" (p. 327).

A second claim emerging from the literature on professional learning communities is when professional activities are developed around subject matter chosen by teachers and last over a long period of time then the community's activities are more likely to be carried out by the teachers in their classrooms (Darling-Hammond et al., 2009; Graham, 2007). Darling-Hammond and colleagues' examination of research on teachers' professional relationships suggested that "schools where teachers were relatively more involved in educational decision-making [and were granted] blocks of time to meet and plan courses and assignments together" (p. 11) were more successful at their teaching and at solving problems of practice, thus providing evidence of the potential impact of professional learning communities.

The Mathematics Excellence Partnership

The *Mathematics Excellence Partnership* (MEP) involved a professional learning community undertaken by a university and Hayfield High School (HHS), which utilized a school-based, bottom-up, collaborative design focusing on mathematics curricula and student learning. In this section, we present the theoretical framework that informed the project, an overview of the project and its activities, a description of the school context, and a description of our development of a shared vision among project partners.

Theoretical Framework

Two theoretical perspectives served to inform our work: sociocultural practices and cognitive and social development. First, the professional development context for this work utilized sociocultural practices as originally advocated by Vygotsky (1978, 1994) and later by the work of Davydov (1990, 1995), Goos (1999), Kozulin (1998), and Wells (1999). Their research suggests that social practices need to be developed to engage learners, teachers, and students in activities that not only promote knowledge acquisition, but also to engage them in activities that further their intellectual development. Therefore, during the professional development sessions, opportunities were provided for social interaction aimed to benefit the teachers' goals and objectives about what they deemed as effective for improving their learning and understanding (e.g., developing a deeper knowledge-base of slope and improving student learning and performance on that concept).

Second, research supports the idea that collaborative group work influences cognitive and social development (Cohen, 1994; Jennings & Di, 1996). Research further illustrates that teachers' knowledge and classroom practices are readily influenced by professional development that focuses on content knowledge and active learning (Cohen & Hill, 2001; DuFour et al., 2005; Garet et al., 2001; Hill, 2004).

Thus, critical to the process of establishing a professional learning community at HHS was the employment of group dynamics that fostered interdependence, promoted shared commitment, and incorporated activities and discussions that sustained inquiry and debate (Cohen, 1994; Osana & Folger, 2000). The idea was to provide a professional learning community at the school level that subsequently influenced what happened in the classroom.

Professional Development Context

The Mathematics Excellence Partnership took a multidimensional approach in order to develop and hone a successful university-school collaboration. A major goal of the partnership was for a local university to collaborate with HHS's mathematics teachers and administrators to improve pedagogical practices and student performance. More specifically, this project aimed to achieve three important outcomes: to increase students' performance on the Comprehensive Assessment System exam (CAS) by targeting the mathematics disposition of the teachers and their students; to increase the number of students taking honors-level mathematics courses in grades 9–11; and to increase the number of students taking AP Calculus. In its collaborative role with HHS, the university faculty members monitored student progress, contributed ideas and classroom resources, and provided research-based insights when changes were needed or requested by the participating teachers.

The professional development activities consisted of three different types of sessions conducted over a four-year period: monthly sessions, biweekly sessions, and summer institutes. The monthly sessions and summer institutes involved all 22 mathematics teachers of grades 9-12 at HHS, including special and bilingual educators. These monthly sessions covered general mathematical topics that cut across the various content strands, from developing algebraic thinking to understanding practical applications of Calculus. The biweekly sessions involved a group of six core teachers who taught honors sections and AP classes. These core teachers convened for two hours per session to discuss and develop activities that would improve the teaching and learning of mathematics for high-performing students. The monthly and biweekly sessions were activities suggested by the collective group of teachers based on their perceptions of their pedagogical practices and needs. The summer institute consisted of a three-day mathematics and technology-based seminar, which included some general pedagogical topics such as classroom management and collaborative grouping. Often, the institute seminars were completed in collaboration with the district's instructional technology department and the university business school, as well as the university mathematics department. The summer institute sessions were driven by suggestions from the teachers, the project's collaborators (see Table 1), and the school district technology support specialists.

Table 1. Overview of Project Collaborators and their Contributions

Collaborators	Contributions
School of Management (SOM)	SOM finance and operations faculty instructed HHS teachers in uses of mathematics in business during the Summer Institute. Sessions included investing in stocks, real-world business problems that involved linear programming, and financial applications.
School of Arts and Sciences Mathematics Department	The faculty members provided expertise as advisors to the Partnership and were instructors during the Summer Institutes. Sessions focusing on hands-on geometry and advanced number sense were a popular request of HHS teachers.
Undergraduate Mentors	Students of color from the School of Management mentors assisted honors-level mathematics teachers in teaching and motivating HHS students throughout the school year. Mentors played a role in improving students' attitudes toward mathematics and understanding its connection to business. Mentors assisted in the after school program to provide homework help in mathematics, other subjects, and SAT Prep.
Project GEARUp	Graduate Assistants and mentors worked with the students throughout the year as well as in the after school program tutoring and assisting students with mathematics homework and SAT Prep.
School District's Office of Instructional Technology	The Office of Instructional Technology provided technological assistant to the teachers during the academic year as well as during the Summer Institutes, focusing on a district-wide initiative to integrate technology into mathematics.

Appendix A presents selected examples of the professional development activities in which the teachers participated during the sessions. As illustrated in the appendix, the mathematical topics and activities fell into three broad categories: analysis of assessment data, mathematics content, and pedagogical activities, both general and content specific.

School Context

HHS is an accredited public secondary school in an urban area and noted for having earned awards and recognition from state and national organizations. One of these acknowledgments was the Bronze Medal for "America's Best High Schools" ranking from *U.S. News & World Reports*. During the course of our project, the student body consisted of 1200 students, as well as 110 staff members, 80 of whom were teachers. Of the students, 42% were African American, 46% were Latino, 6% were Caucasian, 6% were of Asian descent, and about 1% was of Native American descent. The staff had very different demographics, as almost 66% of them were Caucasian. The remaining staff consisted of 18% African Americans, 15% Latinos, and 2% Asian Americans. Approximately 18% of the students were enrolled in special education, while 12% received bilingual education.

The *No Child Left Behind Act* of 2001 (NCLB, 2002) required that all schools make *adequate yearly progress* (AYP) towards all students becoming proficient in the core subject areas of English/Language Arts (ELA) and mathematics. At the time of this project, HHS had recently achieved a *Performance Rating* of "High" in ELA and "Moderate" in mathematics as well as an overall *School Improvement Rating* of "On Target" with the school's Restructuring Status goals.

During the course of the project, HHS subdivided into three small learning communities in order to provide more personalized attention to its students. Within this structure, the school operated on block scheduling and offered six Advanced Placement (AP) courses. Preparation for the CAS, the state's graduation proficiency assessment, and for the SAT was offered after school through various tutoring programs. In order to further prepare students for college and future careers, the school offered academic pathways in business and technology, health professions, media, arts and communication, law and government, and education.

Development of a Shared Vision

During the first phase of the project, we worked judiciously with HHS teachers and administrators to establish dialogue and a shared vision. This vision involved creating a professional learning community, which would support the teachers' aspirations for improving their knowledge of teaching mathematics that subsequently influences student learning and performance. This process consisted of identifying and combining the activities necessary to realize the vision, which included building credibility and trust, establishing benchmarks to target progress, recruiting college student mentors, and then identifying responsibilities. For the second phase of the project, we concentrated on the development of pedagogical content knowledge, and implementation of pedagogical strategies and techniques, which included the analysis of CAS performance data. A common thread throughout this process was the enhancement of a positive and more collaborative disposition or attitude toward mathematics teaching and learning. As effective practices and techniques emerged, whether they were centered on student performance or on successful teacher implementation of the use of technology tools, we worked to further improve them. When we found techniques that did not work, we modified them until we achieved a level of satisfaction agreed upon by the participating teachers.

Developing the Professional Learning Community

The teachers at HHS wanted opportunities for active participation and learning in designing their professional learning community, while lessening the disjointed arrangement that existed in the past. For example, in the past an expert for a particular topic (e.g., mathematics academic language) would provide a workshop on site with little, if any, follow-up, continuation, or discussion among the teachers. Furthermore, the decision of topics to be covered did not take into account the opinions and ideas of the mathematics, special education, and bilingual teachers. The participants of this project envisioned a professional learning community whose cultivated activities would be inclusive of their voices in which they worked together to build a culture of collaboration to make certain that their students learned mathematics. This practical perspective is consistent with a common theme highlighted in studies about effective professional development, which suggests that it is essential for teachers to engage in characterizing their professional needs (Darling-Hammond et al., 2009; DuFour et al., 2005). An important notion is to attach professional development activities to student performance and instruction, and to implement strategies that are fundamentally associated with the day-to-day practice of teaching and learning (Marzano, 2003; Marzano, Waters, & McNulty, 2005; Hawley & Valli, 1998). It seems that the teachers' decision to focus on student performance data influenced their teaching practices in content and pedagogy. The next section demonstrates this insight.

A Glimpse into the HHS Professional Learning Community

During the third year of the project, six professional development monthly sessions focused on analysis of student performance on district and state examinations. During the first sessions, the analysis focused on the district's previous academic year's final exams. These examinations were based on the mathematics students engaged in their courses for the entire academic year. A standard item analysis report was given to each teacher by content (e.g., Algebra 1 or Geometry). Analysis included information about how HHS students performed across the content as compared to the district performance. Low scoring items were identified as any test item in which less than 50% of students taking the test received the correct answer. Those items were then cross referenced according to correct answer, mathematics standard, topic, concept or skill, and the pacing guide and textbook chapter in which the concept or skill was addressed. The teachers self-selected to meet in small groups clustered around their learning communities. In their small groups, they identified specific skills and knowledge that students lacked, determined why students were unable to master these skills, and assessed and developed strategies to achieve instructional change that would help students make sense of the concept(s). Several of the teachers reported that this format allowed them to address basic-skill errors and to think through how the skill might be presented to assist students in making sense of the mathematics. They were able to immediately review the items and discuss strategies regarding why the students might have achieved the incorrect answer, as well as examine what they considered to be the appropriate teaching strategy to implement in the classroom.

For the next five sessions, data analysis included a comparison of HHS performance on the CAS across three years of the project. Data analysis for these sessions centered on student performance on the multiple choice, short answer, and open response items of the test. Each teacher received copies of the test and graphs that compared student performance at HHS, the District, and the State, which included Item Number vs. Percent Correct on Multiple Choice, Item Number vs. Percent Correct on Short Answer, and Item Number opposed to Average Score on Open Response. Each graph highlighted items whereby student performance was below the district and state results. Teachers examined those items in regards to content skill assessed and item complexity (i.e., cognitive demands and language complexity), developed descriptions that might explain student performance, and discussed and recommended a primary and alternative teaching strategy or technique to assist students in developing their understanding of the concepts inherent in the item. This aspect also included thinking through what the implications for teaching might be at other grade levels or subject areas. These sessions seemed to be constructive for many of the teachers. Several of the small groups continued to work after our departure.

We believe that our bottom-up, collaborative approach was key to developing the professional learning community. By focusing on teachers' attitudes about teaching mathematics with a critical eye on improving performance, a consistent effort was made in addressing how to improve students' performance on the CAS. As indicated, the teachers themselves initiated analysis of the test. The PD sessions, thus, rallied around the teachers' efforts and provided them with methods to translate the trends in the exam data into more effective instructional practices.

Emerging Tensions Around Teaching

It is important to note that the professional learning community overall functioned well as a culture group; however, on a few occasions tensions emerged. The tensions were not among the teachers, but rather between the teachers and the researcher/teacher educator. The researcher concentrated on larger scale needs of developing mathematical thinking and reasoning; for example, trying to think about how problem solving facilitates understanding of basic skills and increases performance on tests, in general. By contrast, the teachers focused more on the content, trying to relate it to what was being assessed on the tests, so that they could ensure that students would understand test questions and perform well. Also, the school district's pacing guides directed what was taught and when it was taught in the classroom. Often, this drove teachers' ideas of what they wanted for professional development.

Eventually, through discussion that focused on how to best meet the needs of their students, a common goal was reached: to improve student performance in mathematics as measured by CAS, the district's semester and final exams, and their classroom assessments.

From this discussion, we settled on a set of instructional approaches. The teachers agreed to examine students' work samples and the teaching implications, think more thoroughly about content knowledge, and allow their peers and the researchers to observe their teaching for critical comments that would improve their pedagogical practices. Some specific approaches were designing questions to understand students' mathematical thinking and reasoning and to develop a better sense of students' misconceptions and errors; rewriting textbook problems to be more open-ended and multi-layered, which included discussing the underlying mathematical structure; incorporating at least one problem on their weekly quizzes or tests that required students to explain in writing or through a drawing how they did the work, providing justification for their solution to the problem; and using concrete and visual manipulatives for mathematical representations that would assist students in their mathematical sense making and thinking. The teachers worked together to learn how to develop these approaches through conversations about mathematics educational research and professional literature and through demonstrations and modeling of mathematics concepts and ideas.

Conclusion

Over the four years of the project, the MEP team worked to create credibility and trust with the HHS professional learning community. A change in teachers' disposition and attitudes was observed from what once may have been skepticism to one that was completely engaged in teaching and learning, striving for new levels of excellence. Research has found that it is essential for teachers to be engaged in characterizing their professional needs for professional development to be effective, and this partnership confirmed this claim. HHS teachers appreciated the opportunity to be actively engaged and to have a voice, which led to a more cohesive sequence of professional development activities, focusing on pedagogical content knowledge, student performance, instruction, and the implementation of strategies that are fundamentally associated with the day-to-day practice of teaching and learning. The structure of the professional learning community created "contexts for teacher collaboration, provide[d] a focus for the collaboration, and provide[d] a common frame for interacting with other teachers around common problem. When teachers have opportunities to continue to participate in communities of practices that support their inquiry, instructional practices that foster the development of mathematical [disposition] can more easily be sustained" (NRC, 2001, p. 397).

One theme that was consistent between this project and similar ones is the realization that it often takes more than a program change to sustain improvement in academic achievement. "Educators can create professional learning communities, but there are no easy shortcuts for doing so. It will require a staff to find common ground and to exert a focused coherent consistent effort over time" (DuFour et al., 2005). Built on teacher leadership and university collaboration, the professional development discussed in this paper can support others in thinking about how to develop professional learning communities. Program change necessitates a change in disposition, attitudes, and relationships that calls stakeholders to commit to engaging each other in reform efforts in which the main goal is to improve the academic success of all students. ✪

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APPENDIX A

SELECTED EXAMPLES OF PROFESSIONAL DEVELOPMENT ACTIVITIES

Monthly Sessions	Core Teacher Biweekly Sessions	Summer Institutes
ASSESSMENT • Understanding CAS – analyzing CAS data to determine where students have gaps and discussing strategies to improve their performance • Understanding Trend Analysis of CAS results • Examining CAS data over three years to see gains and areas that continue to need improvement **MATHEMATICS CONTENT** • Developing Algebraic reasoning • Geometry **PEDAGOGICAL ACTIVITIES** • Using strategic questions to scaffold mathematical learning • Analysis of teacher-generated questions to promote mathematical learning • Multi-step problem solving – strategies to help students solve problems that require interpreting word problems • Using software that creates interactive visual representations of difficult mathematical concepts to improve student understanding	**ASSESSMENT** • Analysis of opened-response item analysis **MATHEMATICS CONTENT** • Analysis and discussion of student algebraic and geometry work **PEDAGOGICAL ACTIVITIES** • Research topics such as Beyond the Numbers • Reflecting on possible low expectations of students as written in the Atlantic Monthly article "Stereotype Threat," (Steele, 1999). • What makes for effective professional development • Developing mathematical thinking with effective questions • Analyzing teacher-generated questions to promote a positive disposition to do mathematics • Motivating student learning through strategic questions • Analyzing students' responses to teacher-generated questions to promote positive disposition • Classroom observations • Graphing calculator activities • Hands-on sessions, using Internet-based mathematical resources	**MATHEMATICS CONTENT** • Mathematical analysis of how to invest in the stock market • Linear Regression Analysis of stock market data • Developing algebraic thinking • **PEDAGOGICAL ACTIVITIES** • Mathematical representation with concrete models • Hands-on geometry • Assessing problem solving • Developing mathematical thinking with effective questions • Classroom management • Collaborative grouping **TECHNOLOGY-BASED PEDAGOGICAL ACTIVITIES** • Supporting Algebra and Geometry with Technology • Virtual Manipulative Activities (Algebra and Geometry): Web-based mathematical resources • Tour of online resources • One computer classroom management strategies • Graphing calculator activities • Graphing quadratic equations in vertex form

Using Participant Responses to Video of Coaching Practice to Focus Mathematics Coaching Programs

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Abstract

This article addresses what practices coaching experts and school-based coaches observed and did not observe when watching the practice of another coach. A coach is broadly defined as a person who works collaboratively with a teacher to improve that teacher's practice and content knowledge, with the ultimate goal of affecting student learning. Definitions of coaching knowledge, coaching texts, and standards for mathematics specialists identify three primary aspects of knowledge for coaching: developing teacher content knowledge, promoting reflection by the teacher, and negotiating professional relationships. When we asked school-based coaches and coaching experts to assess the practice of a novice coach depicted in a video-recorded coaching session, surprisingly few of the respondents commented explicitly on these three areas of coaching practice. This indicates that professional development for mathematics coaches can focus specifically on how these three big ideas *for coaching are enacted in practice. We offer recommendations for mathematics programs for focusing professional development with respect to these three practices.*

Coaching has become an increasingly popular mechanism used by school districts to improve mathematics instruction and, ultimately, student learning and achievement. Coaches are recognized as a particular type of mathematics specialist (National Mathematics Advisory Panel, 2008) whose work is defined, in part, by what model the specialist uses (e.g., Cognitive Coaching, Content-Focused Coaching, Instructional Coaching) and, in part, by how the specialist works with teachers in schools (e.g., being assigned as a full-time coach or a peer teacher who takes on some coaching duties). At present, standards and definitions for coaching knowledge and practice are just emerging, and the primary sources of information have been coaching books written by professional development providers that advocate for one approach or another.

Recently, Sutton, Burroughs, and Yopp (2011) published definitions for Mathematics Coaching Knowledge based on a study conducted with coaching experts as "a starting point for further analysis of mathematics coaching knowledge" (p. 14). These definitions cover eight domains of coaching knowledge: Assessment, Communication, Leadership, Relationships, Student Learning, Teacher Development, Teacher Learning, and Teacher Practice. Another resource is *Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and*

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Degree Programs (AMTE, 2009), aimed at identifying the "particular knowledge, skills, and dispositions needed by elementary mathematics specialists (EMS)" (p. iii). The authors of that document included mathematics coaches as a type of EMS (although it is worth noting that because the EMS Standards are intended more broadly, they do not address coaches specifically within the context of specialist). Both of these documents attend to the relationship between coach knowledge and coach practice by writing their definitions and descriptions in action form (e.g., "a coach knows how to . . ." and "a coach uses . . .").

We asked school-based coaches and coaching experts to assess the practice of a novice coach and write a brief summary of their opinions. It was our view that analyzing these reflections would allow us to understand how to focus the professional development in mathematics coaching that we were to offer a group of school-based coaches. When we analyzed these reflections, it was apparent that a minority of the respondents commented on three aspects of coaching practice consistently identified by leading coaching texts, definitions, and standards: developing teacher content knowledge, promoting teacher reflection, and promoting professional relationships. This was surprising because we believed that coaches would naturally notice and comment on these *big ideas* in coaching.

In what follows, we describe our methodology for gathering and analyzing these data, elaborate on our identification of the three areas for coaching focus, and provide suggestions for ways that supervisors and professional development providers can address these aspects of mathematics coaching.

Methodology

Participants

Data were gathered from two groups of participants: school-based, practicing coaches and coaching experts. Each of these groups represented a sample of convenience and will be described separately in the paragraphs that follow. We chose to include school-based coaches in this study because we anticipated that practicing coaches might develop views on coaching unique from those expressed in coaching texts.

School-based practicing coaches. The 21 school-based, practicing elementary mathematics coaches in this study were part of the Examining Mathematics Coaching (EMC) project, a research project that examined how a coach's knowledge influences coached teachers' knowledge and practice. School-based coaches completed the coaching assessment discussed in this article prior to participating in the EMC coaching knowledge professional development workshop. Approximately 16 months prior to taking this assessment, all school-based coaches received a one-hour orientation to the EMC project and to its coaching model.

Table 1 provides a description of the coaching backgrounds reported by these participants. Their experiences ranged from zero to 130 hours of training in coaching, involving multiple models of coaching. All participants had at least two years of coaching experience, except two, as noted, who had no years of coaching experience in the project.

Coaching experts. Six coaching experts were purposefully selected for participation in this study. These experts were chosen to represent different coaching perspectives. Two of the experts were authors of widely used coaching books. Other experts included a mathematics specialist researcher with numerous publications in the area; a mathematics specialist policy maker and author of numerous articles; a professional development researcher who had implemented coaching in several projects; and a professional development provider who had provided training to coaches across the nation.

Assessment

In June 2011, we asked school-based, practicing coaches to complete an online assessment featuring video of a novice coach interacting with two teachers, who were co-planning a lesson on stem-and-leaf plots that would be taught by the teachers as a team. The video shows the novice coach conducting a prelesson conference with the two middle school mathematics teachers as well as the postlesson conference that occurred after the lesson was taught. After viewing the video, the school-based coaches responded to the following prompt: "Please assess this coach's practices as depicted in the video and write a brief summary (under 200 words) of your opinion." Following this activity we asked the coaching experts to reflect on the same video.

Data Analysis

We analyzed the responses to the assessment prompt separately for the school-based coaches and the coaching experts. Using grounded theory (Corbin & Strauss, 2008), we developed concepts within each data set. The emergent themes from the two data sets were then compared and integrated to form overarching themes. Differences and

Table 1. Reported hours and types of coach training

Project Coach Code	Cognitive Coaching (hours)	Instructional Coaching (hours)	Content-Focused Coaching (hours)	Other Coaching Trainings (hours)	Total Training (hours)
1			55		55
2					0
3*					0
4		24			24
5	40	90			130
6			18		18
7				24	24
8			3		3
9					0
10					0
11*					0
12		12			12
13			3		3
14			15	10	25
15		12			12
16					0
17				40	40
18		12	40	10	62
19	40				40
20					0
21			15		15

**Project coaches 3 and 11 had 0 years of coaching experience at the time of this study.*

similarities in how the school-based coaches and the coaching experts viewed the coaching practice of the novice coach were then noted.

After the themes were identified in the data and the data were sorted, we reflected on the themes that are expressed in coaching texts, articles, and standards and compared the way our participants discussed the themes to the way they are discussed in coaching literature. We then reflected on the frequency in which our participants mentioned the themes. We report the culmination of whether or not a participant mentioned a particular theme and how the theme was discussed.

The following subsections of results follow a three-part format for each of three themes. First, we establish that a particular practice (theme) is expressed in coaching texts, articles, and standards. Second, we present the results from our participants under this theme. Third, we offer recommendations for coaching programs that wish to address this aspect of coaching practice.

Results

Developing Teacher Content Knowledge

The issue of developing teacher content knowledge is addressed in several leading coaching texts (and the distinct coaching models they describe), although the texts and models are not consistent in the way they suggest addressing it. *Cognitive Coaching* (Costa & Garmston, 2002) relies heavily on reflective questions to encourage teachers to refine knowledge bases. Instructional Coaching (Knight, 2007) suggests structured co-planning intended to help the teacher make connections among concepts. Content-Focused Coaching (West & Staub, 2003) features a coach who at times takes a more direct approach, actually pointing out important content to the teacher. *A Guide to Mathematics Coaching* (Hull, Balka, & Harbin-Miles, 2009) discusses a scenario in which a teacher who had not acquired an adequate background was coached on effective use of manipulatives with a focus that "not only improved the teacher's knowledge of instructional strategies but also increased her content knowledge" (p. 34).

Some of the differences in how coaching texts recommend addressing teachers' understandings of content result from assumptions about the knowledge base of the coach. The distinct models of instructional coaching (Knight, 2007) and cognitive coaching (Costa & Garmston, 2002) make no assumptions that the coach is more knowledgeable about the content than the teacher being coached. In contrast, the content-focused coaching model (West & Staub, 2003) and the mathematics coaching model (Hull et al., 2009) assume that the coach has a high level of content knowledge and is more experienced than the teacher being coached.

How a coach approaches teachers' understandings of content is also influenced by the various models' assumptions about relationships. Instructional coaching (Knight, 2007) emphasizes equality and reciprocity in learning among coaches and teachers. Similarly, cognitive coaching (Costa & Garmston, 2002) takes pains to caution against coach actions that resemble evaluation, supervision, and mentoring. The concern is that a coach who directly addresses content misconceptions runs the risk of being perceived as an authority in a hierarchy above the teacher. West and Staub (2003) point out that the relationship between coach and teacher is collegial, but the interaction "will not be symmetrical" (p. 17). In a case study of West's actual experience coaching a new teacher, West shows a willingness to give receptive teachers direct feedback and assistance. West does note the tension between refining a teacher's content and not undermining the coach-teacher relationship, and consequently West is careful to situate the discussion in the development of the lesson or in student learning to deflect some of the tension and avoid direct criticism.

EMS Standards (AMTE, 2009) address teacher content knowledge as the pedagogical knowledge needed for teaching mathematics. The EMS professional must know how and be able to:

- Utilize and build upon learners' existing knowledge, skills, understandings, conceptions, and misconceptions to advance learning.
- Create social learning contexts that engage learners in discussions and mathematical explorations among peers to motivate and extend learning opportunities.
- Use questions to effectively probe mathematical understanding and make productive use of responses. (AMTE, 2009, p. 6)

In this knowledge area, "learner" is defined to be either students or teachers (see footnote 2, AMTE, 2009, p. 3). The standards also suggest that specialists "diagnose mathematical misconceptions and errors and design appropriate interventions and decide whether, how and how far, to utilize specific oral or written responses from learners" (AMTE, 2009).

Established definitions of coaching knowledge (Sutton, Burroughs, & Yopp, 2011) convey the importance of developing teacher content knowledge, as shown in the following excerpts from six of the eight domains:

- **Assessment:** A coach knows how to assess teachers' needs—personal, instructional, content, and management—and how to assess and use teacher content knowledge and pedagogical content knowledge to inform and support teachers. (p. 16)
- **Communication:** A coach knows how to communicate in problem-resolving conversations. (p. 16)
- **Leadership:** The coach uses this vision and knowledge to inform her or his work with other school leaders, to bridge the gap that may exist between teachers' beliefs and their ability to implement instruction that reflects those beliefs, to earn trust with teachers and administrators, and to enhance teachers' content knowledge. (p. 16)
- **Teacher Development:** A coach knows how to ascertain a teacher's understanding of mathematics, teaching, and learning and is able to differentiate experiences to support an individual teacher's learning. (p. 18)
- **Teacher Learning:** A coach knows the myriad ways teachers know and understand mathematics content and the teacher's pedagogical and pedagogical content needs, which may or may not be recognized by the teacher. (p. 18)
- **Teacher Practice**: A coach knows how to discern teacher beliefs about mathematics teaching practice and holds a depth and breadth of knowledge of all types of practice and instructional resources for effective management and mathematics learning. (p. 18)

Our review of these coaching materials suggests that developing teacher content knowledge is an important coaching practice. The coaching models discussed here assert explicitly that a coach should take specific actions to uncover teachers' mathematics content knowledge and understandings and take action to improve or refine that knowledge and understanding. The variation in the models is largely based in how a coach addresses content deficiencies among teachers. The differing approaches are due, in large part, to differences in how the coaching models describe the coach-teacher relationship.

Participant responses in video assessment: Content knowledge. Only three of the 21 school-based coaches and three of the six coaching experts commented on the content knowledge of the teachers in the novice coaching video. The following is representative:

> (Coaching Expert A) It seems to me these teachers were not particularly knowledgeable about the math they teach, and the coach did not add much to their knowledge base or even expose the fact that their knowledge was not as robust as it may need to be.

Similar subthemes of uncovering teachers' understandings of content and advancing teachers' understandings of content were found in several of the participants' comments. For instance, Coaching Expert B noted that the novice coach "did not draw out or advance the mathematical or pedagogical understandings of the teachers."

In addition to noting the content issues, several participants revealed insights into what a coach would need to know to assess teacher content understanding. For example, the teachers coached in the video worked a stem-and-leaf plot task prior to the coaching session and compared their responses during the prelesson conference. Coaching Expert C noted this moment and wrote, "I became aware [that the teachers were not on the same page] when the teachers realized they had very different stem-and-leaf plots." School-Based Coach Z also made note of this realization, writing, "The commentary on decimals used in stem-and-leaf plots raised my interest here, and I wondered if more study was needed." These statements illustrate that teachers' understandings (and misunderstandings) about content can be exposed when the teachers discuss their solutions to the lesson's main task during prelesson conferences.

Some participants also suggested that the coach has a role in advancing the teachers' knowledge of the task's mathematical content. School-Based Coach Y noted, "The teachers seemed to debate [about the task], but the coach didn't address their misconceptions." In some instances, participants suggested what the novice coach might do in response to the teachers' understandings exposed by the discussion. For example, after noting that the teachers produced different and possibly inaccurate responses to the task, Coaching Expert C asserted, "I kind of wanted the coach to be more transparent in any concerns." Alternatively, Coaching Expert A offered a less direct approach to dealing with the teachers' inconsistent solutions, stating, "I think the lesson planning sessions need to be much richer and probably include some kind of 'rehearsal' to make sure all the players are clear about what the math concepts are."

Eighteen of the 21 school-based coaches and three of the six coaching experts did not mention the teachers' lack of content knowledge or the coach's lack of attention to the issue. While we cannot say that these respondents did not notice the issue, we can say that they, for whatever reason, did not include it in their assessment of the coach's practice. We argue that because teacher mathematical content knowledge is viewed as critical to effective mathematics instruction and because developing teacher content knowledge is central to several leading coaching models, the topic is fundamental to coaching practice.

Suggestions for coaching programs that wish to focus on developing teacher content knowledge. The results of our analysis and review of coaching texts, definitions, and standards offer specifics about what a coach might do to perform tasks associated with diagnosing and improving teacher content knowledge. Working the upcoming task with teachers and discussing solutions could be a particular way to create social learning contexts conducive to teacher content learning. Collaborating on a task and discussing solutions to the task offer neutral ground where the coach and teacher can learn together as colleagues.

In assisting coaches to recognize this big idea in coaching practice, professional development and support can be focused on how a coach can diagnose and attend to teacher content misconceptions or content deficiencies. Coaches can also be encouraged to engage with teachers in solving the problems that are central to the lesson and helping teachers identify the key mathematical concepts and learning objectives of the lesson. Teacher responses to coach questions give insights into the depth of a teacher's content knowledge and are starting points for attending to teacher content knowledge needs. Coaches can be encouraged to ask questions likely to reveal a teacher's depth of content knowledge.

Promoting Teacher Reflection

Reflection is also recognized as a key component of the coaching process through its inclusion in coaching texts, definitions, and standards. The coach is responsible for engaging the teacher in the process of reflection, although the purpose of this reflection varies across the different coaching models. For example, in cognitive coaching (Costa & Garmston, 2002), a coach's primary objective is supporting teachers in gaining skills in self-directed learning. Instructional coaching (Knight, 2007) states a similar objective with an emphasis on empowering the teacher to make decisions regarding the appropriateness and/or effectiveness of specific teacher actions. In contrast, the goal of reflection in content-focused coaching (West & Staub, 2003) is to "focus on what the teacher can do to assist the students' content-specific learning" (p. 17). Mathematics coaching (Hull et al., 2009) has a related goal of yielding appropriate interventions to support student learning. Despite these differences in purpose, the various models clearly communicate the importance of teacher reflection in the coaching process.

Beyond the purpose of reflection, the authors describing these models also give considerable attention to the coach's role in supporting this process. For example, Knight (2007) states, "[Instructional coaches] don't tell teachers what they should believe; respecting their partners' professionalism, they provide them with enough information to make their own decisions" (Knight, 2007, p. 47). According to Knight, "reflection is only possible when people have the freedom to accept or reject what they are learning as they see fit" (p. 47). Knight's assertions mark clear distinctions between a mentor, who might give specific feedback or praise for actions deemed appropriate or effective by the mentor, and an instructional coach, who facilitates teacher reflection on whether or not the teacher deems the actions appropriate or effective. Facilitating teacher reflection involves mediating the teacher's thinking and beliefs (Costa & Garmston, 2002), a process that can be enhanced through the coach's personal reflection prior to the post-lesson conference (West & Staub, 2003) and through a prepared list of reflective questions (Hull et al., 2009).

With regard to reflective questioning, not all coaching authors express the same insights. West and Staub (2003) assert that beginning the postlesson conference by asking the teacher to reflect (using questions like "How do you think it went?") is "generally a good move" (p. 34) for three reasons. First, this coaching move allows the teacher to express feelings and raise concerns. Second, it allows the coach to focus attention on areas of agreement that are genuinely important to the teacher. Third, this move encourages the teacher to develop habits of self-monitoring and self-reflection, a goal similar to that expressed by Costa and Garmston (2002). Hull et al. (2009) provide a word of caution, however, stating that such a move may be problematic if the teacher has not gained skills of self-awareness and the ability to be critical of one's own practice.

Recognizing the importance of this reflective process, the EMS Standards also address reflection, stating that EMS professionals must be able to "support teachers in systematically reflecting and learning from practice" (AMTE, 2009, p. 7). From our analysis, the EMS professional in the role of a coach must be prepared to move beyond judging a lesson based on teacher actions or the behavior of the students and move the teacher toward critical reflection regarding lesson outcomes and student learning.

Established definitions of coaching knowledge (Sutton, Burroughs, & Yopp, 2011) convey the importance of setting goals, collecting evidence, and using reflective questions to support teacher learning and self-reflection, as shown in the following excerpts from four of the eight domains:

- **Assessment:** A coach knows how to use data and assessment of student thinking to inform her or his work with teachers. A coach knows how to help the teacher learn how to set goals and assess lesson effectiveness. . . . The coach knows how to help teachers interpret and use assessment data to make informed decisions about instruction and student learning. (p. 16)

- **Communication:** A coach knows how to mediate a conversation, by pausing, paraphrasing, probing, and inquiring. A coach knows how to ask reflective questions. (p. 16)

- **Leadership:** A coach knows how to strategically identify, define, and communicate specific goals and objectives that relate to student success and teachers' professional growth, and align with the institution's vision for mathematics. (p. 16)

- **Teacher Learning:** A coach knows how to support teacher learning through reflective practice and self-directed goal-setting. (p. 18)

The collective review of this coaching literature indicates that engaging in reflective coaching conversations is a big idea in coaching practice, and those conversations should include individualized, shared goals with teachers. Such conversations are likely to be best received and most effective if they are based on evidence of student learning collected during the lesson, particularly student work. Moreover, a coach should use reflective questions, including sample questions, which the coach can carefully use to navigate a reflective conversation. Because such conversations can be difficult to navigate, given their personal nature, mathematics coaches need to be able to set aside personal opinions and beliefs so as to entrench these conversations in lesson artifacts, such as student work.

Participant responses in video assessment: Promoting teacher reflection. Only five of the 21 school-based coaches and two of the six coaching experts noted reflection in their assessment of the novice coach's practice. Among those who noted reflection, their assessment of the practice was mixed. Specifically, two of the school-based coaches gave responses indicating that the novice coach successfully engaged the teachers in reflection. Their statements follow.

> (School-Based Coach X) This coach was skillful in getting these teachers to be reflective on their practice.

> (School-Based Coach W) During the post-conference, she . . . guided the teachers into evaluating their own teaching. . . . She offered suggestions where necessary, but like the teachers she was watching, she guided the teachers to reflect.

Although both of these school-based coaches indicated that reflection occurred within the debriefing session, neither critiqued the reflection. In contrast, the remaining three school-based coaches, as well as the two coaching experts, indicated that the novice coach failed to engage the teachers in reflection. Two sample responses follow.

> (Coaching Expert A) It was great that the coach took notes, but the notes are not specific enough or at least not shared in specific ways that lead to deep reflection of practice. . . . I think the coach has lots of potential, good instincts, but needs to get clear about her purpose, the goals for these teachers, and learn to gather specific evidence that will prompt deep reflection.

> (School-Based Coach V) I didn't see deep reflection on the part of the teachers about the mathematics and their students' success or struggles. I thought the coach

might ask the teachers what went well and have some questions ready to reflect, but the conversation was more about what the coach approved of in the lesson.

From these two responses, it would appear that the respondents expected the novice coach to move the conversation beyond a personal perception of strengths of the lesson toward areas for improvement. One should also note the differences that appear between these two responses in terms of the focus of reflection: teaching practice and student learning. The need for depth and focus within the reflection was clearly articulated by other participants as well, who called for "a reflecting conversation that focused on student performance" (Coaching Expert D) and a "reflective 'what could you have improved' conversation" (School-Based Coach U). Of the five statements indicating a lack of reflection, three focused on improving practice of the lesson and two focused on student performance. Improving teacher practice and improving student performance are not necessarily on the same level of influence; however, the ultimate goal of any coaching session is improving student learning.

Sixteen of the 21 school-based coaches and four of the six coaching experts did not comment on the coach's efforts to get the teachers to reflect on their practice. We are not asserting that these respondents did not notice these aspects of the novice coach's practice, but, for whatever reason, the majority of our participants did not mention this aspect when asked to assess the novice coach's practice depicted in the video. Because reflection is central to all the coaching literature reviewed, we include it as a big idea in coaching practice that professional development for coaching should address.

Suggestions for coaching programs that wish to focus on teacher reflection. Coaching programs that focus on promoting teacher reflection can include a discussion of ways to support teachers in becoming self-critical. All of the coaching models discussed in coaching texts promote the use of questioning techniques as a means for moving teachers toward deep reflection of lesson outcomes and student learning. Coaches can be encouraged to use student work and other lesson artifacts as a means for discussing the impact of a lesson on student achievement and improving teacher practice. Though coaches work within a local vision for coaching, they can be reminded that it is appropriate to focus attention on individualized teacher goals that may be unique among other school- or district-based goals.

Promoting Coaching Relationships

Most coaching texts address the coach-teacher relationship, and some coaching models feature coach-teacher relationships as the cornerstone of effective coaching. West and Staub (2003) identify "establishing trusting working relationships among principal, coach, and teachers and building organizational structures within schools so that coaching can take place" (p. 3) as prerequisites to coaching that can help teachers design and implement successful lessons and reflect on issues that are relevant for student learning. Similarly, Knight (2007) as well as Costa and Garmston (2002) both identify the need for relationships as a starting point in bringing about change. Knight (2007) emphasizes that coaches begin the relationship by listening to and respecting the teachers with whom they are interacting. Furthermore, he states that coaches should communicate that they are teachers who are willing to help improve practice and support student learning. To build relationships and get around teacher defensiveness, instructional coaches "can share stories, laugh and empathize, offer positive comments, discuss personal issues, and listen with great care during interviews" (Knight, 2007, p. 94). To this end, Costa and Garmston (2002) lay out useful communication and relationship-building tools that coaches can employ to help change beliefs that lead to changes in behavior.

Recognizing the importance of relationship building, Hansen (2009) suggests that "empathetic coaches with effective communication skills create trusting and respectful relationships with their peers" (p. 37). This relationship can move from casual conversations and informal classroom walkthroughs to more formal observations and interactions over time. In establishing collaborative working relationships, it is important to establish clear norms for how the coach and teacher can interact. Hansen (pp. 39–41) identifies three types of relationships that can take place: a resource relationship (in which the coach is a resource to the teacher); a modeling relationship (in which the coach models standards-based instruction); and a collaborative relationship (in which the coach and teacher share the same pedagogical beliefs about teaching and learning).

The Math Coach Field Guide (Felux & Snowdy, 2006) advocates making good relationships with teachers a priority. According to the authors, a coach can establish a good relationship by helping teachers understand that the coach values the work they are doing with students. At a time when many teachers will feel vulnerable to outsiders coming into their classrooms, it is important to establish good rapport early while gradually moving toward the collaborative coach-teacher relationship.

Cultivating a Math Coaching Practice (Morse, 2009) focuses on building collaborative relationships in support of learning goals. The author indicates that coaches' leadership skills originate in the ability to align teacher skills and coaching goals through relationships and collaboration with others.

In *A Guide to Mathematics Coaching* (Hull et al., 2009), an entire chapter emphasizes the importance of building rapport with teachers. As the authors state, "A collaborative relationship enables a coach to help teachers develop deep mathematical content knowledge and effective research-based instructional strategies" (p. 24). The authors note that rapport is a foundational aspect of goal attainment, which is accomplished through trust developed through "positive relationships" over time.

Each of these authors recognizes the role of relationships in the coaching process, though there is some disagreement worth noting. Knight (2007) poses the question, "What good does it serve students if an [instructional coach] and teacher work together in a healthy relationship but their friendly conversation has no impact on the quality of the teacher's teaching?" At the end of that passage, however, Knight asserts, "If we are viewed in such a way [considered as any other teacher], and teachers come to see us as colleagues they can trust, there is a good chance that together we can make a difference in the way teachers teach and students learn in schools" (p. 52).

West and Staub (2003), on the other hand, do not view content-focused coaches as "any other teacher," but instead assert that the relationship between coach and teacher, while collegial, "will not be symmetrical" (p. 17). Killion (2009) draws clear distinctions between coaches who coach light and coaches who coach heavy. Killion (2009) asserts that "coaching light results in coaches being accepted, appreciated, and even liked by their peers" (p. 22), but that such actions result in "coaches who are valued, although may not be needed" (p. 22). In contrast, coaching heavy occurs when coaches ask thought-provoking questions and have fierce and often difficult conversations. "Coaching heavy causes [teachers] to feel on edge, questioning their actions and decisions" (p. 24).

The EMS Standards address relationships as well:

> *EMS professionals must be able to:* Use leadership skills to improve mathematics programs at the school and district levels, e.g., develop appropriate classroom- or school-level learning environments; build relationships with teachers, administrators and the community; develop evidence-based interventions for high- and low-achieving students; collaborate to create a shared vision and develop an action plan for school improvement; partner with school-based professionals to improve each student's achievement; mentor new and experienced teachers to better serve students. (AMTE, 2009, p. 8)

Established definitions of coaching knowledge (Sutton, Burroughs, & Yopp, 2011) convey the importance of negotiating professional relationships. They include related domains of Communication and Leadership, but also address the domain of Relationships directly:

> *Relationships:* A coach knows that the coaching relationship is grounded in content and how to use the relationship to support self-directedness in teachers. A coach knows how to communicate professionally with a variety of audiences, and knows how to establish and maintain rapport and credibility with teachers and other stakeholders based on trust, empathy, mutual understanding, and confidentiality. A coach knows about environments where positive relationships take place, including challenging and safe learning environments for teachers and students, collaborative working environments, and environments where people share common beliefs and goals with honest reflection. The coach knows how to work within the specific culture of the district and school. The coach knows how autonomy, issues of authority, and socio-cultural aspects of class, race, and gender for students and teachers influence relationships and influence perceptions and models of help and authority. (p. 17)

Participant responses in video assessment: Promoting coaching relationships. After viewing the video of the novice coach, only seven of the school-based coaches and coaching experts commented on the characteristics of the relationship between the novice coach and the teachers: five of the 21 school-based coaches and two of the six coaching experts. Four of the school-based coaches and one of the coaching experts mentioned favorable aspects of the relationship, as in these examples: "It was obvious to me that there was a rapport and trust relationship between the three women" (School-Based Coach T), and "The coach was not intimidating; the teachers seemed comfortable with her and conversed with her" (Coaching Expert B). One school-based coach and one coaching expert mentioned unfavorable aspects of the relationship, as in this response from Coaching Expert E: "The planning process appeared stilted and uncomfortable for the coach and teachers."

One coaching expert made specific comments about the purpose of the coaching relationship: "This level of coaching may get 'relationships' developed . . . but it doesn't dive deep enough into content and doesn't challenge practice" (Coaching Expert A). Rather than focusing on the existence of a relationship between a coach and a teacher, this coaching expert's comment suggests that it is the focus of the relationship that is important in coaching.

Sixteen of the 21 school-based coaches and four of the six coaching experts did not comment on the coach-teacher relationship. We are not asserting that these respondents did not notice the coach-teacher relationship, only that the majority of our participants did not comment on it. Moreover, we noted that those who commented on the coach-teacher relationship did not do so in a consistent manner (e.g., some expressed a favorable view of the coach-teacher relationship and some criticized the relationship). Given that the coaching texts, definitions, and standards we reviewed make explicit reference to relationships, we believe that this aspect is a big idea in coaching practice. We believe that there is enough diversity in the how coach-teacher relationships are described in the literature that professional development can address various specific relationship characteristics.

Suggestions for coaching programs that wish to focus on coach-teacher relationships. The coaching literature supports a variety of possible relationship structures and addresses their consequences. This literature suggests several ways to establish and maintain relationships, as well as ways to move relationships beyond superficial discourse. Because some schools adopt a volunteer coaching program in which teachers have a choice as to whether or not to invite the coach into their classrooms, failure to appropriately address coaching relationship issues can result in teachers' closing their doors to coaches.

Local visions for mathematics coaching would benefit from explicitly defining the coach-teacher relationship prescribed by their coaching models. Given the consistency with which relationships are conveyed as a cornerstone of successful coaching, and the variation in what constitutes an "effective" relationship, it would be advantageous for mathematics coaching programs to provide specific guidance and direction regarding how to establish, build, and nurture relationships in a variety of contexts with teachers at different stages along the continuum of change. For example, some programs might focus on how to ensure that the coach-teacher relationship is collegial. Other programs might focus on how to ensure that the relationship is grounded in professional discussions and centered on content and curriculum or student learning. Other programs might ask how to ensure that teachers readily participate in pre-and postlesson discussions. All programs will likely focus on establishing trust in the coach-teacher relationship.

Discussion

We asked a sample of school-based coaches and coaching experts to assess the practice of a novice coach featured in a video-recorded coaching session. Our results indicated little consistency across participants' comments, especially regarding three coaching aspects that are big ideas in coaching practice: developing teacher content knowledge, promoting reflection by the teacher, and negotiating professional relationships. Our own efforts to focus a professional development course in mathematics coaching were enhanced by our use of this video-observation assessment as a precursor to the course.

Summary suggestions for local coaching programs. There was little consistency in comments about the three big ideas in coaching practice among both the practicing coaches and the coaching experts, indicating that our findings are not limited to the particular characteristics of the coaches enrolled in our program. Instead, the lack of consensus among coaching experts emphasizes to us that a coaching professional development program or local vision for coaching would do well to focus on these three areas. Despite the widespread availability of texts that address mathematics coaching, at present there is not a shared understanding of what coaching practice that focuses on these three areas looks like. With the available material, however, local coaching programs can decide how to focus a coaching program that addresses these three areas by consulting these available resources.

How we used the study results to shape our own work with coaches. We acknowledge that our method has limitations that may have influenced our data collection or analysis. Using the open prompt "assess" allowed for our participants to comment on what they found to be most important, but may have limited their responses in a way that more specific prompts about these three big ideas would not have. Also, our prior work with teaching professionals has taught us how reluctant teachers and coaches can be to critique another professional, so it is possible that our participants noticed these aspects of coaching practice but chose not to comment on them. However, having focused our professional development course on these three aspects, we received comments from those who attended that its focus provided them the structure they needed to understand the aspects of developing teacher knowledge, promoting reflection, and negotiating professional relationships in coaching practice. We conclude that this assessment correctly allowed us to target our mathematics coaching professional development, and we encourage other coaching programs to focus, at least in part, on these three aspects of coaching practice. ✪

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