NCSM Journal

of Mathematics Education Leadership

SPRING 2015 VOL. 16, NO. 1

National Council of Supervisors of Mathematics www.mathedleadership.org

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The Perspectives of Teacher Leaders on Mathematics, Learning, and Teaching: Supporting Reform-Oriented Instruction

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Abstract

We conducted a qualitative study investigating the perspectives of mathematics teacher leaders on mathematics, learning, and teaching throughout a mathematics teacher leadership program. Data sources included nine mathematics teacher leaders' work on three application essays and four assignments across the leadership program. Through a template analysis, we applied a perspectives framework to characterize the views of teacher leaders in transition toward reform-oriented mathematics instruction. Findings revealed all the mathematics teacher leaders entered the program with a view of mathematics as connected and logical with desires to provide active mathematical experiences for students to develop understanding. Approximately half of the teacher leaders enhanced this view to begin to incorporate students' different views on mathematics into instruction, while the other teacher leaders appeared to continue to view mathematics as an objective discipline, independent of students' constructions. Implications include ideas for supporting mathematics teacher leaders in enhancing their views of mathematics, learning, and teaching.

Introduction

The beliefs and perspectives of teachers play a critical role in their mathematics instruction (e.g., Handal & Herrington, 2003; Philipp, 2007; Sztajn, 2003). Mathematics teacher leaders (MTLs) need to be aware of and able to support mathematics teachers in enriching these beliefs and perspectives. It is expected that MTLs' own views on mathematics, learning, and teaching are one aspect, among others, that influence how they interact with teachers around such beliefs and conceptions of mathematics teaching. Yet, little is known about the perspectives of MTLs in these areas. The purpose of this study was to describe the perspectives of MTLs on mathematics, learning, and teaching throughout a Mathematics Teacher Leadership Program (MTLP).

To analyze the perspectives of MTLs, we drew upon a framework developed by Simon, Tzur, Heinz, Kinzel, and Smith (2000). It was originally created for characterizing the perspectives of mathematics teachers in transition from traditional to reform-oriented teaching (National Council of Teachers of Mathematics [NCTM], 1991). At the left end of the continuum, a teacher with a *traditional perspective* emphasizes rules and procedures while teaching focuses on transmitting knowledge. In the middle, a teacher with a *perception-based perspective* views mathematics as logical, understandable, and connected, yet still existing as an objective reality. From this perspective, learning mathematics with understanding requires first-hand experiences, and the teacher should provide opportunities for students to perceive mathematical relationships. At the right end of the continuum, a teacher with a *conception-based perspective* allows for viewing mathematical interpretations as dependent on one's current conceptions. The role of the teacher therefore is to make sense of students' thinking so that instruction may proceed from students' interpretations.

One strength of the perspectives framework is its deliberate connection of teachers' perspectives with their pedagogical practices. As Simon et al. (2000) explained, "Through hypothesis-generating empirical research, we have attempted to understand the overall coherence of teachers' practices, including the conceptions that drive their practices" (p. 580). As such, the framework has been used to compare the perspectives and pedagogical practices of mathematics teachers from different countries (Jin & Tzur, 2011) as well as to understand the interpretations prospective teachers form of their standards-based mathematics instruction in teacher education programs (Chamberlin, 2013). In 2001, Tzur used the framework as part of a self-reflective analysis on his development as a mathematics teacher educator. As he concluded, "The work with beginning teacher educators requires, first, an analysis of their ways of thinking about how people learn mathematics and on the teacher's role in promoting such learning" (p. 278). Due to the utility of the perspectives framework for characterizing mathematics teachers' perspectives and Tzur's precedence for its use with mathematics teacher educators, we felt the framework appropriate for analyzing the perspectives of the MTLs within our Mathematics Teacher Leadership Program.

Perspectives and Beliefs of MTLs

Despite the need for MTLs to be cognizant of and support mathematics teachers' beliefs and perspectives, little is known about the perspectives of MTLs. Spillane and his colleagues investigated the perspectives of district leaders, which included administrators and lead teachers that often worked in several subject areas in addition to mathematics. Spillane (2000a) investigated the views that district leaders constructed from the mathematics reform movement (NCTM, 1989; 1991). District leaders tended to focus on the logistics of implementing mathematics reform rather than the central aim of changing what counts as knowing and doing mathematics. Leaders also tended to generalize reform across subject areas to the point of de-mathematizing the reforms (e.g., using cooperative learning in general rather than considering specific implications for mathematics instruction). Both of these tendencies were accompanied by a perception of mathematics as consisting of procedural knowledge.

Spillane and his colleagues (Burch & Spillane, 2003; Spillane, 2005) also investigated how school subject influences leadership practice, revealing perceptions of district leaders about literacy versus mathematics. In general, district leaders felt both subjects were core to the curriculum, but believed that a) mathematics should be taught in a particular sequence, b) expertise external to the school setting is needed for leading mathematics reform, and c) improving mathematics instruction depends on teachers following the curriculum so students may perform well on standardized tests.

Finally, Spillane (2000b) examined district leaders' perceptions of teacher learning, which he classified into three groups as quasi-behaviorist, situative-sociohistorical, and cognitive (neo-Piagetian). Of the 40 district leaders included in the study, 85% expressed views aligned with quasi-behaviorist, 12.5% were situated-sociohistorical, and one leader was cognitive. In sum, Spillane and his colleagues provide grounding information about the views of district leaders, including leaders associated with various subject areas. The intent of this study was to extend such results by more specifically examining the perspectives of *mathematics* teacher leaders.

In contrast to Spillane's work, Perry, Howard, and Tracey (1999) more directly examined the beliefs of lead mathematics teachers. Specifically, they surveyed head mathematics teachers from Australian secondary schools about their beliefs on the learning and teaching of mathematics. The data included a 20-item questionnaire and follow-up interviews. In comparing head mathematics teachers' beliefs with those of mathematics teachers, the head mathematics teachers held beliefs somewhat more in line with reform efforts (e.g., Australian Education Council, 1991; NCTM, 1989). These results provided an important but limited examination of MTL beliefs. Due to the possible multiple interpretations of the survey items and the self-report data, further inquiry into the perceptions of MTLs was warranted.

Perspectives Framework

Simon et al. (2000) developed their three-perspective framework from examining mathematics teachers in transition toward reform-oriented pedagogy (NCTM, 1991). Each perspective includes descriptors of the teacher's beliefs about mathematics, about how students learn mathematics, and about how to teach mathematics. Through accounts of teachers' practice (Simon & Tzur, 1999), the authors derived the perception-based perspective, which falls between the traditional perspective and the conception-based perspective (Simon et al., 2000; Tzur, Simon, Heinz, & Kinzel, 2001).

The traditional perspective is generally based on direct instruction of how to perform a mathematical task. "Students passively receive mathematical knowledge by listening to and watching others, usually mathematics teachers, and by reading about mathematics (in textbooks)" (Simon et al., 2000, p. 593). This approach typically emphasizes student development of computational skills and factual knowledge while minimizing a more conceptual understanding of mathematics (NCTM, 2000). Teachers holding this perspective believe that mathematical relationships exist as part of an external world, independent of student activity.

A teacher holding a perception-based perspective believes that mathematics is logical, interconnected, and understandable. Mathematical understanding, then, relies on seeing connections between mathematical ideas, representations, and procedures. Such a teacher views mathematical understanding as coming from what students have the opportunity to perceive in their environment; thereby he or she desires to provide opportunities for students to experiment and perceive the mathematics that is "out there" to be discovered (Simon et al., 2000, p. 594). For students to learn mathematics with understanding, they need first-hand and direct experiences of mathematical concepts. This teacher also believes that, like the traditional-perspective, mathematics exists independent of human activity. The mathematics to be learned is viewed as the same for all individuals.

A conception-based perspective is based on the relative view that an individual has no way of accessing a reality independent of his or her own way of experiencing it. Mathematics is seen as a human activity, dependent on one's current conceptions. This view allows one to realize that another person's perceptions of mathematics may be different from his or her own perceptions. A teacher holding a conception-based perspective sees mathematical understanding developing as a result of personal interpretations, rather than simply perceived as in the perception-based perspective. The teacher interacts with students as a participant in the negotiation of constructed mathematical understandings. This role begins with eliciting and making sense of students' thinking so that instruction may proceed from students' current understandings to the intended mathematics. "What is different about the conception-based perspective is that individuals who have developed that perspective have the possibility, at any time, to step back from this assumption of a universally accessible reality to question the differences in learners' experiential realities" (Tzur et al., 2001, p. 249). Thus, although the perception-based perspective holds promise for students' learning by emphasizing mathematical understanding and active experiences, it falls short when students do not learn in anticipated ways. Without realizing that students' current conceptions influence what they learn, teachers are at a loss for helping students construct meaning other than trying to provide more experiences that reveal the mathematics. In contrast, a conception-based perspective allows a teacher to recognize, consider, and incorporate students' current conceptions into instructional decisions.

Since the original report of the Perspectives Framework, studies by Jin and Tzur (2011) have prompted the consideration of another perspective between the perception-based and the conception-based perspectives. Based on mathematics pedagogy utilized by Chinese teachers, the proposed perspective is characterized by the explicit linking of new knowledge to material that has already been mastered. This linking integrates the teacher-directed aspects with the students' individual understandings. Jin and Tzur referred to this as the progressive incorporation perspective. For consistency with the other perspectives and to emphasize our interpretation of a teacher attempting to incorporate students' ideas but with a result toward the teachers' mathematical view, hereafter we refer to this view as the *incorporation-based perspective*. This perspective emerged as we examined the MTLs' products from the MTLP. Specifically, as we coded their work, there were a significant number of instances where the MTLs revealed a discernible propensity to incorporate student ideas into instruction, moving beyond a perception-based perspective, but still indicating a universal view of mathematics.

A teacher that holds an incorporation-based perspective sees an objective mathematical reality and considers his or her role as providing the activities and opportunities that will help students understand that reality. At the same time, he or she has knowledge of students' prior or current understandings, and as such, can anticipate and elicit student thinking regarding the topic and plan accordingly. For instance, a teacher that is presenting the standard equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$, may rely on the students' familiarity with the Pythagorean Theorem to generate the standard equation. Students are then encouraged to share their ideas about the mathematics, question the understandings of others, and create an interpretation that is consistent with their previous knowledge while simultaneously acquiring the intended (objective) mathematics as determined by the teacher. Table 1 provides a summary of the original three perspectives (Simon et al., 2000) along with our proposed incorporation-based perspective.

Table 1: Extended Perspectives Framework

Perspective	Nature of Mathematics	Learning Mathematics	Teaching Mathematics
Traditional	Independent of knower (objective reality) Emphasis on facts, rules, and procedures without focus on understanding	Passive reception of knowledge Listening to the teacher or reading the textbook	Transmitting knowledge by lecture or demonstrations followed by student practice Emphasizes learners' mastery of procedures and producing answers quickly
Perception-Based	Mathematics is logical (understandable) and can be perceived by all learners Mathematics is part of an external world independent of the learner Everyone sees the same mathematics	Students see the mathematics that is out there and it enters through their senses Students need first-hand and direct experiences to see mathematics for themselves	Providing opportunities for students to perceive the mathematics in the environment Emphasizes collaborative activities using concrete representations and manipulatives
Incorporation-Based	Multiple avenues exist to lead to the teacher's view of the mathematics Still an objective view of mathematics	Students learn by active participation and by making connections to previous material	Acknowledging and eliciting different ways that students think about mathematics Using the students' approaches to guide students toward the teacher's perception or understanding
Conception-Based	Another person's perceptions may be different from our own (relative view of reality) Math is a human activity, dependent on one's ways of knowing	Modifying existing ideas Building on current conceptions and interpretations	Eliciting and making sense of students' thinking Proceeding from current student understandings to intended mathematics

Research Questions

We expect that MTLs' perspectives impact their interactions with teachers around beliefs and conceptions of mathematics teaching. This expectation in conjunction with the limited literature on the perspectives of MTLs led us to investigate the following research question: What perspectives on mathematics, learning, and teaching do mathematics teacher leaders exhibit on assignments throughout their participation in a Mathematics Teacher Leadership Program?

The Mathematics Teacher Leadership Program

The MTLP is offered jointly by two mid-sized universities in the Rocky Mountain region and is funded through the National Science Foundation. It is a graduate-level program intended for experienced mathematics teachers as well as MTLs in formal leadership positions. The purpose of the program is to provide opportunities to learn, develop, and implement leadership skills related to the improvement of the teaching and learning of grade K-12 mathematics. The four goals of the program include helping participants develop leadership skills, deepen their mathematical knowledge for teaching, learn to work with teachers, and analyze interactions among culture and mathematics teaching and learning. It is a two-year program through which participants may earn 24 credit hours. The primary instructors consist of a mathematician with extensive work in mathematics teacher education, a mathematics educator who was formerly a secondary mathematics teacher, and two retired teachers with extensive mathematics coaching and classroom experience. The four authors served as researchers for this study and did not serve as instructors for the program.

The MTLP consists of face-to-face as well as on-line components. Each summer includes two residential one-week institutes while each fall and spring semester includes one on-line class and one weekend retreat. The two summer institutes focus on all four of the MTLP goals, while the on-line classes tend to focus on a specific topic (e.g., coaching, assessment, or motivation and change) from the view of a teacher as well as from the view of a teacher leader. The weekend retreats focus on issues of equity and diversity. Three cohorts have completed the program, including 30 participants.

Participants

To examine the MTLs' perspectives across the MTLP, we selected participants from Cohort 1 as they were the only cohort to have completed the program at the time of research. Cohort 1 began in summer 2010, finished in spring 2012, and included nine participants. The participants included two elementary teachers responsible for teaching all subjects, two middle grade teachers instructing mathematics and other subjects, three high school mathematics teachers, a district math coordinator, and a Response to Intervention (RTI) coordinator. Both coordinators had served previously as high school mathematics teachers. The educational experience of the group varied from 9 to 29 years.

Data Collection

Throughout the MTLP, the participants completed several assignments, reflections, and projects. We selected five such products for their potential in revealing the participants' perspectives on mathematics, learning, and teaching. The first data source consisted of three 2-page essays submitted as part of the participants' application packets in spring 2010. The topics for the essays included: an ideal mathematics class, how their approach to mathematics teaching had evolved, and their interest in the MTLP. The second data source was the Pedagogical Content Knowledge (PCK) assignment, completed in summer 2010. The participants selected a mathematical task and completed an associated PCK analysis, which included providing:

- the learning objectives;
- the standards and practices addressed (Common Core State Standards Initiative, 2010);
- helpful materials and technology;
- at least two solution methods along with affordances and limitations for each method; and
- at least three difficulties or misconceptions students may encounter along with associated instructional responses.

The third data source was the Instructional Strategy Reflection, completed in fall 2010. For this assignment, the participants selected one of the instructional strategies highlighted during the on-line class, implemented their selected strategy, and reflected on the implementation.

The fourth data source consisted of the W&G Project, based on the work of Wlodkowski and Ginsberg (1995) concerning culturally responsive teaching. The participants first surveyed some of their students about their perceptions of the four W&G framework conditions (i.e., establish inclusion, develop positive attitude, enhance meaning, and engender competence). Using this information, the participants implemented modest instructional changes to address one or more of the four conditions. Then, they again surveyed their students to assess any change. The participants reflected upon the overall process and turned in a written product at the end of the fall 2011 semester. The final data source was the Lesson Study or Lesson Experiment Reflection, completed in spring 2012. The participants were asked to conduct either a lesson study (e.g., Lewis & Tsuchida, 1998) or a lesson experiment (Hiebert, Morris, & Glass, 2003). For the lesson study, MTLs were directed to:

1. Form a lesson study team.
2. Plan for one cycle (teach the same lesson twice) of lesson study as a participant or facilitator.
3. Select a research theme or goal.
4. Decide on a lesson focus.
5. Design the lesson.
6. Teach/observe the lesson.
7. Debrief and revise the lesson.
8. Teach/observe the revised lesson.
9. Debrief on the revised lesson and the lesson study process.

For the lesson experiment, participants were directed to:

1. Plan for the lesson experiment: Determine a rich task(s) for the lesson experiment, identify the learning outcomes, plan the lesson and the collection of data to capture student thinking, and develop a lesson hypothesis which links the instruction of the task to student learning.
2. Teach the lesson experiment: Document any changes to the task or instruction during implementation, collect the intended student data, and reflect on the taught lesson as soon as possible after completion.
3. Analyze and reflect on the evidence: Test the lesson hypothesis against the students' work to examine the links between instruction and student learning, and record any new conjectures about student thinking and learning.
4. Revise the lesson experiment: Revise the lesson objectives, the student data collection, and the lesson.

The participants then prepared a reflection on how they executed their respective project and what they learned.

Data Analysis

Our qualitative data analysis consisted of a template analysis (Crabtree & Miller, 1992) using the Perspectives Framework. In a template analysis, researchers rely on *a priori* codes (e.g., a template) to apply to the data. These codes may be revised as analysis continues. For us, our *a priori* codes consisted of traditional, perception-based, and conception-based, while we developed and revised the code of incorporation-based as we engaged in the analysis. We began by examining the application essays from three participants. All four of us participated in two cycles of individual coding, collective discussion, and revision of coding. At least two team members then coded individually and met to resolve differences on the remaining application essays and MTL assignments. We then prepared written summaries about the perspectives of each participant across the MTLP. From these individual written summaries, we developed a table to view the change of all participants across the program, thereby addressing our research question of what perspectives on mathematics, learning, and teaching do MTLs exhibit throughout their participation in a MTLP.

Results

The participants' perspectives on mathematics, learning, and teaching fell into two subgroups: five participants showed movement along the continuum toward reform-oriented teaching (NCTM, 1991; 2000; 2007), while three participants appeared to remain stable in the perception-based perspective. We placed one participant, Julieta, in an *other* category. As she was in a formal leadership position throughout the MTLP, she only discussed working with K-12 students in two of the five documents, leaving us unable to discern changes in her perspective across the MTLP. Table 2 (on following page) provides an overview of the participants' perspectives.

Growth in Perspectives

Here we describe the perspectives of Melinda and Pat as examples of growth in the perspectives of five of the participants.

Melinda. Melinda conveyed a perception-based perspective throughout the first-year assignments, while revealing more of an incorporation-based perspective in the second year of the program. At the beginning of the program, Melinda held the view that students learn by participating in and being exposed to mathematics through various activities: "Students must be active participants in the learning process. They should be exposed to meaningful mathematics through a variety of instructional methods which gives every student exposure to the material in a method that best suits their learning style" (application essay). Exposing students to mathematical activities aligns with providing opportunities for students to 'perceive' the mathematics. In addition, Melinda indicated that the teacher may need to guide students to a specific result or modify activities to be less open-ended if students are resistant or do not arrive at the results that the teacher expects.

Table 2: Participants' Perspectives across the MTLP

	Entrance Essays	PCK Assignment	Instructional Strategy	W&G Project	Lesson Study/ Experiment
GROWTH IN PERSPECTIVES					
Pat	PP	IP	IP	IP	IP
Shelby	PP	PP	PP	PP	IP
Candice	PP	PP	PP	IP	PP
Alana	TP	PP	PP	PP	PP
Melinda	PP	PP	PP	IP	IP
STABLE PERCEPTION-BASED PERSPECTIVE					
Jeannie	PP	PP	PP	PP	PP
Ediva	PP	PP	PP	PP	—
Cathrin	PP	PP	PP	—	PP
OTHER					
Julieta	PP	Not discussed	PP	Not discussed	Not discussed

Note: TP = traditional perspective, PP = perception-based perspective, IP = incorporation-based perspective, and CP = conception-based perspective. A dash indicates the assignment was not turned in or not available.

> The students would have rather been told how to do the constructions and grew frustrated with the process of having to 'discover'. They did much better when I guided them toward a specific theorem rather than the general instructions I gave them at the beginning. (Instructional Strategy Reflection)

Inherent in these two examples as well as her other assignments, Melinda tended to reveal an objective view of mathematics, often making instructional decisions that seemed to be based in the perception that all individuals (herself and students included) perceive mathematics in the same way. For example, in her Instructional Strategy Reflection she wrote:

> I withheld the portion of the unit on constructing parallel lines until students had been exposed to all the appropriate theorems. I then asked students to work collaboratively to determine two ways to construct parallel lines (using compass and straightedge) using the theorems about parallel lines.

Here, we see Melinda incorporating an active mathematical experience for students but not until after they had received instruction in material that she believed would be necessary for them to understand the mathematics as she did.

In assignments from the second year of the program, Melinda's view of her role and how students learn seemed to move more toward a reform-oriented view with an incorporation-based perspective. She prioritized allowing students to engage in mathematical discussion and sharing of ideas. In the W&G project, she explained that she implemented a warm-up activity in her daily lessons. "I began class each day with an open-ended short answer question as a warm up . . . I asked one person from each group to share what someone else from their group had said." In her lesson experiment, she seemed to acknowledge that students could approach problems in different ways and conveyed a desire for students to build upon each other's ideas. She wrote, "Students are expected to share their thinking within their groups and build ideas upon each other's thinking. The teacher's role is to encourage this group process through appropriate interactions with each group." Melinda seemed to expand her role from one who provides mathematical experiences to more of a facilitator, providing opportunities for students to share their ideas about mathematics and using student ideas to develop student understanding. However, Melinda still seemed to convey a view of mathematics as objective and independent of the learner, designing instruction in such a way that students could follow her prescribed learning trajectory. In sum, Melinda seemed to move from a perception-based perspective to an incorporation-based perspective as she began to account for students' different approaches to the mathematics and to encourage students to build upon each other's ideas.

Pat. Pat entered the MTLP conveying a perception-based perspective. In his application essays, he described providing opportunities for students to see the mathematics for themselves but did not mention building on students' current understandings or incorporating students' various problem-solving approaches.

> I needed to give them opportunities to be active learners. I needed to create lessons that harnessed their energy rather than relying on my energy to drive the work . . . I needed to show them that what we were learning was important and useful.

In contrast, throughout his later assignments, Pat showed evidence of an incorporation-based perspective. For example, in his lesson study assignment, his first goal was to build on student ideas for computing the volume of solids of revolution around the x and y axes. Prior to the lesson, he had students collaboratively work on review-type problems, intending for students to draw upon volume formulas they had learned in their previous experiences. Then in teaching the lesson, he used visual aids and formulas from the review problems. His second goal was for students to make meaning of the processes involved. Specifically, he wanted students to develop their own procedure for finding volumes of solids of revolution. As he described after the lesson, "Their [the students'] conversations during the collaborative portion of the lesson were focused on making sense of the process rather than the process itself."

However, while Pat wanted to proceed from student understanding, he tended to do so in a certain way. He seemed to try to make sure that the students were starting at the same point, and he monitored their actions to see that the students moved in a certain direction. Consider his following statement in which he described using this lesson in the future, "Planning the lesson led to adjustments in lessons that preceded the lesson study. I made sure to solidify certain concepts so that I could build on them later." In conclusion, we interpreted that Pat had moved from a perception-based perspective in his application essays to an incorporation-based perspective throughout the bulk of the program.

Stable Perception-Based Perspectives

Three participants exhibited a perception-based perspective of mathematics, teaching, and learning throughout the MTLP. These participants valued the use of manipulatives and collaborative learning to ensure that students were able to experience the mathematics for themselves. They also viewed mathematics as logical and independent of the learner. Finally, they felt their role in teaching was to provide opportunities for students to interact with the mathematics that they as teachers perceived to be inherent in the objects or representations.

Jeannie, for example, displayed a non-traditional view beyond the traditional perspective throughout the MTLP in that she described implementing collaborative learning and experiential activities. In her application essays, she mentioned the use of various instructional methods. For example, she wrote, "Discovery activities allow students to work at their own level, and still be cognitively challenged and experience growth," and "I included more hands-on approaches and manipulatives, visuals, multiple representations, focused on building deeper connections between concepts." In the W&G assignment, Jeannie implemented cooperative groups and established a "student leader" who then served as a "source of increased access to prerequisite skills and getting timely help." We did not classify her perspective as incorporation- or conception-based, however, because she did not appear to elicit or build from student understandings. Rather, she described giving students examples of different teacher-generated solution methods so that they could see the connections and ideas she desired. Associated classroom discussions seemed to typically focus on students' preferences and opinions on which solutions they liked best or found most difficult.

Problematizing Learning

Upon describing the perspectives of MTLs, a natural question to ask is: "What types of experiences might cause perturbations in MTLs' views of mathematics, learning, and teaching?" (Heinz, Kinzel, Simon, & Tzur, 2000, p. 105). Thus, we now address aspects of the assignments that may have prompted or allowed for enhancing the participants' perspectives. It was beyond the scope of this study to examine larger factors from the MTLP that may have impacted the MTLs' perspectives. However, we did examine the analyzed assignments for potential catalysts and describe those here.

Tzur et al. (2001) suggested "as a description of teachers' perspectives on mathematics teaching, the construct [the perspectives framework] can inform teacher education instructional design by specifying broad understandings of teachers — understandings they can use to engage in teacher development tasks" (p. 249). One task they recommended was helping teachers problematize learning, in particular realizing that perceiving mathematical relationships may be problematic for students and beset with individual differences. "If teachers can come to explore why seeing particular relationships is problematic for some students and not for others, they may begin to develop understandings of assimilation that can support a conception-based perspective" (Tzur et al., 2001, p. 248). Upon examining the assignments in which the participants revealed incorporation-based perspectives, we found evidence of the potential for this process of problematizing learning. When the participants acknowledged or dealt with the challenges of their students' learning, they revealed a) distinctions between their own mathematical thinking and that of their students and b) instructional plans that attempted to build upon or respond to their students' conceptions, both actions that aligned with more of a conception-based perspective.

We now provide three such examples. First, the PCK assignment required the participants to provide multiple solution approaches, encouraging them to distinguish between their own and their students' mathematical thinking. Furthermore, the assignment required the participants to explain instructional strategies specifically targeted toward students' difficulties, thus increasing the potential for instruction that responded to students' conceptions. Indeed, this was the case for Pat on the PCK assignment. His task for the assignment was as follows:

> Given $f(x) = x^5 + 2x - 1$, find the slope of the inverse function, $f^{-1}(x)$, at $x = 2$. Demonstrate the appropriateness of your answer in a variety of ways. Extension: Find a formula for the slope of an inverse function at a given value ($x = a$).

Pat considered the different approaches of his students by listing the following possible solution methods along with affordances, limitations, and emphases for each (see Table 3).

Table 3: Pat's Solutions Methods along with Affordances, Limitations, and Emphases

Solution Method	Affordances, Limitations, Emphasis
Numeric: Students look at tables of values, switch x- and y-values to create table for inverse, use difference quotients to approximate value of derivative	Demonstrates understanding of key ideas; will find answer quickly, but answer is not likely to be exact; doesn't provide platform for generalizing. Emphasis on meaning of derivative and rate of change.
Graphically: Students graph function and inverse, find slope of function at x = 1, use reciprocal of slope for inverse function	Demonstrates understanding of key ideas; will establish visual understanding of relationship between function and inverse; provides platform for generalizing (making rule), but may not provide exact answer; difficult to check appropriateness of answer. Emphasis on visual representation of derivative and slope.
Symbolic: Students attempt to find algebraic equation for inverse, use derivative to find slope	Demonstrates ability to do algebraic manipulation and use calculus, but creates no connection to the numeric or graphical representations of function and inverse; few of my students have developed their algebraic skills enough to use this method correctly; this is unlikely to lead to generalization. Emphasis on algebra and calculus skill and procedural understanding.
Combination: Students find slope of original function using derivative, then reference graph and identify the reciprocal as the correct slope of the inverse function	Demonstrates a thorough understanding of derivative and ability to apply knowledge to a graphical representation. This is most likely to achieve the desired learning outcomes and find a formula for the slope of an inverse function. Students may find the slope at the wrong point of the original function, leading to the wrong slope of the inverse.

Pat then continued to demonstrate the potential for instruction that might respond to his students' interpretations by offering difficulties or misconceptions students might encounter along with his instructional responses. For example, he included the following two student difficulties:

1. *Misunderstanding of inverse functions:* Students have forgotten the properties of inverse functions, and
2. *Poor or misleading graphs:* Students draw graphs that limit their ability to find the connection between a function and its inverse or limit their ability to find slopes of the function and inverse.

His instructional responses included the following:

- Review the properties of inverse functions before giving task. Perhaps ask students to brainstorm a list of properties of inverses or create multiple representations of inverse functions to post around the room during the task.
- Ask students to come up with real world examples of inverse functions, ex: temperature conversion, 32 degrees F is 0 degrees Celsius. Students should be encouraged to write examples in function notation: $F(0) = 32 \rightarrow C(32) = 0 \rightarrow$ F and C are inverses.
- Provide graph paper, rulers, and colored pencils to encourage students to create accurate, meaningful graphs. Practice graphing equations and inverses on the calculator in a variety of windows. Help students understand that a quality graph can enhance our understanding of calculus topics.

These instructional responses work from students' current ideas, conceptions, and representations to draw out additional mathematical ideas that students may build upon, rather than presenting Pat's perspective of the topic or his demonstration of another way to 'perceive' the intended mathematics.

A second assignment that appeared to hold particular promise for helping participants problematize learning was the Lesson Experiment Reflection, especially its requirements to state a lesson hypothesis and then collect and examine actual student data for revising the lesson. Both Melinda and Candice recognized the problems and challenges involved in student learning as a result of completing their Lesson Experiment Reflection. Melinda's lesson experiment centered on the Going Shopping task (see Figure 1), designed to address the mathematical concepts of compositions of functions and inverses.

FIGURE 1.
The Going Shopping Task

Going Shopping!!! Spring is here; time to update your wardrobe with some nice new spring fashions! Your favorite store is having a 20% off storewide sale and in addition they are giving out "mystery" coupons that can be used in addition to the storewide sale.

1. You found the perfect pair of shorts for $24.99.
   a. Your mystery coupon is for $10 off any one item. What is the total price of your purchase? Remember sales tax is 6%.
   b. What would be the final price for **ANY** item given the above circumstances?
2. Your friend received a mystery coupon for an additional 15% off any one item.
   a. How much does your friend have to pay for the same pair of shorts? Who gets the better deal?
   b. What would be the final price for **ANY** item given the above circumstances?
3. You and your friend decide to not buy the shorts after all. You each choose different items.
   a. Your total is $28.40. What was the original price of the item you purchased?
   b. How could you determine the original price for **ANY** total amount you pay?
   c. Your friend's total price was $36.03. What was the original price of their item?
   d. How could you determine the original price for **ANY** total amount your friend pays?
4. The store manager runs after you as you are leaving the store saying that the clerk made a mistake. She says the mystery coupon MUST be applied first, then the 20% discount. Does this change the total you and your friend would pay? How much would you each pay for the shorts under this circumstance?

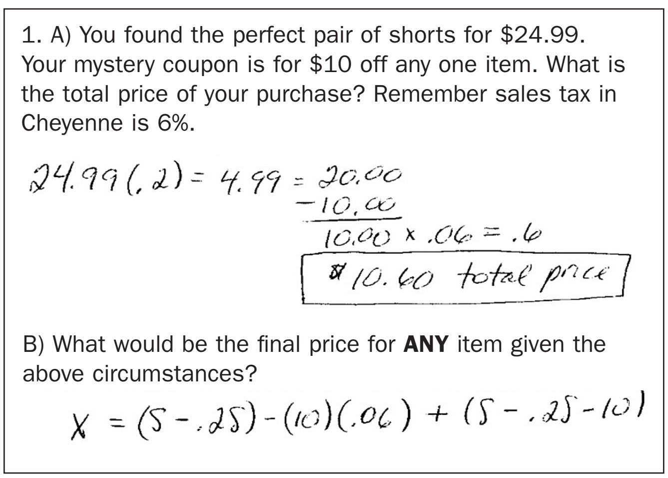
Students worked on the Going Shopping activity in groups of three while Melinda served as a facilitator. Before the lesson, Melinda hypothesized that students may "question the order of the compositions within the application" and that "the inverse functions will be difficult for students to write." However, she did not realize the degree to which the students' computations on the numerical portions (1a, 2a, 3a, and 3c) would influence their attempts to generate the functions on items 1b, 2b, 3b, and 3d. She noted:

> While students knew to change the percent to a decimal and subtract this amount from the price, students were not able to simplify this process to multiply by one minus the decimal amount in order to simplify their process. This led to difficulty when writing functions, particularly the inverse function.

Furthermore, in analyzing the work from a specific student, she wrote,

> As predicted, the students had a much easier time with the numerical examples than the explicit formulas. This is demonstrated on problem 1 from student B (see Figure 2). She correctly figured the final price but her 'round about' method made it difficult to arrive at a simplified formula.

FIGURE 2.
Student B's work on #1 from the Going Shopping activity.



1. A) You found the perfect pair of shorts for $24.99. Your mystery coupon is for $10 off any one item. What is the total price of your purchase? Remember sales tax in Cheyenne is 6%.

24.99(.2) = 4.99 = 20.00
−10.00
10.00 × .06 = .6
$10.60 total price

B) What would be the final price for **ANY** item given the above circumstances?

X = (5 − .25) − (10)(.06) + (5 − .25 − 10)

In revising the lesson, Melinda decided to require students to evaluate equivalent equations before moving on to questions about compositions and inverses. Her intent was to make sure to build from the students' thinking on the numerical portions, but better use such thinking to support the eventual instructional aims of understanding compositions of functions and inverses. In her revised lesson, she explained, "I will implement a jigsaw instructional approach and include a discussion component. The purpose of this is to allow students an opportunity to refine their equations prior to looking at the compositions and inverses." Specifically, after completing the Going Shopping task,

> Students within the groups will number off 1, 2 or 3. Three larger groups will be formed. Students will be given 15 minutes to compare their equations for one of the equations in parts 1b, 2b, 3b, 3d and 4. Each group will select a different equation to discuss. The guiding question for the groups will be to determine if various forms of the equations are equivalent and to justify any differences within the context of the problem. Each group will be asked to agree upon their "favorite" representation and their justification and will select one member to present one of their findings to the class.

Thus, the lesson experiment helped Melinda recognize the challenges students faced in working on the task and plan for instruction that leveraged students' thinking for facilitating intended learning outcomes.

Candice also appeared to problematize the learning of students through the Lesson Experiment Reflection. The learning objective for her lesson experiment was for students to generate and use the binomial theorem. Her lesson included an exploration worksheet through which students looked for patterns in binomial expansions of increasing powers followed by class discussion and presentation time. Analyzing her students' work allowed Candice to recognize the difficulties students encountered with this topic. For example, she described the following challenges her students encountered:

- The students missed the fact that Pascal's triangle first row is actually the binomial raised to the 0th power, so the coefficients are actually produced by the $(n + 1)^{st}$ row.
- Student A did not use Pascal's Triangle, but instead multiplied the binomial out. The student was struggling with conceptualizing the process of using Pascal's Triangle to expand the binomial.
- This shows the student didn't understand the concept that the term included both the coefficient and the variable and they both were to be raised to the power.
- I suspect Student B was finding the 6th term of the expansion rather than the x^6 term.

This analysis helped Candice distinguish between her mathematical thinking and that of her students. Furthermore, it helped her move beyond examining students' work for whether they 'got it' but instead what types of challenges they experienced and how. In her words,

> Analyzing the student work in stage three offered good insight into the errors my students were making. . . Often in my grading I am rushed to get the papers graded and move on to the next item on the to-do list. . . . The analysis of student work is something I don't believe I put enough time into. Too often I think in generalities — most of the students scored well, many of the students missed this problem. The important information is really in the details of what the students are thinking and why the mistakes were made. Using the information gleaned from the specific student errors should inform what we do on a daily basis.

Conclusion

The intent of this study was to provide a descriptive picture of the perspectives on mathematics, learning, and teaching that MTLs exhibit throughout their participation in a MTLP. For this particular cohort, we found that nearly all participants entered with a perception-based perspective on mathematics, learning, and teaching. Slightly more than half of these participants enhanced their perspectives and moved to incorporation-based perspectives, while the others appeared to remain stable within a perception-based perspective. Our study further demonstrated the potential for the recommendations of Simon et al. (2000) and Tzur et al. (2001) to help teachers problematize student learning. In particular, to assist MTLs in enhancing their perspectives on mathematics, learning, and teaching, it appears promising to provide them with opportunities to reflect on the problematic and individualized nature of students learning mathematics. Doing so helped the MTLs begin to appreciate the relative nature of mathematical thinking and the need to design instruction intended to build from students' current conceptions.

The existence of participants who appeared to remain stable in a perception-based perspective throughout the MTLP was not necessarily surprising. Those with a perception-based perspective have a "tremendous capacity for assimilating new experiences, making transformation from that perspective difficult to promote" (Simon et al., 2000, p. 599). Specifically, the shift from a perception-based perspective to a conception-based perspective requires an epistemological shift from an objective view of reality to a relative view. However, as noted here, one potential for helping MTLs enhance their perspectives is to provide them with opportunities to problematize the mathematical learning of students.

We acknowledge two limitations of the study. First, we limited our study to providing a descriptive account of the MTLs' perspectives on mathematics, learning, and teaching and how such views may have been impacted by the four selected assignments. It was beyond the scope of this present study to examine how other aspects of the MTLP impacted the MTLs' perspectives. For example, how were their perspectives impacted by their course experiences, the retreats, and other leadership activities undertaken within the context of the program? Second, we analyzed data from a small sample specific to our MTLP and only from our first cohort of MTLs.

Thus, many avenues exist to expand our work here. First, what additional aspects of a mathematics teacher leadership program influence the perspectives of MTLs and how? What more can we learn about helping MTLs problematize the mathematical learning of students? What other types of tasks might engender perturbations in the views of MTLs? In addition, none of the MTLs here revealed a conception-based perspective on the assignments examined. Is engendering such a viewpoint feasible? How? Second, we propose examining the perspectives of additional teacher leaders on mathematics, learning mathematics, and teaching mathematics. For example, would other cohorts in our MTLP reveal similar perspectives? What about the perspectives of other teacher leaders – those enrolled in formal leadership programs, those not enrolled in formal leadership programs but serving in titled leadership positions, and those working as leaders through teacher positions?

Such work will help us further understand the views of MTLs. From the extensive literature base and work of mathematics educators, we know of the importance to attend to mathematics teachers' beliefs and perspectives. Clearly, a similar need exists to attend to the views and perspectives of MTLs as well. ✪

This material is based upon work supported by the National Science Foundation (NSF) under Grant No. DUE0832026. Any opinions, findings and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF.

Author Note: At the time of this work, Reshmi Nair was a postdoctoral fellow at the University of Northern Colorado.

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