NCSM Journal

of Mathematics Education Leadership

SPRING 2016 VOL. 17, NO. 1

National Council of Supervisors of Mathematics www.mathedleadership.org

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all —levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We seek your reactions, questions, and connections to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Angela Barlow, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

***Note:** Information for manuscript reviewers can be found at the back of this publication.

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM)

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Angela T. Barlow, *Middle Tennessee State University*
Travis A. Olson, *University of Nevada, Las Vegas*

"Research and practice can and should live in productive synergy, with each enhancing the other." (Schoenfeld, 2014, p. 408).

To this end, this issue features three articles aimed at supporting the work of mathematics education leaders. Although varied in focus, each article represents a unique blend of research and practice, serving as an opportunity to reflect on our work as mathematics education leaders.

With a focus on meeting the expectations of the Technology Principle (National Council of Teachers of Mathematics [NCTM], 2000), the first article, authored by Ivy and Franz, provides insight into two high school mathematics teachers' technological pedagogical content knowledge or TPACK (Niess, 2008). Through classroom observations, surveys, and interviews, the authors describe the interplay between beliefs and pedagogical content knowledge and the influences of each on the participating teachers' TPACK. Although several implications for mathematics education leaders are offered, particular attention is given to the need to tailor professional development to the specific needs of teachers.

This call to meet the individual needs of teachers in professional development settings is the impetus for the second article by Brodesky and colleagues. Grounded in their work with mathematics teachers, general educators, and special educators, the authors offer an in-depth description of their differentiated professional development (DPD) model, which was created through an iterative design process. The DPD model consists of three components: core activities, choice points, and self-assessment opportunities. In addition to descriptions, the authors provide examples and helpful insights for each component of the DPD model to support the reader in utilizing it in a variety of contexts.

Finally, Lischka and colleagues present a summary of the outcomes of the Scholarly Inquiry and Practices Conference on Mathematics Education Methods. This conference focused on the preparation of prospective teachers and gave attention to the different theoretical perspectives that inform this process. Although not typically envisioned as part of the role of mathematics education leaders, the authors utilize this information to support readers in reflecting on their roles in the preparation of prospective teachers as well as on how teacher preparation can support the work of mathematics education leaders.

Taken collectively, these three articles offer insight into the research and practice that composes the work of mathematics education leaders. As you reflect on the ideas expressed, it is our hope that these ideas will support the synergy within your work. ✪

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Exploring the Technological Pedagogical Content Knowledge (TPACK) of High School Mathematics Teachers: A Multiple Case Study

Jessica T. Ivy and Dana P. Franz, *Mississippi State University*

Abstract

The Technology Principle highlights the opportunities offered to enhance instruction through technology integration. With the advent and increased availability of new technologies, access has become less of an issue, yet widespread integration of instructional technologies in ways that support learning are not necessarily observed in classrooms. In this article, the barriers to technology integration are considered, with a particular emphasis on pedagogical content knowledge and its role in development of technological pedagogical content knowledge (TPACK). This interplay of beliefs about student learning and practices when teaching with technology is explored through the cases of two secondary mathematics teachers with common backgrounds but contrasting levels of TPACK.

Introduction

Instructional technologies introduce novel opportunities for student learning in secondary mathematics classrooms (National Council of Teachers of Mathematics [NCTM], 2000). The promise and vision for technologies is exemplified in the Technology Principle (NCTM, 2000), which states that technology has the potential to offer access to multiple representations and deepen mathematical understandings through exploring mathematical patterns, making conjectures, and testing those conjectures in ways which are only feasible with the technology. In this way, the quality use of technology does not suggest a replacement of paper-and-pencil calculations, but rather offers complimentary opportunities for students to make more generalizations, engage in symbolic transformations, and more accurately examine graphical representations (National Research Council, 2001). Most recently, NCTM (2014) affirmed their call for quality technology use in *Principles to Actions*. "An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (p. 5).

The identified potential for enhancing student learning has led to widespread attention to instructional technology in the mathematics classroom (Cuban, Kirkpatrick, & Peck, 2001). The issue, however, is found in the ways in which technology tools are implemented in the classroom. Is the technology being used to take advantage of the opportunities described in the Technology Principle (NCTM, 2000) or by the National Research Council (2001)? Or, is technology a different tool used in traditional types of teaching? Research by Ertmer and Ottenbreit-Leftwich (2009) indicated that the latter might be the case. Although the availability of instructional technology is clearly essential to its implementation, availability does not guarantee implementation, much less quality implementation. Therefore, understanding teachers' knowledge and decisions regarding implementation of technology is essential. With this in mind, this study examined two juxtaposed case studies and provided insight for mathematics teacher leaders who aim to support teachers as they integrate technologies for the teaching and learning of mathematics.

Specifically, this research study examined teachers' beliefs and practices along with their implications on technology integration through the lens of teacher knowledge. Through the construct of technological pedagogical content knowledge (TPACK), the researcher explored the specialized knowledge that two high school mathematics teachers possessed, the evidence of this knowledge, and the implications for classroom practices. The following research questions were posed.

1. How do the two teachers studied perceive their use of instructional technologies?

2. How do these perceptions compare to indications from the analysis of other data gathered by the researcher? What is the role of second-order barriers to technology integration with regard to the two teachers' practices?

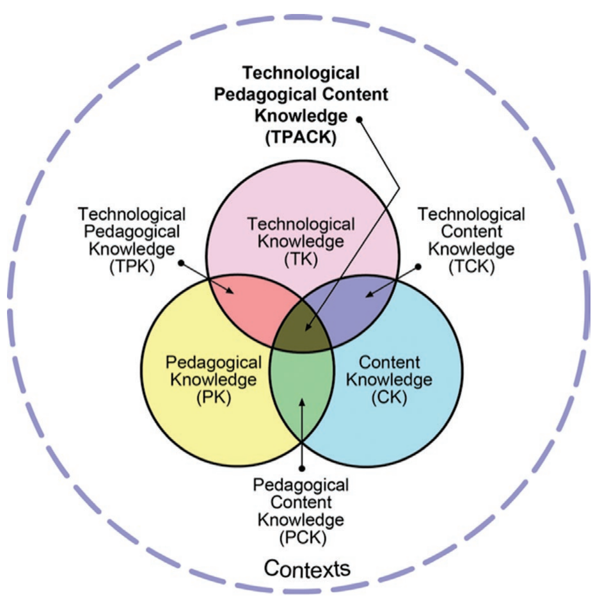
Background Literature

Technological Pedagogical and Content Knowledge

It is well established that mathematical content knowledge is required, but not sufficient, for being an effective mathematics teacher (Ball, Thames, & Phelps, 2008; Cuban, Kirkpatrick, & Peck, 2001; Hill, Rowan, & Ball, 2005; Shulman, 1986). Shulman (1986) described a special knowledge that must accompany teacher content knowledge to promote learning, referred to as pedagogical content knowledge (PCK). Many studies since Shulman's definition have confirmed the need for PCK (e.g., Ball et al, 2008; Grossman & Shulman, 1996; Shulman, 1987) indicating that teachers must understand appropriate pedagogical techniques specific to the subject matter content. Without PCK, teachers possess few tools for establishing an environment conducive to learning. Unfortunately, research studies have indicated that teachers rely on strategies they experienced as learners, utilize lecture-based strategies, or use repetitive examples during instruction (Darling-Hamond, 2006; Feiman-Memser, 1983; Lorti, 1975). With continued development of teachers' PCK, however, they become more successful in identifying the needs of their students, interpreting students' error patterns, engaging students in learning that leads to conceptual understandings, and possessing an awareness of the interconnected nature of mathematical concepts (NCTM, 2000, 2014; van Es, 2011).

With the overwhelming proliferation of instructional technologies, it has become apparent that possessing PCK and mathematical content knowledge is insufficient for ensuring effective mathematics instruction in this era of technology-enhanced classrooms (NCTM, 2014; Neiss, 2005; Pierson, 2001). Recognizing this deficiency, researchers defined a new type of PCK necessary for teaching mathematics with technology (Mishra & Koehler, 2006; Niess et al., 2009). Koehler and Mishra (2008) initially described this technology-driven knowledge. From their perspective, teachers needed to understand: technological content knowledge (TCK) or how to use the technology; technological pedagogical knowledge (TPK) or how to effectively teach the technology; and pedagogical content knowledge (PCK) or how to anticipate the learning needs of students to promote conceptual learning through the use of technology. Further discussions yielded a new construct: technology, pedagogy, and content knowledge (TPACK), which included these three realms of knowledge and the dynamic interactions among these realms (Niess, 2008). The TPACK construct takes into account the interplay of curricular decisions, assessment practices, teaching practices, and learning practices associated with student and teacher use of instructional technologies and is represented by a Venn diagram to demonstrate this interplay (see Figure 1). Using

FIGURE 1.
The TPACK Model and its components. Reproduced with permission of the publisher © 2012 by tpack.org.



a progressive model, Niess and colleagues interpreted the TPACK construct specifically for the mathematics classroom (Niess et al., 2009). This model defines the development of mathematics teachers' TPACK, across four themes (i.e., curriculum and assessment, learning, teaching, and access) with five teacher use levels within each theme (i.e., recognizing, accepting, adapting, exploring, and advancing). The next section provides descriptions of these levels.

Levels of TPACK

Teachers' beliefs about the use of technology generally fall within one of five levels of the model defined by Neiss and colleagues (2009). At the ***recognizing*** level, teachers believe that technology is a distraction from learning. Teachers at this level limit the use of technology to checking computations or reinforcing previously taught concepts. When teachers begin to incorporate technology into lessons, they progress to the ***accepting*** level. These teachers tend to plan lessons that integrate technology as supplemental lessons, which are taught in a teacher-centered fashion with no opportunities for students to select their own strategies. As teachers begin to view technology as a learning tool they enter the ***adapting*** level. At this level, teachers continue to use technology to reinforce previously learned concepts in teacher-led lessons, but these teachers have a clear vision for integrating technology as a tool for student learning. The fourth level of TPACK is the ***exploring*** level. At this level, teachers integrate technology as a tool for student-led explorations of high-level thinking tasks that may be technology-dependent. These teachers use inductive and deductive strategies with technology by planning engaging questions for instruction. The highest TPACK level is the ***advancing*** level, in which technology is consistently used as a tool for the teaching and learning of mathematics. These teachers are often recognized by their colleagues for their specialized knowledge and pedagogy regarding instructional technology.

The TPACK Development Model differentiates teachers who integrate technology seamlessly into daily instruction from those who use technology as a supplement to traditional teaching (Pape et al., 2012). It should be noted that a teacher might be at different levels for different themes and for different technologies (Miller, 2011). The recognizing level is the lowest level, but it is assumed that teachers who meet the criteria for the recognizing level or fall below those criteria are classified at the recognizing level.

Teachers with available instructional technologies may experience a failure to progress through the levels of the TPACK Development Model due to a variety of barriers to technology integration (An & Reigeluth, 2011; Boling & Beatty, 2012). Barriers to integration will be examined in the following section.

Identified Barriers to Technology Integration

Although the availability of technology is essential to its implementation, availability alone does not guarantee implementation (An & Reigeluth, 2011; Ertmer, 1999). Ertmer stated, "Integration is better determined by observing the extent to which technology is used to facilitate teaching and learning" (p. 50). A synthesis of research by Dunham and Hennessy (2008) suggested that although availability of instructional technologies had increased dramatically, technology was still not adequately integrated into the teaching and learning of mathematics. Given the possibilities of enhancing student learning noted by Dunham and Hennessy, as well as in the Technology Principle (NCTM, 2000) and *Principles to Action* (NCTM, 2014), it is essential that teachers are afforded opportunities to gain the knowledge necessary to take advantage of instructional technology (Machado, Laverick, & Smith, 2011).

Researchers have sought to identify barriers to appropriate instructional technology integration in the mathematics classroom (e.g., Ertmer & Ottenbreit-Leftwich, 2009; Hew & Brush, 2007; Norton, McRobbie, & Cooper, 2000; Swan & Dixon, 2006). Ertmer (1999) classified barriers based upon their relationships to teachers. The researcher called barriers external to teachers *first-order barriers* and barriers internal to teachers *second-order barriers*. First order barriers included receiving inadequate training opportunities, experiencing problems with hardware, having small student-to-technology ratios, lacking time to work on planning and applications, and having problems making technology purchases due to district guidelines. Second-order barriers included teachers' attitudes, beliefs, knowledge, skills, and practices. Understanding these barriers will help academic leaders understand how best to assist with quality technology integration and facilitate teachers' development of TPACK, which supports the significance of this study. Although first- and second-order barriers both existed, first-order barriers were outside of the participants' control. Thus, this research focused on second-order barriers to technology integration.

Methodology

Research Overview

The study from which this data was gathered was a qualitative study, consisting of data from seven secondary mathematics teachers in a southeastern state in the United States (Ivy, 2011). To identify potential participants, the researcher sent a Call for Participants to a list-serve of secondary mathematics teachers and selected a sample of seven teachers whose responses to the call indicated varied levels of instructional technology integration. Two of the seven teachers were selected for inclusion in this article because their similarities in setting and experience contrasted notably with their differences in instructional technology integration and pedagogical practices. Yin (2014) described the use of multiple case design through theoretical replication to consider cases with commonalities which can yield compelling and robust results. This methodology follows the replication, rather than sampling, techniques described by Yin for the purpose of introducing theoretical interest, extending beyond the similarities and differences of the cases.

To gain a vision of the level of instructional technology integration of each participant, the researcher conducted an initial interview, observed a classroom lesson that included the use of graphing calculators, conducted a follow-up interview, collected a sample lesson, and collected a completed TPACK Development Survey. The qualitative data were analyzed using deductive analysis to align data pieces (i.e., statements from participants, observations, sample lessons) to fit within the existing levels of the TPACK Developmental Model. Deductive analysis is described by Patton (2002) as the use of an existing framework to consider qualitative data. Brief descriptions of the instruments used in data collection are provided below.

Instruments

To gain insight into the beliefs and practices of the participants, the researcher in collaboration with a colleague created the TPACK Development Model Self-Report Survey and Interview Protocol. Each of these instruments along with information regarding classroom observations will be described in the paragraphs that follow.

TPACK Development Model Self-Report Survey. As the colleague's research interests also focused on in-service teachers' TPACK, the collaboration between the colleague and the researcher led to the development of the research instruments. The TPACK Development Model Self-Report Survey (see Appendix A)[1] included statements that pertained to the themes identified by the TPACK Development Model. For this study, responses to the items related to the Teaching and Learning themes and their subthemes were considered. Subthemes of the Learning theme include mathematics and conceptions of student thinking. Subthemes of the Teaching theme include mathematics learning, instruction, environment, and professional development. These subthemes originated in the work of Niess and colleagues (2009), who described the mathematics specific TPACK Development Model. These subthemes resulted in six separate categories with five statements per category. Each of the five statements corresponded to a particular development level. The order of the statements on the survey corresponded to their levels, with the lower levels provided first.

Although this instrument was created in collaboration with the aforementioned colleague, the TPACK Development Model Self-Report Survey was also submitted to Margaret Niess, who is one of the foremost experts in the study of mathematics teachers' TPACK. The colleague and the researcher used feedback from Niess to further refine the survey prior to using it as an instrument in this study. Because the survey was a newly developed instrument, statements and details from the study were examined by colleagues in the field who provided insight and opportunities for further revision and ensured construct validity.

Prior to completing the survey, participants were provided with oral instructions. They were instructed to select one instructional technology (e.g., graphing calculator) that they used regularly and to check all statements that were true for them when considering their experiences with this self-selected type of technology. Statements provided by participants were examined to ensure alignment with appropriate levels of the TPACK Development Model.

Interview protocol. The Initial Interview Protocol included broad questions regarding technology integration to offer participants an opportunity to share information about instructional technology use in their classrooms. The Interview Protocol included eleven items. Three

[1] The first author would like to express gratitude for the collaborative contributions of Julie Riales in creating and refining the TPACK Development Model Self-Report Survey.

of the items were administrative, seeking either background information or scheduling of observation time. The eight remaining items assessed multiple subthemes of the TPACK Development Model. The interview questions were designed to solicit information pertinent to each participant's levels within the TPACK Development Model. The focus of the Interview Protocol was on the Learning and Teaching themes. The follow-up interviews consisted of individual rather than standardized protocols. Questions asked during these discussions were written to seek clarification and additional details.

Field notes. An organizational tool was used to collect field notes during classroom observations. The primary purpose of the field notes was to gather insight into teachers' practices using a method that did not introduce the bias of the self-reported data. Observation field notes focused on teacher actions with particular attention given to actions described in the TPACK Development Model. Classroom observations were utilized to validate the assignment to the levels when conflicting evidence surfaced in interviews and surveys.

Researcher as an instrument. The first author served as the primary researcher, collecting and analyzing the qualitative data. Therefore, the researcher served as an instrument (Patton, 2002). In this capacity, the researcher collected field notes and other data while practicing reflexivity, that is keeping a conscious note of ideologies and biases which could influence findings, as recommended by Patton. Due to these practices, as well as professional experiences studying the use of technology in the classroom, working with teachers to increase technology integration, and teaching mathematics lessons with technology, the researcher effectively served as an instrument throughout the study.

Qualitative Analysis Considerations

To adhere to the constructs of qualitative inquiry, the researcher integrated assurances of credibility, transferability, dependability, and confirmability into the research design (Shenton, 2004). Miles, Huberman, and Saldana (2014) specifically noted three key recommendations for achieving internal validity, credibility, and authenticity, which were used by the researcher in the analysis of study data: triangulation between complementary data sources to reach converging conclusions combined with methods to reconcile the differences between conflicting conclusions; use of meaningful, context-rich descriptions; and linking of data to existing theories or constructs. In consideration of dependability concerns of qualitative research, the interviews were conducted using a guided conversation style. Audio recordings of the interviews were transcribed and transcriptions were analyzed by the primary researcher with final data analysis reviewed by a credible critic. Documentation of dependability was established through an audit trail kept through researcher notes and reflections constructed throughout the duration of the study. Confirmability was ensured through the aforementioned triangulation, as well as being reflexive in consciousness (Patton, 2002).

Results

Both participating teachers, Ms. Thomas and Ms. James (pseudonyms), taught at high schools in which approximately 60% of students qualified for free or reduced lunch and with a racial makeup of approximately two-thirds of the students were Caucasian and slightly less than one-third were African American. They both had been teaching approximately 25 years at the time of the study. In addition, both taught a variety of high school mathematics courses and had access to instructional technologies, notably graphing calculators and mathematical software. Despite these similarities, data collected from the two participants painted contrasting pictures of instructional technology integration and equally different pedagogical practices. Descriptions of each case and the relevant data collected are provided in the following paragraphs.

MS. THOMAS

Initial interview. During the initial interview, the researcher asked Ms. Thomas a series of questions to gain an understanding of her practices and beliefs regarding instructional technologies. Then the researcher analyzed Ms. Thomas' responses and noted responses that were relevant to her TPACK levels for the teaching and learning themes.

When asked to describe her feelings about technology, Ms. Thomas responded, "I do think it's important for the kids to learn basic skills before they get loose on the calculator because they get really dependent on the calculator even just to do basic functions." She provided an example of how she used the calculator to introduce parallel lines, through carefully controlled students' experiences, and maintained that paper-and-pencil procedures should precede explorations involving technology. "When they get through and they understand the concept that they have the same slope, then, they could take a problem and work it out. And then they could check it with the calculators and see that they're parallel."

There were several notable components to Ms. Thomas' statement. First, Ms. Thomas stated that students should learn concepts prior to using instructional technology especially noting the overdependence for simple calculations. This statement corresponded to the recognizing level for the Teaching theme. Ms. Thomas provided an example of how technology could be used to display a representation during the introduction of the concept of graphing parallel lines; however, the use she described was limited to using the technology as a teaching tool, indicating that she was at the recognizing level for the Learning theme. During the interview, Ms. Thomas expressed that she incorporated technology into her lessons partly out of a fear of "getting left behind." She also stated that she had resisted integrating the technology into her teaching, but had recently "jumped on that idea that we have to use technology . . . [because] I can get left behind or I can jump on and go."

Ms. Thomas described her participation in professional development opportunities related to instructional technology integration. She stated that she would occasionally structure her lessons to model things she had learned during these professional development sessions, suggesting she could have been moving toward the accepting level for the Teaching theme, which is demonstrated when teachers mimic aspects of professional development in their teaching (Niess et al., 2009).

When asked to describe the role technology played in her classroom on a daily basis, Ms. Thomas made a reference to using technology to introduce real-world concepts; however, she did not provide an example of this practice when asked to do so during a follow-up question. Based on this response, Ms. Thomas limited students' opportunities with instructional technologies to using the calculators for computations and occasional graphing, which was characteristic of the recognizing level for the Teaching theme. She also expressed that she limited the availability of technology during the formative phase of concept development, further indicating the recognizing level for the Teaching theme and advancing beyond the recognizing level for the Learning theme.

The analysis of the interview data revealed that Ms. Thomas was at the accepting level for the Teaching theme of TPACK (Niess et al., 2009). At this level, a teacher "merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies" (p. 22). The researcher made this classification despite Ms. Thomas' connections to the recognizing level for this theme. Ms. Thomas' occasional technology use for concept exploration, and her participation in technology-related professional development enabled her to be rated at the accepting level for the Teaching theme. For the Learning theme, interview data revealed that Ms. Thomas was at the recognizing level of TPACK (Niess et al., 2009). At this level, a teacher "views mathematics as being learned in specific ways and that technology often gets in the way of learning" (p. 21).

Observation. The researcher observed an Algebra I lesson in Ms. Thomas' classroom approximately two weeks after the initial interview. When the researcher entered the classroom, it was noted that desks were arranged in rows. The teacher's desk was located near the front of the room, and an electronic whiteboard was located at the front of the room.

As students entered the room, Ms. Thomas instructed them to retrieve calculators from a designated area. When class began, Ms. Thomas distributed graded exams to students and read the solutions to the exam aloud. She instructed students to rework the problems they missed for homework. Next, Ms. Thomas displayed an equation and asked students to graph the equation in their graphing calculators. Ms. Thomas used the SmartView program to display the graph on the electronic whiteboard. After noting the slope and y-intercept of the line, the participant asked students to graph a second equation. The two lines were parallel to each other. Ms. Thomas asked, "What do you notice about their slopes? What do you notice about their y-intercepts? Why are they parallel?" Ms. Thomas allowed less than a minute for discussion and quickly moved to a second example. In the second example, the two lines intersected but were not perpendicular to each other. She verbally provided the procedures necessary for using the calculator to find the point of intersection. The focus of the instruction was on the sequence of keys that students should push, without a discussion as to why this was appropriate.

As the lesson continued, Ms. Thomas provided six additional examples similar to the first two. The final example asked students to consider two equations. Students noticed that these two equations were equivalent. Ms. Thomas instructed students to write in their notes, "If they share the same line, they have infinitely many solutions. If they intersect, they have one solution, and if they're parallel, they have no solutions." Ms. Thomas concluded the lesson by informing the class they would return to this topic the following day.

The researcher noted that during the observed lesson, Ms. Thomas limited students' use of instructional technology to graphing linear equations and using a calculator application to find the point of intersection. Students did not use technology in ways that embodied the Technology Principle (NCTM, 2000). Specifically, students were not using technology to access mathematics that they would not otherwise have been able to access, nor did they explore new concepts with the technology. Calculator use was reserved for performing a series of procedures after the teacher determined the skill had been "mastered" by students using paper and pencil. Furthermore, this use of technology limited students' opportunities to develop conceptual understanding of the mathematics by focusing on memorized procedures rather than concepts and connections. Data from the observed lesson indicated that Ms. Thomas was at the recognizing level for both the Teaching and Learning themes of TPACK (Niess et al., 2009).

Follow-up interview. The follow-up interview for Ms. Thomas occurred immediately after the observed lesson. Data analyzed from the initial interview and the observation provided conflicting levels for the teaching theme. During the follow-up interview, the researcher sought to gather data to better understand Ms. Thomas' practices and beliefs regarding instructional technology integration. During this interview, Ms. Thomas stated that she often used technology to allow students to make connections to the real world; however, she did not provide an example of tasks that she had used in this way. The researcher asked Ms. Thomas if she ever fostered discussions about explorations from the calculators. Ms. Thomas responded with a simple affirmative response, but declined to elaborate. Ms. Thomas also stated that her students' engagement increased when they had access to the graphing calculators because "they're more apt to try stuff on it than they would if they were just using pen and a [paper], I think." Her response suggested that her students' use of the graphing calculators was limited to tasks that could be performed quickly with the calculator, such as performing operations.

As the interview continued, the researcher asked Ms. Thomas whether she engaged students in projects with instructional technology. Ms. Thomas stated that she did not do this because of a lack of time. She specifically referenced time concerns due to high-stakes testing. The researcher noted that her responses generally suggested that she did not view technology as a tool that was useful for exploring new mathematical topics. Ms. Thomas' responses during the follow-up interview suggested she was at the recognizing level for the Teaching and Learning themes of TPACK (Niess et al., 2009).

Self-report survey. Ms. Thomas' responses to the TPACK Development Model Self-Report survey indicated her perceptions about her TPACK levels to be mixed for the various themes when considering her use of graphing calculators. Responses to the self-report survey indicated that Ms. Thomas generally perceived herself to be at a higher TPACK level than that suggested by other data collected during the study. Six items from the survey aligned with the Teaching and Learning themes, with two items for the Learning theme and four items for the Teaching theme (see Appendix A). Ms. Thomas' responses are summarized in Table 1.

Table 1: Summary of Ms. Thomas' Survey Responses

Theme	Survey Statements*	Level alignment to TPACK Developmental Model
Learning	3	Adapting
	9	Exploring
	10	Advancing
Teaching	14	Exploring
	15	Advancing
	18	Adapting
	21	Recognizing
	24	Exploring
	27	Accepting

* Survey statement numbers correspond to the survey items found in Appendix A.

Summary. An initial interview analysis indicated accepting and recognizing levels for the Teaching and Learning themes, respectively. In contrast, self-report survey data indicated Ms. Thomas' TPACK for the Learning theme to be between the adapting, exploring, and advancing (highest) levels for the Learning theme and at all levels for the Teaching theme. Subsequent observation and follow-up interview data provided indications of the accepting levels for both themes. Self-report bias and the alignment of non-survey data led to the conclusion that Ms. Thomas' TPACK levels for both the Teaching and Learning themes of TPACK were within the recognizing and accepting (lowest) levels.

MS. JAMES

Initial interview. The researcher was particularly interested in Ms. James because she, along with a colleague, went to such efforts to acquire technological resources for her classroom. During the initial interview, the researcher asked Ms. James a series of questions to gain an understanding of her beliefs and practices related to instructional technology integration. The researcher analyzed the interview data to make connections to the teaching and learning themes of TPACK (Niess et al., 2009). Interview data relevant to Ms. James's TPACK levels for the Teaching and Learning themes will be discussed in this section.

When asked to describe her feelings about teaching with technology, Ms. James responded, "I just think about how I taught before we got technology. And I just think about how it wouldn't have made sense to me. Math wouldn't have made sense to me if I were in those classes." She elaborated, "I don't see how math makes sense without seeing a picture of it and using graphing calculators or technology. . . . Concepts were probably lost with kids that needed a visual to see why things work and how they're connected."

Ms. James' response indicated a vision of instructional technology use as a tool for teaching and learning. She related her feelings toward the learners' experiences. This statement connected to the conception of the student thinking descriptor at the exploring level for the Learning theme of TPACK (Niess et al., 2009). Also, Ms. James made references to the NCTM Process Standards of connections and representations in this statement. Additionally, Ms. James' comment suggested that she used technology as a teaching tool in her classroom.

Ms. James had extensive teaching experience with technology. She was able to recall in detail her acquisition of instructional technologies. Her statements demonstrated a certain internal motivation to incorporate instructional technologies while teaching mathematics. The researcher asked Ms. James to recount how she learned to use the graphing calculators. Ms. James responded, "This colleague of mine, she and I just taught each other how to use it, and that's the way we've done with everything." She described her current practices including visits to professional conferences. "I'll try to go in sessions and learn as much as I can. . . So whatever we do, we just figure out on our own." She also described learning from her students. "They can teach me a lot. . . Like with the [TI-89] graphing calculator. . . They take one home, and they have one with them all the time. They come back, and they show me what it does."

Ms. James' statements suggested that she actively sought out the knowledge necessary to integrate instructional technologies. She communicated that she used the resources that were available, including workshops, conferences, colleagues, and students. Her statements connected with the professional development descriptor at the exploring level for the Teaching theme of TPACK (Niess et al., 2009). During the interview, Ms. James expressed that she was continuing to grow as a learner and a teacher. She spoke about plans to integrate dynamic geometry software into her calculus instruction. The researcher noticed that although Ms. James was a proficient user of multiple instructional technologies, she continued to seek out additional technologies and strategies for incorporating them in her classroom. Ms. James had a certain motivation that she made reference to during the interview. She described her experiences of becoming comfortable with using the TI-Navigator system in her classroom. "You just have to dig your heels in and say, 'I'm going to use it' because, you know, too much good comes out of it." She elaborated, "The kids are all engaged when you're using the Navigator system, but on the other hand, they may not stay on task. . . . when they realize that technology does so much, and they want to show off." She concluded with an example. "If I ask them to send equations that do a certain thing, then . . . And it may not be anything like we were looking for. . . You have to take the good with the bad."

There were two notable components to these statements. First, Ms. James expressed an internal motivation to succeed at implementing the Navigator technology. Based on data obtained during the interview, this motivation seemed to apply to other technologies as well and had shaped her teaching and learning strategies. The second notable aspect to this response was the idea that when using technology you have "to take the good with the bad." This was notable because Ms. James viewed herself as a technology supporter, yet she still acknowledged misuse and challenges associated with technology integration.

During the interview, the researcher asked Ms. James to describe the factors that influenced her decision to incorporate instructional technologies into daily lessons. Ms. James' responded with laughter and stated, "I don't ever think about not using it. It's an everyday thing." Technology had become an essential component to Ms. James' class, so much so that she referred to technology as "like your child or your husband" while emphasizing the role it played in her classroom. Data gathered during the initial interview suggested Ms. James was at the exploring level for the Teaching and Learning themes of TPACK (Niess et al., 2009).

Observation. The researcher observed a Pre-Calculus lesson in Ms. James' room three weeks following the initial interview. At the beginning of class, Ms. James summarized the previous section in a few sentences and procedurally worked through an item from the homework assignment. This item required students to consider the graphs of two equations (i.e., a circle and a line) and determine the intersections of their graphs. Ms. James led a discussion about graphing a circle on the calculator, determining where the graphs intersected, and changing the graph so the top half of the circle was not visible. Ms. James also asked students to consider why the circle did not "look like a circle" when it was graphed in the calculator with the default window setting.

After reviewing the homework item, Ms. James distributed a task sheet. Ms. James introduced the task by first asking students, "How many of you have iPods?" This conversation continued into a discussion of the history of recorded audio that related to the task. Ms. James asked one student to sit in her chair and operate the SmartView software so the students could confirm their steps as they worked through the task. Using the data from the worksheet, students entered information into lists in the calculator. Ms. James anticipated technical difficulties with the technology that students would have, and she worked quickly to overcome these issues as they arose. Specifically, Ms. James anticipated that some students would initially not be able to view the data because they would be using the default window. She encouraged students to discuss these types of issues. Ms. James led the class through graphing the data in a scatter plot. Throughout the lesson, she often asked students to make predictions about what the graph would look like or how they would expect the data to look if the graph continued. Ms. James challenged students to write an equation of a line that fit a specified set of data on the scatter plot. The class discussed whether it was reasonable to interpret this data linearly. When Ms. James asked students to tell what they noticed about the data, they reported that, based on the data provided, the number of individual songs purchased increased while compact disc sales decreased.

A subsequent class discussion focused on how students would predict when the sale of digital albums would overtake the sale of CDs. Other questions were used to guide students' interpretations of the data. The lesson was teacher-led but solicited active participation from the students. Due to the prescribed nature of the task, students were offered few opportunities to make decisions about how to proceed. This lesson integrated multiple topics that the students had previously studied and did not introduce any new concepts. This suggested that Ms. James was at the adapting level for the Teaching theme of TPACK (Niess et al., 2009). The focus of the use of technology during the observed lesson was to enhance and assess student understanding of the concepts. Based on the observation data, Ms. James was at the exploring level for the Learning theme of TPACK (Niess et al., 2009).

Follow-up interview. The follow-up interview with Ms. James occurred immediately after the observed lesson. In the follow-up interview, Ms. James stated that her lessons were usually teacher-led, although once or twice a week she implemented a student-led lesson. Ms. James acknowledged that the observed lesson was more teacher-led than she would have liked, but attributed this to having a visitor in the classroom. She discussed how she could adapt the lesson in the future. "I can see that activity being easily student-led or at least be done in small groups first and then do a whole group discussion on it. Then students lead that as presentations or carousels or something like that."

This response was indicative of Ms. James' continual desire to improve her teaching strategies. She also described how students used technology to engage in projects and decision-making tasks. She described a challenge she had assigned that day based on a student's suggestion. Students were challenged to find piece-wise graphs that made a Christmas tree shape. This was a task that was not planned but rather an extension task used to further explore the concept from the daily lesson. The follow-up interview data indicated that Ms. James was at the exploring level for the Teaching and Learning themes of TPACK (Niess et al., 2009). This analysis was based upon statements that indicated that Ms. James integrated instructional technology into all aspects of her teaching, took instructional risks with technology, and sought out professional development opportunities.

Self-report survey. Ms. James' responses to the TPACK Development Model Self-Report survey indicated her perceptions about her TPACK levels to be high for the various themes when considering graphing calculators. Self-report survey data suggested that Ms. James' perceptions of her Teaching and Learning TPACK levels were slightly higher than the levels suggested by other data the researcher obtained. Ms. James classified herself to be primarily at the advancing and exploring levels for the Teaching and Learning themes, respectively. The researcher deduced, however, that Ms. James was at the exploring level for the Learning theme due primarily to the teacher-guided structure of her lessons and use of task sheets, which provided little opportunity for students to use technology in an exploratory way. Further, the researcher classified Ms. James as transitioning from the adapting level to the exploring level for the Teaching theme of TPACK due to her tendency to rely on one primary technology (graphing calculators) and the limited way in which calculators were used to explore new concepts (Niess et al., 2009). Ms. James' responses are summarized in Table 2.

Table 2: Summary of Ms. James' Survey Responses

Theme	Survey Statements*	Level alignment to TPACK Developmental Model
Learning	5	Advancing
	10	Advancing
	14	Exploring
Teaching	20	Advancing
	24	Exploring
	29	Advancing
	30	Exploring

* Survey statement numbers correspond to the survey items found in Appendix A.

Summary. An initial interview analysis indicated that Ms. James was at the exploring level for the Teaching and Learning themes. In contrast, self-report survey data indicated Ms. James' TPACK for the Teaching and Learning themes to be between the exploring and advancing (highest) levels. Further, subsequent observation and follow-up interview data provided indications of the exploring levels for both themes. Self-report bias and the alignment of non-survey data led to the conclusion that Ms. James' TPACK levels for both the Teaching and Learning themes of TPACK were at the exploring level.

Discussion

Based on interviews, it seemed that the participants had similar PCK based on consistent statements describing the use of technology to explore mathematics concept; however, classroom observations and additional data suggested otherwise. Ms. Thomas described fostering an environment conducive to developing mathematical understanding through exploration of concepts, but her classroom instruction was lecture-based and teacher-centered with few opportunities for students to make decisions about how to proceed or to problem solve. Alternatively, Ms. James was less structured in her approach to teaching, yet employed techniques that allowed students to control the flow of the lesson within reason. She used questions to guide students toward generalizations and encouraged participation through requiring students to lead the class. Ms. James' actions indicated that she viewed the role of the teacher as a facilitator.

The participants also held different views about the role of instructional technology in the classroom. Ms. Thomas used technology out of a fear of "being left behind," while Ms. James used technology because she believed it held promise for deepening mathematical understandings. Through the data collected from these two participants with similar years of teaching experience and teaching settings, it seemed that limited PCK may have been the single most important barrier to overcome with regard to instructional technology integration. This claim is based on the second order barrier that comes from a PCK deficiency. That is, a lack of understanding of how to teach well will certainly prevent an understanding of how to teach well *with technology*.

Despite the apparent differences in PCK, data from both participants demonstrated inconsistencies between their perceptions of their instructional practices and observed instructional practices. Although the researcher crafted and revised the instruments based on extensive feedback from peers and an expert in educational technology, there was evidence to indicate that the participating teachers did not consistently communicate in ways which aligned with their practices, perhaps even misinterpreting questions and survey items based on their misunderstandings of academic language. Ms. Thomas and Ms. James referenced engagement of students in NCTM's Process Standards; however, this was not consistently present during observed lessons, particularly in Ms. Thomas' class. Both participants frequently used educational jargon such as *conceptual understanding, problem solving*, and *connections*. These ideas were often referenced using vague phrases and without providing details to substantiate the claims. A misunderstanding of these terms links to the explanation for the lack of alignment among the themes identified, the results of the TPACK Development Model Self-Report Survey, and other data collected. Participants' misinterpretation of words used in the survey could have affected their responses. Regardless, it was clear from the data collected that Ms. Thomas and Ms. James both envisioned their technology integration to be exemplary.

Implications for Leaders

As stated previously, the TPACK Development Model consists of five levels for each theme. These levels, from lowest to highest, are recognizing, accepting, adapting, exploring, and advancing. As described in the literature, a lack of pedagogical content knowledge (PCK) will prevent progression through this model (Neiss et al., 2009; Pape et al., 2012). In particular, a teacher with low PCK may have difficulty progressing past the adapting level for the Teaching and Learning themes, though this may cause some teachers to not progress beyond the recognizing level. It is useful to consider the descriptors and examples from the TPACK Development Model in this explanation and to relate these ideas to the concept of PCK.

Pedagogical practices that indicate low PCK link to the unproductive beliefs toward the teaching and learning of mathematics found in *Principles to Actions* (NCTM, 2014). These unproductive beliefs include a focus on procedures and memorization over reasoning and conceptual understanding, mastering a set of basic skills prior to exploring and solving contextual problems, and a focus on step-by-step procedures to minimize classroom struggle. Further, *Principles to Actions* identifies unproductive beliefs about tools and technology, which align with lower TPACK levels. Unproductive beliefs include restricting technology use until a skill or procedure is mastered without the technology, viewing technology as solely an efficient way to get or confirm computational solutions, using technology with only certain groups of students, and limiting experiences with technology to individual activities or videos. The unproductive beliefs about teaching and learning certainly support unproductive beliefs about the use of tools and technology for teaching mathematics.

An awareness of the influence of low PCK and unproductive beliefs on teachers' TPACK has implications for mathematics education leaders, particularly in terms of planning for professional development and other areas of teacher support. Considering the Learning theme, a teacher at the accepting level "has concerns about students' attention being diverted from learning. . . mathematics to a focus on the technology" (Niess et al., 2009, p. 21), whereas at the adapting level a teacher "begins to explore, experiment and practice integrating technologies as mathematics learning tools" (p. 21). At the exploring level a teacher "uses technologies as tools to facilitate the learning of specific topics" (p. 21). Academic leaders can use this information to foster exploration of new instructional technologies with specific attention to the ways the technologies represent specific concepts, situated within the context of existing course structures. Professional development needs to be focused and as much as possible individualized, if teachers are going to implement technologies in ways that result in increasing student understanding and achievement.

In evaluating the descriptors of the Teaching theme, similar indications are observed. A teacher at the adapting level "uses technology to enhance or reinforce mathematics ideas that students have learned previously" (Niess et al., 2009, p. 21) as seen in Ms. Thomas' classroom, while a teacher at the exploring level "engages students in high-level thinking activities for learning mathematics using technology as a learning tool" (p. 23) as demonstrated in Ms. James' classroom. It is equally relevant to note that teachers with low levels of TPACK and unproductive beliefs about technology view instructional technology as a supplement to instruction, whereas teachers with higher levels of TPACK and productive beliefs about technology envision instructional technology as a valuable tool for enhancing learning opportunities for students (NCTM, 2014). This idea of using available resources to improve learning opportunities also ties to the concept of PCK by indicating that a teacher with higher levels of TPACK views mathematics teaching as a dynamic system in which tools can improve opportunities, and teachers with lower levels of TPACK envision mathematics teaching as unchanging and algorithmic.

Of further use to mathematics education leaders is the use and inadvertent misuse of educational jargon. The realization that mathematics teachers may inadvertently use common education terminology in ways that inaccurately represent their classroom practices highlights the need for mathematics teacher leaders to gain insight into classroom practice from a variety of sources. According to Davis and Simtt (2003), learning systems are complex to study due, in part, to the lack of consistency in language or jargon. It is essential, then, that within a school system leaders ensure teachers and teacher leaders clearly define the jargon used to ensure that the vision and interpretation is consistent and clear.

Through the examination of these two participants and the TPACK Development Model descriptors, it is suggested that significant PCK serves as an impetus to effective instructional technology integration. Likewise, a lack of PCK presents a second-order barrier to quality instructional technology integration. Although prior research has clearly identified barriers to instructional technology integration (Ertmer, 1999; Ertmer & Ottenbreit-Leftwich, 2009; Hew & Brush, 2007), the role of PCK has not been identified as is suggested in this study. Clearly PCK is an essential component of TPACK, yet the interplay of these two types of knowledge deems further exploration.

Conclusion

This research highlights opportunities for increased exploration of secondary mathematics teachers' perceptions of technology integration through the lens of the TPACK Development Framework. However, TPACK exploration and implementation introduces implications for policies, research on best practices for teaching with technology, and teaching professional development for implementing instructional technologies (Trouche, Drijvers, & Sacristan, 2013). With consideration of individual teachers' PCK, professional development should be built around the needs of teachers rather than limited to rapid introduction of new technologies or prepared lessons with technology. It seems that if meeting a teacher's needs for PCK improvement is expected, then a teacher's TPACK could progress and thus impact student learning with technology. In our current era of the integration of the Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010), we must recognize the imperative nature of engaging students in the Standards for Mathematical Practice. Notably, students should engage in a learning environment, which fosters the "use of appropriate tools strategically" (p. 7). Ms. Thomas' students used tools appropriate for the mathematics being studied, but were not given the opportunity to use these tools in a strategic fashion. In contrast, Ms. James required her students to develop strategies for solving problems, and her students used technology to carry out these strategies and explore concepts. Teachers' beliefs about how to effectively facilitate student learning directly impact their classroom practices, and this study demonstrated how instructional technology integration is not immune to this effect. In a brief conversation with these two participants, it would seem that they had similar beliefs about teaching and learning. Further analysis, however, highlighted stark differences in their beliefs and practices toward the use of technology. Though these findings provide insight into these cases, it is important to acknowledge that with such a small sample, large generalizations are not possible. While Ms. Thomas' beliefs about technology integration lacked depth and reinforced purely procedural uses of calculators, Ms. James' view of technology was much closer to achieving the vision set forth by the Technology Principle (NCTM, 2000). The phenomena that allowed Ms. James to overcome the second-order barriers that continued to plague Ms. Thomas necessitate further inquiry. ✪

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APPENDIX A.

TPACK Development Model Self-Report Survey

(Teaching and Learning Items only)

Specific to ______________________________ (technology)

Please place a check in the box to the left of each statement that describes your beliefs and/or integration of technology in your classroom. You may give additional information in the spaces provided to clarify your selections or if none of the statements describe your beliefs/integration.

	1.	I believe that if my students use this technology too often, they will not learn the math for themselves.
	2.	I am afraid that if I try to introduce a new topic with this technology, that my students will be too distracted by the technology use to really learn the mathematics. I want them to learn how to do it on paper first, and then they can use the technology.
	3.	I have allowed my students to explore a few topics using this technology even before the topics are discussed in class.
	4.	My students explore several topics for themselves using this technology to help them develop a deeper understanding. Sometimes the students' thinking guides their explorations in directions other than what I had planned.
	5.	I design my own technology lessons. When I plan my lessons, I really think about how to integrate the technology to help the students better understand the mathematics. After the lesson, I reflect on the lesson and how it could be changed to increase student understanding using this and/or other technologies.
Use this space for any additional information related to the statements above.		
	6.	I might show my students how this technology relates to the topic, and I don't mind if my students use this technology outside of class, but I do not plan to allow class time for the students to use this technology.
	7.	If my students use the technology to explore a new topic, they won't think about and develop the mathematical skills for themselves.
	8.	I try to use this technology to promote my students' thinking, but have not had a lot of success.
	9.	I often use pre-made technology activities to engage my students in their learning. I reflect on my students' thinking, communication and ideas during the technology use to make decisions about any changes that need to be made in the design of the lesson.
	10.	I cannot imagine my classes without this technology! Using this technology is a vital piece of facilitating my students' learning and helps promote their thinking to more advanced levels.
Use this space for any additional information related to the statements above.		
	11.	This technology might be useful, but before I could use this technology, I would have to teach my students about the technology and how it works. I have too many objectives to cover to do that.
	12.	I use this technology occasionally, such as between units or at the end of the term. The technology use doesn't necessarily tie with the mathematical goals of the class.
	13.	I use this technology to reinforce concepts that I have taught earlier or that my students should have learned in a previous class. I do not use it regularly when teaching new topics.
	14.	I use this technology as a learning tool to engage my students in high-level thinking activities (such as projects or problem-solving).

	15.	I use this technology to present mathematical concepts and processes in ways that are understandable to my students. I actively accept and promote use of this technology for learning mathematics. Other teachers come to me as a resource for ideas of how to help their students use the technology to promote understanding.
Use this space for any additional information related to the statements above.		
	16.	My students and I use this technology for procedural purposes only.
	17.	I have led my students through a few simple ideas of how to use this technology that I learned during professional development.
	18.	I have led my students through uses of this technology that I learned during professional development, but I changed the activities to meet the needs of my students.
	19.	When my students explore with this technology, I serve as a guide. I do not direct their every action with the technology.
	20.	On a regular basis, I use a wide variety of instructional methods with this technology. I present tasks for my students to engage in both deductive and inductive strategies with the technology to investigate and think about mathematics to deepen their understanding.
Use this space for any additional information related to the statements above.		
	21.	In my class, the focus is on the mathematics first. I can imagine that perhaps this technology might be used to reinforce those mathematical ideas only after the students have shown they can perform the skills on paper.
	22.	I allow my students to use this technology to assist them with their skills. I direct my students step-by-step to use this technology.
	23.	I use some exploration activities with this technology, but I usually guide my students through the steps to save class time.
	24.	I have explored a variety of instructional methods with this technology, to allow my students to engage both inductively and deductively.
	25.	I use this technology in a student-led environment, where the students explore with the technology both individually and in groups. When working in groups, all members of the group are actively involved.
Use this space for any additional information related to the statements above.		
	26.	I would consider attending a workshop demonstrating the use of this technology, but only if it is local.
	27.	I am interested and would be likely to attend workshops or professional developments to learn more about how to use this technology to further mathematics education.
	28.	I am likely to attend professional developments related to technology use in mathematics education and to share those ideas with other teachers in my building, but I am likely to focus on learning one type of technology integration at a time.
	29.	I have made contact with others who are using this technology and plan to meet and work with them throughout the year to integrate this and other technologies appropriately into our mathematics curriculum.
	30.	I believe it is time to transform our mathematics curriculum to one that utilizes 21st century technologies! I have found organizations and workshops that I can attend to learn more about how to integrate this and other technologies into my math curriculum. I plan to share what I learn with others in my district.
Use this space for any additional information related to the statements above.		

This instrument was created by Julie Riales and Jessica Ivy.

Moving Beyond One-Size-Fits-All PD: A Model for Differentiating Professional Learning for Teachers

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Education Development Center

Abstract

This article describes an innovative model for differentiating professional development to address teachers' wide range of content knowledge, experiences, and interests. The model has three components: core activities *that all participants experience;* choice points *that allow teachers to choose options to individualize their learning; and* self-assessment opportunities *to help teachers reflect on their knowledge and identify areas to strengthen. To elucidate the workings of each component and the overall model, we present design principles and examples from a differentiated professional development sequence on fraction multiplication. To support the application of the model, we share implementation findings and offer suggestions to help mathematics education leaders plan and facilitate professional development that is differentiated for their teachers' needs.*

Introduction

> Raising achievement in mathematics for every student and effectively implementing the CCSSM in every classroom requires extensive and ongoing opportunities for teachers to enhance their own professional learning and build their capacity to reach all students. (National Council of Supervisors of Mathematics [NCSM], 2014, p. 44)

As districts strive to implement the Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010) with rigor and equity for all students, mathematics education leaders play a critical role in providing teachers with much-needed professional development (PD) and support. The challenge for teachers is not only to enact an ambitious set of mathematics standards and practices, but also to do so in ways that are accessible, meaningful, and effective for a wide range of students. Just as students have diverse learning needs, teachers themselves vary greatly in their own prior content knowledge, experiences, and strategies for meeting this challenge, and yet PD programs often use a one-size-fits-all approach.

In this article, we describe a model for differentiated professional development (DPD) that enables teachers to play an active role in tailoring PD to meet their varied professional learning needs. We created the model using an iterative process of design, testing, and revision, and applied it to different mathematics topics and teacher audiences from the upper elementary and middle grades. We begin our discussion with a brief rationale for differentiating professional learning, followed by an overview of the DPD model. To elucidate how the model works, we offer an in-depth look at the model's three components, accompanied by examples and a summary of implementation findings. We conclude with suggestions and planning tools to help mathematics education leaders, PD developers, and facilitators use our DPD model to address their teachers' needs and goals. These suggestions draw on three authors' perspectives as developers/facilitators of differentiated PD and one author's role as a researcher.

Why Differentiate PD?

The case that has been made for differentiating instruction for students (Huebner, 2010; Tomlinson, 2001) also applies to teachers. According to Tomlinson (2005), staff development needs to be differentiated to address the "reality that teachers themselves differ in readiness, interest, and learning profile, [and] will do so throughout their professional lives" (p. 12). In their professional contexts, educators also differ in their roles and in the uses they expect to make of their takeaways from PD. Teachers, however, have limited opportunities to make choices about which PD to attend (Bill & Melinda Gates Foundation, 2014) or to individualize their experiences within PD. One study that surveyed over 10,000 teachers reported, "Many teachers' complaints about their professional development appear to stem from a sense that it is not customized to fit their needs" (TNTP, 2015, p. 26). It is concerning that teachers are typically expected to differentiate instruction for their students but rarely experience differentiation firsthand in their own professional learning.

The need to differentiate PD for teachers is supported by research on adult learners. According to Knowles, adults want what they learn to be directly relevant to their work situations, roles, goals, and interests (as cited in Kenner & Weinerman, 2011). Because they have limited time available for dedicated learning experiences, adult learners want to have choices in what and how they learn. Giving teachers choices in PD helps increase their ownership and investment in their own learning, which are critical components for adult learners. Increasing teachers' investment in *career-long professional growth* reflects the recommendations of the Professionalism Principle of the National Council of Teachers of Mathematics (NCTM, 2014).

> As professionals, mathematics teachers recognize that their own learning is never finished and continually seek to improve and enhance their mathematical knowledge for teaching, their knowledge of mathematical pedagogy, and their knowledge of students as learners of mathematics. (p. 99)

The benefits of differentiating PD extend to the district level. With the implementation of CCSSM, districts have an increased need for PD, but the time available has typically remained the same or decreased. This lack of time for PD has been identified by both teachers and administrators as one of the top barriers to effective PD (Bill & Melinda Gates Foundation, 2014). Differentiating professional learning provides a way for districts to maximize the available time by allowing teachers to make choices to customize the PD to directly respond to their individual needs.

Overview of DPD Model

We designed our DPD model during a five-year research and development project funded by the National Science Foundation (NSF) (DRL-1020163), *Differentiated Professional Development: Building Mathematics Knowledge for Teaching Struggling Learners*. Our central design challenge was how to create PD that would achieve our goal of building *all* participating teachers' mathematics content knowledge, diagnostic approaches, and instructional practices, while differentiating the learning experience so that *each* teacher would have opportunities to meet his or her professional learning needs. To address this challenge, we built three main components into the DPD model: core activities, choice points, and self-assessment opportunities. *Core activities* are expected of all participants because they cover essential content and provide a common ground for building a learning community. *Choice points* provide the opportunity for teachers to choose options to customize their learning. For example, a choice point might invite teachers to select from a set of activities, choose their own starting point or path within a given activity, or select the level of challenge of mathematics problems to solve. *Self-assessment opportunities* help teachers assess their level of understanding, reflect on their progress towards the learning goals, and identify areas to strengthen. This information, in turn, helps teachers to select topics or activities on which to focus in the choice points. The model's three components work together to provide a comprehensive and flexible PD approach that allows teachers to collaborate on common goals while also making choices to individualize their learning. In the sections that follow, we provide a closer look at these components.

Our DPD Model in Action

In this section, we present examples from a PD session on fraction multiplication to illustrate how we use the DPD model to build an understanding of key topics from the CCSSM standards. We have found that teachers vary in their levels of prior experience exploring fraction multiplication at a conceptual level that goes deeper than performing the algorithm. Therefore, this session engages teachers in a variety of activities to strengthen their understanding of what it means to multiply fractions and to build flexibility with representing the operation visually, verbally, and numerically. Teachers also learn about common student difficulties and misconceptions, such as *multiplication always makes larger* (i.e., the incorrect assumption that the product is always larger than the factors), as well as ways to address these misconceptions. For this PD session, we differentiate by creating a sequence of core activities and choice points, which are described in the following sections. Although this example is from a face-to-face PD session, we later describe ways to create choice points for online settings.

Core Activities

To launch the topic of representing the multiplication of fractions, we use a series of core activities to engage teachers in common experiences that motivate further exploration and serve as a shared reference point for later activities (see Figure 1). Our initial goal is for teachers to make sense of a fraction multiplication situation by creating their own ways to represent it visually. All of the teachers work on the same word problem (see Figure 2) and individually create a picture or diagram to represent the problem. Then, they share their representations with the group, discussing the similarities and differences. This core activity showcases a variety of approaches and provides us with formative information on teachers' prior knowledge on which to build in the subsequent activities.

FIGURE 1.
Core activities.

Overview of Core Activities	
1.	Teachers come up with their own visual representations for a word problem and share approaches.
2.	Teachers watch and discuss a classroom video in which students draw representations for the same word problem that the teachers worked on. The video shows a few common difficulties.
3.	Facilitators demonstrate how to use one area model approach for fraction multiplication, and they connect it to teachers' approaches in the initial activity.
4.	Teachers solve a few problems using this area model approach.

After sharing their approaches, teachers watch a video of a fifth grade classroom in which pairs of students work on the same word problem that the teachers completed. The video shows a few examples of students having difficulty making sense of and representing the problem. As teachers watch, they take notes and then discuss their observations with the group. Because the video is new to all participants, making this a core activity is a straightforward decision. This shared experience builds awareness of student difficulties with fraction multiplication and motivates teachers' interest in learning ways to provide support. One instructional strategy is to help students use an area model to represent a variety of problems and see firsthand how the size of the product relates to the size of the factors. This visual approach helps address a common student misconception that products are always larger than the factors, by showing that "multiplying a given number by a fraction

FIGURE 2.
Sample problem from Core Activity 1.

Make Drawings to Represent and Solve Problems
1. Jodi is decorating a cake for a party with her friends. She knows that her friends have different tastes and she wants everyone to get what they like. She frosts 1/2 of the cake with chocolate frosting and the other 1/2 of the cake with vanilla frosting. Then, she puts rainbow sprinkles only on 1/3 of the chocolate-frosted part. What fraction of the whole cake will have chocolate frosting AND rainbow sprinkles? *Make a drawing to represent and solve this problem.*

FIGURE 3.

Sample problem from Core Activity 3.

Use An Area Model for Fraction Multiplication

1. Celia got a block of clay to use for a school project. After she finish the project, she had 1/4 of the block of clay left over. She gave 2/3 of the leftover clay to her brother. What fraction of the whole block of clay did Celia give to her brother?

 A) Represent the situation with an area model.

 B) Answer: Celia gave ________ of the whole block of clay to her brother.

 C) How would you represent the situation with words and a number sentence?

 - Words: _____ group of _______
 - Number sentence: ____________________________________

less than 1 results in a product smaller than the given number" (CCSSI, 2010, p. 36).

Next, we demonstrate an area model for fraction multiplication and ask participants to explore it themselves in the role of learners. Although the model is new to some teachers and familiar to others, we feel that it is important for everyone to see the same demonstration to provide a shared reference point for further work and discussion. After the demonstration, teachers use the model to represent and solve several problems (see Figure 3). Because of teachers' varied prior knowledge of the model, we keep this section relatively short and follow it with a choice point to offer teachers an additional opportunity for support, exploration, or challenge.

Choice Point Options

We want teachers who are new to the area model approach for fraction multiplication to have the opportunity to immerse themselves in using the representation and for experienced teachers to be able to stretch their knowledge. Therefore, we next provide a variety of choice points that allow teachers to customize their learning experience. When we designed these choice points, we considered what professional learning needs teachers might have after completing the core activities and what learning experiences would address those needs. Figure 4 presents the list of teachers' varied needs and the options that we brainstormed for the choice points.

FIGURE 4.

Planning choice points by identifying teachers' varied needs and possible options.

Consider Teachers' Varied Needs ***What might teachers be thinking after the core activities?***	**Brainstorm Choice Point Options** ***What are ways to address these needs?***
This is new to me. I never thought about fraction multiplication except with the algorithm. I need more time getting to know the model.	Provide these teachers with more opportunities to work with the model by starting with problems like the ones in the demonstration. Then provide a variety of problems that progress in difficulty.
Representing mathematical ideas visually is hard for me. I'm having difficulty with the model.	Provide these teachers with more instruction on the model by having a facilitator work with a small group. Give teachers the opportunity to use an applet that helps set up the model and gives immediate feedback.
I feel like I have a good grasp of how to use the model myself. I'm ready for more challenge.	Provide more challenging problems that involve more difficult fractions and creating products that are larger or smaller than a given number.
I have experience teaching this model. I want to think about ways to improve how I use this model.	Ask teachers to create their own problems for using the model with their students.

Working from our brainstormed list, we selected three options to offer at the choice point (see Figure 5). For face-to-face sessions, we find that limiting the number of options to two or three works well, because having more choices can be overwhelming for teachers to decide among and challenging for facilitators to implement. To help teachers make a choice, we describe the options and instruct participants to reflect on their prior knowledge/experience. Teachers move to different tables for their selected option and work individually or in pairs.

At the Option B table, one facilitator provides a short introduction to the applet to get teachers started with using it. Then both facilitators circulate among the tables to provide support as needed. We encourage teachers to focus in-depth on their chosen option and work at their own pace. We also provide teachers with copies and/or links to all of the options for later use, so that they do not try to rush through all options at the session for fear of missing out. Figures 5a, 5b, and 5c show sample problems for each of the options.

FIGURE 5.
Overview of choice point options in the fraction multiplication sequence.

Choice Point: Area Model		
Directions: Reflect on your prior knowledge/experience with the model. Based on your experience and self-assessment, choose an option to move your learning forward.		
Option A	**Option B**	**Option C**
If you want to focus on getting to know the model: Work on a series of problems designed to build your understanding and fluency with using the model. Tip: Choose this option if you have no or little prior experience teaching the model.	*If you want to try an applet approach to setting up the visual representation:* Use an applet, Field of Fractions, that provides support in using the approach by drawing and dividing the parts.	*If you feel ready for more challenge:* Work on more challenging problems and create your own problems.

FIGURE 5a.
Sample problems for Option A.

Get to Know an Area Model

Problem	Describe in Words	Draw an Area Model	Product
1. $\frac{1}{5} \times \frac{1}{3}$	______ group of ______		Product: __________
2. $\frac{5}{6} \times \frac{1}{2}$	______ group of ______		Product: __________
3. $\frac{2}{3} \times \frac{4}{5}$	______ group of ______		Product: __________

4. Write a problem that has a product **greater than** the product in #3 but is less than 1.

Represent Sequences of Fraction Multiplication Problems

1. Multiplying proper fractions: Represent each problem by drawing an area model.

a) $\frac{2}{3} \times \frac{3}{4}$ b) $\frac{3}{4} \times \frac{3}{4}$ c) $\frac{4}{5} \times \frac{3}{4}$ d) $\frac{4}{5} \times \frac{4}{5}$

2. Look at the area models for the different problems in **#1**. How is the size of the products related to the size of the factors?

FIGURE 5b.
Screenshots from the Field of Fractions *Applet (http://tube.geogebra.org/m/40736) for Option B.*

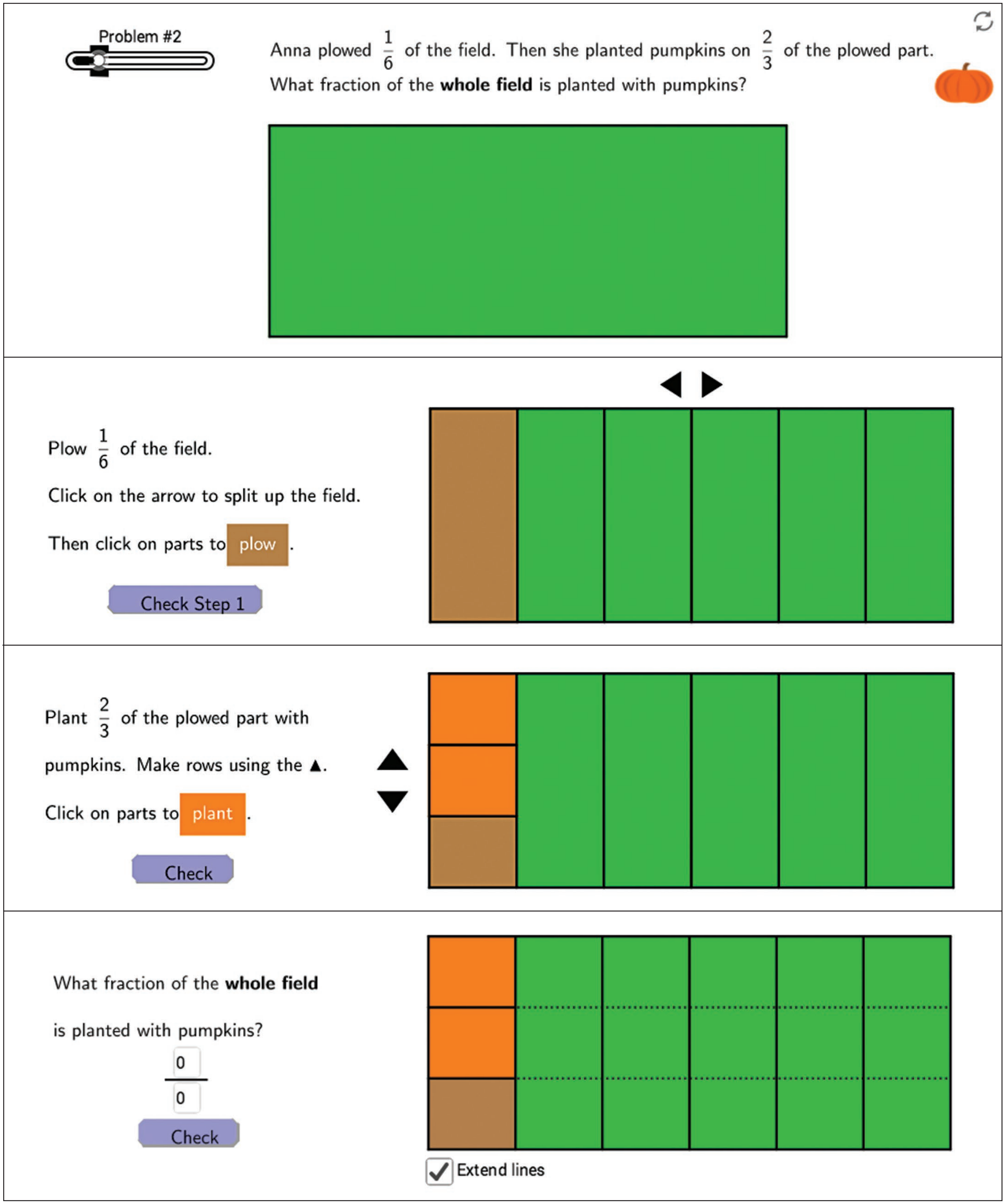

FIGURE 5c.
Sample problems for Option C.

Create Larger and Smaller Products

1. Task: Find all the ways to get products that are **greater than 1/2 but less than 1**

Possible digits: Use 2, 3, 4, 5 or 6 to fill in the blanks.

a) $\frac{?}{5}$ of $\frac{2}{?} > \frac{1}{2}$ but < 1

Show all the ways by drawing area models or explain why it is not possible.

b) $\frac{5}{?}$ of $\frac{?}{2} > \frac{1}{2}$ but < 1

Show all the ways by drawing area models or explain why it is not possible.

Create Your Own Problems

2. Create your own problem by using a similar format.

a. Decide which type of problem you want to create:

________ a problem with two or more ways to get a product less than $\frac{1}{2}$.

________ a problem for which it is not possible to get a product less than $\frac{1}{2}$.

b. Fill out the starting information for the problem.

Task: Find all the ways to make products that are less than $\frac{1}{2}$.

Available Digits: ________________

Starting Expression. (Put 1 or 2 digits in the blank boxes to start.)

$$\frac{\square}{\square} \textit{ of } \frac{\square}{\square} < \frac{1}{2}$$

c. Prepare the solution for your problem by using area models to show all the possible ways to get the target product. Explain how you know that you found all the ways. Or explain why the problem is impossible.

d. Reflect on your experience. What important mathematical ideas did you use to create and solve the problem?

To wrap up the choice point section, we bring all teachers back together for a shared discussion about themes that cut across all three options. One challenge of having teachers work on different activities is designing and facilitating discussions to bring together ideas from their different experiences. We strive to create discussion questions that are applicable to each option and allow all participants to contribute. In this example, we engage teachers in first discussing the area model from their experiences as learners and then from a teaching perspective. We ask them to share considerations and suggestions for using the area model to build their students' understanding of fraction multiplication, with particular attention to addressing the common misconception described above. Sample discussion questions include:

- What was your experience like using the area model as a learner? What important ideas did it bring out about fraction multiplication?
- What are the strengths and limitations of this model for building understanding of what it means to multiply fractions? What are the model's strengths and limitations for solving problems?
- One potential pitfall is that the model could be used in only a procedural way. What are ways to use the model to build conceptual understanding of fraction multiplication?

For this choice point, we use a culminating whole-group discussion because all of the options focus on the area model. When the options are more disparate, we may use separate small-group discussions and then ask each group to share a few ideas with the whole group. For some choice points, we include a few common mathematics problems in each option to facilitate the sharing of approaches in the subsequent whole-group discussion. In designing choice points, it is important to consider not only how to create each option but also how to bring together ideas across options to move learning forward for the whole group as a learning community.

Self-assessment Opportunities

We present the learning goals, such as *build understanding of and flexibility with visual representations for fraction multiplication*, at the beginning of a face-to-face session. Then, during the sequence of core and choice point activities, we pause periodically to ask teachers to reflect on and self-assess their learning. Figure 6 is a PowerPoint slide from the sample session, showing the kind of prompts we use to engage teachers in this process.

FIGURE 6.
Sample reflection prompt for fraction multiplication sequence.

Reflection Opportunity

Directions: Think about your progress towards the goal and write notes on the Reflection Handout.

Learning Goal: Build understanding and flexibility with visually representing fraction multiplication.

Reflect on Progress: To what extent are you able to use an area model to visually represent fraction multiplication and explain what your visual representation means? What questions do you have?

During these intervals of reflection and self-assessment, teachers write their thoughts on a Reflection Handout. As they continue to work on fraction multiplication, they refer back to the handout to identify areas to strengthen and inform their decision making at subsequent choice points.

Two Design Decisions for the DPD Model

The three DPD model components that we have demonstrated here (i.e., core activities, choice points, and self-assessment opportunities) work together to create a robust, flexible approach to differentiating professional learning. In designing this model, we made two key decisions about the differentiation:

1) not everything would be a choice; and when choices were offered, teachers would decide for themselves what options to select. As we have described, the reason for the first decision was that we wanted some content to be required for all teachers in order to provide a shared experience with these topics/activities. These core activities serve as a foundation for further learning and help to build a community of learners. For our differentiated PD courses, we create a combination of activities that is about 60% core activities and 40% choice points.
2) The second decision was to allow teachers to choose an option rather than having the differentiated activity selected for them, such as basing the assignment on their test results. We believe that giving teachers choices promotes ownership and investment in their own

learning and respects their professionalism. A potential downside of this decision is that, as teachers are learning to make these choices for their professional learning, they may overestimate their own understanding or, conversely, choose topics that they are already comfortable with instead of ones that they need. We strove to mitigate these issues and support teachers in their decision-making through the use of reflection and self-assessment opportunities as well as through the facilitator's role in building a supportive learning community that encourages participants to take risks and stretch themselves in their learning.

A Closer Look at the Model's Components

The fraction multiplication sequence above illustrates how our DPD model's three components work together to foster differentiated and group learning in our PD courses. Here we offer a closer look at each component and share the decision-making process we use to interweave the components to meet the professional learning needs of teachers.

Core Activities

To decide which topics should be addressed in core activities, we consider the learning goals and participants' professional learning needs, including their prior knowledge and experiences with the topic, as well as its relevance to their roles and work with students. If the topic is new to all participants, our decision to create a core activity is straightforward. When there is a lot of variation in participants' prior knowledge, such as large groups of new and experienced participants, we tend to use choice point formats. For other situations, we weigh the pros and cons of offering options to differentiate the learning experience versus keeping the group together for a shared activity.

When we choose a core activity format, we consider ways to design a common experience that takes into account participants' varied needs. Even when all participants are new to a topic, they may vary in other ways, including comfort with doing mathematics. It is important to design activities to be accessible and engaging to a range of learners, such as by using a low threshold, high ceiling approach and by giving teachers the opportunity to use multiple strategies. We strive to provide entry points that allow all participants to get started and immerse themselves in the tasks. In addition, we plan ways to draw in and motivate teachers who may have low interest in a topic because they are not responsible for teaching it at their grade level.

In some cases, we decide to use a core activity format for topics in which participants vary greatly in prior knowledge because we want to provide a shared experience on which to build in future activities. In doing so, however, we risk the potential of frustrating participants who are new to the topic by moving too quickly or those who are experienced with the topic by moving too slowly or spending a long time on a familiar topic. In light of these concerns, we aim to design a streamlined core activity that engages all participants and then moves quickly to a choice point.

Choice Points

Choice points are our model's central vehicle for differentiation. In developing the model, we explored a variety of ways to create choice points to allow teachers to make decisions based on different factors: prior knowledge or experience, mathematics topic, type of mathematics problem, desired level of challenge, preferred mode of getting information, and type of activity. We describe each type of choice point and provide examples in Figure 7. Our intention is not to suggest that someone use all of the different types in one PD program. Instead, we encourage the selection of one or more that best match participating teachers' needs.

Prior knowledge and experience. Teachers, who are new to a topic or approach, benefit from introductory activities and from moving at a slower pace with more support than those who have extensive experience teaching the topic to students. Experienced participants need opportunities to build on their prior experiences, stretch their knowledge, and view the topic in new ways. In this type of choice point, we ask teachers to reflect on and self-assess their prior knowledge and experience and choose accordingly.

Mathematics topic. These choice points allow teachers to choose a topic on which to focus in more depth and extend their learning from the core activities. Teachers may select a topic to strengthen their own knowledge and/or focus on content that is applicable to their grade level standards. This type of choice point is helpful for designing PD for teachers from different grade levels.

Type of mathematics problem. We give teachers options to work on different types of problems, such as word problems, numeric (non-word) problems, or estimation problems for the same mathematics topic, such as fraction multiplication. They might select a type of problem that they find more challenging themselves or that they want to strengthen in their work with students.

FIGURE 7.
Types of choice points.

Make Choice Based On:	Examples
Prior Knowledge and Experience	***Use prior experience with fraction circle manipulatives to choose a starting point:*** A. If you have *no or little prior experience*, start on page 1 with an introductory exploration of fraction circle manipulatives and unit fractions. B. If you have *some prior experience*, start on page 3 to use fraction circles to find equivalent fractions. C. If you have *a lot of prior experience*, start on page 5 to use fraction circles to compare fractions. ***Use prior experience to choose to focus on one manipulative or compare two:*** A. *If you are new to using manipulatives for fraction addition*, choose *one* of the following to explore and analyze: Fraction Circles, Fraction Bars, or Pattern Blocks. B. *If you have experience using manipulatives for fraction addition*, choose *two* manipulatives. Compare the strengths and limitations of the manipulatives for building understanding of fraction addition. Consider the ways in which using the manipulatives might support mathematics practices 2 and 7.
Mathematics Topic	***Choose problems based on specific mathematics content:*** A. Multiplication problems with whole number times fraction (Grade 4 standard). B. Multiplication problems with fraction times fraction (Grade 5 standard). C. Mix of multiplication problems. ***After focusing on core fraction division activities, choose to focus in more depth on one of the following topics:*** A. Solve word problems for building understanding of division of whole numbers by fractions and vice versa. B. Use visual models for representing and solving fraction division problems with remainders. C. Further investigate why the fraction division algorithm works.
Type of Mathematics Problem	***Choose type of problem:*** A. Word problems. B. Numeric/symbolic (non-word) problems. C. Estimation problems.
Level of Challenge	***After solving a set of core problems, choose to:*** A. Continue solving problems at the same level. B. Solve problems at an easier or more foundational level. C. Solve more challenging problems.
Preferred Mode for Getting Information	***Build background knowledge of the number line representation for fractions by choosing to:*** A. Watch a video. B. Read an article. C. Explore resources on a website.
Type of Activity	***Choose what kinds of mathematics activities to do:*** A. Paper-and-pencil mathematics activity. B. Interactive mathematics applet with online feedback. C. Collaborative mathematics game to play with colleagues.

Desired level of challenge. This format begins with all participants working on the same problems to gain a sense of the level of difficulty. After finishing the initial problem set, they have the opportunity to adjust the level of challenge for the subsequent problems by continuing to work on problems at a similar level, moving to more foundational problems, or skipping to more challenging problems. Teachers might want to work on more foundational problems because they are having difficulty themselves or because they would like to provide those types of problems to their struggling students. In a face-to-face session, teachers make their choices by moving to different pages in a packet of handouts. In the online environment, participants use interactive menus to branch to their chosen level of challenge.

Preferred mode for getting information. We offer participants choices of getting background information in various formats, such as watching a video or reading an article. They select the format based on their learning preferences (e.g., visual, auditory, verbal) or to serve different purposes. For example, a teacher may prefer to watch a video to first learn about student misconceptions with the number line representation and then use a reading as a reference later on.

Type of activity. These choice points offer a selection of instructional activities, such as a collaborative game, computer applet, or paper-pencil activity. Our goal is to help teachers expand their repertoires of instructional activities for use with their students. We encourage teachers to try new activities and approaches or consider new ways to apply them in their classroom practice.

Choice Point Formats for Face-to-Face and Online Settings

We have created and tested choice points for both face-to-face and online sessions; our intent was to explore ways to leverage the unique features of each environment to support differentiation. We use several strategies to differentiate PD in the *face-to-face* setting. For example, we offer participants a packet of mathematics activities that has a *choose your own adventure* format. That is, as teachers work on the activity handouts in the packet, they come to choice points with options, such as skipping to more challenging problems or moving to a different type of problem (see Figure 8). Teachers work on different activities that are all related to the same mathematics topic so that connections can be made in subsequent whole group discussions. Another differentiation strategy is to use centers, stations, or breakout rooms to allow teachers to move to the topics or activities on which they want to focus. In addition, we offer options for different ways to work on mathematics problems, such as a choice of manipulatives or models.

FIGURE 8.
Example of face-to-face choice point: Choose Your Own Adventure format.

Choice Point

Directions: Reflect on your experience solving fraction addition and subtraction word problems on pages 1-2. Choose an option to move your learning forward.

A. To solve more word problems, work on pages 3-4.
B. To write and solve your own word problems, go to page 5.
C. To use pattern blocks to solve problems (not word problems), go to page 7.

In the *online* sessions, we offer participants similar opportunities to explore representations and build upon the core content by choosing options from choice point menus (see top example of Figure 9). In addition, the power and flexibility of the online environment provides different ways for teachers to customize their learning. Because the online environment allows easy access to materials in various media, online choice points allow teachers to build background knowledge of the same topic by watching a video, reading an article, or working with an applet (see bottom example in Figure 9). To foster teachers' use of diagnostic approaches, we give them a choice of examining student work samples or watching videos of mathematics interviews so they can decide on which type of evidence to focus. For the videos of the mathematics interviews and other topics, teachers can choose to view them multiple times to take a closer look at students' approaches.

FIGURE 9.

Two examples of online choice point menus.

Choice Point: Compare Decimals by Using Different Representations

Directions: Choose two representations. We encourage you to choose representations that are less familiar to you.

A. Use base-ten blocks to build decimals and compare them.
B. Shade grids to represent and compare decimals.
C. Use number lines to locate and compare decimals.

Choice Point: Build Background Knowledge of Number Line Representation

Directions: Choose at least *one* option to build background knowledge on these key questions.

A. Why is the number line an important representation for fractions? (Video)
B. How does the number line representation help students build understanding of fractions as numbers? (Reading)
C. What are key fraction/number line concepts from the Common Core State Standards? (Reading)

Self-assessment Opportunities

For teachers to make choices that are a good match to their learning needs, they need to clearly understand the goals for which they are aiming and have opportunities to regularly self-assess their understanding, pinpoint their strengths and weaknesses, and gauge their progress toward achieving the goals. Articulating learning goals, including success criteria for meeting the goals, and providing reflection opportunities are essential parts of our DPD model. As described in the example above, during face-to-face sessions, we begin with the learning goals and then pause at key points during the day to ask teachers to reflect on their progress towards the goals. Teachers also have opportunities to consider their learning during discussions and to write down "ideas to take away." Similarly, in the online sessions, we incorporate *reflect on progress prompts* (like those in Figure 6) to engage teachers in taking stock of their learning, as well as opportunities to discuss their experiences in the discussion forums. In addition, we provide online *self-check and reflect* activities that include questions about the central mathematics concepts, accompanied by immediate feedback (see Figure 10). These activities are designed for self-assessment purposes and are non-evaluative. Their main purpose is to help teachers identify areas to strengthen and to inform their selection of options at the choice points.

FIGURE 10.

Sample question from a Self-Check and Reflect *activity.*

Self-Check and Reflect Activity

Directions: Read the two problems. Can each problem be solved by using the calculation 1/4 x 2/3?

Problem I: After the party, Sue brought home 2/3 of a cake. She ate 1/4 of the leftover cake. What fraction of the whole cake did she eat?

Problem II: Tomas made an apple pie for the picnic. He ate 1/4 of the pie and Chris ate 2/3 of it. What fraction of the whole pie did they eat?

Select one:
○ **a.** Problem I only
○ **b.** Problem II only
◉ **c.** Both problems I and II
○ **d.** Neither problem

Your answer is incorrect.

Problem I is a fraction multiplication situation but Problem II is not. In Problem I, you need to find 1/4 of 2/3 to determine what part of the whole cake was eaten, so it makes sense to multiply. For Problem II, 1/4 and 2/3 should be added to determine what part of the whole pie was eaten altogether.

Suggestion: If you want to solve more word problems with mixed operations, go to Session 6, Tab 5.

Implementation Findings

A team of researchers gathered information on the implementation of the DPD model as part of extensive field tests for three differentiated courses developed during our NSF-funded project. Overall, 148 mathematics teachers, general educators, and special educators from 21 school districts completed one or more of the courses. Teachers were asked to complete several instruments that explored their experience with the DPD model, including course evaluation surveys and telephone interviews. Here, we share findings from the fractions course because it was the largest, with 104 participants from 16 districts.

Teachers' Perceptions of Usefulness

There were many indications that participants found the PD to be useful, high quality, and a good match for their professional learning needs. For example, using a scale from 1–not useful to 5–very useful, participants gave the overall PD experience a mean rating of 4.8. There were no significant differences in usefulness ratings of the fractions course by participants with different types of certification (i.e., general education/mathematics, special education, or dual), different roles (i.e., general educator, mathematics teacher, special educator), or those with and without post-secondary study of mathematics or mathematics education. These findings demonstrated that the PD was viewed as useful by participants with different professional backgrounds, years of teaching experience, and roles, which reflected positively on the DPD model.

Teachers' Feedback on the Choice Points

Participants gave high ratings to the choice points in both the face-to-face and the online sessions. The research team asked participants how well the choice points met their needs, and the average rating was 3.6 out of 4 for the face-to-face and 3.5 for the online sessions (rating scale: 1–not at all; 2–a little; 3–some; and 4–a lot). When asked to explain their ratings, participants' most common reason for a high rating was the opportunity to customize the learning experience for one's needs, including permission to start at one's current level of knowledge or comfort, challenge oneself, or concentrate on specific topics. As one teacher wrote:

> In courses/workshops I am always feeling others know more than me so at times I am uncomfortable with the idea of everyone working on the same assignment. I was relieved when I could pick my level which lessened my performance anxiety and at times was pleasantly surprised that I could choose a more challenging assignment.

Another teacher wrote, "I loved that we could work at our own pace. [It] really allowed me the opportunity to use the manipulatives and gain an understanding of the concepts."

Other reasons for high ratings included feeling ownership and appreciating the freedom to make choices. In the words of one teacher, "[Choice points] gave me the freedom to pick things that I knew I either needed some practice/support with, or things that I knew I would be able to use in the classroom. It was nice to have that freedom."

Many teachers said they chose options that were new to them or would expand their own learning. As one participant said, "If you are going to take a course like this I think it is important to really dig into what you don't know or have not had much exposure to." Although the PD placed a strong emphasis on having teachers build their own content knowledge, many participants' decisions of which activities to select were heavily influenced by the potential for classroom application. Like other adult learners, teachers want their learning to be relevant, so it is natural that the participants would choose options that they felt they could ultimately use with students. Many participants were interested in learning effective ways to teach concepts and skills that they considered particularly important or difficult for students, or methods suitable to specific needs of their students. As one teacher wrote, "I wanted to work with word problems as my students have language-based learning disabilities."

Although the primary intent of the choice points was to meet participants' varied needs, experiencing differentiation firsthand gave some participants a sense of its benefits for students. One teacher wrote, "I liked the choices because they engaged me. If I enjoyed the choices, my kids will enjoy them. . . . We still covered the agenda we had to get through. We can now provide choices for our kids." Another commented, "I thought you really exemplified HOW to differentiate through using these [choice points]."

Reasons for Low Ratings of Choice Points

Although the majority of participants gave the choice points high ratings, it is important to consider the explanations given for a small number of low ratings. The most common reason was the desire to complete *all* of the activities. Some participants who were new to the content wanted to work on everything; therefore, having options did not increase the usefulness of the program for them. One participant wrote, "Basically, I wanted to try all the problems. So I always started at the beginning rather than jumping into the middle or end." Some participants seemed to have difficulty trusting that they could make choices and still learn all they needed. As one put it, "I tried to do all of the choices because I felt like I was missing out if I didn't." Our perception is that many participants were new to making choices in professional development, and thus it was not surprising that some would feel unsure about the process.

Issues with Choice Point Decisions

For the most part, choice points allowed participants to customize their learning experience in beneficial ways. In a few cases, participants' interviews and survey responses indicated that they made choices for expedience or to avoid taking risks in learning. Examples included: deciding not to select activities with a particular visual model because they found it confusing themselves and thus would not use it with students; choosing an activity they already knew because they were hesitant to step outside of their comfort zone; or choosing the first activity in the list just to fulfill the requirements, without considering all the different options.

Suggestions for Mathematics Education Leaders, PD Providers, and Facilitators

Just as differentiating instruction for students is more complex than teaching everyone the same way, differentiating PD requires a different type of planning and facilitation on the part of mathematics education leaders, PD developers, and facilitators. In the following sections, we first provide an overview of the planning process and then offer suggestions for designing, implementing, and facilitating differentiated PD using our model.

Overview of the Planning Process

The planning process begins with guiding questions that are essential for designing any PD program: *What are the professional learning goals?* and *What are participants' needs?* In the DPD model, deciding where and how to differentiate involves asking additional questions: *What content will be core for all participants?* and *What content will be differentiated to address teachers' varied needs?* Figure 11 incorporates these questions in the second and third columns to provide a differentiation lens for the planning process.

This expanded set of guiding questions helps strengthen the overall PD design by closely aligning the goals and activities with participants' needs. Considering what will be core and what will be differentiated provides a useful lens for clarifying what is most important for all participants to learn and experience, and identifying where there are openings for individualization.

We have also created a *Differentiated PD Planning Tool* that incorporates the main guiding questions into an agenda format (see Figure 12). To use the tool, we suggest filling out the first two rows and columns of the agenda as you would for any PD program. Then, examine the agenda several times through a differentiation lens. In the second column, star (*) the activities/topics for which teachers have particularly varied needs. Look over topics/activities to make an initial decision about which activities might be core and which might be differentiated; label them with a "C" or "D" in the third column. Next, consider all topics with "D's" to decide which ones are the top priorities to differentiate and write down ideas for creating choice point options in the fourth column. This approach can be adapted for use with existing PD agendas; start with the prior agenda and add columns with the questions on differentiation.

FIGURE 11.
Guiding questions for DPD Model.

PD Planning Questions	What content will be *core* for all participants?	What content will be *differentiated?*
What are the professional learning goals?	What is essential for everyone to learn?	In what ways do the goals vary for different groups of teachers (by role, grade level, etc.)?
What are participants' professional learning needs?	For which areas do participants have a lot of consistency in their professional learning needs?	For which areas do participants have a lot of variation in their professional learning needs? What is the distribution of needs?
What activities will you use to address the learning goals and participants' needs?	How important is it for all teachers to experience this activity for building knowledge and/or providing a shared experience?	What are ways to differentiate the activity to address teachers' varied needs? What choices might you offer?

FIGURE 12.
Differentiated PD Planning Tool.

What are the professional learning goals?			
What are participants' learning needs?			
Time	**What are the topics & activities?** Fill out as you would for any agenda. Then star (*) topics/activities for which participants have particularly varied needs.	**What might be core (C) or differentiated (D)?**	**What will you differentiate? How?** Look over the topics/activities that you marked 'D'. Which of these are high priorities to differentiate? Choose a few and brainstorm ways to differentiate by using a choice point or other methods. Write down ideas below.

Suggestions for Designing and Implementing Differentiated PD

Prior to the PD, the following steps are recommended.

1. Identify the professional learning goals.
2. Conduct a needs assessment.
3. Analyze the findings to identify areas of variation.
4. Decide what will be a core activity and what will be differentiated. *Tip:* Start small by choosing one section of an agenda to differentiate.
5. Plan ways to address logistical constraints, such as available space for dividing into groups and the number of instructors available to lead simultaneous activities.

During the PD, consider the following steps.

6. Gather ongoing information from participants to fine-tune the differentiation.
7. Gradually add choice points to multi-session PD programs.

Finally, after the PD, evaluate and revise the differentiation.

Suggestions for Facilitating Differentiated PD

If you will be facilitating differentiated PD, the following suggestions will help alert you to challenges you may encounter and give you strategies to overcome potential obstacles.

Help teachers make choices and feel comfortable with them. As discussed in the Implementation Findings section, some teachers may feel unsure about which choices to pick or may select "safer" options because they are reluctant to move outside their comfort zones. To address these issues, facilitators need to be careful to describe all of the choices clearly and equitably, without placing value on one over another, and to set a comfortable tone for making the decisions.

In face-to-face sessions, teachers may feel more self-conscious about their choices. In planning sessions, consider how different formats for choice points might affect teachers' comfort in making choices. One option is to have teachers stay at the same table and choose to work on different but related activities in a packet of handouts (using the *choose your own adventure* format previously described). Because teachers do not need to move to a new location, this option lets them choose in greater privacy and also allows them to change direction more inconspicuously if a choice does not meet their needs. Alternatively, there are many benefits to having teachers move so that everyone at a table is working on the same activity.

The anonymity of the online environment reduces some of the concerns described above for the face-to-face sessions; teachers may feel more comfortable making choices and taking risks away from the eyes of their colleagues. Teachers can preview choices to decide if an activity will meet their learning needs and can easily switch activities. Another benefit of the online environment is that the number of choices offered is not constrained by the availability of meeting rooms and instructors.

Although the online environment offers great potential for differentiating learning, it also poses some challenges. Because it is easy to switch from activity to activity online, teachers may skim the options at a choice point without delving into them. Also, while teachers may feel more comfortable in the anonymity of making choices online, they may also feel less motivated, connected, and accountable because they are not working with colleagues. As an online facilitator, you can help by placing a high priority on fostering interaction, building community, and making connections across participants who are working on different activities. These design and facilitation principles are integral to implementing online professional development in general and have particular importance for differentiated programs.

Foster collaboration. A potential downside of differentiation is that the experience may become too individualized. Facilitators need to be attentive to differentiating in ways that support, rather than detract, from building a learning community. We recommend starting differentiated sessions with core activities to provide shared experiences and community building to lay the groundwork for ongoing collaboration. For choice point sections, plan and facilitate discussions to engage teachers in sharing ideas from their different experiences and bringing out crosscutting themes.

Set clear expectations and build in accountability. While establishing expectations and accountability is essential for all PD programs, there are specific issues that need to be addressed in facilitating differentiated programs. One potential issue is that teachers may think of the choice point activities as optional, or as less important than the core activities, and thus decide to skip or skim them.

Another concern is that teachers may feel less accountable because different people are working on different activities as opposed to everyone working on the same one. It is important to explain that everyone is responsible for focusing in-depth on his/her selected activity and to clarify expectations by setting an end goal, such as being prepared to share ideas with the whole group.

Implications for Mathematics Education Leaders

Differentiating professional development offers important benefits to both teachers and their school districts. Our DPD model, with its combination of core activities, choice points, and reflection opportunities, allows teachers to work together on common goals while also making choices to individualize their learning. It gives teachers greater ownership of their professional learning, as each practitioner chooses to focus on the knowledge or skills that he or she needs to strengthen. For districts, this approach for customizing offerings to teachers' varied needs helps to optimize the limited amount of time available for professional development. Although the examples in this article focus on fractions, the DPD model lends itself to mathematics topics across the standards and grade levels. We invite mathematics education leaders, PD providers, and facilitators to use the model to differentiate professional development to address their districts' specific mathematics goals and their teachers' diverse learning needs. As the model is applied, we encourage the exploration of new directions, the sharing of approaches, and research that investigates the impact of differentiated PD on teacher and student learning. ✪

This material is based upon work supported by the National Science Foundation under Grant No. DRL-1020163. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors(s) and do not necessarily reflect the views of the National Science Foundation.

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Seeking Bridges between Theory and Practice: A Report from the Scholarly Inquiry and Practices Conference on Mathematics Methods Education

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Abstract

This paper reports on the NSF-funded Scholarly Inquiry and Practices Conference on Mathematics Education Methods (Grant No. 1503358), held in Atlanta, Georgia on September 30 through October 2, 2015. Conference participants from three different theoretical perspectives (sociopolitical, cognitive, and situative) discussed the goals and activities of methods courses with a focus toward developing more scholarly inquiry and practices in the work of methods instruction. Conference participants discussed ways of building partnerships between K-12 schools and university teacher education programs. Implications of the conference for mathematics supervisors and leaders are provided.

Introduction

The elements involved in the preparation of prospective K-12 mathematics teachers and the support of in-service mathematics teachers are complex and managed by a variety of interested parties, from teacher educators in university settings to mathematics education leaders and mentor teachers in school settings. The collective work of leaders in mathematics education is generally guided by the goal of improving student achievement in the learning of mathematics for all students (National Council of Supervisors of Mathematics [NCSM], 2014; National Council of Teachers of Mathematics [NCTM], 2014). In order to progress, multiple efforts in improving teaching for both practicing and prospective teachers are warranted. In his Judith Jacobs Lecture at the 2016 Annual Conference of the Association of Mathematics Teacher Educators [AMTE], Skip Fennell challenged the members of AMTE to broaden the scope of the organization to include all who participate in the preparation and development of teachers. In addition, he encouraged teacher preparation programs and school districts to envision teacher preparation as a shared responsibility. As this collective work toward improved mathematics instruction and learning is carried out in both universities and K-12 settings, it is important to attend to the ways in which the workers and activities in each setting can learn with and support the other.

At the university level, the preparation and support of mathematics teachers often includes both content-specific and pedagogically focused activities, offered in a variety of ways. Teacher preparation programs include field experiences, mathematics content courses, and methods courses that typically focus on developing the practices and pedagogy of teaching mathematics. However, variation in teacher preparation and professional learning programs can be seen in the structure of and courses in different programs (Seago, 2008). Research has also shown variation in the structure, content, activities, and goals of mathematics methods courses (Harder & Talbot, 1997; Kastberg, Tyminski, & Sanchez, in press; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999). Amid this variation, mathematics teacher educators are urged to move toward more scholarly practices, which are "adapted from empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (Lee & Mewborn, 2009, p. 3), as they design curriculum and experiences for prospective mathematics teachers during their preparation programs.

Toward the goal of developing scholarly practices and the scholarly inquiry (Lee & Mewborn, 2009) that supports the design of such practices in mathematics methods courses, the Scholarly Inquiry and Practices [SIP] Conference for Mathematics Education Methods, funded by the National Science Foundation (Grant No. 1503358), was convened September 30 through October 2, 2015 in Atlanta, Georgia. Conference participants were university mathematics teacher educators and researchers, many of whom teach methods courses and provide teacher professional development in K-12 schools and classrooms. The purpose of this article is to review the conference discussions and outcomes that highlighted the need to situate experiences in K-12 settings and illuminate the importance of partnerships with mathematics education leaders and K-12 faculty that are needed to build scholarly inquiry and practice.

Description of Conference Events

The SIP conference included 53 participants from 29 states who were either university mathematics education faculty or mathematics education doctoral students. Participants were selected, in part, because of their experiences working with practicing teachers in professional development, supervising prospective teachers' field experiences, and/or developing partnerships with K-12 schools.

The activities of the conference were focused on six goals:

1. Discuss important goals for methods [courses] based on theoretical orientations, the participants' experiences, and the literature;
2. Identify the nature of activities that might be useful in methods [courses] to meet important goals;
3. Discuss the evolution of methods instructors' practices within and across individuals;
4. Discuss and suggest protocols for research and reporting practices that would make the literature more useful for building scholarly practices in methods [courses];
5. Discuss and establish a research agenda for improving and determining the impact and residue of methods courses; and
6. Form working or writing groups to progress the research agenda and an action plan for creating and disseminating the agenda. (Sanchez, Kastberg, Tyminski, & Lischka, 2015, p. 9)

Activities were also informed by three theoretical perspectives, which undergird much of the research in mathematics education: socio-political, cognitive, and situative. The socio-political perspective is based on critically examining the process of schooling and its capacity to educate all learners equitably. In order to prepare prospective teachers to examine their own contexts using a critical lens, mathematics teacher educators need to help prospective teachers develop a knowledge base about and skills for promoting equitable learning environments for their future students. The cognitive perspective questions what it means to learn mathematics, both individually and through social interactions, and processes through which this learning occurs. The situative perspective stresses the importance of teacher preparation being conducted within increasingly authentic school contexts, with prospective teachers learning ambitious teaching, which "requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions" (Kazemi, Franke, & Lampert, 2009, p. 1). Conference participants self-identified with one of these perspectives and then worked within perspective groups for three of the four breakout sessions that comprised the conference.

Keynote Addresses

Next, we report on the content of keynote addresses delivered by leading researchers in each of the three perspectives, along with outcomes of subsequent breakout sessions that composed the remainder of the working time of the conference.

Socio-political perspective. Representing the socio-political perspective, Rochelle Gutiérrez challenged conference participants to recognize the political nature of education and to attend to the development of political knowledge for teaching or *political conocimiento*. In her previous writings, Gutiérrez (2013) described characteristics of teachers who have such knowledge.

> Among other things, political conocimiento involves: understanding how oppression in schooling operates not only at the individual level but also the systemic level; deconstructing the deficit discourses about historically underserved and/or marginalized students; negotiating the world of high-stakes testing and standardization; connecting with and explaining one's discipline to community members and district officials; and buffering oneself, reinventing, or subverting the system in order to be an advocate for one's students. (p. 11)

Such knowledge supports mathematics teachers' role as "identity workers" (Gutiérrez, 2015) who situate ways in which mathematics is reproduced and therefore contribute to how learners position themselves in school and society.

Teacher education programs that support the development of political knowledge for teaching begin with strong partnerships with schools, professional development within those schools, and opportunities for prospective teachers to interact with learners in both school and non-school settings. In addition, prospective teachers need opportunities to envision, practice, and reflect on challenging situations. Gutiérrez described two such opportunities. "In My Shoes" (Gutiérrez, 2012, 2015) is a task that allows prospective teachers to envision situations where they might want to challenge a political notion (e.g., a discussion on tracking in mathematics during a faculty meeting) and then practice responses they might give in those situations. This structured practice allows prospective teachers to build language and ways of interacting that support the growth of political knowledge for teaching. As a second example, "The Mirror Test" (Gutiérrez, 2015) asks an educator to reflect by asking, "Am I doing what I said I wanted to do in education when I set out to be in this profession and, if I'm not, what am I going to do about that?" Gutiérrez argued that these opportunities and others empower prospective teachers to see teaching as a profession where colleagues and community work together to advocate for learners.

Cognitive perspective. Representing the cognitive perspective, Martin Simon began his keynote by challenging the conference participants to consider whether current practices in teacher preparation were fostering induction into the current system or supporting "new teachers [to] be eventual leaders of a different way of teaching mathematics" (Simon, 2015). Simon identified barriers to enacting change in mathematics teaching, including structures of mathematics content courses, time allotted for methods and content courses, insufficient support structures for field experiences, and a lack of knowledge of teacher development. Simon further argued that the key issue to be addressed in the preparation of teachers is the identification of a model of teaching and challenged participants with the question, "If I ask you, how do you help somebody learn something that they don't already know or understand — are you prepared with an answer?" (Simon, 2015).

To explore this question, Simon described two major assimilatory structures prospective teachers develop over years of experiences: perception-based structures and conception-based structures. Teachers who have a perception-based structure interpret learning of new ideas (mathematical or pedagogical) as observing characteristics of phenomena. Problems and models to teach mathematics are chosen because they allow a learner to see a mathematics concept (i.e., the use of base-ten blocks to see relationships between powers of ten). Teachers with a conception-based structure view existing knowledge as impacting what learners know and how they make sense. "Knowledge affects what we see and the sense we make. . . What we know affects what we pay attention to, what we see, and the sense we make of what we see" (Simon, 2015). In addition, the conception-based structure includes the notion that "we learn by building on prior knowledge. We don't take it in from materials, we don't take it in from somebody else, but rather we have to work with what we have" (Simon, 2015). Simon argued that changing prospective teachers' approaches to mathematics teaching and learning involves changing their major assimilatory structures. According to Simon, this change can result in the re-conceptualization of teaching and mathematics learning.

To support teacher development, Simon proposed that mathematics educators consider *pedagogical concepts* such as the negotiation of classroom norms or the meaning of developing a new mathematical operation (Simon, 2015). Simon defined a pedagogical concept in the context of teaching prospective teachers as "the particular understandings we want our [prospective teachers] to come away with" (Simon, 2015). He advocated for the clear articulation of pedagogical concepts and an exploration of how these concepts are learned. Simon returned repeatedly to asking the fundamental question of how someone comes to know something they did not know before. Mathematics education researchers have made substantial progress addressing this question for learning mathematics, but the process of learning to teach mathematics has been largely un-theorized.

Situative perspective. Elham Kazemi began her discussion of the situative perspective by posing three questions:

1. How do you make school a worthwhile place to be (for both teachers and students)?
2. What kinds of learning environments get you inside practice, with others, to pay careful attention to the content and to students as learners and as people?
3. How can you design and carry out powerful ways to learn together as adults?

Kazemi espoused a broader goal of schooling, beyond determining what students "can and can't do in life [and] how and why and what they contribute to society" (Kazemi, 2015). Drawing from the work of Greeno (2006), Kazemi adopts a situative perspective in which understandings are shaped by an activity system. Activity systems are collections of people and other systems, within which we study interactions and relationships between actors in the systems (Kazemi, 2015). Such systems are dynamic and involve the development of collaborative discourse, positioning of all actors within the system, and knowledge that is visible in representations of practice. Research in activity systems shapes Kazemi's work within methods instruction.

Kazemi described the activity system in which she works with colleagues, prospective teachers, and teachers in schools to develop and support her methods instruction. Tenets of her methods course that make the practice of teaching public and provide opportunities for learning include:

- Teachers must position students as sense-makers and knowledge generators who desire to invest and succeed in school;
- Teaching is both intellectual work and a craft;
- Teachers must design equitable learning environments in which all children are engaged in robust and consequential learning;
- Teachers' instruction and student learning is always conducted within the context of larger social systems, structures, and hierarchies; and
- What we do and say matters and must be analyzed. (Kazemi, 2015)

The course is situated in an elementary school context and involves time divided between academic course instruction and interactions with students and school personnel. Activities in the course are designed to incorporate playfulness and build community through sharing and practicing the work of teaching.

The major work of her methods course is focused on planning and enacting lesson activities selected by Kazemi and supervised by a network of teacher educators and mentor teachers. Kazemi emphasized this as an opportunity to "put ourselves in situations where we can learn together instead of thinking we have to wait for the perfect mentor-teacher in order for our [prospective teachers] to have good experiences out in the field" (Kazemi, 2015). Prospective teachers rehearse lessons, question each other, provide critiques, and then enact lessons with a group of students. Prospective teachers are urged to use learners' reactions to mathematical experiences as a lens into their thinking about the mathematics. In this way, the prospective teachers examine the complex work of teaching and have opportunities to develop and reflect on their practices.

After sharing videos of activities from the methods course, Kazemi explained how she works to challenge typical structures of teacher preparation to build connections between university preparation programs and the schools with which they work. She argued that situating methods courses in school settings and including teachers enables teacher educators to grow a profession that is "connected rather than isolated" (Kazemi, 2015).

Summary. The three keynote addresses encouraged participants to focus on ways in which a perspective influences methods course goals and activities, as well as what constitutes evidence of teacher development. Participants referred to ideas drawn from the keynote addresses throughout the conference discussions. In three of the four breakout sessions that structured the conference activities, participants worked within their selected perspective group. The outcomes of these sessions are described next.

Table 1: Goals for Methods Courses Identified by Each Perspective Group

Perspective Group	Methods Course Goals Identified
Socio-Political	• Develop strategies for disrupting current mathematics education norms and agency for pushing back • Become aware of and draw on knowledge of context in which prospective teachers work, including families and communities • Develop a critical orientation to mathematics • Critique discourses of education (schools are failing, achievement gap is really about achievement) • Critically analyze and develop personal mathematics teacher identity
Cognitive	• Enable prospective teachers to become learners from their practice ∘ Develop prospective teachers' abilities to anticipate student responses, based on prior analysis of student thinking. Knowledge required to anticipate student responses includes: the mathematics concept (the discipline), task, and students' prior knowledge.
Situative	• Develop skills, knowledge, and dispositions for building on student thinking using an asset mindset to meet students where they are • Facilitate meaningful mathematical discourse and communication • Plan, enact, and reflect on a lesson that focuses on student thinking and promotes reasoning and problem-solving • Reframe personal relationship with mathematics • Identify evidence that supports being able to say what students understand and do not understand • Learn specific classroom structures, routines, and activities • See the role of teaching and learning mathematics in addressing issues of educational inequity and opportunity

Note: All goals are drawn from slides presented during the conference (Cognitive Perspective, October, 2015; Situative Perspective, October, 2015; Socio-political Perspective, October, 2015).

Learning Goals for Methods Courses (Breakout Session 1)

In the first breakout session, participants were asked to identify learning goals associated with methods courses. The socio-political and situative groups developed extensive lists of goals, a selection of which is provided in Table 1. The cognitive group focused on one overarching goal with one identified sub-goal and sought to clarify how existing knowledge and assimilatory structures might be changed.

Although the goals across groups differ in significant ways, there are commonalities. For example, each group attended to the "learners' mathematics and context as an asset" (Kastberg, Lischka, Tyminski, & Sanchez, 2015). Differences in the three perspectives, however, influenced language of the goals and brought different emphases to the foreground. The socio-political group emphasized knowledge of student culture; whereas the cognitive group emphasized knowledge of student thinking. In contrast, the situative group described knowledge of students as a prerequisite for instruction within a community of learners, thus emphasizing the role of the activity system in the work of teacher preparation. Participants' discussions of the goals highlighted ways in which each perspective influenced the goals identified by the group.

Activities for Methods Courses (Breakout Session 2)

In Breakout Session 2, participants returned to their perspective groups (i.e., socio-political, cognitive, situative) and considered activities that provided opportunities to address the previously identified learning goals. The socio-political group discussed role-play or rehearsal activities (e.g., In My Shoes (Gutiérrez, 2012, 2015)), to develop practices that support social justice goals. Activities that engage prospective teachers in building understanding of and empathy for diverse learners were discussed in order to attend to the group's second stated goal (Table 1). For example, participants discussed tasks that involved teachers experiencing instruction in a language other than English to develop empathy for English language learners. Community walks (e.g., Koestler, 2012), in which prospective teachers walk around the school and neighborhood with a student in order to learn about students' lives outside of school, were also discussed as an activity that can build prospective teachers' understanding of diverse learners and also attend to the second goal stated by this group.

The cognitive group focused on designing an activity to develop prospective teachers' abilities to anticipate learner responses based on prior analysis of learner thinking. The activity involved several tasks. First, prospective teachers solve a mathematics task designed for mathematics learners. Second, they analyze learner conceptions as represented in provided learner responses. After analyzing and discussing learner responses, prospective teachers anticipate learner responses on a similar task. This cycle of completing the task, analyzing learner responses, and then predicting responses on another task attended to the group's goal of developing prospective teachers' abilities to anticipate learner thinking.

The situative group, which was the largest group, divided into six sub-groups, each focusing on a single activity supporting one of the identified goals. Across the sub-groups, an emphasis on approximations of practice (Grossman et al., 2009) was evident. One sub-group focused their discussion on rehearsals, in which prospective teachers practice specific pedagogical moves. A second sub-group focused on analysis of curriculum materials, building prospective teachers' knowledge for choosing resources to meet the needs of learners. Yet another sub-group discussed the use of videos or other approximations of practice to develop prospective teachers' noticing of questioning techniques. Each of these activities focused on the desire to provide opportunities for prospective teachers to experience teaching activities in controlled situations. One sub-group extended this idea and discussed the ways in which providing methods instruction in K-12 school settings could enrich the approximations of practice that prospective teachers experience. This sub-group explored the evolution of relationships with schools and situating methods courses in schools. They described this evolution in levels from interacting with after-school groups to moving the methods course into an actual classroom of learners for a portion of each day.

Across the perspective groups, activities described as attending to goals for methods courses required prospective teachers to think about and interact with K-12 mathematics learners. Participants discussed the importance of developing experiences that approximate important components of practice through which prospective teachers can build their knowledge, skills, and dispositions for teaching mathematics.

Common emphases across perspectives, however, should be interpreted carefully because similar language does not necessarily imply similar understandings. For example, the project's external evaluator observed,

> It is interesting to note that different [perspective] groups identified similar activities (such as rehearsal) for different purposes, suggesting that there could be some value in cross-perspective discussions. This commonality also suggests the need for practitioners and scholars to be explicit about their perspectives as people with different theoretical orientations might think they are talking about the same idea because they use the same term (such as *rehearsal*) when, in fact, they are talking about very different ideas. (D. Spangler, personal communication, October 3, 2015)

This observation demonstrates a need for a common language and shared understandings of central ideas relevant to mathematics teacher preparation. Moreover, working across the complicated boundaries between K-12 schools and universities introduces even more opportunities for different interpretations of similar sounding ideas.

Researching Effectiveness of Activities (Breakout Sessions 3 and 4)

In Breakout Session 3, participants considered the types of evidence that would indicate teacher growth in the direction of the stated learning goals. Following this, writing teams formed with the purpose of developing a chapter for a potential publication disseminating the work of the SIP conference (Breakout 4). Across both breakout sessions, participants focused discussions on the ways in which mathematics education researchers could learn from and report on their practice to support mathematics teacher educator development and scholarly practice.

The effectiveness of methods course activities was discussed using the ideas of experience, impact, and residue (Kastberg, Sanchez, Tyminski, Lischka, & Lim, 2013). The experiences of an activity are the concepts or ideas taken up by prospective teachers as a result of their interaction with the course activity. The impact of a course activity is the evidence of prospective teachers' use of the concepts developed in the stated activity within other aspects of the methods course. Residue refers to evidence of prospective teachers' continued use or application of concepts from an activity after the course has been completed. In many cases, prospective teacher performance in a subsequent course activity in relation to an initial activity was identified as a way to assess impact. For example, the situative perspective group assessed prospective teachers' internalization of a task analysis framework by observing their use of curriculum materials in later lesson planning activities. Alternatively, participants proposed that research on residue take place during student teaching or in the induction phase of teaching. Research of this type will require collaboration with and access to learners and teachers in schools. As mathematics education researchers work to gain evidence of impact and residue of methods course activities, universities and schools will need to form partnerships that extend beyond traditional field experience components of teacher education programs.

Important Conference Conversations for Mathematics Education Leaders

Reflecting on the events of the conference, three themes emerged as relevant for all stakeholders involved in the successful preparation and ongoing support of mathematics teachers and mathematics education leaders. First, ***The preparation of teachers is best enacted by and within a community.*** Elham Kazemi identified this potential, sharing: "What's interesting about the way this [conference] is organized is that I think our perspectives and our work actually all need each other" (Kazemi, 2015). Participants expressed the desire to work across perspectives, explaining that a researcher working from a situative perspective might implement an activity designed to address goals identified by cognitive or socio-political groups. The need to extend the community of educators working with prospective teachers to more fully include school-based personnel, including practicing teachers and mathematics education leaders, was frequently expressed by participants. Through the discussions of methods activities that build on student thinking and focus on interactions with students, the value that practicing teachers, mathematics education leaders, and students bring to the process of teacher preparation was noted. In describing the community impact of teacher preparation, Rochelle Gutiérrez said, "It's not just what you learn in a pre-service teacher education program, but it's actually how you learn it that matters" (Gutiérrez, 2015). Immersing prospective teachers in the professional culture of teaching while providing access to the community of schools and learners was deemed essential to progress in methods education.

The second theme from the discussion was, ***Teachers need opportunities to attend to learning from and within practice.*** Kazemi described,

> It is intellectual work to teach — to actually be interested in learning is intellectual work and it requires specialized knowledge. It's more than just being a student yourself of the subject matter. It is about being a student of your students. (Kazemi, 2015)

Across all three perspectives, conference participants viewed teaching as an evolving practice from which educators and prospective teachers should learn. Simon (2015) began his address by explaining that he has taught methods courses for 25 years and is still dissatisfied with his approaches. He further explained that he is still learning about how prospective teachers learn, specifically through the lens of major assimilatory structures, and that mathematics teacher educators should continue in investigations of the learning of prospective teachers. Many of the activities described by participants provide opportunities for prospective teachers to engage in approximations of practice (Grossman et al., 2009) and reflect on their actions to more clearly understand them. In some cases, the approximations are made more relevant by enacting them with learners of mathematics in K-12 settings. In any form, the importance of learning from and within practice was highlighted and extends to the learning of all involved: practicing teachers, prospective teachers, mathematics education leaders, university faculty, and K-12 learners.

The final theme draws from the first two: ***Partnerships between teacher preparation programs and the K-12 schools they serve are essential for engaging in scholarly inquiry that supports the development of scholarly practice.*** The authors of *It's TIME* (NCSM, 2014) emphasized the need for mathematics education leaders to "cultivate connections with the postsecondary mathematics and mathematics education communities" (p. 17). In her keynote address, Kazemi described a model of methods instruction that takes place in schools and encourages learning on the part of all involved: university faculty, classroom teacher, and prospective teachers. She explained that they “invite the mentor teachers to be part of that process with us when we're in their classrooms and we'll invite the supervisor and the principal at that school” (Kazemi, 2015). In this way, they have forged a partnership with potential benefits for all participants. Rather than experiencing field placements as disconnected from university coursework, the prospective teachers experience learning in the context of multiple perspectives on teaching and learning. In this program, the faculty and school system acknowledged, “It's those little kinds of ways that we grow a profession that is better connected rather than isolated.” (Kazemi, 2015).

Gutiérrez described a similar picture and argued for the blending of professional work with both prospective and practicing teachers.

> If I'm learning through rehearsals and out-of-school spaces, if I’m attending conferences and movies with veteran teachers and novices, if I'm debriefing with others, it means that I'm not going to expect to do this work on my own as a teacher. It also means that I’m going . . . to want to debrief with other people. It means that I'm not just going to look to textbooks or professors or peers and that I will imagine that I'll continue to do this work in community with a diverse group of people. (Gutiérrez, 2015)

These statements create a vision of learning to teach as a collaborative practice with practicing teachers, prospective teachers, and university faculty.

Building Bridges between Theory and Practice

The themes and discussions from the SIP Conference echo the leadership framework set forth in *It’s TIME: A Leadership Framework for Common Core Mathematics* (NCSM, 2014). In particular, this document sets forth imperatives for mathematics education leaders, which contain elements of the three perspectives undergirding the SIP Conference. Authors of *It’s TIME* stated, “The beliefs teachers have about students, society, and education can result in certain populations of students having limited access to the high level of rigor, depth of mathematics content, and breadth of practice” (NCSM, 2014, p. 13). Leaders are charged with helping mathematics teachers develop productive beliefs about all learners and to expect higher order thinking from all learners. Gutiérrez’s ideas about political conocimiento and her suggestions about activities to help develop such knowledge are useful to mathematics education leaders in their work which aims to affect teachers’ beliefs about learners and schooling. Role-playing scenarios such as Gutiérrez’s (2012) “In My Shoes” activity are meaningful during mathematics teacher professional development. Teachers need support as they develop ways of interrogating institutional structures and deficit discourses that are counterproductive to helping all learners reach their full potential.

In addition to knowing, advocating for, and having high expectations for their students, teachers also need knowledge of their students’ mathematical thinking. NCSM (2014) explained, “Pedagogical content knowledge includes an understanding of what makes concepts easy or difficult to learn and which models or representations work best for individual students” (p. 23). They also stressed that an effective mathematics curriculum can only be delivered if teachers “develop and deepen understandings of learning progressions” (p. 24). When a teacher deeply knows how his or her learners think and what they know, that teacher is better positioned to help learners build new understanding based on current knowledge. Based upon these assertions, Simon’s ideas about *perception-based* and *conception-based* major assimilatory structures can be useful for mathematics education leaders to consider in work with mathematics teachers aimed at developing and using knowledge needed to realize the vision in *It’s TIME*.

Kazemi also provided insights useful for mathematics education leaders. In her keynote address at the SIP conference, she emphasized, “What we do and say matters and must be analyzed” (Kazemi, 2015). Therefore, she structures her methods courses in schools where prospective teachers are provided the opportunity to learn about a routine, practice it with students, and then reflect on their work. Echoing this sentiment, the *It’s TIME* authors (NCSM, 2014) stated, “It is critical that teachers possess knowledge and understanding that support [the mathematical practices] as well as the ability necessary to first envision them and then translate them into actions” (p. 29). Mathematics education leaders can support mathematics teachers’ uses of these practices by setting up structures for collaboration, observation, rehearsal, and reflection.

Finally, *It’s TIME* authors asserted, “The surest way to limit one’s impact is to attend to only one piece of a system. . . without regard to how it affects the other pieces and systems" (NCSM, 2014, p. 9). Goos (2015) conceptualized the space of community boundaries, in this case the boundaries between school communities and teacher preparation programs, as a space that is "generative of new practices – and therefore, new learning" (p. 276). It is in this space that both university mathematics teacher educators and school-based mathematics education leaders can achieve their separate goals, where each is a knowledgeable other and offers learning opportunities to strengthen the work of the other. Consideration of the ways methods course activities might enrich the work of mathematics education leaders and the ways in which mathematics education leaders can contribute to methods courses can encourage knowledge growth in both communities and the boundaries between them. Throughout discussions at the SIP conference, participants explored the connections between teacher preparation and school settings as a boundary where learning can and should occur for all parties involved. ✪

This material is based upon work supported by the National Science Foundation under Grant No. 1503358. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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