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Is there a Common Pedagogical Core? Examining Instructional Practices of Competing Models of Mathematics Teaching

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Abstract

Debates concerning which ideas should be included in the K-12 curriculum, how they are learned, and how they should be taught are longstanding. Although the adoption of the Common Core State Standards for Mathematics largely resolves content-*focused aspects of the debates,* pedagogical *decisions remain open to interpretation. The National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics have attempted to address this issue with recent calls to action, promoting particular instructional practices that represent a shared vision of the goal for every mathematics classroom. We examine these practices from the perspectives of two competing approaches to mathematics instruction—*dialogic *and* direct*—to ask whether a shared vision is sometimes inaccurately presumed, and to press for a common pedagogical core that includes not only specifications of observable practices, but also their underlying rationales in terms of equitably supporting all students in coming to know and do mathematics.*

Introduction

Since the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989, the National Council of Teachers of Mathematics (NCTM) has worked to build and promote a consistent vision for learning and teaching mathematics that focuses on thinking, reasoning, and communicating rather than almost exclusively on memorization and procedural fluency. During that time, research, standards documents, policy statements, and curricular materials have provided further support for and refinement of this vision. For almost as long, though, this vision has been met with resistance. Criticism has been lodged on both mathematical and pedagogical grounds, leading to longstanding, divisive debates concerning which ideas should be included in the K-12 curriculum, how they are learned[1], and how they should be taught (Klein, 2003; Schoenfeld, 2004).

Recently, *content*-focused aspects of the debate have been largely resolved. The latest standards document, the Common Core State Standards for Mathematics (CCSSM; Common Core State Standards Initiative [CCSSI], 2010), represents an unprecedented agreement across previously divided parties regarding K-12 mathematics content[2]

[1] For more on this topic, we refer the reader to Donovan and Bransford (2005).

[2] The consensus to which we refer is primarily among mathematics educators and mathematicians. We acknowledge that in political and popular arenas, the CCSSM have recently come under increased scrutiny. But even there, only a handful of states have not adopted the Standards, and recent polling suggests that a majority of adults still support the Standards (Henderson, Peterson, & West, 2015), with any decline in support varying along political lines, which suggests that concerns are likely less about the Standards' content than implementation.

(Conference Board of the Mathematical Sciences, 2013; NCTM, 2013). *Pedagogical* decisions, however, remain open to interpretation: "[t]he standards themselves do not dictate curriculum, pedagogy, or delivery of content" (CCSSI, 2010, p. 84). This is to say that the CCSSM specify *what* but not *how* mathematics should be taught in schools.

Professional mathematics education organizations are trying to address this issue regarding how mathematics should be taught. Since the release of the standards, these organizations have argued that the CCSSM "will enable teachers and education leaders to focus on improving teaching and learning, which is critical to ensuring that all students have access to a high-quality mathematics program and the support that they need to be successful" (NCTM, 2010, p. 1). Moreover, the focus on improving teaching and learning and ideas about what counts as high-quality mathematics instruction have recently been reinforced in two publications: *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014) and *It's TIME: Themes and Imperatives for Mathematics Education* (National Council of Supervisors of Mathematics [NCSM], 2014). Each includes a set of instructional practices that are meant to define the kind of high-quality instruction that represents the goal for every mathematics classroom, and of reform and professional development efforts (mapped onto each other in Figure 1). The ways that such documents and their respective lists are interpreted, however, will be influenced by individuals' current practices, perspectives, and institutional settings (EEPA, 1990). Consequently, these new documents run the risk of being interpreted as merely providing new labels (and perhaps clearer definitions) for what one presumes that s/he already does, which can present challenges for those charged with effecting and supporting instructional change and improvement (Cohen, 1990).

The purpose of this article is to make the case that specifications of professional practices, such as those offered by

FIGURE 1.

NCSM's (2014) "Research-affirmed instructional practices" mapped onto NCTM's (2014) "Mathematics teaching practices"

"Mathematics teaching practices" (NCTM, 2014)	"Research-affirmed instructional practices" (NCSM, 2014)
Establish mathematics goals to focus learning	
Implement tasks that promote reasoning and problem solving	Embed the mathematical content they are teaching in contexts to connect the mathematics to the real world
Use and connect mathematical representations	Provide multiple representations—for example, models, diagrams, number lines, tables and graphs, as well as symbols—of all mathematical work to support the visualization of skills and concepts
Facilitate meaningful mathematical discourse	Create language-rich classrooms that emphasize terminology, vocabulary, explanations and solutions
Pose purposeful questions	Respond to most student answers with "why?," "how do you know that?," or "can you explain your thinking?"
Build procedural fluency from conceptual understanding	
Support productive struggle in learning mathematics	Elicit, value, and celebrate alternative approaches to solving mathematics problems to that students are taught that mathematics is a sense-making process for understanding why and not memorizing the right procedure to get the one right answer
Elicit and use evidence of student thinking	Devote the last five minutes of every lesson to some form of formative assessments, for example, an exit slip, to assess the degree to which the lesson's objective was accomplished
	Conduct daily cumulative review of critical and prerequisite skills and concepts at the beginning of every lesson
	Take every opportunity to develop number sense by asking for, and justifying, estimates, mental calculations and equivalent forms of numbers
	Demonstrate through the coherence of their instruction that their lessons—the tasks, the activities, the questions and the assessments—were carefully planned

the NCTM (2014) and NCSM (2014), should be viewed not as collections of what are often referred to as instructional strategies or best practices, but rather as representing approaches to teaching mathematics that are coherent and consistent with respect to perspectives on what it means to know and do mathematics and how children learn it (Donovan & Bransford, 2005). In so doing, we raise the question of whether the achievement of a shared instructional vision is sometimes inaccurately presumed, and offer suggestions for avoiding that pitfall. It is our view that making the CCSSM a reality in our nation's classrooms will require establishing a genuine, common pedagogical core among all members of the educational system, which includes not only specifications of observable practices, but also their underlying rationales in terms of equitably supporting all students in coming to know and do mathematics.

Over the last few years we have sought to better understand and clarify the distinctions between two competing models of instruction: dialogic and direct. Both are coherent and consistent with respect to particular commitments to students' learning; but, in our view, of the two, only dialogic instruction aligns with the vision promoted by NCTM and NCSM. After describing our process for specifying distinct instructional models, we present and compare the resulting models. Then, we turn to the recent calls to action noted previously to consider them from the perspectives of these competing approaches to mathematics instruction, concluding with suggestions for mathematics education leaders and other stakeholders.

Methods

We sought to specify distinct models of mathematics instruction, beginning with different commitments to what it means to know and do mathematics, theories of learning, and perspectives on teaching. We did so with an eye toward an eventual comparative research study of the effectiveness of different instructional models, but first and foremost to understand—and draw clear distinctions between—viable alternatives to mathematics teaching.

To aid in this effort, we convened five meetings that brought together 26 mathematicians, educators, psychologists, and learning scientists, each time separated into two groups representing different perspectives on learning and instruction (see the appendix for a list of participants). Each meeting focused on some aspect of preparatory work for the eventual study. Two meetings were devoted to defining what it means to know and learn mathematics and specifying distinct instructional models—which, as a result, we came to refer to as dialogic and direct. By focusing the initial meetings on the articulation of the theories of learning and teaching on which the two instructional models are built, subsequent discussions of curriculum and assessment, professional development, and implementation could then be framed in terms of the models' underlying theories.

Each meeting consisted of a combination of simultaneous small group discussions among proponents of the same model and whole group discussions in which each group shared the essence of their discussion with members of the other group—not with the goal of reaching consensus, but of identifying exactly how their perspectives *differ*. All meetings were audio recorded and all artifacts created for and during the meetings were archived. Following each meeting, a summary was produced and vetted by the authors. The summary was then shared with participants, feedback was solicited, and a revised version of the summary was created.

Instructional Models

Based on the input of the experts at the meetings we convened, we specified two distinct mathematics instructional models. Below we provide abbreviated descriptions of what teaching entails in each, preceded by brief summaries of the perspectives on knowing and learning mathematics that underpin the respective pedagogies, and followed by a discussion of their similarities and differences. (Complete descriptions of the models are available upon request. A fuller description of this work is reported in Munter, Stein, and Smith, in press.)

Knowing and Learning

In general, advocates of both models viewed two prominent consensus documents—the National Research Council's five strands of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001) and the CCSSM (both content and practice standards)—as reasonable representations of *knowing and doing* mathematics, but emphasized different aspects of those strands and practices. For example, the direct instruction model does not emphasize the communication aspect of the third Standard for Mathematical Practice (SMP). Although a good student may have an internal dialogue concerning other aspects of that standard, communicating effectively with others is not a necessary capability. In the dialogic model, communicating effectively with others is fundamental to knowing (and learning). Similarly, in the direct instruction model, to “make conjectures and build a logical progression of statements to explore the truth of their conjectures” (CCSSI, 2010, p. 6) is limited to trying strategies for solving a problem posed to the students; student questions that drive instruction or lead to new mathematical investigations are not emphasized as they are in the dialogic model.

Although their goals are similar, the two models attempt to achieve them by offering different *learning opportunities* to students. In the direct instruction model, when students have the prerequisite conceptual and procedural knowledge, they will learn from (a) watching clear, complete demonstrations of how to solve problems, with accompanying explanations and accurate definitions; (b) practicing similar problems sequenced according to difficulty; and (c) receiving immediate, corrective feedback. Whereas in the dialogic model, students must (a) actively engage in new mathematics, persevering to solve novel problems; (b) participate in a discourse of conjecture, explanation, and argumentation; (c) engage in generalization and abstraction, developing efficient problem-solving strategies and relating their ideas to conventional procedures; and, to achieve fluency with these skills, (d) engage in some amount of practice. The *pedagogies* by which these opportunities are afforded are described separately in the next sub-section.

Pedagogy

Direct instruction. In the direct instruction model, typical lessons include (a) the teacher’s descriptions of an objective, motivating reasons for achieving the objective, and connections to previous topics; (b) presentation of requisite concepts; (c) demonstration of how to complete the target problem type; and (d) scaffolded phases of guided and independent practice, accompanied by corrective feedback.

During guided practice, the teacher invites the class to solve similar problems (perhaps with some students working them at the board), answering students’ questions, and correcting errors. In order to transition into independent practice, the teacher might begin by priming students’ work through minimally prompted presentation (e.g., completing the first two steps in solving a problem), and gradually withdraw that support. During independent practice, the teacher’s feedback should focus on how strategies need to be corrected (rather than emphasizing that mistakes have been made), and should not interrupt students’ thinking. For example, after a student has solved a problem, the teacher might tell the student what s/he did accurately, and what needs to be modified in order to achieve a complete, accurate solution.

Across these phases, lessons should be captivating, which can be accomplished through keeping a brisk instructional pace, inviting group unison responses to questions, and providing focused praise. Lessons should also be interactive. For example, after students have solved a number of fraction multiplication problems using number lines and area models, the teacher could draw attention to the rule, $\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$. To do so, teachers might invite students to state whether they have noticed a pattern, since it is likely that in solving the progressively difficult problems one or more students will have developed an efficient algorithm. Interacting with students in such a way is good for classroom relationships, keeping students on task, and making the environment more interesting. However, who articulates such a pattern is not important, only *that it gets articulated* (by someone).

Dialogic instruction. In the dialogic instruction model, although instruction will not fit a particular pattern within every lesson, it should, over time, provide coherent sequences of opportunities for students to engage in tasks that have been carefully designed to surface particular mathematical ideas and to build new understandings from previous knowledge. This requires teachers to:

a) have access to and be able to make use of learning progressions—sensible (preferably research-based) paths by which students are likely to reach a set of explicit learning goals given a particular instructional sequence;

b) engage students in two main types of tasks: 1) tasks that initiate students to new ideas and deepen their understanding of concepts, and 2) tasks that help them become more competent with what they already know (with type 2 tasks generally not preceding type 1 tasks);

c) orchestrate productive discussions that make mathematical ideas available to all students and steer collective understandings toward the mathematical goal of the lesson;

d) introduce tools and representations that have longevity (i.e., can be used repeatedly over time for different purposes, as students' understanding grows); and

e) sequence the necessarily varied types of classroom activities in a way that consistently positions students as autonomous learners and users of mathematics, each an agent who has and is developing mathematical authority in the classroom.

A key aspect of this model is the flexible use of multiple representations, which should be used by students to *think with* rather than being limited to illustrate concepts. Equally important to the effective use of multiple representations is encouraging discussion that translates between representations, making explicit the relations between them, including those that are considered standard. Along these lines, with regard to coordinating the use of representations with instructional goals, there are times when it is beneficial for students to be able to choose which representation to use and other times when constraining students to the use of a particular representation will better accomplish the learning goals (with the former more often the case early in the development of a new topic).

An inherent challenge of this model is affording learning opportunities that are emergent through instruction that is *systematic* (see the description in Figure 2 about creativity). This seeming contradiction is reconciled by ensuring that the paths that any given group of students' learning take eventually lead to (at least) the mathematical goals of a particular instructional sequence or grade level. By flexibly following students' reasoning, the teacher can build on their initial thinking to move toward ideas important to both students and the discipline.

Similarities

Specifying and comparing these two models has revealed both differences and similarities. Regarding the latter, we found that in both models, both conceptual understanding and procedural fluency are not only valued as important forms of knowledge, but are viewed as being developed together. Additionally, we found that both models emphasize using carefully designed, purposefully sequenced, mathematically rigorous tasks; closely monitoring students' reasoning; and providing regular opportunities for practice—although the purpose and nature of those tasks, those student diagnoses, and that practice may differ between the models.

Differences

Previously, we alluded to differences between the two models with respect to classroom talk, group work, learning progressions, mathematical tasks, representations, and the role and timing of feedback. In Figure 2, we summarize these differences as well as three additional areas of distinction: students' classroom roles and mathematical creativity; the introduction and role of definitions; and the purpose of diagnosing student thinking. Although abbreviated, we present the differences in table form to allow for more direct comparisons conceptually, and to provide a tool for teachers' and teacher leaders' reflection and conversation.

(Re)Considering "High-Quality" from Competing Perspectives

As alluded to previously, NCTM and NCSM, two prominent professional organizations in mathematics education, have each recently published calls to action (NCSM, 2014; NCTM, 2014), including lists of research-based instructional practices that represent the goal for how mathematics should be taught in classrooms (see Figure 1). Not surprisingly, there is considerable overlap in the lists, which symbolizes the consensus that has developed by these organizations over time. However, advocates of different approaches to instruction would, at least in name, likely embrace a majority of these practices. In some cases, it may be that an instructional practice transcends pedagogy. For example, "establish[ing] mathematics goals to focus learning" (NCTM, 2014, p. 12) and enacting "carefully planned" lessons (NCSM, 2014, p. 30) are important in both dialogic and direct approaches to instruction, and for similar reasons.

In other cases, however, the summaries presented in Figure 2 suggest that very different instructional models may employ similar practices, but in different ways and for different purposes. For example, related to the NCTM's practice of "implement[ing] tasks that promote reasoning and problem solving" (2014, p. 17), the NCSM (2014) authors suggested, specifically, that teachers should "embed the mathematical content they are teaching in contexts to connect the mathematics to the real world" (p. 30). From a dialogic perspective, one key purpose of this practice is to provide opportunities to mathematize familiar contexts (Putnam, Lampert, & Peterson, 1990), quantifying relations in order to solve problems by distilling the mathematical essence of a situation and deciding

FIGURE 2.
Major distinctions between dialogic and direct mathematics instruction

Dialogic Instruction	Distinction	Direct Instruction
Fundamental to both knowing and learning mathematics. Students need opportunities in both small-group and whole-class settings to talk about their thinking, questions, and arguments.	The importance and role of **talk**	Most important during the guided practice phase, when students are required to explain to the teacher how they have solved problems in order to ensure they are encoding new knowledge.
Provides a venue for more talking and listening than is available in a totally teacher-led lesson. Students should have regular opportunities to work on and talk about solving problems in collaboration with peers.	The importance of and role of **group work**	An optional component of a lesson; when employed, it should follow guided practice on problem solving, focus primarily on verifying that the procedures that have just been demonstrated work, and provide additional practice opportunities.
Dictated by both disciplinary and developmental (i.e., building new knowledge from prior knowledge) progressions.	The **sequencing of topics**	Dictated primarily by a disciplinary progression (i.e., prerequisites determined by the structure of mathematics).
Two main types of tasks are important: 1) tasks that initiate students to new ideas and deepen their understanding of concepts (and to which they do not have an immediate solution), and 2) tasks that help them become more competent with what they already know (with type 2 generally not preceding type 1 and both engaging students in reasoning).	The nature and ordering of **instructional tasks**	Students should be given opportunities to use and build on what they have just seen the teacher demonstrate by practicing similar problems, sequenced by difficulty. Tasks afford opportunities to develop the ability to adapt a procedure to fit a novel situation as well as to discriminate between classes of problems (the more varied practice students do, the more adaptability they will develop).
Students should be given time to wrestle with tasks that involve big ideas, without teachers interfering to correct their work. After this, feedback can come in small-group or whole-class settings; the purpose is not merely correcting misconceptions, but advancing students' growing intellectual authority about how to judge the correctness of one's own and others' reasoning.	The nature, timing, source, and purpose of **feedback**	Students should receive immediate feedback from the teacher regarding how their strategies need to be corrected (rather than emphasizing that mistakes have been made). In addition to one-to-one feedback, when multiple students have a particular misconception, teachers should bring the issue to the entire class's attention in order to correct the misconception for all.
Students' learning pathways are emergent. Students should make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures (CCSS-M-SMP 3), asking questions that drive instruction and lead to new investigations.	The emphasis on **creativity**	Students' learning pathways are predetermined and carefully designed for. To "make conjectures and build a logical progression of statements to explore the truth of their conjectures" (CCSS-M-SMP 3) is limited to trying solution strategies for solving a problem posed to them.
Students' thinking and activity are consistent sources of ideas of which to make deliberate use: by flexibly following students' reasoning, the teacher can build on their initial thinking to move toward important ideas of the discipline.	The purpose of **diagnosing student thinking**	Through efficient instructional design and close monitoring (or interviewing), the teacher should diagnose the cause of errors (often a missing prerequisite skill) and intervene on exactly the component of the strategy that likely caused the error.
Students participate in the defining process, with the teacher ensuring that definitions are mathematically sound and formalized at the appropriate time for students' current understanding.	The introduction and role of **definitions**	At the outset of learning a new topic, students should be provided an accurate definition of relevant concepts.
Representations are used not just for illustrating mathematical ideas, but also for thinking with. Representations are created in the moment to support/afford shared attention to specific pieces of the problem space and how they interconnect.	The nature and role of **representations**	Representations are used to illustrate mathematical ideas (e.g., introducing an area model for multi-digit multiplication after teaching the algorithm), not to think with or to anchor problem-solving conversations.

when mathematical modeling is appropriate. In a direct instructional approach, however, this instructional practice is likely employed to give students opportunities to develop the ability to adapt a procedure to fit a novel situation as well as to get better at discriminating between types of problems. The goals that underlie the use of real world problems have implications for how a lesson is structured. What may be used in dialogic instruction to *initiate* an idea through mathematizing may be used in direct instruction to solidify an idea and support the development of adaptability.

Similarly, both direct and dialogic instruction advocates would likely agree that teachers should "elicit and use evidence of student thinking . . . to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (NCTM, 2014, p. 53), perhaps even with an exit slip during "the last five minutes of every lesson" (NCSM, 2014, p. 30). However, as the descriptions in Figure 2 suggest, the reasons for employing such practices differ across competing instructional models. In direct instruction, the teacher should consistently work to diagnose the cause of students' errors (e.g., a missing prerequisite skill) and intervene on exactly the component of the strategy that likely caused the error, which is typically achieved through efficient instructional design and close monitoring or interviewing. Alternatively, teachers taking a dialogic approach treat students' thinking and activity as sources of ideas on which they, the classroom community, can build to move toward important mathematical ideas. In this case, the emphasis is on not only whether but also how students understand an idea.

Both of these recent calls to action refer to a "shared vision" of high-quality mathematics instruction, which they represent with lists of "practices." The two examples above, however, illustrate how particular instructional practices can be interpreted differently and enacted for different purposes, depending on the instructional approach in which they are being used. This fact calls into question what it is that is "shared" when we refer to a shared vision. More importantly, though, it points to the importance of talking about, attempting, and reflecting on such practices in terms of the underlying goals we have for mathematical activity in the classroom and children's learning, a point to which we return in the discussion section.

Discussion

In this article, we have presented abbreviated versions of two instructional models, identified differences in the models' goals for students' learning and the ways by which the models are intended to achieve their goals, and examined currently promoted instructional practices from the perspectives of those competing models. To be clear, we do not claim that the two models we have described are *the* two, only that they are different. But their differences are not evidenced simply by the instructional practices that they employ: teachers in dialogic classrooms may very well demonstrate some procedures, just as students in a direct instruction classroom may very well engage in project-based activities. Our conjecture is that it is not a matter of the particular instructional practices, necessarily, but rather when the practice is used, the purpose for employing a particular practice, and how the practices within each model fit together into a cohesive whole that is important. For example, a teacher in a dialogic classroom may demonstrate a procedure, but only *after* students have developed an understanding of the concept and are able to connect the procedure to its underlying mathematical meaning. Hence the practice, while on the surface may be similar to what you might find in a direct instruction classroom, potentially leads to a very different learning outcome.

Identifying high-quality instructional practices helps to clarify and solidify what we are working to achieve in every mathematics classroom; but identifying distinctions between competing instructional models—even idealized versions—helps to clarify *why* teachers might employ those practices. Thus, we argue that specifications of high-quality instruction must include the identification of both instructional practices and the underlying rationales for employing those practices.

Our call for a more complete specification of high-quality instruction has implications for multiple stakeholders. For example, although it is as yet unclear whether it is possible or necessary to pursue a shared instructional vision across an entire school district, recent research suggests that, for those who choose to initiate district-wide improvement efforts, a coherent, well-articulated instructional vision is foundational (Cobb & Jackson, 2011). Without well-communicated and agreed-upon goals for students' learning, along with the specification of and rationale for particular instructional practices for achieving those goals, the basis for leaders' decisions will be tenuous. For example, leaders may select instructional materials, district- and school-based professional development, formative assessments, or interventions for struggling students that match the superficial features of dialogic instruction but that are aligned to a different underlying theory of how students learn. To maximize the coherence of the system, each of the above decisions must align with and support the enactment of a clear instructional vision. If we begin with a specification such as those provided in this article, the adequacy of decisions regarding all other aspects of an instructional system can be measured against that vision.

Considering the distinctions we have drawn can also be helpful to teachers and those directly supporting teachers. Articulating the rationales underlying our instructional choices can help get beyond the promotion of particular, so-called teaching strategies or best practices to careful reflection on how and why particular strategies or practices are used. We offer two suggestions for doing so. First, we echo numerous other educators and researchers in recommending an emphasis on the CCSSM SMP as the kind of mathematical activity in which we want to support students in participating. But within that emphasis, we recommend that teachers and leaders approach the SMP as both an end and a means—not just the goal for what students will eventually do, but the kind of activity in which they need to engage now so that they can learn mathematics. In addition, paralleling the holistic interpretation of NCTM's (2014) and NCSM's (2014) instructional practices that we have promoted, we recommend that teachers and leaders avoid the temptation to emphasize some SMP to the exclusion of others, and instead treat the practices as interrelated parts of a whole—all necessary to define authentic disciplinary engagement.

Second, as stated previously, a majority of the practices identified in the NCTM (2014) and NCSM (2014) reports would likely be embraced by advocates of different instructional approaches—but not all. For example, as indicated by the distinctions in Figure 2, at least two of the eight practices identified in the NCTM report would not be emphasized by advocates of direct instruction: "support productive struggle in learning mathematics" and "facilitate meaningful mathematical discourse" (p. 10). In direct instruction, corrective feedback is provided as soon as possible so that students are not left to struggle; and, although interaction is encouraged, participating in mathematical discourse is not emphasized as a goal or valued as a strong learning support as it is in dialogic instruction. Because these two practices are incompatible with a direct instruction approach, they stand apart from the others in their potential as anchors for developing and promoting a particular instructional vision. For example, professional development efforts could focus specifically on affording opportunities for productive struggle in solving complex tasks (Stein, Grover, & Henningsen, 1996; Smith & Stein, 1998) or on orchestrating productive mathematical discussions (Smith & Stein, 2011; Stein, Engle, Smith, & Hughes, 2008), and make explicit how the other practices are in service of, or at least related to, those two key practices.

Conclusion

The authors of the CCSSM (CCSSI, 2010) were intentionally silent on the topic of pedagogy. Since that time, researchers and practitioners have been converging on a definition of high-quality mathematics instruction, as comprised of particular instructional practices. These efforts have recently been amplified by calls to action by the NCTM (2014) and the NCSM (2014). The models described in this article represent two distinct perspectives on how instructional practices characterized as high quality might be interpreted and enacted. Examining and reflecting on our goals, teaching, and professional development efforts through these lenses can help us move past the presumption of a shared vision to the work of establishing a genuine, common pedagogical core. ✪

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Appendix

Participants in the Meetings Hosted at the University of Pittsburgh*

Participant	Area	Institution
Sybilla Beckmann	Mathematics	University of Georgia
Jo Boaler	Mathematics education	Stanford University
Diane Briars	Mathematics education	Past President, National Council of Supervisors of Mathematics (NCSM)
Richard Clark	Educational psychology	University of Southern California
David Cordray	Psychology	Vanderbilt University
Mark Driscoll	Mathematics education	EDC
Janet Fender	Professional development	My Direct Instruction Consultant LLC
Anne Garrison	Mathematics education	Vanderbilt University
James Greeno	Learning sciences	University of Pittsburgh
James Hiebert	Mathematics education	University of Delaware
John Hollingsworth	Classroom instruction	President, DataWORKS Educational Research
Mary Ann Huntley	Mathematics education	Cornell University
Ken Koedinger	Cognitive psychology	Carnegie Mellon University
William McCallum	Mathematics	University of Arizona
John Opfer	Psychology	The Ohio State University
Randolph Philipp	Mathematics education	San Diego State University
Frank Quinn	Mathematics	Virginia Tech
Anna Sfard	Mathematics education	University of Haifa, Israel
Alan Siegel	Computer science	New York University
Edward Silver	Mathematics education	University of Michigan
Jon Star	Educational psychology / Mathematics education	Harvard University
Marcy Stein	Education	University of Washington Tacoma
W. Stephen Wilson	Mathematics	Johns Hopkins University
Michael Winders	Mathematics	Worcester State University
Hung-Hsi Wu	Mathematics	University of California at Berkeley
Judith Zawojewski	Mathematics education	Illinois Institute of Technology

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*Although all participants reviewed the full descriptions of the instructional models, inclusion of an individual's name on the above list is not to imply that the individual necessarily agrees with the additional assertions made in this paper. Information listed was current at the time of the meetings.