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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Brief commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We want to hear about your reactions, questions, and connections you are finding to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Linda Ruiz Davenport, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM)

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***Note:** Information for manuscript reviewers can be found on the inside back cover of this publication.

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editor

Angela T. Barlow, *Middle Tennessee State University Murfreesboro, Tennessee*

In reviewing the contents of this issue, you might immediately recognize the diversity of ideas presented. Close examination, however, reveals that each article has as its purpose to support mathematics education leaders in addressing the following imperative.

> Raising achievement in mathematics for *every* student and effectively implementing the CCSSM in *every* classroom requires *extensive and ongoing opportunities for teachers to enhance their own professional learning and to build their capacity to reach all students.* (National Council of Supervisors of Mathematics, 2014, p. 44)

At the core of this imperative is professional learning. In the first article, White describes Math Teachers' Circles as a means for supporting professional learning. Through her descriptions, she provides the reader not only with insight into how to form a Math Teachers' Circle but also with a vision of the types of activities with which participating teachers engage. Sample problems are provided along with evidence regarding the outcomes of the program.

Similarly, Edgington and colleagues describe their work in a professional learning setting. In their project, participating teachers examine student work as a means for developing understandings of students' mathematical thinking. Supporting teachers in productive discussion of student work, however, proved to be challenging. As a result, Edgington and colleagues share the norms they developed to guide teachers' discussions of student work. Sample quotes are provided to demonstrate the shift in discussions that occurred as a result of establishing the discussion norms.

As teachers gain skill in identifying students' mathematical thinking, they are primed to begin thinking about the role of formative assessment in reaching *all* students. To this end, Petit and Bouck describe the *essence* of formative assessment, noting that formative assessment includes not only soliciting and interpreting students' thinking but also the purposeful use of the information for making instructional decisions. In their article, they share five assessment strategies and use classroom examples to support the reader in understanding the essence of the strategies.

The ultimate goal of professional learning is to raise mathematics achievement through the implementation of effective instructional practices. As teachers' views of what constitutes effective instructional practices change, mathematics education leaders often desire to have a tool for documenting the changing views. The remaining two articles in this issue provide such tools. In the first, Munter describes the development and use of a rubric for describing and tracking teachers' evolving visions of the teachers' role in the mathematics classroom along three dimensions: lesson structure, classroom discourse, and mathematical authority. In the second article, Chamberlin and colleagues demonstrate how they used a perspectives framework for characterizing the views of teacher leaders as they transitioned towards reform-oriented instruction. Both articles provide a means for describing the paths that teachers take as they engage in professional learning.

Collectively, these five articles provide mathematics education leaders with useful information for not only supporting the achievement of the previously stated imperative but also documenting the progress teachers are making in this area. I hope that you will find these ideas useful as you apply them to your work. ✪

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Math Teachers' Circles as a Form of Professional Development: An In-depth Look at One Model

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Abstract

In this paper, I discuss one chapter of a growing professional development program that aims to improve the quality of mathematics education for students by developing middle level mathematics teachers' content knowledge and problem-solving skills, as well as their facility with applying the Standards for Mathematical Practice. I provide context within the national program, then discuss how the local chapter emerged, describe in detail the summer workshop and associated academic year sessions, discuss outcomes from the program, and provide information regarding how to start a similar program.

Introduction

In the summer of 2009, the University of Colorado Denver and the St. Vrain Valley School District in Longmont, Colorado began a partnership designed to strengthen the problem-solving skills and mathematical habits of mind of middle school mathematics teachers. We wanted teachers to have a venue to work on their mathematical problem solving and to develop their mathematical habits of mind while simultaneously building their capacity to implement rich mathematical tasks in the classroom. After the initial summer workshop, the project expanded to include teachers from a variety of school districts, mostly in the Denver metropolitan area.

For over four years now, this program, known as the Rocky Mountain Math Teachers' Circle Program, has provided professional development for teachers in which they engage in the process of doing mathematics with guidance from university mathematicians. We recognize that an unfortunate side effect of how mathematics has traditionally been taught at both the K-12 and collegiate levels is that many teachers have never had the opportunity to truly explore mathematics using the same disciplinary-specific habits of mind that research mathematicians use on a daily basis. That is, they have not had the opportunity to explore, question, conjecture, create examples, generalize, and communicate mathematically (Conference Board of the Mathematical Sciences [CBMS], 2012).

Yet with the introduction of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) and the Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSI], 2010), teachers are being asked to develop skills (e.g., the Standards of Mathematical Practice) in their students that they themselves may never have had the opportunity to develop (CBMS, 2012). As such, we take the approach that teachers will benefit in the classroom from focusing on their own development of these skills as learners. Since we know that teachers' instructional practices often reflect their own learning experiences, we provide "the opportunity to experience firsthand a form of teaching that facilitates and supports learning" (Smith, 2001, p. 43).

Since inception, the Rocky Mountain Math Teachers' Circle Program has served approximately 150 teachers from 25 districts, including numerous mathematics coaches and others with informal leadership roles (e.g., mathematics department chairs). Each summer, we hold a one-week summer immersion workshop. In addition, academic year sessions are held on Saturday mornings approximately once per month. Graduate credit or continuing education units are available to participants.

Background

The Rocky Mountain Math Teachers' Circle Program is part of a national network of Math Teachers' Circles organized through the American Institute of Mathematics (AIM), headquartered in Palo Alto, California. AIM is one of eight mathematics research organizations in the United States. While AIM predominantly provides a venue and structured format for research mathematicians to come together to further their own mathematical research, outreach via Math Teachers' Circles (MTCs) is a key part of AIM's efforts. The first MTC began at AIM in 2006, as an offshoot of a local math circle for K-12 students (American Institute of Mathematics, n.d.). Since then, MTCs have spread rapidly, thanks to the extensive efforts of AIM and a core group of dedicated mathematics professionals. There are now approximately 57 active MTCs across the country, with approximately 12 new teams attending training each summer at one of the two weeklong training workshops.

The first MTC was intended solely to provide a venue for teachers to explore exciting mathematics, much as they were observing their students do in the math circles for students (Donaldson, Nakayame, Umland, & White, 2014). This initial intent remains, but MTCs have evolved substantially over time, and they now align more closely with many fundamental tenets of mathematics professional development, including those of Desimone (2009) and Guskey (2003).

The Rocky Mountain Math Teachers' Circle Program began in the summer of 2009, when a five-member leadership team attended a weeklong training session entitled "How to Run a Math Teachers' Circle." It was organized by AIM, sponsored by the National Science Foundation, the National Security Agency, and the Mathematical Association of America (MAA), and hosted at the MAA's headquarters in Washington D.C. The leadership team was intentionally diverse, consisting of a mathematician and statistician from the University of Colorado Denver, and the district mathematics coordinator, a high school teacher, and a middle school teacher from the St. Vrain Valley School District.

Throughout the week, mornings were spent engaging in problem-solving activities (i.e., sample MTC activities and sessions) with seven different leadership teams, while afternoons were spent in structured planning for each team's local MTC. Although all MTCs share certain basic properties, each MTC is also tailored to meet the needs of the community that it serves.

There was ample time for the team to bond and get to know one another over meals and evening excursions in the D.C. area. This training was the first time that the entire team was together, and for some, the first time that they were meeting each other. In hindsight, this opportunity to spend an intense week immersed in training and preparing to run a MTC was pivotal in shaping the program and developing the professional and personal relationships to work effectively together and to develop a high quality MTC. Following this initial meeting, the team continues to meet for monthly planning over dinner at a central location to work further on fundraising, recruiting, specific activity planning, and other logistics.

The Rocky Mountain Math Teachers' Circle Program has evolved from this initial partnership with one district to a stand-alone professional development program open to teachers of mathematics from any district. While we focus primarily on middle-level teachers (i.e., grades 5-9), we have some dedicated high school teachers who attend, as well as an occasional elementary teacher. Rationale for attendance varies, with some teachers reporting that they attend because they feel like their district does not provide the math-specific professional development that they need or want. Others simply love to engage with and explore the mathematics. Several regular attendees teach in non-traditional settings (e.g., juvenile rehabilitation facilities, credit recovery alternative schools, charter schools). However, most teach in a traditional middle school setting.

Although the entire leadership team played important roles in developing the initial workshop and program, the high school teacher and the mathematician have emerged as the program co-directors. The high school teacher, with her expertise on assessment and on leading professional development, has taken the lead on the overall structure of the workshop, to include community building, establishing group norms, and various logistics related to the physical set-up of the space. Given her role at the university and her content expertise, the mathematician has assumed overall leadership of the project as program director, handling local logistics such as graduate credit, fundraising outside the initial partner district, and overall content planning.

Summer Immersion Workshops

The summer immersion workshops, held each summer since 2010, last five full days and have each supported 12 to 25 teachers. Most have been held on the University of Colorado Denver campus, and have been fully funded through grants from the National Science Foundation and other foundations. With the exception of the first summer, all workshops have been widely advertised and open to teachers from any district. We estimate that approximately half of the participants attend a single summer workshop only, with the other half attending for at least one year, to include both academic year sessions and additional summer workshops.

Typically there are 3-5 facilitators who lead sessions throughout the week, providing participants with a variety of different styles of facilitation as well as a selection of diverse topics. These facilitators typically have significant experience working with teachers and a substantial mathematics content background. Most are mathematicians, though other members of the leadership team have led sessions throughout the week as well.

On the first day of the workshop, we openly acknowledge that the participants are in a dual role as both learners and teachers of mathematics. As Tassell and colleagues (2011) noted, they are engaged in "teacher learning through a bifocal lens" (p. 44). Specifically, we ask them to spend the first four days of the workshop focusing on their role as active learners of mathematics, and assure them that on the last day, we will connect what they have learned throughout the week to their role as teachers of mathematics. A general description of the week is provided in the sections that follow.

Day 1 - Morning

The first morning is spent setting the tone and developing the community for the week. We aim to create an atmosphere where all participants feel that their mathematical thinking is valued and that the other participants support and respect their personal learning. This is particularly important as participants' background can vary tremendously, from someone trained initially as an elementary teacher to someone with a bachelor's degree in mathematics.

To accomplish this, we complete two carefully constructed activities. First, after a brief introduction to the facilitators and announcements about logistics for the week, participants spend approximately 45 minutes creating community agreements for the week. These are developed through the following three questions:

> What are the characteristics of a problem solver?
>
> What are the characteristics of an effective group?
>
> What are the characteristics of an active listener?

After this, we begin our second activity that focuses on building community and trust amongst the participants. To accomplish this, we pose the first mathematical problem of the week. One problem that we have used repeatedly follows.

> In a crazy New York apartment building there are seven elevators, each stopping at no more than six floors. It is possible to get from any one floor to any other floor without changing elevators. What is the maximum number of floors in the building? (Konhauser, Velleman, & Wagon, 1996, p. 42).

In selecting the initial problem we are careful to choose one that is easy to state and understand, requires minimal mathematics background needed to begin to explore it, and allows for a variety of approaches that can lead to significant progress toward an answer. In the case of this particular problem, trial and error readily provides lower bounds for the number of floors, but finding the maximum number possible is considerably more challenging.

From the start of the workshop, we want group members to initially develop their own mathematical approaches and ideas, articulate them and have them heard by others, and examine various approaches. Thus, participants are asked to work individually for at least 20 minutes, before the facilitator has them share out within the others at their table using a round robin format. Each table group, generally consisting of four people, is asked to make a poster that includes the original thinking of all of the group members.

A random choice of presenter for each group then shows all participants that there is an expectation that each individual listen to and absorb the various ideas from others.

Day 1 Afternoon – Day 4: Main Workshop Sessions

Beginning with the first afternoon and continuing through Days 2-4, the workshop focuses on providing participants with experiences to engage with intense, cognitively demanding mathematics. To ensure that participants experience lots of different approaches to mathematics throughout the week, groups are randomly assigned each day.

Mornings are composed of one long mathematical exploration (3-3.5 hours), while afternoons are spent with two shorter sessions (1.5 hours each). At least one, sometimes two or more, topics are intentionally developed across multiple days. The reasons for this include: developing participants' comfort level with leaving problems unanswered; supporting participants in recognizing that often in mathematics there is no quick answer; and helping participants to realize that struggle and exploration are ongoing.

During these sessions, we intentionally choose topics and problems from across the mathematical spectrum, ensuring that we include diverse areas such as geometry, probability and statistics, discrete mathematics, and topics related to number systems. Often topics overlap several areas of mathematics. Sometimes sessions start with a specific question, such as one of the following:

1. Write numbers from 1 to 100 on the board. Select any two of the numbers, erase them, and write on the board the sum plus the product of the two numbers. For example, if you erased 2 and 5, the sum plus the product is 7 plus 10, or 17, and so you write a 17 on the board. Now there are two 17s, but that's OK. Repeat this process of selecting two numbers and replacing them with their sum plus their product. What are the possible outcomes?

2. A 3 x 3 x 3 cube is made up of 27 smaller 1 x 1 x 1 cubes. Each of the smaller cubes are painted such that the 27 cubes can be assembled to create an all blue larger cube. Then, they can be reassembled so that they can create an all red larger cube. Finally, the 27 little cubes can be taken apart and reassembled to create an all white larger cube. How could the 27 little cubes be painted in order for this to happen?

Other times, sessions surround a specific topic, for example, combinatorial games. A variety of games were introduced over several days, with participants given time to explore each game and work toward finding a so-called winning strategy (i.e., a strategy whereby if both they and their opponent make the best possible move at each turn, then they are guaranteed to win).

Another popular extended topic that has been used in several of the workshops is *Exploding Dots*, which investigates many of the basic ideas of place value and standard algorithms for arithmetic and algebra in a novel way. Tanton (n.d.) has a wonderful video exposition of this topic. Several shorter topics have investigated diverse topics such as symmetries of plane figures, ways to tile the plane, logic puzzles, and topics from probability and statistics.

Last Day

The last day of the summer immersion workshop begins with returning to the mathematical problem from the first day. Participants are provided additional time to work on it in their small groups for the day, and then report out to the large group. They are amazed to see how far they have progressed from that first day, and how much more versatile they are in their mathematical thinking.

As a way to encourage participants to reflect on their experience and learning from the week, the second and final activity of the morning is for the participants to make a poster, containing only symbols and pictures, which represents their journey for the week. Each small group presents their poster to the larger group.

The afternoon is dedicated entirely to connecting what participants have learned to their classrooms. Participants discuss the Standards for Mathematical Practice (CCSSI, 2010), which have been referenced throughout the week, as well as the 21st Century Skills (Partnership for 21st Century Skills, 2008). They watch a TEDxNYED video (Meyer, 2010) and read a chapter from the book, *The Courage to Teach* (Palmer, 1997).

Transitioning to concrete plans, participants are asked to choose three things from the week that they would most like to infuse in their classroom during the first semester of the upcoming academic year. Choosing one, they describe how it would look at the end of the semester, the end of the first quarter, and the first three weeks of the semester. They then make a to-do list of things that they need for the plan. These could include a timeline, prompts to use, manipulatives, professional development, or support from others.

Summer Immersion Workshop Outcomes

Our hope was that, at the end of a weeklong immersion workshop, participants would have increased comfort level with open-ended mathematical problems, increased self-efficacy related to mathematical problem solving, increased content knowledge, increased mathematical knowledge for teaching, and a stronger desire to implement more student-centered mathematics into their classroom. Measuring all of this has proven to be a challenge. We have used three primary sources for evaluative data, which are described in the sections that follow.

Workshop Surveys

To measure the outcomes of the workshop, we have used end-of-workshop surveys. On these, self-report participant gains can be loosely separated into gains as a *learner* of mathematics and gains as a *teacher* of mathematics. In their role as learners, many commented that they were challenged by both the content and problem solving, and that they had not previously been asked to work collaboratively to this extent on mathematics. They also commented that they felt incredibly supported by the various facilitators throughout the week and that they see the value in observing how the various facilitators lead sessions, taking ideas or even specific mathematics problems back to their own classroom setting or to their mathematics club.

In their role as teachers, participants commented that they intend to require more justifications and explanations from students. They also plan to incorporate more group work, more open-ended problems and problems requiring exploration, and more mathematical discussions into their classrooms. They reported that they were able to learn teaching strategies such as effective questioning techniques by observing the instructional practices that the facilitators modeled. Although efforts to conduct case studies of participants' classrooms are ongoing (e.g., Donaldson et al., 2014), it should be noted that this self-report data alone is insufficient for drawing conclusions about the classroom teaching practices of MTC participants.

Content Assessment

For two years, we administered a pre-post assessment known as the Learning Math for Teaching assessment (Hill, Schilling, & Ball, 2004), which measures aspects of what is referred to as *mathematical knowledge for teaching* (Ball, Thames, & Phelps, 2008). This phrase refers to the mathematics specifically needed to teach mathematics, as opposed to the mathematics commonly needed in other professions that use mathematics, like science and engineering (Hill et al., 2004). Teacher performance on this instrument has been linked to student achievement (Hill, Rowan, & Ball, 2005). Both years, our participants showed statistically significant gains on the version that we gave, which focused on number concepts and operations at the middle level (White, Donaldson, Hodge, & Ruff 2013).

Facilitator Observations

Finally, the facilitators and co-directors debrief at the end of each day and at the end of the week. Our observations indicate that participants are developing perseverance, an openness to try problems that may have intimidated them before, communication skills, and a trust in their own mathematical reasoning.

Academic Year Workshops

Recognizing that teacher professional development needs to be sustained (Darling-Hammond & McLaughlin, 1995), we have offered approximately 7-9 sessions each academic year. Each meets on a Saturday morning for approximately 3.5 hours, with a free lunch immediately following. These sessions follow the same spirit as those of the summer, with a variety of different facilitators throughout the academic year.

Participants report that their ongoing participation helps keep them thinking mathematically throughout the year. There are several schools in which multiple teachers have committed to attending as a team for at least a semester. They then report co-planning and discussing what they have learned together at their schools. This is an area that we would like to study in more depth, as one of the tenants of effective professional development is collective participation (Desimone, 2009). Overall, most participants who attend more than one or two workshops attend regularly for approximately two years, with a few outliers having attended almost all four years of academic year sessions and summer workshops.

Effective Professional Development

The MTC model addresses the five criteria for effective professional development identified by Desimone (2009). A description of each follows.

Content Focus

MTC activities are centered on rich, open-ended problems with multiple entry points. Although the problems can be stated in such a way that a middle or high school student could understand them, some are rich enough that aspects of them are the subject of active mathematical research. Mathematicians are centrally involved in selecting problems and leading sessions to ensure participants' access to deep content contextualized within the mathematical process.

Active Learning

Participants are involved in active problem solving for the majority of each MTC session, with small group work and whole group discussions occupying the majority of each mathematics session.

Coherence

The activities of a MTC are designed to directly support participants' development of the habits of mind described in the Standards for Mathematical Practice (CCSSI, 2010). MTCs intentionally support participants in developing at least six of the eight standards, including the ability to (1) make sense of problems and persevere in solving them, (2) reason quantitatively and abstractly, (3) construct viable arguments and critique the reasoning of others, (5) use appropriate tools strategically, (6) attend to precision, and (7) look for and make use of structure. While the specific content addressed in any given MTC varies, the focus on one or more of these critical mathematical practices is always present.

Duration

Participants can attend MTCs for multiple years. Each participant engages in approximately 35 hours of professional development during each intensive summer workshop and between 21-28 hours during each academic year of participation.

Collective Participation

The MTC model builds a community among participants, provides a natural way for mathematicians to become involved in K-12 education and form meaningful long-term partnerships with teachers, and engages participants in the larger mathematical community.

Forming a Math Teachers' Circle – A National Community of Support

There is now a well-formed support network for those interested in starting a Math Teachers' Circle. AIM runs the national Math Teachers' Circle Network (mathteacherscircle.org) and provides resources both for existing MTCs and those interested in starting a new MTC. They can help connect interested district personnel with mathematicians at a local institution of higher education to explore forming a team to attend the weeklong "How to Run a Math Teachers' Circle" training workshop.

Each morning during this training workshop, experienced national MTC facilitators lead sample MTC sessions with the workshop participants acting as learners. In doing so, the teams explore a variety of problems that they could, in turn, use for their own MTC sessions, as well as see highly qualified MTC facilitators model sample sessions. This is especially important for those mathematicians who may have minimal, if any, prior experience working with teachers, as there can be a steep learning curve associated with learning to lead sessions effectively. Partnering with a teacher from the leadership team to co-develop sessions is one way in which some teams have worked together to develop and implement sessions effectively.

During the afternoons, teams work together to develop their own logistical plan for their MTC, including to define roles, learn about and plan for funding opportunities, write their own mission and vision statements, and make a concrete plan for starting their own MTC.

These training workshops have been quite successful, with over 85% of teams who have attended a training workshop successfully starting their own MTC. A variety of smaller seed grants ranging from \$1500-\$2000 have been available for the past few years to help new MTCs get started, and most have been able to find state or private foundation funding as well. In some states, Math-Science Partnership Grants or Improving Teacher Quality Grants ranging from \$30,000 to \$90,000 have been awarded for various MTC programs. The aforementioned website included a wide variety of materials that can be used to aid MTCs in securing funding, leading sessions, and gathering evaluation data. Facilitators from various MTCs regularly visit other MTCs to act as guest facilitators, thereby further spreading knowledge and experience. There is also a national listserv open to all MTC leadership teams that is used as a forum for communication and sharing of information.

The national MTC community is welcoming and growing, with no shortage of interested and experienced people willing to help support new MTCs.

Conclusion

The purpose of the Rocky Mountain Math Teachers' Circle Program is to improve the quality of mathematics education for students, specifically by developing middle level mathematics teachers' content knowledge and problem-solving skills, as well as their facility with applying the Standards for Mathematical Practice (CCSSI, 2010). The program supports teachers by providing a variety of experiences for teachers to engage in learning mathematics though this active approach with authentic engagement in mathematical problems under the direction of professional mathematicians.

It is our hope that the impact of this program goes beyond participating teachers and their students, and that teachers take lessons learned back to their schools and districts to share with colleagues. In that way, they become informal teacher-leaders and the impact of the program is magnified. Research on the program at the national level is ongoing (e.g., Donaldson et al., 2014; White et al., 2013; White & Yow, in press), and other local Math Teachers' Circles are beginning to disseminate their programs and outcomes as well (e.g., Geddings, White, & Yow, 2015). Preliminary data analysis shows that the program, both at the local and national level, does have this effect on some teachers.

The program has evolved over time, and a variety of supplemental workshops have been developed to help further connect the mathematical learning with participant's classroom teaching. After four years, the Rocky Mountain Math Teachers' Circle program is still going strong, and we hope to report back in several years with more successes and lessons learned. ✪

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Norms for Teachers' Discussions of Students' Mathematics in Professional Development Settings

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Abstract

In this paper, we share our experiences using student work to engage teachers in learning about students' mathematical thinking and the need to develop norms for talking about students' mathematics in professional development settings. In such settings, it can often be challenging to maintain productive perspectives that focus on students' mathematics. We describe our experiences facilitating a professional learning task designed to support teachers' participation in discussions about students as mathematics learners. We share discourse norms that can be used by teacher leaders to focus teachers' discussion on students' mathematical thinking and a set of questions that teachers may use to reflect on their students' mathematical thinking as they engage in discussions with colleagues about students' mathematics.

Introduction

Consider two contrasting statements from teachers' discussions about their students' mathematical work:

"He's in the low group, so I thought this task is way too hard for him."

"He started counting from six. He used his fingers to show 'seven, eight, nine.' So I think he can count on."

What might each statement reveal about the student's understanding? What knowledge and opportunities can teachers leverage? What information does each statement provide about the student's prior knowledge that teachers can use in responding to the student? Further, how does each statement portray the student as a doer of mathematics?

Students' mathematical work is often a central focus of discussions in professional development settings, grade level meetings, and professional learning communities (Sowder, 2007; van Es & Sherin 2008). Student work in the form of video cases, classroom videos, and written work is often used to foster teachers' discussions about teaching and learning by reflecting on specific aspects of students' mathematical thinking (Kazemi & Franke, 2004). Though these discussions can lead to insights into students' successes and struggles, maintaining a productive perspective that focuses clearly on the mathematics of the student can be challenging (Battey & Chan, 2010). In this paper, we consider norms that can be used when teachers are engaged in discussions about students' mathematical thinking.

We share the statements above to illustrate the challenge of focusing professional discussions on students' mathematical thinking. Both statements come from a professional development project in which elementary teachers learned about students' learning trajectories in mathematics. These trajectories, which served as the basis for development of the Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010), represent levels of students' mathematical thinking as they progress from less to more sophisticated over time (Daro, Mosher, & Corcoran, 2011).

Throughout our professional development, teachers watched a variety of videos of interviews with children. As the project unfolded, we learned that explicit norms for talking about students supported teachers' participation in productive discussions about students' mathematical thinking. For example, discussing students in ways that mask their mathematical understanding by focusing on non-mathematical factors, such as classroom management, ability grouping, or grade-level expectations, promoted an image of students as mathematics learners that was not based on their mathematical thinking. This way of talking about students left teachers with little recourse in supporting students' mathematical development. Alternatively, describing what students can do and making hypotheses about their thinking based on evidence contributed to an image of students as doers of mathematics, where the role of the teacher is to design instructional experiences that build from students' current conceptions to move learning forward.

In this paper, we share our initial experience facilitating a professional learning task designed to engage teachers in discussions about students' mathematical thinking represented in student work. We discuss how the task unfolded and how we altered the task to support teachers in participating in productive discussions about students as mathematics learners. We conclude with a set of questions that teachers may use to reflect on their students' mathematical thinking and to engage in discussions with colleagues about students' mathematical thinking. These questions support both teachers' individual reflections and their professional discussions in learning communities.

Professional Norms

Those who design and study professional development have noted the importance of teachers' studying the practice of teaching (Sowder, 2007). As a consequence, there is increased attention to utilizing practice-based professional learning tasks (Ball & Cohen, 1999) to support teachers' development of their mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). Professional learning tasks also foster a "disposition of inquiry" (p. 27) for teachers to learn in, from, and around their practice. In particular, professional learning tasks that utilize student work samples, video, and narrative cases bring the work of teaching into a setting that allows teachers opportunities to inquire about their practice. The use of classroom videos and students' written work in these professional learning tasks has been connected to improvements in teachers' classroom instruction (Kazemi & Franke, 2004; Sherin & van Es, 2009).

The use of artifacts from practice in professional development settings for teachers has resulted in increased awareness of the norms necessary to cultivate teacher learning (Nemirovsky, DiMattia, Ribeiro, & Lara-Meloy, 2005; Van Zoest & Stockero, 2012). Seago, Mumme, and Branca (2004) proposed the idea of *professional norms* — a set of norms needed to support teacher learning from practice. They recognized that teachers talk about mathematics teaching as much as they talk about mathematics itself, and explained that professional norms were patterns of behaviors specific to talking about teaching. Seago and colleagues (2004) developed a set of professional norms in conjunction with their video cases to help teachers learn to analyze instructional decisions. These norms included: listening to others' ideas, adopting a tentative stance towards practice (i.e., wondering versus certainty), providing evidence, and being critical yet respectful.

Van Zoest and Stockero (2012) incorporated the norms outlined by Seago et al. (2004) into their work with teachers to help foster teachers' mathematical knowledge for teaching. Although they did not explicitly discuss these norms with the teachers, they purposefully worked to develop the norms in professional discussions. For example, when examining student thinking and teaching in videos, facilitators encouraged participants to provide specific evidence for the claims they made. According to the researchers, introducing such norms early in the teachers' discussions supported teachers' learning of mathematics with understanding and learning from practice.

In our professional development, we fostered similar professional norms regarding the use of practice-based artifacts. However, we focused not only on the ways teachers talked about mathematics teaching, but also attended to the ways teachers talked about students. Just as teacher learning can be supported by norms for discussions about teaching, we argue that similar learning can result from constructive discussions about students as mathematics learners.

In what follows, we describe how we revised a professional learning task we used in the Learning Trajectory Based Instruction project with the goal of promoting norms for talking about students' mathematical thinking in ways that attend to their current mathematical understanding instead of pre-determined, fixed expectations based on factors such as grade level or achievement.

The Learning Trajectory Based Instruction Project

Learning Trajectory Based Instruction (LTBI) is a professional development project that engages teachers in learning about students' mathematical thinking and an instructional model in which student thinking provides guidance for teachers' instructional decisions. Our work is based on the concept of learning trajectories, which use research on student learning to clarify the intermediate steps students take as learning proceeds from informal understanding to more sophisticated concepts over time (Clements & Sarama, 2004; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). LTBI utilizes professional learning tasks that emphasize students' mathematical thinking, the use of open instructional tasks, and pedagogical practices that build on and centralize student thinking. As we considered the practice-based artifacts used in the professional development, we prepared to address norms for talking about teaching by emphasizing the need to be critical yet respectful when talking about videos that share teaching and interactions with students. Yet, as we discuss in this paper, we found that these norms concerning mathematics teaching were insufficient to keep discussions of mathematics learners productive and focused.

A Professional Learning Task Focused on Students' Mathematical Thinking: An Example

The initial professional learning task we used in LTBI engaged teachers in discussing when they were surprised by the mathematical thinking a student displayed. Our goal was to encourage teachers to consider the need to listen to students in order to understand their mathematical thinking. We used videos of interviews of three students engaged in fair sharing problems. We first described the problem students were solving, which involved sharing 24 coins fairly among three pirates and sharing a round birthday cake fairly among six friends (Wilson, Edgington, & Confrey, 2010). We asked teachers to anticipate how the students would likely solve the problem, and asked them to make notes as they watched the videos using the following guidelines: 1) monitor what the children were doing as they solved the tasks, 2) compare the students' strategies and solutions to what was anticipated, and 3) think about what was surprising about the students' mathematical thinking.

After viewing the videos, teachers discussed their observations in whole group. As this discussion progressed, we noted that despite our efforts to focus on students' mathematics, most of the discussion attended to the ways the questions were posed in the interviews or the materials the students were using. Little attention was given to teachers' speculations of the students' current mathematical understandings using evidence from the videos. The following quotes summarize the discussion that emerged during this professional learning task.

> *Yeah, at first I thought she was guessing. Oh, she just got lucky, you know? And then she explained it. But with the 3rd grader, the proctor, she said, "Well, why don't we put these back together and then divide them." And I wonder what would have happened if they stayed in those four groups and she said, "What if one of these pirates went away, how could you share these?" I thought the results could have been a lot different.*

> *I think she was told to because she didn't know how to get started.*

I expected the third grader to maybe, initially my thought would have been that at least she would have been putting them over in groups of two. So I was really surprised that she was just one, one, one.

My last thought was that the interviewers . . . these hands would come into the field of vision and he's like doing stuff. And I was just wondering if [the student] is going off her intuition and a little, you know, not sure of herself, how much that particular factor might have thrown her? Just like in her case, that might have been a big influence.

These comments indicate that the discussion of the video focused on the wording of questions, grade level expectations, luck of the student, or the influence of the interviewer as opposed to the mathematical understandings exhibited by the students in the videos. The discussion was not as focused on the students' mathematical thinking as originally intended in the professional learning task. Although teachers adhered to professional norms put in place, further norms were needed to guide discussions about students' mathematical thinking. It is this set of norms that we aim to share. As we revised the professional learning task, we sought to be more purposeful about setting norms to guide teachers' conversations about students' mathematical work.

Norms for Discussing Students' Mathematical Work

Building from professional norms used for teachers to talk about teaching, we developed four guidelines for teachers to consider as norms for talking about students (Figure 1). While these are similar to other professional norms (Seago et al., 2004; Van Zoest & Stockero, 2012), they are specific to discussing *students' mathematical thinking*. Moreover, these norms can be purposefully shared with teachers in the context of analyzing students' written work or watching videos of students engaged in mathematical tasks. They aim to encourage teachers to use evidence from representations of students' work to consider the students' mathematical understandings, focusing on what the students can do as opposed to what they cannot do. In the sections that follow, we describe each norm, including its purpose towards supporting teachers' focus on students' mathematical thinking.

FIGURE 1.
Norms for discussing students' mathematical thinking

- Describe what students can do
- Provide evidence for your claims about what students do or do not know
- Develop hypotheses about the mathematical reasoning for the work students do
- Recognize when statements are speculations or judgments

Describe what students can do. To focus on students' mathematical thinking, it is important to describe what students are doing, withholding any judgments or expectations. This is in contrast to statements that speculate about a student's capabilities based on what is known about the student's grade level, achievement, or previous work. Since student-centered instruction builds from students' prior knowledge and current understandings, identifying and articulating what students are doing mathematically may lead to building meaningful instruction.

Provide evidence for claims about what students do or do not know. When discussing students as mathematics learners, providing evidence for claims is key. Stating evidence assists in avoiding unwarranted speculations or judgments that detract from a focus on the mathematics. Moreover, evidence provides details about students' mathematical thinking that can be leveraged when considering future instructional moves.

Develop hypotheses about students' mathematical reasoning. Once what students are doing is identified, more accurate hypotheses about students' possible understandings or alternate conceptions can be made. It is important, however, to remember that when discussing videos or samples of student work, what we have are hypotheses, not certainties, about students' understandings. We can consider what instructional experiences might provide us with opportunities to confirm or revise our hypotheses.

Recognize when statements are speculations or judgments. Often, when discussing students' mathematical work, we may speculate what students are or are not capable of doing. Recognizing when statements are speculations or judgments allows for the examination of assumptions or expectations one may carry about students as learners of mathematics. Such recognition focuses on children as mathematical thinkers rather than other factors such as behavior, race, or gender.

Discussion

We have continued to use the same professional learning task described earlier with teachers in order to support them in focusing on students' mathematical thinking. However, we share the norms for discussing students' mathematics as shown in Figure 1 with the participants prior to watching the videos. As teachers examine students' mathematical thinking, we explicitly encourage them to focus on what students are doing, provide evidence for their claims, make hypotheses instead of certainties, and understand when statements are speculations. During professional discussions, when teachers discuss students using predominantly non-mathematical characteristics, or are unclear about evidence or expectations, we openly challenge them to apply these norms to focus their discussion in productive ways. Though occasionally the use of language and expectations associated with students' grade level or perceived ability occurs, we are finding that teachers recognize when these labels are not useful for considering students' mathematical thinking. As one teacher commented:

> *I think more about where they are in their learning as opposed to 'we're at the end of third grade and this is what you should be doing.' It's more so, at the beginning of third grade, he was doing things on this level or this level, but look at the progress he's made. He's now dabbling in place value and he's really strong in counting on.*

Further evidence of teachers' use of the norms to describe students' mathematical thinking from subsequent iterations of the LTBI professional development can be found in the following quotes.

> *Her understanding right now is that you take the smaller numbers from the larger numbers, so she was moving from the top number, you know…I think she would get it but she is missing that link. But that's just something she hasn't been taught yet.*

> *She knows she can't take something away from zero.*

> *She could look at the rod and cover up 3 and see it was 7 and say, "Oh, this is 47."*

> *I am speculating that maybe she has never been taught, she doesn't have the language to describe what she just did. Basically, she regrouped…she didn't know that's what she was doing…I wonder if she just has never been officially taught regrouping.*

Conclusion

Based on these experiences and the positive outcomes of the norms for discussing students as mathematics learners, we conclude by offering a set of questions that may assist teachers and teacher leaders in agreeing upon productive ways to carry out professional discussions of students' mathematical thinking.

1. What is the student able to do mathematically?
2. What evidence do I have?
3. What does this reveal about the student's understanding?
4. What are some potential instructional moves based on the student's current understandings?

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The Essence of Formative Assessment in Practice: Classroom Examples

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Abstract

Formative assessment involves the eliciting of students' understanding for the purpose of informing instructional decisions. In this paper, we present an overview of formative assessment strategies. We include classroom examples that capture the essence of formative assessment and conclude with questions intended to engage teachers and teacher leaders in reflecting on the teacher actions necessary to support effective implementation of formative assessment strategies.

Introduction

In a classroom that uses assessment to support learning, the divide between instruction and assessment blurs. Everything students do—such as conversing in groups, completing seatwork, answering and asking questions, working on projects, handing in homework assignments, even sitting silently and looking confused—is a potential source of information about how much they understand. (Leahy, Lyon, Thompson, & Wiliam, 2005, p. 19)

One emphasis in education today is using formative assessment to inform instruction and learning. In a Web search on the topic, one finds a tremendous amount of available information. On the day we looked, there were approximately 1,110,000 results. When we narrowed our search to "formative assessment math," there were approximately 946,000 results. Although this search suggests there exists enough information for teachers and other mathematics leaders about formative assessment, Popham (2012) indicated that the essence of formative assessment is being lost in classrooms.

Formative assessment is not a test or activity. Rather, the essence of formative assessment is "the relentless attention to evidence of student thinking" (p. ix) and the systematic and intentional use of this information to inform instruction (Popham, 2012). Formative assessment is a planned process, used by teachers during instruction to adjust teaching or by students to adjust their current strategies and tactics in an effort to improve students' achievement of intended instructional outcomes (Popham, 2013).

This paper is designed to provide a series of classroom examples featuring formative assessment in action. The situations portrayed in each have been selected for their potential to reveal the complexity inherent in the use of formative assessment and to make visible the essence of this powerful instructional tool. The examples come from observations in U.S. classrooms and are examples of what is possible. If you had the opportunity to speak with the teachers from these classrooms, they would tell you that their current practice evolved over multiple years, after being part of on-going professional development on the use of formative assessment. Some of the classroom examples are based on observations made across several classrooms and synthesized into one example. Others are based on observations made in single classrooms. This will be indicated at the beginning of each classroom example. As you read through each classroom example, think about *the actions* the teacher takes to engage all students in the mathematics and learning experience and to gather, interpret, and act on evidence of student thinking.

At the end of this paper, questions are provided to help mathematics education leaders, coaches, and teachers analyze formative assessment practices in the classroom. These questions are also meant to guide discussions or reflections that, in turn, can be applied to lesson plans to move the effective use of formative assessment forward in an effort to increase the likelihood of moving student learning forward.

Gathering, Interpreting, and Acting on Evidence at the End of a Lesson

Background Information

This section describes general practices observed across multiple classrooms and synthesized into a single classroom example. The specific pieces of student work and actions taken are examples of these practices and are included to make an important point about interpreting and acting on evidence, as well as what it means to provide actionable feedback to students.

One formative assessment strategy used by many teachers to collect evidence to inform instruction and student learning is the practice of asking students to solve a problem and/or explain reasoning at the end of the lesson, often referred to as exit cards. The question is based on the goal of the lesson and the evidence in the students' work is to help inform one's lesson planning for the next day. What follows is an example of the use of exit cards in a classroom focused on developing strategies for comparing and ordering fractions. Students in this classroom were given the question in Figure 1.

FIGURE 1.
Fraction comparison exit card
(The Ongoing Assessment Project, 2013).

Which fraction is closest to 1?
Show your work or thinking.

$\frac{7}{3}$ $\frac{7}{5}$ $\frac{7}{6}$ $\frac{7}{12}$

Classroom Example 1

Since administering exit card questions at the end of a lesson is a regular part of Ms. Brown's practice, the students understand that the responses will not be graded. Rather, the teacher will use their work to inform her instruction. Ms. Brown approaches the analysis of her student work with the following questions (Petit, Hulbert, & Laird, 2012) in mind.

1) What are evidences of developing understandings that can be built upon?
2) What are issues, misconceptions and/or errors of concern?
3) What are potential next steps based on the evidence?

Figures 2 and 3 provide examples of students' work typical of what Ms. Brown reviews. Notice that all of these responses have the correct answer: $\frac{7}{6}$.

However, Ms. Brown is interested in more than right answers. She uses her knowledge of how students develop understanding and identifies errors or misconceptions that may be interfering with learning new concepts or solving problems, with the goal of identifying the next step needed to move students' learning (Heritage, 2007)

FIGURE 2a.
Kelyn's exit card response (Petit, Hulbert, & Laird, 2012).

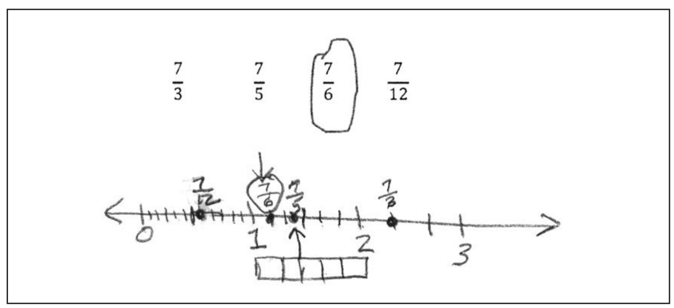


FIGURE 2b.
Abdi's exit card response (Petit, Hulbert, & Laird, 2012).

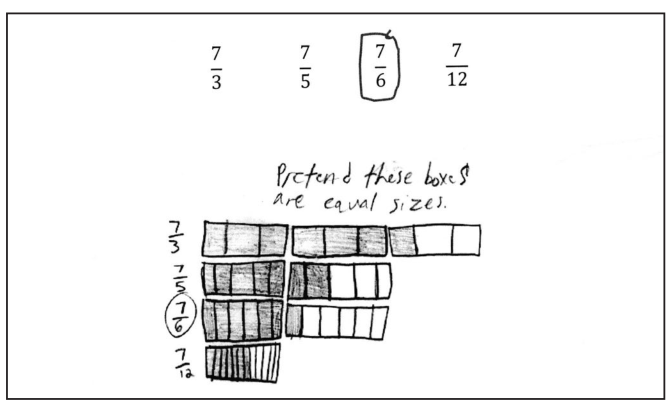
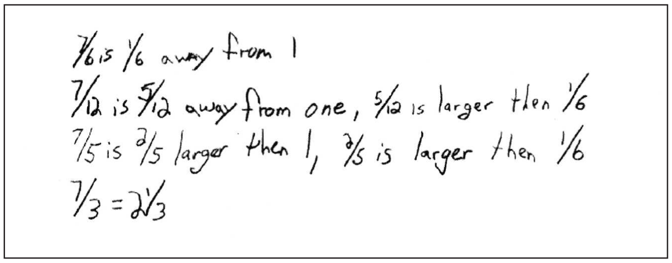


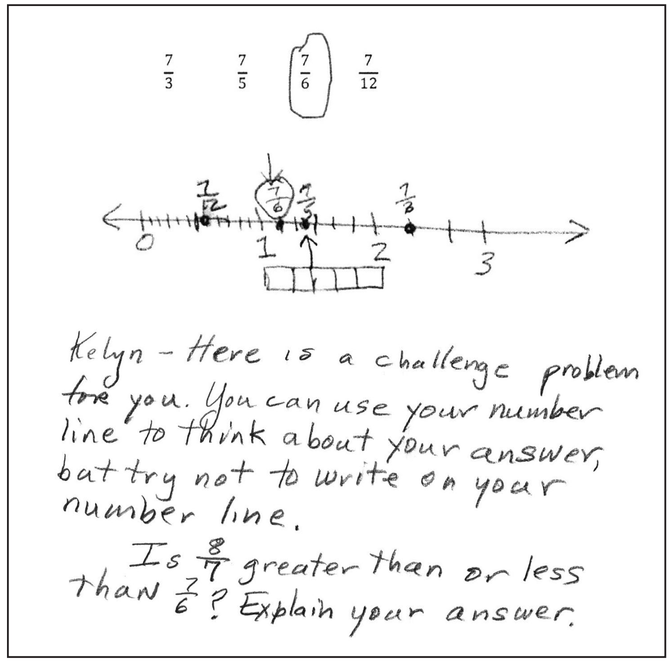
FIGURE 3.
Sam's exit card response (Petit, Laird, & Hulbert, 2014).



In looking at the evidence, one would likely note that the visual models that Kelyn and Abdi (Figure 2) used are important steppingstones to more efficient reasoning strategies when comparing fractions. Based on the findings, Ms. Brown decides to focus her instruction for the next couple of days on helping students build their understanding from visual models to reasoning without a visual model, as evidenced in Sam's response (Figure 3). Starting the next lesson with Kelyn's, Abdi's, and Sam's responses, Ms. Brown engineers a class discussion designed to help students investigate how Sam's reasoning is reflected in the Kelyn's and Abdi's visual models. Using the evidence elicited from this initial discussion, Ms. Brown follows with a series of fraction comparison questions focused on understanding the impact of partitioning in their visual models to advance their unit fraction and benchmark reasoning.

Ms. Brown also provides feedback to her students. As a regular part of her practice, Ms. Brown's feedback often appears in three forms: whole class oral feedback; individual oral feedback; and individual written feedback on students' papers. When she gives feedback to students, she knows that providing "comments like 'think' or 'try again' or 'good work' do not result in increased motivation or raising goals and therefore do not result in increased student achievement" (Wiliam, 2011, p. 127). Instead, she works hard to design questions that ask students to think and take action on their work (Wiliam, 2011), as exemplified in Figure 4. Based on evidence from Kelyn's exit card, as well as other work, Ms. Brown had noticed that Kelyn was consistently relying on the number line to compare fractions instead of transitioning to benchmark and unit fraction reasoning. The written feedback on her response (Figure 4) is an attempt to move Kelyn's thinking to a new level.

FIGURE 4.
Ms. Brown's feedback to Kelyn (Petit, Hulbert, & Laird, 2012).



Ms. Brown also considers next steps to push Sam's learning. She understands that changing the context of a problem or even the numbers in a problem can influence a student's ability to solve problems with similar mathematics. With this understanding she decides to engage Sam and the other students with similar responses in a new problem (see Figure 5), engineered to elicit additional evidence of their unit fraction reasoning.

FIGURE 5.
Follow-up problems (Petit, Laird, & Hulbert, 2014).

Answer the following two questions and consider if the reasoning you used in yesterday's exit problem can be used to solve these? Why or why not?

a) Isaac said $\frac{1}{125} > \frac{1}{57}$ Is Isaac correct? Why or why not?

b) Sheila believes that the inequality below is a true statement. Is she correct or incorrect? Explain your reasoning.

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} > \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

Reflection

This intentional and systematic analysis and use of evidence of student thinking by Ms. Brown is what Popham (2012) referred to as the *essence* of formative assessment. Ms. Brown selected an exit card that provided her evidence based on the goals of her lesson. Then, based on the evidence in the student work, she made instructional decisions about the instruction for all her students, focusing on the needs of individual students.

Formative Assessment Strategies

Formative assessment is much more than implementing exit cards at the end of a lesson. As previously stated, it can be everything students do if teachers use the information. To this end, five overarching strategies have been identified for supporting the use of formative assessment (Leahy et al., 2005). These strategies have been published in many documents, including the Joint Position Paper by the National Council of Supervisors of Mathematics and the Association of Mathematics Teacher Educators (2014) and in a National Council of Teachers of Mathematics Research Brief (Wiliam, 2007). The five assessment strategies are:

- clarifying and sharing learning intentions and criteria for success;
- engineering effective classroom discussions, questions, and learning tasks;
- providing feedback that moves learners forward;
- activating students as the owners of their learning; and
- activating students as resources for one another. (Leahy et al., 2005, p. 20)

It is important to note that these strategies support the effective use of formative assessment. That is, each strategy is not a formative assessment itself. The essence still remains that the evidence must be "elicited, interpreted and used by both teachers and students" (Wiliam, 2011, p. 43) to inform instruction and learning.

Although the strategies described by Leahy et al. (2005) look like a list of separate activities and events, they are not. Consider the example above of Ms. Brown and the exit cards. The activity of using the cards does not mean that formative assessment took place. Rather, it was the combination of the teacher posing a problem to her class for the purpose of gathering evidence on her students' reasoning when comparing fractions to a benchmark, analyzing the evidence, and then using it to inform her planning, targeted at moving her students' thinking forward, that made the event formative assessment at its essence.

Clarifying and Sharing Learning Goals

Background Information

This classroom example was based on an observation of a 5th grade teacher. It provides an example that is typical of how this teacher engages his students in learning goals. It also is an example of how the strategies stated above are interrelated and represents one way that a teacher might clarify and share learning goals. One can go into many classrooms and see teachers posting the goal of a lesson, or even the mathematics standard that is to be addressed that day. In Mr. Phillips's classroom, however, one sees the strategy, *clarifying and sharing learning intentions and criteria for success*, at its essence.

Classroom Example 2

Class starts with the students opening their mathematics notebooks, dating a page, and writing the goal at the top of the paper. Mr. Phillips has the following goal for the day posted on the white board.

> *Goal:* Use visual models to understand how to use benchmark and unit fractions reasoning when comparing and ordering fractions.

Mr. Phillips asks someone to read the goal for the lesson. Where his lesson departs from the norms of other classrooms and captures the *essence* of formative assessment is when he asks students to individually think for a minute and then talk with their partners about what the goal means and how it is connected to what they have been working on. He also asks them to identify any words they do not understand. It is apparent from the student interaction that this analysis of the goal is a regular part of the practice in this classroom. As the students talk with their partners, Mr. Phillips circulates the room, listening into conversations (but not talking) for what sense students are making of the goal, evidence of understanding the goal, and connections students are making to previous lessons. Next, Mr. Phillips leads a whole-class discussion.

> **Mr. Phillips:** Richard, what have you and your partner been discussing?

Richard: It looks like we will be comparing and ordering fractions, but we were not sure if we would be allowed to draw visual models anymore or if we needed to use other strategies.

On the front board, under the goal, the teacher writes "comparing and ordering fractions."

Mr. Phillips: Kim, did you and your partner have a similar discussion or can you add to their thoughts?

Kim: Like them we saw that we would still be comparing and ordering fractions, but we thought the goal means that we will use our visual models to understand and use these new strategies?

Under the goal the teacher writes "Use visual models to understand."

Robert: Sally and I thought the same thing. We thought we might start using our drawings in the beginning of the lesson and then move away from using them like we had done before in other lessons.

Under the goal the teacher writes "use visual models in the beginning and move away from them."

Richard: Oh, I get it. I remember when we first drew visual models to understand that a fraction is made up of unit fractions. After a while we stopped having to draw the model to know that

$$\tfrac{3}{4} = 3(\tfrac{1}{4}) \text{ and } \tfrac{3}{4} = \tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}.$$

Is that what we are doing today?

Mr. Phillips: Yes, that is like what we are doing over the next couple of days. How many other people saw the goal in this way? (Many students raised their hands.) I have a few more questions. Did any groups discuss what it meant to compare to a benchmark? Or compare using unit fractions reasoning?

Several hands are raised.

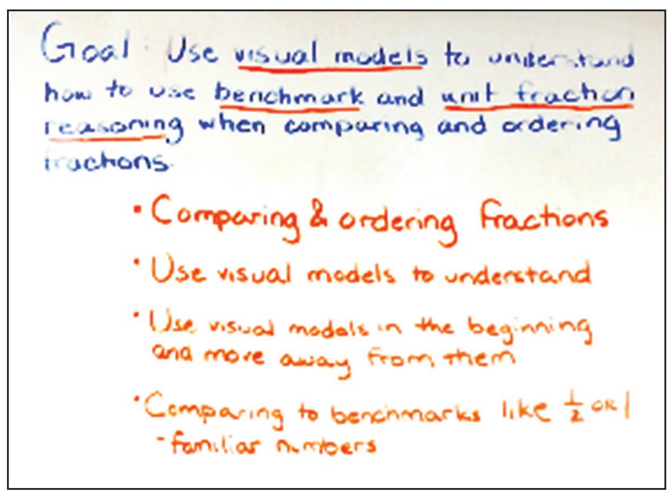
Caitlyn: We remembered in grade 3 comparing $\tfrac{1}{3}$ and $\tfrac{2}{3}$ to $\tfrac{1}{2}$. (Others shake their heads remembering this.) We remembered that a benchmark number is like $\tfrac{1}{2}$ or 1 – something that is familiar.

Gavin: We remembered that as well. We also discussed what a unit fraction was but were not sure what it means to compare fractions using unit fractions reasoning.

Next, Mr. Phillips wraps up the discussion and uses this to transition to the lesson.

Mr. Phillips: Comparing fractions using unit fraction reasoning is a new idea that we will work on today as well as moving away from using our models all the time to compare fractions. You have gotten very good at using both rectangles and number lines to compare fractions, but sometimes the fractions we need to compare can't easily be compared using a visual model, and there are more efficient ways to compare fractions than always drawing a picture. Starting today and for the next couple of days we are going to work on comparing different kinds of fractions using more efficient strategies. We will keep our mathematical goal, with your thoughts and interpretations about it, posted (see Figure 6). If you would like to add anything to the clarification of the goal, please let me know.

FIGURE 6.
Goal with student descriptions of understanding.



At the end of the lesson, Mr. Phillips' students open their mathematics notebook and respond to his exit question written on the board (see Figure 7). Note how the question was specifically designed to elicit benchmark and unit fraction reasoning. That is, a student may reason as follows: $\tfrac{3}{4}$ is $\tfrac{1}{4}$ greater than $\tfrac{1}{2}$; $\tfrac{5}{12}$ is $\tfrac{1}{12}$ less than $\tfrac{1}{2}$. Since $\tfrac{1}{12}<\tfrac{1}{4}$, $\tfrac{1}{12}$ is closer to $\tfrac{1}{2}$ than $\tfrac{1}{4}$. In this way, the problem connects directly to his original goal for the lesson. A careful review and analysis of students' responses provides evidence of their understandings and guidance to Mr. Phillips' lesson planning and instruction for the next lesson.

FIGURE 7.
Mr. Phillips's exit card (Petit, Laird, & Hulbert, 2014).

Which fraction is closest to 1/2?
Explain your thinking.

$$\frac{7}{6} \text{ or } \frac{5}{12}$$

Reflection

This vignette exemplifies many of the strategies described by Leahy et al. (2005), including *engaging all students in the discussion, asking questions that revealed student thinking,* and *providing opportunities for students to be resources to each other* as they worked in small groups. However, its real value is in the teacher's intentionality about assuring that students understand the mathematical goal of the lesson. He knew from experience that if students do not understand the goal (i.e., to move to a new level of understanding — from models to reasoning based on the use of benchmarks and unit models), they may continue to rely on earlier strategies.

Connectedness of Formative Assessment Strategies

Background Information

This vignette describes an observed lesson in a single classroom belonging to Ms. Gibson. In her classroom, one can regularly find a teacher *engineering effective classroom discussions, questions, and learning tasks*. On most days, one sees her making an ongoing effort to continually gather, interpret, and use multiple sources of information to understand what her students know so that she can make on-going adjustments to her instruction.

Part way through a unit on fraction operations, and as an introduction to multiplication of fractions, the instructional materials Ms. Gibson uses asked the students to work on a task (see Figure 8) before any introduction to formal procedures for multiplying fractions.

In asking the students to draw a picture of the transactions, with an expectation that one will explain their answer, an opportunity is provided in the materials for the students to

FIGURE 8.
The Brownie Problem
(Lappan, Fey, Fitzgerald, Phillips, & Friel, 1998).

The school is having a carnival. One of the booths is selling brownies. The brownies were made by the school kitchen staff in large square pans. Individuals can buy a whole pan or they can buy part of a pan.

Mr. Schmidt stops by the brownie booth and buys 1/3 of a pan. Ms. Cady comes up right after and wants to buy 1/2 of what is left in that brownie pan.

- Draw a picture to show what happened with these two transactions.
- Be prepared to tell how much of a pan Ms. Cady bought and how you arrived at your answer.

share their thinking (Heritage, 2007). Thus, by Leahy and colleagues' (2005) description of formative assessment strategies, this task has the potential to provide a teacher with insights into student thinking about fractions and multiplication before any formal procedures are presented.

Although the instructional materials provide a lesson plan for the task, Ms. Gibson expands what is provided in the plan, including additional questions she wants to ask of the class at the start of the lesson, when students are working in groups, and at the end of the lesson when she is working with the class to analyze and summarize the ideas of the lesson. Her questions incorporate what she knows about her students' knowledge and understanding of the topic that she has gathered from previous lessons, common misconceptions students often have on the topic, overall goals of the lesson, and where the particular lesson fits in the learning progression for the topic. Her detailed lesson planning is all part of her effort to create an opportunity for high quality discussion designed to elicit student thinking while building important mathematical understandings.

Classroom Example 3

Ms. Gibson launches the lesson by asking students to summarize what they have been working on during the past few lessons. Her students share how they have been solving problems with fractions and ways to add and subtract fractions. She listens carefully, asking students to say more, if they agree with what another student has said, and/or to add to what has been said. She shares the goal of this lesson, stating that they are going to continue to work on problems involving fractions. She does not state or hint that the lesson involves problems that can be solved by multiplying fractions. That goal is for a near future lesson.

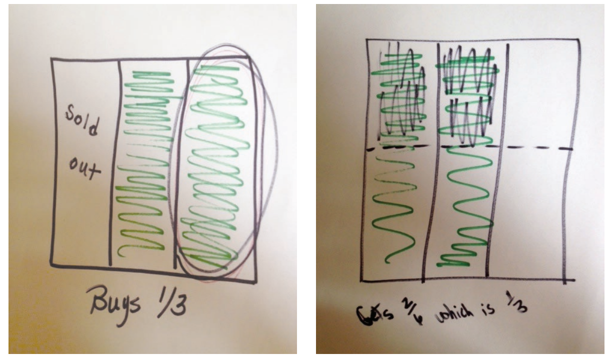
Ms. Gibson presents the task (see Figure 8) and asks students to individually think about the task and ways they might try and answer the questions. After approximately five minutes of individual time, she moves the students into groups of three to share their ideas and work together to answer the questions. Groups do their work on larger sheets of poster paper, which are displayed around the room when completed. As students work in groups, Ms. Gibson moves around the room listening and noting strategies and struggles encountered by the students in the groups. These conversations, along with the posters created by the groups, provide the teacher with insights into her students' thinking.

As posters are being completed and displayed, Ms. Gibson asks the students to review other group's posters, individually, looking for ways that the solutions are alike and ways that they are different. When all posters are up, she asks students to talk in their small groups about what they noticed. Again, she listens to students' conversations, analyzing what sense they are making of the mathematics on the poster and where the students are in their thinking and understanding. She compares the reasoning strategies she hears to the ones she anticipated when planning the lesson to help her make decisions about selecting and sequencing work samples and to adjust some of the questions she anticipated asking. For example, in analyzing the visual models on two of the posters (see Figure 9), she notes how the approaches are different, and considers how she would use these pieces, as well as what she heard the students saying, to help her make decisions on the best way to debrief the mathematics in the posters.

With this evidence in mind, Ms. Gibson starts the whole class discussion by calling on different groups to share what they noticed. She is intentional in selecting students and/or groups so as to get all ideas, right or wrong, in the open for the whole class to think about and consider. Yet, this does not mean that a student from each group is called upon to share her/his group's discussion, as some groups had the same or very similar ideas.

Based on what is shared during the analysis of posters, Ms. Gibson selects some groups to explain their work. She is strategic about which group she calls on to present

FIGURE 9.
Posters from Ms. Gibson's class.



first, second, and third, as she works to use the mathematics on the posters to move students' understanding along the learning progression. Based on her plans and student work/comments, she starts with a poster that shows the least movement in the learning progression and ends with a poster that shows the greatest. Although this is not the only way one might sequence the presentation of strategies, it was deliberately and intentionally chosen by this teacher for this class on this day as a means of further developing student understanding by making connections among the different strategies presented. In another situation, she might have used a different approach, such as looking for patterns across the solutions or to compare and contrast solutions in an effort to debate and question the solutions being presented (Smith, Bill, & Hughes, 2008).

As Ms. Gibson's students share their ideas, she uses the questions she anticipated when planning the lesson, sometimes asking a particular student what s/he thinks about another's explanation and other times asking the students to think about what was shared and then discuss it with the others in their group, using a—think, pair, share—technique in the middle of a whole class discussion. Through out this lesson's summary discussion, the teacher works to make the mathematics transparent, helping students see and connect ideas, and engaging all students in the discussion.

Reflection

With this lesson, like all of her lessons, Ms. Gibson works hard to move students' understanding of important mathematics (e.g., fractions) along a learning progression. Her incorporation of at least three formative assessment strategies (i.e., *engineering effective classroom discussions, questions, and learning tasks*; *providing feedback that moves*

FIGURE 10.
Evidence for self-assessment (Eley, 2012).

	This category means...	Pieces in this category might include...	You will need...
Growth/ Progress	You will show how a concept has grown for you over the course of a week.	• a piece of classwork that shows growth on an idea • an exit book goal and question showing understanding • proof of applied feedback on a tast or assignment	**3 pieces in this category**
Questioning	You asked questions of yourself and others to seek help or extend thinking.	• examples of times a concept didn't make sense and you sought help • examples of times a concempt made sense and you took on a challenge • extensions to a problem you explained • questions you asked of yourself or others	**3 pieces in this category**
Group Work/ Participcation	You worked with others in as a (cannot read) mathematician in math classroom.	• work that shows you changed your thinking based on someone else's ideas • proof work that shows collaboration with others in class • patience with others in math thinking	**3 pieces in this category**

learners forward; and *activating students as resources for one another*) into this lesson, and all five into her class routine on a regular basis, indicates her knowledge of these tools that she knows can help her in her effort. The point of her lessons and her work with her students, like the other examples in this paper, is not to "do formative assessment" but rather to use the tools/strategies of formative assessment to help students learn mathematics.

Activating Students as the Owners of their Learning

Background Information

The importance of activating students as the owners of their learning cannot be underestimated. Multiple studies have shown that strategies used to help students regulate their learning have had significant positive impact on performance (Wiliam, 2011). This classroom example, which was observed in a single classroom, demonstrates one way to accomplish the *essence* of this strategy.

Classroom Example 4

Building on his intentional and systematic approach to engaging students in goals of lessons, exemplified in Classroom Example 2, Mr. Phillip's students complete a weekly self-assessment of their progress. Given a set of criteria (see Figure 10), students are asked to take *ownership of their learning* by providing evidence of their progress in three categories: concept growth/progress; questioning; and group work/participation. Students assess their progress in each of these areas based on the stated criteria and provide concrete evidence of their learning from a range of sources: class work, exit questions, challenge problems, problem extensions, descriptions of changes, and evidence of seeking help on a topic.

Reflection

What is interesting about Mr. Philips's student self-assessment is how the Leahy et al. (2005) strategies are woven into the analysis that the students complete. For "growth and progress," students give evidence of their learning (*activating students as owners of their learning* and *sharing criteria for success*). In the section on "questions," students give evidence of *asking questions of themselves that moved them forward*. Finally, in the section on "group work," students give evidence that *they were a resource to their peers.*

Asking students to gather and analyze this type of information has the potential to do two things. First, it asks student to *take ownership* for making progress in the development of their mathematics learning. Second, it is another opportunity for Mr. Philips, and the students, to elicit, interpret, and use valuable information about their engagement and understanding of the week's mathematics ideas and concepts in the hopes of guiding instruction and the students next step in their learning.

Conclusion

These classroom examples demonstrate how formative assessment can be part of a teachers' regular practice. They are meant to highlight how formative assessment is more than an activity or an event, more than and different from another test or quiz. The examples are meant to show how the Leahy et al. (2005) strategies can support effective use of formative assessment, but only when teachers' use the strategies for the purpose of gathering information, analyzing student understanding, and influencing student learning. Although the examples demonstrate that the effective use of formative assessment is possible, it should be noted that in all of these teachers' classrooms it took time, effort, and a deep commitment to changing their practice. One cannot expect teachers who are beginning to use formative assessment to incorporate all of these strategies at once. Rather, they should focus on incorporating a few ideas at a time and building their practice over time.

Supporting teachers with expanding or incorporating the *essences* of formative assessment will need to include examining one's practice, broadening one's awareness of the strategies, and also constantly growing in one's ability to effectively analyze and appropriately use information to make instructional decisions. It also means connecting the strategies, as exemplified by the classrooms and teachers above.

Although we do not wish to underemphasize the complexity of this task, one starting point for such work is to use the questions that follow to examine teacher practice. These questions are designed to help one examine the actions that teachers take to gather, interpret, and act upon evidence of student learning in the spirit of implementing formative assessment at its *essence*.

- In what ways is there evidence that there is intentional planning for gathering evidence of student thinking throughout the lesson? What is the evidence?
- In what ways are teachers engaging students in the goal of the lesson so that they can begin to take ownership of their learning? What is the evidence?
- Is the teacher using strategies such as think-pair-share and group work to engage all students in the discussion? What is the evidence?
- Are the tasks used in instruction engineered to elicit student thinking in relationship to the instructional goal? What is the evidence?
- Are teachers using the evidence elicited as the lesson progresses to make instructional decisions? What is the evidence?
- Does the teacher have a strategy to gather evidence to inform the next day's instruction? What is the evidence?
- Does the teacher gather descriptive evidence about students' developing understandings, errors, and misconceptions rather than focusing exclusively on the answers to questions for correctness? What is the evidence?
- In what ways does the teacher help students self-regulate their learning? What is the evidence?

These questions can be used by both mathematics education leaders and teachers in a variety of ways: a) as a platform on which to observe practice and provide feedback to teachers on their implementation of formative assessment; b) in a Lesson Study group, to guide the planning, analysis, and revision to lessons; c) during a Professional Learning Community to focus discussion on formative assessment in practice; and d) as a self-assessment tool by teachers focusing on improving their formative assessment in their practice to name a few. ✪

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Envisioning the Role of the Mathematics Teacher

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Abstract

This article presents a tool for identifying and tracking changes in teachers' (and others') evolving visions of the role of the mathematics teacher through five levels: motivator, deliverer of knowledge, monitor, facilitator, *and* more knowledgeable other. *It includes a brief account of the rubric's development and a description of its use in a large-scale, longitudinal study of mathematics instructional reform efforts in four urban school districts, including an examination of relationships between the ways that teachers envision their roles and the quality of their instruction. The article concludes with a discussion of implications for mathematics education leaders, including a description of how the tool was used in a recent professional development effort.*

Introduction

During the last two decades, mathematics educators have made considerable progress in describing the multiple facets and nuances of the role that effective mathematics teachers play in the classroom to support students in meaningfully participating in classroom mathematical activity (cf. Ball, Sleep, Boerst, & Bass, 2009; Chazan & Ball, 1999; Hiebert et al., 1997; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Lobato, Clarke, & Ellis, 2005; Staples, 2007; Stein, Engle, Smith, & Hughes, 2008). Unfortunately, what has too often been omitted are descriptions of how teachers envision and enact their role along the way to developing such professional proficiency. As Staples (2007) argued, research has focused on describing instruction "*only once the practices have been established*" (p. 164, italics in original), leaving those charged with supporting teachers' ongoing learning without a roadmap for doing so.

In this article, I present a rubric for identifying and tracking changes in teachers' (and others') evolving visions of the role of the mathematics teacher—from less to more sophisticated articulations of classroom practice. After describing the tool's origins, I report on its use in a large-scale, longitudinal study of mathematics instructional reform efforts in four urban school districts. Then, I describe the relationship between ways that teachers envision their roles and the quality of their instruction in order to (a) make a case for the relevance and importance of attending to instructional visions, and (b) provide insight to potential users of the tool. Last, I discuss implications of this work for mathematics education leaders, including a description of how it was used in a recent professional development effort.

Considering Instructional Vision

Underlying the work described in this article are three assumptions about teacher professional development. First, teacher professional development occurs in a variety of settings, with a variety of resources and possible foci, including, among other things, co-planning lessons, examining student work, reading and discussing books or articles, watching and discussing video of teaching, peer observation, coaching cycles, and formal evaluation and feedback. Second, across most of these settings and foci, it is through talk that the bulk of teacher learning is expected to happen. Third, supporting individuals in developing more sophisticated ways of describing aspects of their practice can influence what they see and do in their classrooms. As Sfard (2007) suggested, “We need a discursive change to become aware of new possibilities and arrive at a new vision of things. We thus often need a change in how we talk before we can experience a change in what we see” (p. 575).

I refer to teachers’ and others’ dynamic conceptions and articulations of their (future) practice (Hammerness, 2001; Senge, 2006) as *instructional vision*. It is this notion of instructional vision that is the focus of this article, and the ways that teachers envision their role in the classroom in particular. If we expect teachers’ talk about mathematics instruction to precede their enactments, then we need to attend not only to what teachers do in their classrooms, but also the ways they articulate their role and the vision they have for what they are striving to accomplish. Knowing the end goals or best practices, however, is not sufficient; we need to be able to anticipate the trajectories of growth that teachers’ conceptions and enactments of high-quality mathematics instruction might follow (Sherin, 2001), and then support them in moving along that pathway. Of course, *growth* implies that progress is defined with respect to a particular vision of the teacher’s role, which I summarize next.

The Role of the Mathematics Teacher

Over the last several years, mathematics education research has documented cases of teachers striving to support students in learning mathematics with understanding (cf. Boaler & Humphreys, 2005; Hiebert et al., 1997; Staples, 2007) and in developing the kinds of mathematical practices identified in the Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSI], 2010). Such work has often described the teacher as a *co-participant* in authentic mathematical activity (Cobb & Yackel, 1996; Rogoff, Matusov, & White, 1996). This does not mean that students are engaged in *pure discovery learning* or that the teacher’s job is to merely keep students on task as they spontaneously reinvent the mathematics curriculum. Rather, the teacher plays a crucial role in each phase of a lesson, as well as the planning and reflection that precede and follow the lesson. Although far more complex than can be described here, the role of the mathematics teacher can be at least partially defined with respect to three dimensions drawn from the research literature.

Lesson Structure

The first dimension involves structuring a lesson’s activity by employing a three-phase classroom activity structure (Van de Walle, Karp, & Bay-Williams, 2012). Within this structure, the teacher begins by posing a problem and ensures that all students understand the context and expectations (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Next, students develop strategies and solutions, typically in collaboration with each other. Finally, through reflection and sharing, the teacher and students work together to clarify the mathematical concepts underlying the lesson’s problem (Stein & Smith, 2011; Stigler & Hiebert, 1999).

Classroom Discourse

In considering the role of the mathematics teacher, a second dimension includes influencing classroom discourse, in which the teacher proactively supports students in participating in mathematical conversations (Fraivillig, Murphy, & Fuson, 1999). This influencing includes: eliciting students’ explanations and questions and then using those contributions as lesson content (Lappan, 1993; Staples, 2007; Stein & Smith, 2011); engaging *with* students in mathematical argument (Lampert, 1990); and choosing appropriate moments to share essential information such as conventional rules or symbols and alternative methods (Hiebert et al., 1997).

Mathematical Authority

A third dimension to consider in the role of the mathematics teacher involves sharing mathematical authority with students. This can be evidenced by the teacher consistently treating students as thinkers and decision-makers (Staples, 2007). In addition, the teacher ensures that students share in the responsibility for determining whether mathematical ideas and strategies are valid, rather than relying solely on the teacher or textbook (Simon, 1994).

This vision of the role of the teacher represents the top level of the rubric described below. The rubric, which establishes the goal of instructional reform efforts, was originally developed within a research project involving four large urban school districts. A description of this research project follows.

Methods

The overall goal of the larger study, the Middle School Mathematics and Institutional Setting of Teaching (MIST) project, was to investigate, test, and refine a set of hypotheses and conjectures about organizational support structures that enhance the impact of professional development on middle-grades mathematics teachers' instruction and student achievement. Working for four years with four urban school districts with ambitious goals for reforming math instruction provided opportunities to investigate teachers' (and others') evolving conceptions of high quality mathematics instruction in settings in which leaders were promoting change. To do so, I developed a series of rubrics for assessing visions of high quality mathematics instruction (VHQMI), including teachers' articulations of high quality classroom discourse, mathematical tasks, student engagement, and the role of the teacher. The latter of these is the focus of the rubric presented in this article.

A more thorough description of the development and application of all of the VHQMI rubrics is provided elsewhere (Munter, 2014). Here, I will provide a brief account of the development of the role of the teacher rubric and its use in scoring interviews. Then, I will describe my methods for examining the importance of considering how teachers envision their role.

Rubric Development

I developed the rubric based on analyses of more than 100 interviews conducted during the first two years of the MIST project with middle-grades mathematics teachers, coaches, principals, and district leaders. In those interviews, we asked participants:

> If you were asked to observe another teacher's math classroom, what would you look for to decide whether the mathematics instruction is high quality?
>
> Why do you think it is important to use/do _____ in a math classroom?
>
> Is there anything else you would look for? If so, what? Why?

If the participant had not already described the role of the teacher, we asked:

> What are some of the things that the teacher should actually be doing in the classroom for instruction to be of high quality?

Taking the research-based description of the role of the teacher summarized previously as the top level, I interpreted each interview response against that benchmark, looking for patterns indicating potentially important qualitative distinctions that could help model a developmental trajectory of the ways that teachers' and others' visions of the role of the mathematics teacher might change over time in settings in which instructional reform is being supported.

Especially useful in this analysis was research that has identified important variations in form- and function-relationships within mathematics instructional reform efforts (Saxe, Gearhart, Franke, Howard, & Crockett, 1999; Spillane, 2000). For example, Saxe et al. (1999) described how teachers might employ *new forms* of assessment, such as more open-ended questions, to serve the *old function* of evaluating the correctness of answers, rather than using the questions to diagnose students' thinking. Likewise, Spillane (2000) found that district leaders might describe the need for real-world connections in terms of making mathematics more relevant and engaging for students, failing to emphasize the function of providing meaningful contexts for students' sense-making and mathematical reasoning. Such distinctions proved useful in differentiating between less and more sophisticated descriptions of the role of the teacher. For example, as described further below, I identified differences in articulated functions underlying assertions that the teacher should act as facilitator or refrain from lecturing, emphases on group work, and descriptions of the place and purpose of students' talk.

This analysis resulted in the rubric that is presented in Figure 1. All but the lowest level (Level 0) are defined with respect to the three dimensions identified above: *conception of typical activity structure*, *influencing classroom discourse*, and *attribution of mathematical authority*. Although reading all of the level descriptions is likely necessary in order to understand the qualitative differences that the rubric is intended to capture, a summary of the primary conceptual distinctions, accompanied by sample participant quotes from our interviews, may orient the reader to the rubric's intent.

Motivator. At the lowest level (i.e., Level 0), an individual's description of the role of the teacher is limited to an assertion that the teacher must be energetic and captivating so that students will be sufficiently motivated to learn. "It is more about being an entertainer than it is a teacher." But "making connections [to students]" does not mean that they will learn mathematics.

Deliverer of knowledge. Level 1 descriptions emphasize that the role of the teacher is to teach mathematics. Specifically, at Level 1 an individual's description suggests that the teacher has mathematical knowledge that must be imparted unto students, which requires very clear explanation. For example, according to one participant the "teacher provides clear instructions, clear assignment, examples shown, students being walked through a problem." Others noted that if students have questions, "they should feel free to ask," and that the teacher "should answer all student questions," including "explain[ing] why and how it's used in everyday life, not just formulas."

Monitor. At Level 2, individuals' descriptions of the role of the teacher suggest that students play an active role in working together on mathematical tasks and that affording time to students for figuring out or, more likely, reproducing what the teacher has explained or demonstrated is important. A typical description at this level was that the teacher should "show [students] examples [of] how to do it and why are they doing it, what is the purpose of it. Then, do the facilitation, walk around, see the group work." Whereas at Level 1 the image of students' role is one of receiving knowledge, at Level 2 students play a role in mediating what the teacher has explained. Individuals

FIGURE 1. *VHQMI Rubric: Role of the Teacher* *(continued on pg. 33)*

Level	Description	Potential ways of characterizing teacher's role
4 Teacher as "more knowledgeable other"	Describes the role of the teacher as proactively supporting students' learning through co-participation. Stresses the importance of designing learning environments that support problematizing mathematical ideas, giving students mathematical authority, holding students accountable to others and to shared disciplinary norms, and providing students with relevant resources (Engle & Conant, 2002).	Influencing classroom discourse: Suggests that the teacher should purposefully intervene in classroom discussions to elicit & scaffold students' ideas, create a shared context, and maintain continuity over time (Staples, 2007).
		Attribution of mathematical authority: Suggests that the teacher should support students in sharing in authority (Lampert, 1990), problematizing content (Hiebert et al., 1996), working toward a shared goal (Hiebert et al, 1997), and ensuring that the responsibility for determining the validity of ideas resides with the classroom community (Simon, 1994).
		Conception of typical activity structure: Promotes a "launch-explore-summarize" lesson (Madsen-Nason & Lappan, 1987), in which (a) the teacher poses a problem and ensures that all students understand the context and expectations (Jackson et al., 2013), (b) students develop strategies and solutions (typically in collaboration with each other), and (c) through reflection and sharing, the teacher and students work together to explicate the mathematical concepts underlying the lesson's problem (Stigler & Hiebert, 1999).
3 Teacher as "facilitator"	Focuses on the forms of "reform instruction" without a strong conception of the accompanying functions that underlie those forms: either (a) views the teacher's role as passive, as students discover new mathematical insights as the result of collaborative problem solving (e.g. "romantic constructivism"), or (b) describes a transitional view that incorporates both teacher demonstration or introduction (e.g., at the beginning of the lesson) and "turning it over" to the students (who then make the remaining "discoveries"). Description likely stresses "rules" for structuring lessons, discussion, etc., or describes posing problems and asking students to describe their strategies but does not detail a proactive role in supporting students in engaging in genuine mathematical inquiry (Kazemi & Stipek, 2001).	Influencing classroom discourse: Describes the teacher facilitating student-to-student talk, but primarily in terms of students taking turns sharing their solutions; hesitates to "tell" too much for fear of interrupting the "discovery" process (Lobato et al, 2005).
		Attribution of mathematical authority: Supports a "no-tell policy": Stresses that students should figure things out for themselves and play a role in "teaching." Suggests that if students are pursuing an unfruitful path of inquiry or an inaccurate line of reasoning, the teacher should pose a question to help them find their mistake, but the reason for doing so focuses more on not telling than helping students develop mathematical authority. Is open to students developing their own mathematical problems, but these inquiries are not candidates for paths of classroom mathematical investigation.
		Conception of typical activity structure: Promotes a "launch-explore-summarize" lesson (Madsen-Nason & Lappan, 1987), in which (a) the teacher poses a problem and possibly completes the first step or two with the class or demonstrates how to solve similar problems, (b) students work (likely in groups) to complete the task(s), and (c) students take turns sharing their solutions and strategies and/or the teacher clarifies the primary mathematical concept of the day (i.e., how they "should have" solved the task).

who envision such a role of the teacher might suggest that students who "get it" should be invited to (re)teach their classmates. "Having a kid who's really good at the math, but who's still at their [peers'] level, sometimes they can explain it a little bit better [than the teacher]." Still, at this level, it is the teacher's job to identify and correct students' misconceptions by intervening directly. As one participant suggested, if students are pursuing a solution path that looks like a dead end, "the teacher needs to circle the wagons, regroup, 'Oh guys this is not working out. We need to back up cause, cause we're going the wrong way."

Facilitator. A Level 3 envisioning of the teacher's role marks an important shift in who does the mathematical work in the classroom. At this level, an individual describes the teacher's role as facilitating students' sense making during at least part of the lesson, and this can be done in one of two ways. First, an individual may envision a *passive* role of the teacher, in which students collaborate to discover the lesson's main ideas. "The kids are pretty much teaching themselves; the teacher's just kind of up there facilitating and making sure that their light bulbs are turning on." Alternatively, an individual may describe a

FIGURE 1. *VHQMI Rubric: Role of the Teacher (cont. from pg. 32)*

Level	Description	Potential ways of characterizing teacher's role
2 Teacher as "monitor"	Describes the teacher as the primary source of knowledge, but stresses the importance of providing time for students to work together, to try on their own and make sense of what the teacher has demonstrated, to (first) explain things to each other, and then get help from the teacher.	Influencing classroom discourse: Suggests the teacher should promote student-student discussion in group work.
		Attribution of mathematical authority: Suggests a view of teacher as an "adjudicator of correctness" (Hiebert et al, 1997). Students may participate in "teaching" but only as mediators of the teacher's instruction, adding clarification, etc. If students are pursuing an unfruitful path of inquiry or an inaccurate line of reasoning, the teacher stops them and sets them on a "better" path.
		Conception of typical activity structure: Promotes a two phase, "acquisition and application" lesson (Stigler & Hiebert, 1999), in which (a) the teacher demonstrates or leads a discussion on how to solve a type of problem, and then (b) students are expected to work together (or "teach each other") to use what has just been demonstrated to solve similar problems while the teacher circulates throughout the classroom, providing assistance when needed.
1 Teacher as "deliverer of knowledge"	Describes the teacher as the primary source of knowledge, focusing primarily on mathematical correctness and thoroughness of explanations (i.e., showing all steps). Description suggests that students are welcome to ask questions, but that there is no expectation that the teacher will facilitate student collaboration or discussion.	Influencing classroom discourse: Focuses exclusively on teacher-to-student discourse. Considers quality of teacher's explanations in terms of clarity and mathematical correctness.
		Attribution of mathematical authority: Suggests that the responsibility for determining the validity of ideas resides with the teacher or is ascribed to the textbook (Simon, 1994). (This includes insistence that teachers be mathematically knowledgeable and correct.)
		Conception of typical activity structure: Promotes efficiently structured lessons (in terms of coverage) in which the teacher directly teaches how to solve problems. Periods might include time for practice while teacher checks students' work and answers questions, but this is likely quiet & individually-based with no opportunity for whole-class discussion. Description suggests no qualms with exclusive lecture format.
0 Teacher as "motivator"	Suggests that the teacher must first and foremost be sufficiently captivating to attract and hold students' attention.	

transitional role in which, at most, the teacher introduces the task and does the first part or two with the class before "turning it over" to the students, and then keeps students on the "right path" by asking questions.

In either case, key ideas of the lesson are left to the students to figure out, rather than limiting students to reinterpreting what the teacher has already demonstrated or explained (Level 2). For example, one participant suggested that in a high-quality lesson, students are "not waiting all the time for the teacher [to] come and spoon feed them, but doing investigating on their own, coming up with ah-has on their own or coming up with 'what if this'?" On its surface, such a description may represent a Level 4 envisioning of the teacher's role. But what is key to consider is the function underlying that form. At Level 3, the rationale for actively engaging students in figuring problems out is not that it affords opportunities, for example, to become proficient in the mathematical practices specified in the Common Core (CCSSI, 2010), but rather that it "helps [students] remember it a little bit better than just a teacher up there talking about it." Similarly, a Level 3 envisioning of the teacher's role likely includes a commitment to "not telling" (e.g., the teacher should "answer questions with questions," or, if students are headed "down the wrong path," the teacher should "ask them something else to put them back on the right track"), but the rationale for which is not about supporting students in developing mathematical authority.

More knowledgeable other. At the highest level (described above), an individual describes the teacher's role as proactive, co-participation with students, in which the teacher has a clear image of the instructional goals, and orchestrates, scaffolds, and builds on student contributions to achieve that goal. Distinct from Level 3, envisioning this kind of a role of the teacher requires acknowledging that students will likely not discover all of a lesson's learning goals without purposeful work on the part of the teacher to support them in participating in solving problems and participating in productive discussions about their ideas, questions, and explanations. For example, one participant described how the teacher should play a proactive role in supporting and scaffolding students' talk.

> When [teachers] pose a question and a student answers, they don't say "yes this is how it is always done." They ask the kids to explain how they came up with the answer, ask for other students to explain how they came up with the answer, present all the ideas to the student and ask them if these are good procedures for answering types of problems like this and talk about student preference—"Do you like one way more than another and does this way make sense?"—so that the kids can build their own frame of reference to the material.

Often accompanying such descriptions is a commitment to using students' explanations, responses, questions, and problems as lesson content (Fraivillig et al., 1999): "Students should be involved in the learning process as far as asking questions and being able to maybe actually give examples and working them and talking to the teacher about them." Such a perspective suggests that the teacher's role includes keeping students positioned as thinkers and decision-makers (Staples, 2007), the underlying function of which is to support students' engagement in mathematical practice. As one participant asked, "When kids are getting stuck, are you [the teacher] just pulling them out or are you asking those questions that press students to think even deeper so that they figure out the problem, that they become the problem-solvers?"

Interview Coding

The rubric for assessing individuals' ways of envisioning the role of the teacher was applied to 932 transcripts of interviews conducted over the first four years of the MIST project, including 433 teacher interviews. Each relevant statement was scored according to the levels of the rubric, with a final score determined by the highest score that was assigned. For example, in an entire interview transcript, multiple statements might have been scored at level 2, but if just one statement was scored at level 3, the final role of the teacher score for the participant's interview would have been a 3. The decision to score this way was both practically and conceptually motivated. The practical motivation stemmed from a need to establish rules for achieving sufficient reliability in coding nearly a thousand interviews. (Across all years combined, based on the 16% of transcripts that were double-scored, the overall rate of exact agreement in scoring with this rubric was 0.74.) More important, however, was the conceptual rationale. Because of the way the transcripts were coded, a score can be interpreted as representing the greatest level of sophistication with which a participant *was able to* describe the role of the teacher—not necessarily how the participant *typically* describes the teacher's role (and likely not the role a teacher *actually plays* in her/his classroom).

Of course, this is not the *correct way* to use the tool, only how it was used in one large research study, the analyses of which are reported in the next section. In different settings, such as working to support the learning of a local group of teachers, it would likely be used differently. I return to this notion in the discussion section.

Statistical Analyses

The analyses that are the focus of this article were conducted in order to answer two research questions:

1) Do the ways that teachers (and others, including principals, coaches, and district leaders) envision the role of the mathematics teacher change over time in settings in which leaders are promoting models of instruction aligned with mathematics education research?
2) Is the sophistication with which teachers articulate the role of the teacher related to the quality of their instruction? If so, how?

Given that our participants were in districts in which leaders were actively pursuing change in ways aligned with the vision of mathematics instruction on which the rubric is based, my conjecture was that the sophistication with which teachers described their role in the classroom would increase over time. To determine whether this was the case, I examined both the average scores among all teachers combined for each year and, because any increases in scores could be attributed to changes in teaching staff or study participants, I also examined average scores among just the 44 participants (teachers and others) whose interviews were scored for role of the teacher in all four years.

To answer the second question above, using regression analysis I examined the relationships between teachers' scores on the role of the teacher rubric and an index of instructional quality. The quality of teachers' instruction was assessed with an adapted version of the Instructional Quality Assessment (Boston, 2012). The two primary sections of the IQA are designed to assess the cognitive demand of classroom activity over the course of the lesson (i.e., academic rigor) and specific aspects of discourse during the whole-class discussion after students have had a chance to work on solving the task (i.e., Accountable Talk®). Members of the research team used the instrument to score video-recordings of two consecutive days of classroom instruction for each participating teacher in late winter of each year. Scores from eight IQA rubrics were combined to create two sub-scores, one pertaining to the cognitive demand of the mathematical task as posed and then as implemented, and one pertaining to class discussion. Additionally, these two sub-scores were averaged to create one annual, overall IQA score for each teacher. Each of these three scores—the task and discussion sub-scores, as well as the overall IQA—could range from 0 (low) to 4 (high), which, conceptually, maps roughly onto the range represented in the role of the teacher rubric.

Combining data across years, I calculated mean IQA scores for each level of the role of the teacher rubric. To do so, I used a two-level regression model to adjust for clustering within teachers. Including dummy variables for each level of the role of the teacher rubric and identifying each level as the base in multiple runs allowed me to test for significant differences in IQA scores between (consecutive) levels on the role of the teacher rubric. If the sophistication with which teachers describe their role in the classroom is associated with instructional quality, IQA means should increase with higher levels of the rubric.

Results

As listed in Table 1 *(see page 36)*, the results of using the rubric to score interviews suggest that, on average, the sophistication with which teachers described their role in the classroom increased. In addition, average scores among just those 44 participants whose interviews were scored for role of the teacher in all four years increased as well, with some changes in consecutive years being statistically significant. This suggested that the increase was not attributable solely to fluctuations in district personnel or study participants.

Table 2 *(see page 36)* lists mean instructional quality scores (both overall IQA and IQA sub-scores) by level of the role of the teacher rubric, adjusted for clustering due to repeated observations across some teachers. Additionally, statistically significant increases between consecutive levels are noted. Generally, overall IQA scores increased as role of the teacher scores increased. The difference in IQA scores between teachers with a Level 2 role of the teacher score and those with a Level 3 score was statistically significant, as was the difference in IQA scores between teachers with role of the teacher scores of 3 and 4.

Table 1: Mean role of the teacher scores by year

Year	All teachers combined	Participants with role of the teacher score in all four years [n = 44]
1	1.85 (sd = 0.94) [n = 82]	1.98 (0.88)
2	2.23 (0.96) [111]	2.48** (0.95)
3	2.50 (0.71) [118]	2.75# (0.69)
4	2.63 (0.67) [122]	2.66 (0.75)

Wilcoxon signed rank tests comparing consecutive years' means for participants with role of the teacher score in all four years: ** $p < 0.01$; # $p = 0.05$

Table 2: Mean instructional quality (IQA) scores by role of the teacher level

Role of the teacher score	IQA Task	IQA Discussion	Overall IQA
0	2.50	1.76	2.09
1	2.50	1.62	2.07
2	2.58	1.63	2.10
3	2.69#	1.75	2.22*
4	2.97*	2.01	2.47*

Note: two-level regression analysis to test for differences between consecutive levels of role of the teacher rubric and adjust for clustering within teachers (425 observations across 223 teachers): *$p < 0.05$; # $p = 0.07$

Examining the IQA sub-scores, however, provides at least two additional insights into this relationship. First, participants' task-related scores were, in general, higher than discussion-related scores. This is likely due, in part, to the fact that leaders in each of the districts had attempted to provide teachers with more inquiry-oriented curriculum materials, including the second edition of the Connected Mathematics Project series (CMP2; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) in three of the four districts. Simply using the tasks found in the district-provided curriculum would likely lead to higher task sub-scores. However, this does not explain the significantly higher scores among teachers with level 3 or 4 instructional visions of the teacher's role. As argued by Wilhelm (2014), this difference suggested that teachers with more sophisticated ways of envisioning their role were more likely to maintain a task's *potential* rigor in its *implementation* with students.

Second, although no difference in discussion sub-scores between consecutive levels on the role of the teacher rubric was statistically significant, there was an upward trend, including significant differences between teachers of Level 4 and Levels 1 and 2. Unlike the task sub-score, however, average discussion scores were higher among teachers who envisioned the role of the teachers as being primarily one of motivator (Level 0) than those of teachers with Level 1 or 2 instructional visions of the teacher's role. Although this finding should be treated cautiously, it does suggest that emphases on interpersonal and content-specific aspects of the teacher's role in teachers' instructional visions may relate differently to different aspects of their practice (i.e., those related to task choice and implementation, and those related to classroom discussion). In the following section, I discuss the implications of these results for potential users of the tool.

Discussion

In this article, I have presented a rubric that mathematics education leaders might use to identify and track changes in the ways that mathematics teachers envision their role in the classroom. It is intended to promote developmental approaches to supporting mathematics teachers' (and leaders') learning of high quality forms of practice (Stein & Matsumura, 2008), in that it provides a model for the pathways that the evolution in teachers' instructional visions (and possibly practice) might take. The findings reported in this article speak to the tool's validity. They suggest that the ways that individuals envision the role of the mathematics teacher in the classroom did change in settings in which such change was being promoted and supported. Also, the findings suggested that there is a positive relationship between the ways that teachers articulate their role and the quality of their instruction—an outcome of great importance to many stakeholders, considering the mounting evidence of the relationship between quality of teaching and student outcomes (cf. Nye, Konstantopoulus, & Hedges, 2004; Rockoff, 2004; Wilson et al., 2009). Before discussing the implications for mathematics education leaders, however, I wish to make two points of caution.

First, the rubric presented in this article was developed based on interviews with teachers in particular kinds of settings—four urban school districts whose leaders had formulated and begun implementing comprehensive initiatives for improving middle-grades mathematics instruction district-wide, including providing comprehensive professional development to teachers (and even principals) focused on placing students' reasoning at the center of instructional decision making, and adopting mathematics curricula aligned with such an agenda. The ways that teachers (re)envision their roles are likely highly influenced by the settings in which they work. The tool presented here was appropriately aligned for use in districts that participated in the MIST project; its levels may not align as well with different goals for instruction being promoted in other settings.

Second, the rubric was originally developed as a research tool, applied to transcripts of annual interviews with study participants. Although we observed increases in average scores, in many cases, we did not observe any change across multiple years. The rubric is likely very applicable for those working to support teachers' professional growth, but for those who interact more frequently and directly with teachers, it is important to remember that change takes time. That said, the levels should not be interpreted as rigid beliefs that teachers and others hold, or as developmental stages that instructional visions cleanly progress through one at a time. Instead they should be interpreted as a guide for what teachers currently consider important; are thinking about, looking for, or attempting to achieve in their classrooms; and what, of all of that, they are able to articulate. Finally, following the notion that talk might precede practice, users of the rubric should expect to find discrepancies between teachers' instructional visions and their instruction.

To that last point, the results presented above suggest that teachers' talk about mathematics instruction, indeed, often precedes their enactments. Even in classrooms of those who articulated the most sophisticated descriptions of the teacher's role (Level 4), IQA discussion scores were, on average, around a 2. Such a score represents instances in which students show and describe their work in solving a task, but discussion of that work is limited to procedures followed rather than connections to underlying concepts and/or other strategies. This likely does not come as a surprise to those charged with supporting teachers' learning; most of us who teach probably talk a better game than we play. However, the findings reported here point to the potential for making productive use of such discrepancies —by framing teachers' descriptions as how they *envision* their role, rather than merely inaccurate (or worse, dishonest) descriptions of how they actually teach. In the following paragraphs, I discuss one example of such an approach.

Using the Tool in Work with Teachers

While the motivation for modeling developmental trajectories arose from a need to reliably document change in study participants' articulated visions of high-quality mathematics instruction, the instrument could potentially be useful for those working to support teachers' professional growth in a variety of settings. For example, diagnosing how individuals envision the role of the mathematics teachers could serve as a formative assessment of pre-service teachers' instruction and conceptions of practice; as a pre-post assessment of learning from professional development experiences; or as a means of determining where to begin professional development efforts and of identifying incremental goals over the course of that support.

As an illustration of the last possibility, in a professional development effort that I led with a small group of Algebra 2 teachers in an urban public high school in the northeast, my colleagues and I began by interviewing the teachers and their principal. In addition to inquiring about the setting in which they worked, we asked questions that pertained to their ideas about students and teaching, including the questions previously listed for eliciting their visions of the role of the mathematics teacher. Based on this initial diagnostic interview, we identified the instructional vision of the teacher leader with whom we worked the most as being a Level 3 on the role of the teacher rubric. In her interview, the teacher leader said she would want to see that:

> [s]tudents are doing the work, not the teacher . . . the teacher is advancing the student's thinking by asking those higher level types of question by getting the kids to draw out connections between the math concepts, by getting kids to activate their prior knowledge to do the math . . . in an ideal situation of course, kids would be challenging each other's thinking and listening to each other. And I as a teacher would be monitoring, advancing thought when needed, often asking questions, but in an ideal world, kids would be pushing each other and listening to each other.

In initial observations of this teacher leader, we found that her envisioned role was not the role she was actually playing typically. In follow-up professional development sessions, she expressed her own frustration about this fact. She complained that students "constantly" asked her to tell them whether their answers were correct; that without immediate validation they would stop working. She did not blame the students, however. She suggested that "we as educators must have taught them to do it" through teacher-centered instruction and by making them feel insecure. "They feel like they're always wrong and they don't know what they're doing."

Thus, in our professional development efforts, we attempted to address the discrepancy between the teacher leader's current classroom role and the role she envisioned for herself. She had an idea of what she wanted to see and experience with her students, and we worked to support her in achieving it. But we also aimed to support her in developing a more sophisticated vision for her role (i.e., Level 4), by beginning to identify ways that she might purposefully and proactively scaffold students in taking on responsibility for their learning. In this way, we were simultaneously working to meet her current (envisioned) goal, while supporting her in envisioning new goals for the future.

Conclusion

Recently, Hiebert (2013) argued that "the basic nature of teaching—presenting definitions and rules, demonstrating solution procedures on sample problems, and then asking students to practice the procedures on similar problems—has remained remarkably consistent over the years," and further suggested that "[t]he persistence of the way mathematics is taught in the face of numerous efforts to change it poses a serious and urgent problem for mathematics educators" (p. 45). This problem, he argued, is most productively approached as a problem of supporting teachers' ongoing learning—a gradual process that takes time and requires consistent support. In this process, it is unlikely that all teachers learning paths will look the same (Fennema et al., 1996), or that they are all ready to transition to enacting the kind of role described as the goal in this article. Instead, transitions will likely be incremental, as teachers incorporate new practices into current repertoires.

The tool presented in this article provides a roadmap to what that transition might look like—at least in how teachers' ways of envisioning their role change, if not their actual practice. It provides those charged with supporting teacher learning with a means of diagnosing teachers' current ways of describing practice and then leveraging those instructional visions as both goals to reach and points on which to build.

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The Perspectives of Teacher Leaders on Mathematics, Learning, and Teaching: Supporting Reform-Oriented Instruction

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Abstract

We conducted a qualitative study investigating the perspectives of mathematics teacher leaders on mathematics, learning, and teaching throughout a mathematics teacher leadership program. Data sources included nine mathematics teacher leaders' work on three application essays and four assignments across the leadership program. Through a template analysis, we applied a perspectives framework to characterize the views of teacher leaders in transition toward reform-oriented mathematics instruction. Findings revealed all the mathematics teacher leaders entered the program with a view of mathematics as connected and logical with desires to provide active mathematical experiences for students to develop understanding. Approximately half of the teacher leaders enhanced this view to begin to incorporate students' different views on mathematics into instruction, while the other teacher leaders appeared to continue to view mathematics as an objective discipline, independent of students' constructions. Implications include ideas for supporting mathematics teacher leaders in enhancing their views of mathematics, learning, and teaching.

Introduction

The beliefs and perspectives of teachers play a critical role in their mathematics instruction (e.g., Handal & Herrington, 2003; Philipp, 2007; Sztajn, 2003). Mathematics teacher leaders (MTLs) need to be aware of and able to support mathematics teachers in enriching these beliefs and perspectives. It is expected that MTLs' own views on mathematics, learning, and teaching are one aspect, among others, that influence how they interact with teachers around such beliefs and conceptions of mathematics teaching. Yet, little is known about the perspectives of MTLs in these areas. The purpose of this study was to describe the perspectives of MTLs on mathematics, learning, and teaching throughout a Mathematics Teacher Leadership Program (MTLP).

To analyze the perspectives of MTLs, we drew upon a framework developed by Simon, Tzur, Heinz, Kinzel, and Smith (2000). It was originally created for characterizing the perspectives of mathematics teachers in transition from traditional to reform-oriented teaching (National Council of Teachers of Mathematics [NCTM], 1991). At the left end of the continuum, a teacher with a *traditional perspective* emphasizes rules and procedures while teaching focuses on transmitting knowledge. In the middle, a teacher with a *perception-based perspective* views mathematics as logical, understandable, and connected, yet still existing as an objective reality. From this perspective, learning mathematics with understanding requires first-hand experiences, and the teacher should provide opportunities for students to perceive mathematical relationships. At the right end of the continuum, a teacher with a *conception-based perspective* allows for viewing mathematical interpretations as dependent on one's current conceptions. The role of the teacher therefore is to make sense of students' thinking so that instruction may proceed from students' interpretations.

One strength of the perspectives framework is its deliberate connection of teachers' perspectives with their pedagogical practices. As Simon et al. (2000) explained, "Through hypothesis-generating empirical research, we have attempted to understand the overall coherence of teachers' practices, including the conceptions that drive their practices" (p. 580). As such, the framework has been used to compare the perspectives and pedagogical practices of mathematics teachers from different countries (Jin & Tzur, 2011) as well as to understand the interpretations prospective teachers form of their standards-based mathematics instruction in teacher education programs (Chamberlin, 2013). In 2001, Tzur used the framework as part of a self-reflective analysis on his development as a mathematics teacher educator. As he concluded, "The work with beginning teacher educators requires, first, an analysis of their ways of thinking about how people learn mathematics and on the teacher's role in promoting such learning" (p. 278). Due to the utility of the perspectives framework for characterizing mathematics teachers' perspectives and Tzur's precedence for its use with mathematics teacher educators, we felt the framework appropriate for analyzing the perspectives of the MTLs within our Mathematics Teacher Leadership Program.

Perspectives and Beliefs of MTLs

Despite the need for MTLs to be cognizant of and support mathematics teachers' beliefs and perspectives, little is known about the perspectives of MTLs. Spillane and his colleagues investigated the perspectives of district leaders, which included administrators and lead teachers that often worked in several subject areas in addition to mathematics. Spillane (2000a) investigated the views that district leaders constructed from the mathematics reform movement (NCTM, 1989; 1991). District leaders tended to focus on the logistics of implementing mathematics reform rather than the central aim of changing what counts as knowing and doing mathematics. Leaders also tended to generalize reform across subject areas to the point of de-mathematizing the reforms (e.g., using cooperative learning in general rather than considering specific implications for mathematics instruction). Both of these tendencies were accompanied by a perception of mathematics as consisting of procedural knowledge.

Spillane and his colleagues (Burch & Spillane, 2003; Spillane, 2005) also investigated how school subject influences leadership practice, revealing perceptions of district leaders about literacy versus mathematics. In general, district leaders felt both subjects were core to the curriculum, but believed that a) mathematics should be taught in a particular sequence, b) expertise external to the school setting is needed for leading mathematics reform, and c) improving mathematics instruction depends on teachers following the curriculum so students may perform well on standardized tests.

Finally, Spillane (2000b) examined district leaders' perceptions of teacher learning, which he classified into three groups as quasi-behaviorist, situative-sociohistorical, and cognitive (neo-Piagetian). Of the 40 district leaders included in the study, 85% expressed views aligned with quasi-behaviorist, 12.5% were situated-sociohistorical, and one leader was cognitive. In sum, Spillane and his colleagues provide grounding information about the views of district leaders, including leaders associated with various subject areas. The intent of this study was to extend such results by more specifically examining the perspectives of *mathematics* teacher leaders.

In contrast to Spillane's work, Perry, Howard, and Tracey (1999) more directly examined the beliefs of lead mathematics teachers. Specifically, they surveyed head mathematics teachers from Australian secondary schools about their beliefs on the learning and teaching of mathematics. The data included a 20-item questionnaire and follow-up interviews. In comparing head mathematics teachers' beliefs with those of mathematics teachers, the head mathematics teachers held beliefs somewhat more in line with reform efforts (e.g., Australian Education Council, 1991; NCTM, 1989). These results provided an important but limited examination of MTL beliefs. Due to the possible multiple interpretations of the survey items and the self-report data, further inquiry into the perceptions of MTLs was warranted.

Perspectives Framework

Simon et al. (2000) developed their three-perspective framework from examining mathematics teachers in transition toward reform-oriented pedagogy (NCTM, 1991). Each perspective includes descriptors of the teacher's beliefs about mathematics, about how students learn mathematics, and about how to teach mathematics. Through accounts of teachers' practice (Simon & Tzur, 1999), the authors derived the perception-based perspective, which falls between the traditional perspective and the conception-based perspective (Simon et al., 2000; Tzur, Simon, Heinz, & Kinzel, 2001).

The traditional perspective is generally based on direct instruction of how to perform a mathematical task. "Students passively receive mathematical knowledge by listening to and watching others, usually mathematics teachers, and by reading about mathematics (in textbooks)" (Simon et al., 2000, p. 593). This approach typically emphasizes student development of computational skills and factual knowledge while minimizing a more conceptual understanding of mathematics (NCTM, 2000). Teachers holding this perspective believe that mathematical relationships exist as part of an external world, independent of student activity.

A teacher holding a perception-based perspective believes that mathematics is logical, interconnected, and understandable. Mathematical understanding, then, relies on seeing connections between mathematical ideas, representations, and procedures. Such a teacher views mathematical understanding as coming from what students have the opportunity to perceive in their environment; thereby he or she desires to provide opportunities for students to experiment and perceive the mathematics that is "out there" to be discovered (Simon et al., 2000, p. 594). For students to learn mathematics with understanding, they need first-hand and direct experiences of mathematical concepts. This teacher also believes that, like the traditional-perspective, mathematics exists independent of human activity. The mathematics to be learned is viewed as the same for all individuals.

A conception-based perspective is based on the relative view that an individual has no way of accessing a reality independent of his or her own way of experiencing it. Mathematics is seen as a human activity, dependent on one's current conceptions. This view allows one to realize that another person's perceptions of mathematics may be different from his or her own perceptions. A teacher holding a conception-based perspective sees mathematical understanding developing as a result of personal interpretations, rather than simply perceived as in the perception-based perspective. The teacher interacts with students as a participant in the negotiation of constructed mathematical understandings. This role begins with eliciting and making sense of students' thinking so that instruction may proceed from students' current understandings to the intended mathematics. "What is different about the conception-based perspective is that individuals who have developed that perspective have the possibility, at any time, to step back from this assumption of a universally accessible reality to question the differences in learners' experiential realities" (Tzur et al., 2001, p. 249). Thus, although the perception-based perspective holds promise for students' learning by emphasizing mathematical understanding and active experiences, it falls short when students do not learn in anticipated ways. Without realizing that students' current conceptions influence what they learn, teachers are at a loss for helping students construct meaning other than trying to provide more experiences that reveal the mathematics. In contrast, a conception-based perspective allows a teacher to recognize, consider, and incorporate students' current conceptions into instructional decisions.

Since the original report of the Perspectives Framework, studies by Jin and Tzur (2011) have prompted the consideration of another perspective between the perception-based and the conception-based perspectives. Based on mathematics pedagogy utilized by Chinese teachers, the proposed perspective is characterized by the explicit linking of new knowledge to material that has already been mastered. This linking integrates the teacher-directed aspects with the students' individual understandings. Jin and Tzur referred to this as the progressive incorporation perspective. For consistency with the other perspectives and to emphasize our interpretation of a teacher attempting to incorporate students' ideas but with a result toward the teachers' mathematical view, hereafter we refer to this view as the *incorporation-based perspective*. This perspective emerged as we examined the MTLs' products from the MTLP. Specifically, as we coded their work, there were a significant number of instances where the MTLs revealed a discernible propensity to incorporate student ideas into instruction, moving beyond a perception-based perspective, but still indicating a universal view of mathematics.

A teacher that holds an incorporation-based perspective sees an objective mathematical reality and considers his or her role as providing the activities and opportunities that will help students understand that reality. At the same time, he or she has knowledge of students' prior or current understandings, and as such, can anticipate and elicit student thinking regarding the topic and plan accordingly. For instance, a teacher that is presenting the standard equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$, may rely on the students' familiarity with the Pythagorean Theorem to generate the standard equation. Students are then encouraged to share their ideas about the mathematics, question the understandings of others, and create an interpretation that is consistent with their previous knowledge while simultaneously acquiring the intended (objective) mathematics as determined by the teacher. Table 1 provides a summary of the original three perspectives (Simon et al., 2000) along with our proposed incorporation-based perspective.

Table 1: Extended Perspectives Framework

Perspective	Nature of Mathematics	Learning Mathematics	Teaching Mathematics
Traditional	Independent of knower (objective reality) Emphasis on facts, rules, and procedures without focus on understanding	Passive reception of knowledge Listening to the teacher or reading the textbook	Transmitting knowledge by lecture or demonstrations followed by student practice Emphasizes learners' mastery of procedures and producing answers quickly
Perception-Based	Mathematics is logical (understandable) and can be perceived by all learners Mathematics is part of an external world independent of the learner Everyone sees the same mathematics	Students see the mathematics that is out there and it enters through their senses Students need first-hand and direct experiences to see mathematics for themselves	Providing opportunities for students to perceive the mathematics in the environment Emphasizes collaborative activities using concrete representations and manipulatives
Incorporation-Based	Multiple avenues exist to lead to the teacher's view of the mathematics Still an objective view of mathematics	Students learn by active participation and by making connections to previous material	Acknowledging and eliciting different ways that students think about mathematics Using the students' approaches to guide students toward the teacher's perception or understanding
Conception-Based	Another person's perceptions may be different from our own (relative view of reality) Math is a human activity, dependent on one's ways of knowing	Modifying existing ideas Building on current conceptions and interpretations	Eliciting and making sense of students' thinking Proceeding from current student understandings to intended mathematics

Research Questions

We expect that MTLs' perspectives impact their interactions with teachers around beliefs and conceptions of mathematics teaching. This expectation in conjunction with the limited literature on the perspectives of MTLs led us to investigate the following research question: What perspectives on mathematics, learning, and teaching do mathematics teacher leaders exhibit on assignments throughout their participation in a Mathematics Teacher Leadership Program?

The Mathematics Teacher Leadership Program

The MTLP is offered jointly by two mid-sized universities in the Rocky Mountain region and is funded through the National Science Foundation. It is a graduate-level program intended for experienced mathematics teachers as well as MTLs in formal leadership positions. The purpose of the program is to provide opportunities to learn, develop, and implement leadership skills related to the improvement of the teaching and learning of grade K-12 mathematics. The four goals of the program include helping participants develop leadership skills, deepen their mathematical knowledge for teaching, learn to work with teachers, and analyze interactions among culture and mathematics teaching and learning. It is a two-year program through which participants may earn 24 credit hours. The primary instructors consist of a mathematician with extensive work in mathematics teacher education, a mathematics educator who was formerly a secondary mathematics teacher, and two retired teachers with extensive mathematics coaching and classroom experience. The four authors served as researchers for this study and did not serve as instructors for the program.

The MTLP consists of face-to-face as well as on-line components. Each summer includes two residential one-week institutes while each fall and spring semester includes one on-line class and one weekend retreat. The two summer institutes focus on all four of the MTLP goals, while the on-line classes tend to focus on a specific topic (e.g., coaching, assessment, or motivation and change) from the view of a teacher as well as from the view of a teacher leader. The weekend retreats focus on issues of equity and diversity. Three cohorts have completed the program, including 30 participants.

Participants

To examine the MTLs' perspectives across the MTLP, we selected participants from Cohort 1 as they were the only cohort to have completed the program at the time of research. Cohort 1 began in summer 2010, finished in spring 2012, and included nine participants. The participants included two elementary teachers responsible for teaching all subjects, two middle grade teachers instructing mathematics and other subjects, three high school mathematics teachers, a district math coordinator, and a Response to Intervention (RTI) coordinator. Both coordinators had served previously as high school mathematics teachers. The educational experience of the group varied from 9 to 29 years.

Data Collection

Throughout the MTLP, the participants completed several assignments, reflections, and projects. We selected five such products for their potential in revealing the participants' perspectives on mathematics, learning, and teaching. The first data source consisted of three 2-page essays submitted as part of the participants' application packets in spring 2010. The topics for the essays included: an ideal mathematics class, how their approach to mathematics teaching had evolved, and their interest in the MTLP. The second data source was the Pedagogical Content Knowledge (PCK) assignment, completed in summer 2010. The participants selected a mathematical task and completed an associated PCK analysis, which included providing:

- the learning objectives;
- the standards and practices addressed (Common Core State Standards Initiative, 2010);
- helpful materials and technology;
- at least two solution methods along with affordances and limitations for each method; and
- at least three difficulties or misconceptions students may encounter along with associated instructional responses.

The third data source was the Instructional Strategy Reflection, completed in fall 2010. For this assignment, the participants selected one of the instructional strategies highlighted during the on-line class, implemented their selected strategy, and reflected on the implementation.

The fourth data source consisted of the W&G Project, based on the work of Wlodkowski and Ginsberg (1995) concerning culturally responsive teaching. The participants first surveyed some of their students about their perceptions of the four W&G framework conditions (i.e., establish inclusion, develop positive attitude, enhance meaning, and engender competence). Using this information, the participants implemented modest instructional changes to address one or more of the four conditions. Then, they again surveyed their students to assess any change. The participants reflected upon the overall process and turned in a written product at the end of the fall 2011 semester. The final data source was the Lesson Study or Lesson Experiment Reflection, completed in spring 2012. The participants were asked to conduct either a lesson study (e.g., Lewis & Tsuchida, 1998) or a lesson experiment (Hiebert, Morris, & Glass, 2003). For the lesson study, MTLs were directed to:

1. Form a lesson study team.
2. Plan for one cycle (teach the same lesson twice) of lesson study as a participant or facilitator.
3. Select a research theme or goal.
4. Decide on a lesson focus.
5. Design the lesson.
6. Teach/observe the lesson.
7. Debrief and revise the lesson.
8. Teach/observe the revised lesson.
9. Debrief on the revised lesson and the lesson study process.

For the lesson experiment, participants were directed to:

1. Plan for the lesson experiment: Determine a rich task(s) for the lesson experiment, identify the learning outcomes, plan the lesson and the collection of data to capture student thinking, and develop a lesson hypothesis which links the instruction of the task to student learning.
2. Teach the lesson experiment: Document any changes to the task or instruction during implementation, collect the intended student data, and reflect on the taught lesson as soon as possible after completion.
3. Analyze and reflect on the evidence: Test the lesson hypothesis against the students' work to examine the links between instruction and student learning, and record any new conjectures about student thinking and learning.
4. Revise the lesson experiment: Revise the lesson objectives, the student data collection, and the lesson.

The participants then prepared a reflection on how they executed their respective project and what they learned.

Data Analysis

Our qualitative data analysis consisted of a template analysis (Crabtree & Miller, 1992) using the Perspectives Framework. In a template analysis, researchers rely on *a priori* codes (e.g., a template) to apply to the data. These codes may be revised as analysis continues. For us, our *a priori* codes consisted of traditional, perception-based, and conception-based, while we developed and revised the code of incorporation-based as we engaged in the analysis. We began by examining the application essays from three participants. All four of us participated in two cycles of individual coding, collective discussion, and revision of coding. At least two team members then coded individually and met to resolve differences on the remaining application essays and MTL assignments. We then prepared written summaries about the perspectives of each participant across the MTLP. From these individual written summaries, we developed a table to view the change of all participants across the program, thereby addressing our research question of what perspectives on mathematics, learning, and teaching do MTLs exhibit throughout their participation in a MTLP.

Results

The participants' perspectives on mathematics, learning, and teaching fell into two subgroups: five participants showed movement along the continuum toward reform-oriented teaching (NCTM, 1991; 2000; 2007), while three participants appeared to remain stable in the perception-based perspective. We placed one participant, Julieta, in an *other* category. As she was in a formal leadership position throughout the MTLP, she only discussed working with K-12 students in two of the five documents, leaving us unable to discern changes in her perspective across the MTLP. Table 2 (on following page) provides an overview of the participants' perspectives.

Growth in Perspectives

Here we describe the perspectives of Melinda and Pat as examples of growth in the perspectives of five of the participants.

Melinda. Melinda conveyed a perception-based perspective throughout the first-year assignments, while revealing more of an incorporation-based perspective in the second year of the program. At the beginning of the program, Melinda held the view that students learn by participating in and being exposed to mathematics through various activities: "Students must be active participants in the learning process. They should be exposed to meaningful mathematics through a variety of instructional methods which gives every student exposure to the material in a method that best suits their learning style" (application essay). Exposing students to mathematical activities aligns with providing opportunities for students to 'perceive' the mathematics. In addition, Melinda indicated that the teacher may need to guide students to a specific result or modify activities to be less open-ended if students are resistant or do not arrive at the results that the teacher expects.

Table 2: Participants' Perspectives across the MTLP

	Entrance Essays	PCK Assignment	Instructional Strategy	W&G Project	Lesson Study/ Experiment
GROWTH IN PERSPECTIVES					
Pat	PP	IP	IP	IP	IP
Shelby	PP	PP	PP	PP	IP
Candice	PP	PP	PP	IP	PP
Alana	TP	PP	PP	PP	PP
Melinda	PP	PP	PP	IP	IP
STABLE PERCEPTION-BASED PERSPECTIVE					
Jeannie	PP	PP	PP	PP	PP
Ediva	PP	PP	PP	PP	—
Cathrin	PP	PP	PP	—	PP
OTHER					
Julieta	PP	Not discussed	PP	Not discussed	Not discussed

Note: TP = traditional perspective, PP = perception-based perspective, IP = incorporation-based perspective, and CP = conception-based perspective. A dash indicates the assignment was not turned in or not available.

> The students would have rather been told how to do the constructions and grew frustrated with the process of having to 'discover'. They did much better when I guided them toward a specific theorem rather than the general instructions I gave them at the beginning. (Instructional Strategy Reflection)

Inherent in these two examples as well as her other assignments, Melinda tended to reveal an objective view of mathematics, often making instructional decisions that seemed to be based in the perception that all individuals (herself and students included) perceive mathematics in the same way. For example, in her Instructional Strategy Reflection she wrote:

> I withheld the portion of the unit on constructing parallel lines until students had been exposed to all the appropriate theorems. I then asked students to work collaboratively to determine two ways to construct parallel lines (using compass and straightedge) using the theorems about parallel lines.

Here, we see Melinda incorporating an active mathematical experience for students but not until after they had received instruction in material that she believed would be necessary for them to understand the mathematics as she did.

In assignments from the second year of the program, Melinda's view of her role and how students learn seemed to move more toward a reform-oriented view with an incorporation-based perspective. She prioritized allowing students to engage in mathematical discussion and sharing of ideas. In the W&G project, she explained that she implemented a warm-up activity in her daily lessons. "I began class each day with an open-ended short answer question as a warm up . . . I asked one person from each group to share what someone else from their group had said." In her lesson experiment, she seemed to acknowledge that students could approach problems in different ways and conveyed a desire for students to build upon each other's ideas. She wrote, "Students are expected to share their thinking within their groups and build ideas upon each other's thinking. The teacher's role is to encourage this group process through appropriate interactions with each group." Melinda seemed to expand her role from one who provides mathematical experiences to more of a facilitator, providing opportunities for students to share their ideas about mathematics and using student ideas to develop student understanding. However, Melinda still seemed to convey a view of mathematics as objective and independent of the learner, designing instruction in such a way that students could follow her prescribed learning trajectory. In sum, Melinda seemed to move from a perception-based perspective to an incorporation-based perspective as she began to account for students' different approaches to the mathematics and to encourage students to build upon each other's ideas.

Pat. Pat entered the MTLP conveying a perception-based perspective. In his application essays, he described providing opportunities for students to see the mathematics for themselves but did not mention building on students' current understandings or incorporating students' various problem-solving approaches.

> I needed to give them opportunities to be active learners. I needed to create lessons that harnessed their energy rather than relying on my energy to drive the work . . . I needed to show them that what we were learning was important and useful.

In contrast, throughout his later assignments, Pat showed evidence of an incorporation-based perspective. For example, in his lesson study assignment, his first goal was to build on student ideas for computing the volume of solids of revolution around the x and y axes. Prior to the lesson, he had students collaboratively work on review-type problems, intending for students to draw upon volume formulas they had learned in their previous experiences. Then in teaching the lesson, he used visual aids and formulas from the review problems. His second goal was for students to make meaning of the processes involved. Specifically, he wanted students to develop their own procedure for finding volumes of solids of revolution. As he described after the lesson, "Their [the students'] conversations during the collaborative portion of the lesson were focused on making sense of the process rather than the process itself."

However, while Pat wanted to proceed from student understanding, he tended to do so in a certain way. He seemed to try to make sure that the students were starting at the same point, and he monitored their actions to see that the students moved in a certain direction. Consider his following statement in which he described using this lesson in the future, "Planning the lesson led to adjustments in lessons that preceded the lesson study. I made sure to solidify certain concepts so that I could build on them later." In conclusion, we interpreted that Pat had moved from a perception-based perspective in his application essays to an incorporation-based perspective throughout the bulk of the program.

Stable Perception-Based Perspectives

Three participants exhibited a perception-based perspective of mathematics, teaching, and learning throughout the MTLP. These participants valued the use of manipulatives and collaborative learning to ensure that students were able to experience the mathematics for themselves. They also viewed mathematics as logical and independent of the learner. Finally, they felt their role in teaching was to provide opportunities for students to interact with the mathematics that they as teachers perceived to be inherent in the objects or representations.

Jeannie, for example, displayed a non-traditional view beyond the traditional perspective throughout the MTLP in that she described implementing collaborative learning and experiential activities. In her application essays, she mentioned the use of various instructional methods. For example, she wrote, "Discovery activities allow students to work at their own level, and still be cognitively challenged and experience growth," and "I included more hands-on approaches and manipulatives, visuals, multiple representations, focused on building deeper connections between concepts." In the W&G assignment, Jeannie implemented cooperative groups and established a "student leader" who then served as a "source of increased access to prerequisite skills and getting timely help." We did not classify her perspective as incorporation- or conception-based, however, because she did not appear to elicit or build from student understandings. Rather, she described giving students examples of different teacher-generated solution methods so that they could see the connections and ideas she desired. Associated classroom discussions seemed to typically focus on students' preferences and opinions on which solutions they liked best or found most difficult.

Problematizing Learning

Upon describing the perspectives of MTLs, a natural question to ask is: "What types of experiences might cause perturbations in MTLs' views of mathematics, learning, and teaching?" (Heinz, Kinzel, Simon, & Tzur, 2000, p. 105). Thus, we now address aspects of the assignments that may have prompted or allowed for enhancing the participants' perspectives. It was beyond the scope of this study to examine larger factors from the MTLP that may have impacted the MTLs' perspectives. However, we did examine the analyzed assignments for potential catalysts and describe those here.

Tzur et al. (2001) suggested "as a description of teachers' perspectives on mathematics teaching, the construct [the perspectives framework] can inform teacher education instructional design by specifying broad understandings of teachers — understandings they can use to engage in teacher development tasks" (p. 249). One task they recommended was helping teachers problematize learning, in particular realizing that perceiving mathematical relationships may be problematic for students and beset with individual differences. "If teachers can come to explore why seeing particular relationships is problematic for some students and not for others, they may begin to develop understandings of assimilation that can support a conception-based perspective" (Tzur et al., 2001, p. 248). Upon examining the assignments in which the participants revealed incorporation-based perspectives, we found evidence of the potential for this process of problematizing learning. When the participants acknowledged or dealt with the challenges of their students' learning, they revealed a) distinctions between their own mathematical thinking and that of their students and b) instructional plans that attempted to build upon or respond to their students' conceptions, both actions that aligned with more of a conception-based perspective.

We now provide three such examples. First, the PCK assignment required the participants to provide multiple solution approaches, encouraging them to distinguish between their own and their students' mathematical thinking. Furthermore, the assignment required the participants to explain instructional strategies specifically targeted toward students' difficulties, thus increasing the potential for instruction that responded to students' conceptions. Indeed, this was the case for Pat on the PCK assignment. His task for the assignment was as follows:

> Given $f(x) = x^5 + 2x - 1$, find the slope of the inverse function, $f^{-1}(x)$, at $x = 2$. Demonstrate the appropriateness of your answer in a variety of ways. Extension: Find a formula for the slope of an inverse function at a given value ($x = a$).

Pat considered the different approaches of his students by listing the following possible solution methods along with affordances, limitations, and emphases for each (see Table 3).

Table 3: Pat's Solutions Methods along with Affordances, Limitations, and Emphases

Solution Method	Affordances, Limitations, Emphasis
Numeric: Students look at tables of values, switch x- and y-values to create table for inverse, use difference quotients to approximate value of derivative	Demonstrates understanding of key ideas; will find answer quickly, but answer is not likely to be exact; doesn't provide platform for generalizing. Emphasis on meaning of derivative and rate of change.
Graphically: Students graph function and inverse, find slope of function at x = 1, use reciprocal of slope for inverse function	Demonstrates understanding of key ideas; will establish visual understanding of relationship between function and inverse; provides platform for generalizing (making rule), but may not provide exact answer; difficult to check appropriateness of answer. Emphasis on visual representation of derivative and slope.
Symbolic: Students attempt to find algebraic equation for inverse, use derivative to find slope	Demonstrates ability to do algebraic manipulation and use calculus, but creates no connection to the numeric or graphical representations of function and inverse; few of my students have developed their algebraic skills enough to use this method correctly; this is unlikely to lead to generalization. Emphasis on algebra and calculus skill and procedural understanding.
Combination: Students find slope of original function using derivative, then reference graph and identify the reciprocal as the correct slope of the inverse function	Demonstrates a thorough understanding of derivative and ability to apply knowledge to a graphical representation. This is most likely to achieve the desired learning outcomes and find a formula for the slope of an inverse function. Students may find the slope at the wrong point of the original function, leading to the wrong slope of the inverse.

Pat then continued to demonstrate the potential for instruction that might respond to his students' interpretations by offering difficulties or misconceptions students might encounter along with his instructional responses. For example, he included the following two student difficulties:

1. *Misunderstanding of inverse functions:* Students have forgotten the properties of inverse functions, and
2. *Poor or misleading graphs:* Students draw graphs that limit their ability to find the connection between a function and its inverse or limit their ability to find slopes of the function and inverse.

His instructional responses included the following:

- Review the properties of inverse functions before giving task. Perhaps ask students to brainstorm a list of properties of inverses or create multiple representations of inverse functions to post around the room during the task.
- Ask students to come up with real world examples of inverse functions, ex: temperature conversion, 32 degrees F is 0 degrees Celsius. Students should be encouraged to write examples in function notation: $F(0) = 32 \rightarrow C(32) = 0 \rightarrow$F and C are inverses.
- Provide graph paper, rulers, and colored pencils to encourage students to create accurate, meaningful graphs. Practice graphing equations and inverses on the calculator in a variety of windows. Help students understand that a quality graph can enhance our understanding of calculus topics.

These instructional responses work from students' current ideas, conceptions, and representations to draw out additional mathematical ideas that students may build upon, rather than presenting Pat's perspective of the topic or his demonstration of another way to 'perceive' the intended mathematics.

A second assignment that appeared to hold particular promise for helping participants problematize learning was the Lesson Experiment Reflection, especially its requirements to state a lesson hypothesis and then collect and examine actual student data for revising the lesson. Both Melinda and Candice recognized the problems and challenges involved in student learning as a result of completing their Lesson Experiment Reflection. Melinda's lesson experiment centered on the Going Shopping task (see Figure 1), designed to address the mathematical concepts of compositions of functions and inverses.

FIGURE 1.
The Going Shopping Task

Going Shopping!!! Spring is here; time to update your wardrobe with some nice new spring fashions! Your favorite store is having a 20% off storewide sale and in addition they are giving out "mystery" coupons that can be used in addition to the storewide sale.

1. You found the perfect pair of shorts for $24.99.
   a. Your mystery coupon is for $10 off any one item. What is the total price of your purchase? Remember sales tax is 6%.
   b. What would be the final price for **ANY** item given the above circumstances?
2. Your friend received a mystery coupon for an additional 15% off any one item.
   a. How much does your friend have to pay for the same pair of shorts? Who gets the better deal?
   b. What would be the final price for **ANY** item given the above circumstances?
3. You and your friend decide to not buy the shorts after all. You each choose different items.
   a. Your total is $28.40. What was the original price of the item you purchased?
   b. How could you determine the original price for **ANY** total amount you pay?
   c. Your friend's total price was $36.03. What was the original price of their item?
   d. How could you determine the original price for **ANY** total amount your friend pays?
4. The store manager runs after you as you are leaving the store saying that the clerk made a mistake. She says the mystery coupon MUST be applied first, then the 20% discount. Does this change the total you and your friend would pay? How much would you each pay for the shorts under this circumstance?

Students worked on the Going Shopping activity in groups of three while Melinda served as a facilitator. Before the lesson, Melinda hypothesized that students may "question the order of the compositions within the application" and that "the inverse functions will be difficult for students to write." However, she did not realize the degree to which the students' computations on the numerical portions (1a, 2a, 3a, and 3c) would influence their attempts to generate the functions on items 1b, 2b, 3b, and 3d. She noted:

> While students knew to change the percent to a decimal and subtract this amount from the price, students were not able to simplify this process to multiply by one minus the decimal amount in order to simplify their process. This led to difficulty when writing functions, particularly the inverse function.

Furthermore, in analyzing the work from a specific student, she wrote,

> As predicted, the students had a much easier time with the numerical examples than the explicit formulas. This is demonstrated on problem 1 from student B (see Figure 2). She correctly figured the final price but her 'round about' method made it difficult to arrive at a simplified formula.

FIGURE 2.
Student B's work on #1 from the Going Shopping activity.

1. A) You found the perfect pair of shorts for $24.99. Your mystery coupon is for $10 off any one item. What is the total price of your purchase? Remember sales tax in Cheyenne is 6%.

$24.99(.2) = 4.99 = 20.00$
-10.00
$10.00 \times .06 = .6$
$\boxed{\$10.60 \text{ total price}}$

B) What would be the final price for **ANY** item given the above circumstances?

$X = (S - .2S) - (10)(.06) + (S - .2S - 10)$

In revising the lesson, Melinda decided to require students to evaluate equivalent equations before moving on to questions about compositions and inverses. Her intent was to make sure to build from the students' thinking on the numerical portions, but better use such thinking to support the eventual instructional aims of understanding compositions of functions and inverses. In her revised lesson, she explained, "I will implement a jigsaw instructional approach and include a discussion component. The purpose of this is to allow students an opportunity to refine their equations prior to looking at the compositions and inverses." Specifically, after completing the Going Shopping task,

> Students within the groups will number off 1, 2 or 3. Three larger groups will be formed. Students will be given 15 minutes to compare their equations for one of the equations in parts 1b, 2b, 3b, 3d and 4. Each group will select a different equation to discuss. The guiding question for the groups will be to determine if various forms of the equations are equivalent and to justify any differences within the context of the problem. Each group will be asked to agree upon their "favorite" representation and their justification and will select one member to present one of their findings to the class.

Thus, the lesson experiment helped Melinda recognize the challenges students faced in working on the task and plan for instruction that leveraged students' thinking for facilitating intended learning outcomes.

Candice also appeared to problematize the learning of students through the Lesson Experiment Reflection. The learning objective for her lesson experiment was for students to generate and use the binomial theorem. Her lesson included an exploration worksheet through which students looked for patterns in binomial expansions of increasing powers followed by class discussion and presentation time. Analyzing her students' work allowed Candice to recognize the difficulties students encountered with this topic. For example, she described the following challenges her students encountered:

- The students missed the fact that Pascal's triangle first row is actually the binomial raised to the 0th power, so the coefficients are actually produced by the $(n + 1)^{st}$ row.
- Student A did not use Pascal's Triangle, but instead multiplied the binomial out. The student was struggling with conceptualizing the process of using Pascal's Triangle to expand the binomial.
- This shows the student didn't understand the concept that the term included both the coefficient and the variable and they both were to be raised to the power.
- I suspect Student B was finding the 6th term of the expansion rather than the x^6 term.

This analysis helped Candice distinguish between her mathematical thinking and that of her students. Furthermore, it helped her move beyond examining students' work for whether they 'got it' but instead what types of challenges they experienced and how. In her words,

> Analyzing the student work in stage three offered good insight into the errors my students were making. . . Often in my grading I am rushed to get the papers graded and move on to the next item on the to-do list. . . . The analysis of student work is something I don't believe I put enough time into. Too often I think in generalities — most of the students scored well, many of the students missed this problem. The important information is really in the details of what the students are thinking and why the mistakes were made. Using the information gleaned from the specific student errors should inform what we do on a daily basis.

Conclusion

The intent of this study was to provide a descriptive picture of the perspectives on mathematics, learning, and teaching that MTLs exhibit throughout their participation in a MTLP. For this particular cohort, we found that nearly all participants entered with a perception-based perspective on mathematics, learning, and teaching. Slightly more than half of these participants enhanced their perspectives and moved to incorporation-based perspectives, while the others appeared to remain stable within a perception-based perspective. Our study further demonstrated the potential for the recommendations of Simon et al. (2000) and Tzur et al. (2001) to help teachers problematize student learning. In particular, to assist MTLs in enhancing their perspectives on mathematics, learning, and teaching, it appears promising to provide them with opportunities to reflect on the problematic and individualized nature of students learning mathematics. Doing so helped the MTLs begin to appreciate the relative nature of mathematical thinking and the need to design instruction intended to build from students' current conceptions.

The existence of participants who appeared to remain stable in a perception-based perspective throughout the MTLP was not necessarily surprising. Those with a perception-based perspective have a "tremendous capacity for assimilating new experiences, making transformation from that perspective difficult to promote" (Simon et al., 2000, p. 599). Specifically, the shift from a perception-based perspective to a conception-based perspective requires an epistemological shift from an objective view of reality to a relative view. However, as noted here, one potential for helping MTLs enhance their perspectives is to provide them with opportunities to problematize the mathematical learning of students.

We acknowledge two limitations of the study. First, we limited our study to providing a descriptive account of the MTLs' perspectives on mathematics, learning, and teaching and how such views may have been impacted by the four selected assignments. It was beyond the scope of this present study to examine how other aspects of the MTLP impacted the MTLs' perspectives. For example, how were their perspectives impacted by their course experiences, the retreats, and other leadership activities undertaken within the context of the program? Second, we analyzed data from a small sample specific to our MTLP and only from our first cohort of MTLs.

Thus, many avenues exist to expand our work here. First, what additional aspects of a mathematics teacher leadership program influence the perspectives of MTLs and how? What more can we learn about helping MTLs problematize the mathematical learning of students? What other types of tasks might engender perturbations in the views of MTLs? In addition, none of the MTLs here revealed a conception-based perspective on the assignments examined. Is engendering such a viewpoint feasible? How? Second, we propose examining the perspectives of additional teacher leaders on mathematics, learning mathematics, and teaching mathematics. For example, would other cohorts in our MTLP reveal similar perspectives? What about the perspectives of other teacher leaders – those enrolled in formal leadership programs, those not enrolled in formal leadership programs but serving in titled leadership positions, and those working as leaders through teacher positions?

Such work will help us further understand the views of MTLs. From the extensive literature base and work of mathematics educators, we know of the importance to attend to mathematics teachers' beliefs and perspectives. Clearly, a similar need exists to attend to the views and perspectives of MTLs as well. ✪

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