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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all —levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We seek your reactions, questions, and connections to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Angela Barlow, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

***Note:** Information for manuscript reviewers can be found at the back of this publication.

Inquiries about the ***NCSM Journal of Mathematics Education Leadership*** may be sent to:
Angela T. Barlow
MTSU Box 76
Murfreesboro, TN 37132
Email: ncsmJMEL@mathedleadership.org

Other NCSM inquiries may be addressed to:
National Council of Supervisors of Mathematics
6000 East Evans Avenue
Denver, CO 80222-5423
Email: office@ncsmonline.org • ncsm@mathforum.org

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education

- Fostering inquiry into key challenges of mathematics education leadership

- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice

- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Angela T. Barlow, *Middle Tennessee State University*
Travis A. Olson, *University of Nevada, Las Vegas*

Although the responsibilities of mathematics education leaders can be quite varied, we likely all have a common goal of supporting effective teaching and learning of mathematics. In *Principles to Actions,* the National Council of Teachers of Mathematics (2014) described effective teaching as "teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p. 7). Such teaching is complex and requires ongoing professional development (National Council of Supervisors of Mathematics, 2014; National Council of Teachers of Mathematics, 2014). To this end, this issue features two articles with implications for supporting the work of mathematics education leaders in this area.

In the article "Teaching Reform-oriented Statistics in the Middle Grades: Results from a Case Study," Gerstenschlager describes the case of a sixth-grade teacher implementing a statistical unit of instruction. The focus of the inquiry is two-fold. First, Gerstenschlager describes the implementation fidelity of the unit, including the alignment of implementation to reform-oriented instructional practices. In addition, she considers the deviations from the unit that were introduced by the participating teacher. Second, Gerstenschlager reports the participant's perceived challenges that arose during the implementation of the unit. The results demonstrate the importance of considering both teachers' mathematical perspectives (Jin & Tzur, 2011; Simon, Tzur, Heinz, Kinzel, & Smith, 2000) and knowledge (Ball, Thames, & Phelps, 2008; Groth 2007, 2013) as mathematics education leaders aim to support teachers' implementation of reform-oriented curricula.

Within this context of reform-oriented instruction, many mathematics education leaders find themselves giving attention to the instructional practices that support students' engagement in the Standards for Mathematical Practice (SMP, Common Core State Standards Initiative, 2010). Bostic and Matney report on the potential associations among the Standards for Mathematical Practice in their article, "Leveraging Modeling with Mathematics-focused Instruction to Promote Other Standards for Mathematical Practice." Specifically, they share their results from analyzing lessons that were designed to support Modeling with Mathematics (SMP4), looking for instances of teacher moves aimed at promoting any of the SMPs. Bostic and Matney report that lessons designed to promote SMP4 were associated with promoting behaviors and habits found in the other SMPs. Given this result, the authors suggest that mathematics education leaders who aim to support teachers' work with the SMPs might consider initially focusing on teachers' understanding of SMP4, as it seems to promote practices that support the other SMPs.

With a goal of supporting effective teaching and learning of mathematics, it is our hope that the ideas presented in these two articles will serve to inform your work with teachers. ✪

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Teaching Reform-oriented Statistics in the Middle Grades: Results from a Case Study

Natasha E. Gerstenschlager, *Western Kentucky University*

Abstract

With the Common Core State Standards for Mathematics, statistics has a more influential role in the middle grades curriculum than in the past. However, statistics is generally not a priority in teacher professional development programs leading to teachers' poor content knowledge in statistics and many teachers feeling unprepared to teach statistics. These prove to be barriers to reform-oriented instruction, and some have recommended using lessons created by statistics educators as a way to address these barriers. Unfortunately, simply having these lessons is not enough to ensure that students develop a conceptual understanding of the topic. In addition, even if teachers have those lessons, there is limited research on how well instruction aligns with curriculum expectations when the lessons are implemented in the classroom and how this implementation is related to teachers' mathematical perspectives. Therefore, this descriptive case study examined the implementation fidelity, including deviations and alignment, of a reform-oriented statistics unit in a sixth-grade classroom and challenges the teacher identified regarding the implementation of the unit. Implications of results to the mathematics and statistics education community are included.

Introduction

As data become more prevalent in society, the need for statistically literate citizens who can be critical of the information they are receiving becomes exceedingly more important (Franklin & Mewborn, 2008; Kader & Mamer, 2008). Franklin and Kader (2010) noted that developing statistical reasoning skills takes a significant amount of time and cannot be achieved in one statistics class. As a response to this, Franklin et al. (2007) suggested that statistics education needs to happen for students in a more rigorous manner and earlier in their academic careers. Consequently, statistics education is undergoing a reform that began over 30 years ago at both the pre-K-12 level (Franklin et al., 2007; National Council of Teachers of Mathematics [NCTM], 1989, 2000) and the collegiate level (Aliaga et al., 2005; Garfield, Hogg, Schau, & Whittinghill, 2002). This effort is not limited to the United States (Jacobbe & Horton, 2012), but rather is global, as countries recognize the "importance of statistics in the education of its citizens" (Peck, Kader, & Franklin, 2008, p. 1).

In the United States, this reform effort has produced many influential standards documents including: *Principles and Standards for School Mathematics (PSSM)* (NCTM, 2000), the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework* (Franklin et al., 2007), and the *Common Core State Standards for Mathematics* (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010). Many states have adopted the CCSSM, a set of standards that begins statistics education informally in elementary school and introduces formal standards in middle school. Although the GAISE document is not completely aligned with the CCSSM expectations for statistics education, there is at least one similarity between the two documents: statistics is being suggested at an earlier time in students' academic careers than in previous standards. The GAISE and *PSSM* documents, in contrast to CCSSM, expect more rigorous statistics instruction happening as early as kindergarten.

To meet the expectations within these documents, research in mathematics education literature has demonstrated that teachers need a different type of knowledge to teach effectively, specifically that of pedagogical content knowledge (PCK) (Shulman, 1986) and mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008). Given that many agree statistics contains several non-mathematical areas and is considered to require a different type of thinking (delMas, 2004; Groth, 2007; Hannigan, Gill, & Leavy, 2013), Groth (2007, 2013) reconceptualized MKT into a framework called statistical knowledge for teaching (SKT). Research has shown that teachers' SKT is poor (Stohl, 2005), thus becoming an obstacle towards enacting statistics instruction as envisioned in these documents.

Suggestions have been made for ways to ensure the teaching of rigorous statistics that include ways to overcome teachers' poor SKT and other obstacles. First, the American Statistical Association and NCTM (2013) emphasized the importance of professional development for teachers, specifically in statistics, that models appropriate pedagogies for teaching statistics. Second, the recent *Statistical Education of Teachers* (SET) (Franklin et al., 2015) document includes specific professional development recommendations for in-service K-12 teachers. These recommendations include a structure for professional development that engages teachers in the statistical problem-solving process (Franklin et al., 2007). This four-step process focuses on the role of variability and includes: creating a statistical question; deciding upon a plan to collect data and collecting the data; analyzing the data; and interpreting the results in context. Despite these recommendations, high quality professional development for teachers focused on statistics is still considered a critical need (Shaughnessy, 2007).

In addition to these recommendations, Bargagliotti, Jacobbe, and Webb (2014) suggested that teachers use "K-12 statistics lessons that have been reviewed and written by statistics education experts" (p. 11). These researchers suggested that using these K-12 lessons in teachers' classrooms could provide appropriate teacher training for statistical content. However, simply having lessons is not enough to ensure "meaningful, effective, and connected lesson sequences" (NCTM, 2014, p. 71). There can be a "substantial difference" (Stein, Remillard, & Smith, 2007, p. 321) between what is intended within a curriculum and what actually happens in the classroom. This misalignment between intended (Stein et al., 2007) and enacted curricula (Gehrke, Knapp, & Sirotnik, 1992) is related to the concept of implementation fidelity. Although there does not exist a universal definition for implementation fidelity, the term generally refers to "the extent to which an enacted program is consistent with the intended program model" (Century, Rudnick, & Freeman, 2010, p. 202).

Although much research exists around implementation fidelity in the mathematics classroom (summarized in Stein et al., 2007), similar research is lacking in the context of a statistics classroom and how this implementation might be influenced by teachers' mathematical perspectives. Therefore, this study sought to examine the fidelity of implementation of a middle-grades reform-oriented statistics unit and explore the participant's perceived barriers to implementation. Specifically, the research questions were: How does a sixth-grade teacher implement a reform-oriented statistics unit, and what affected the implementation fidelity as identified by the teacher?

Literature Review

To better understand the phenomenon of implementation fidelity and potential issues surrounding how and why curricula are implemented, the literature on mathematics and statistics teacher knowledge, mathematical perspectives, and implementation fidelity is reviewed in this section. It is important to note that although this is not an exhaustive review of the literature, the ideas explored in this section provided a foundation for the study and its conceptual framework.

Mathematics and Statistics Teachers' Knowledge

In 1986, Shulman introduced PCK as a type of knowledge needed by all teachers to teach successfully. This knowledge, which he described as a blend of pedagogy and content in a way that is specific to each teacher's content area, was further refined by Ball and colleagues (2008) for the specific subject of mathematics. This research resulted in the MKT Framework, which consists of three content knowledge domains (i.e., common content knowledge, horizon content knowledge, and specialized content knowledge) and three pedagogical content domains (i.e., knowledge of content and curriculum, knowledge of content and teaching, and knowledge of content and students). Recognizing the differences between mathematics and statistics as disciplines (Cobb & Moore, 1997; delMas, 2004; Gal & Garfield, 1997; Rossman, Chance, & Medina, 2006), Groth (2007) first conceptualized the SKT framework, which contains many of the same domains as the MKT Framework. In his revised SKT framework, however, Groth (2013) identified key developmental understandings and pedagogically powerful ideas specific to the field of statistics that are crucial for the development of subject matter knowledge and PCK. In this section, I briefly review some research on both MKT and SKT and how these constructs relate to student achievement and instructional practices.

In terms of MKT and student achievement, several studies have shown that teachers' with improved MKT can significantly affect students' mathematics achievement (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Rockoff, Jacob, Kane, & Staiger, 2008). Specifically, Hill and colleagues (2005) found that teachers' with higher MKT engaged in instructional activities that subsequently improved first and third graders' mathematics achievement scores, with the first graders' scores being more significantly affected by their teacher's level of content knowledge. Similarly, Rockoff and colleagues (2008) found that not only was MKT a significant predictor for students' mathematics achievement but that there was also a significant relationship between teachers' self-efficacy, cognitive ability, and their MKT. In a study by Baumert et al. (2010), the researchers went further by examining the potentially different effects on student achievement of PCK and content knowledge, defined as a deep understanding of mathematics content they were expected to teach. Interestingly, the researchers found that PCK was a more significant predictor for student success in mathematics than teachers' content knowledge. Students in lower socioeconomic statuses were more affected by teachers with improved PCK.

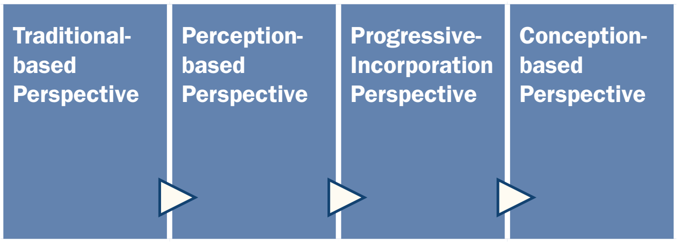
Studies have also examined how teachers' MKT and their SKT affected their instructional practices. Galant (2013) examined how teachers' MKT affected the way they chose and sequenced tasks within their classroom. The researcher found that the participants' weaknesses in their MKT significantly influenced how they selected and sequenced tasks, specifically those with poor MKT had a poor understanding of the "progression and development of mathematical concepts and processes" (p. 46). Similarly, Groth and Bergner (2013) found that when teachers had poor SKT, they were more likely to provide weaker responses to students when asked to analyze students' statistical work. These researchers stated that the participants in their study with improved SKT were more likely to address student misconceptions without telling students how to complete the problems, aligning with reform-oriented philosophy.

Although dealing with pre-service teachers, Leavy (2015) similarly found that participants' issues within SKT led to perplexed responses to students' misconceptions with data handling. In another study on MKT and instructional practices, Copur-Gencturk (2015) found that as participants' common and specialized content knowledge improved so did their ability to implement lessons that aligned with the inquiry-based philosophy and developed students' conceptual understanding of the content. One can see that MKT and SKT have been shown to affect teachers' instructional practices, which subsequently affect students' achievement. Hence, these constructs are important to explore as potential barriers to implementation fidelity.

Mathematics Perspectives

Researchers have found that teachers sometimes find it difficult to "participate effectively in reforming mathematics teaching" (Simon, Tzur, Heinz, Kinzel, & Smith, 2000, p. 579). As a means to help understand why, Simon and colleagues explored teachers' perspectives (i.e., meaning-making structures) that potentially influence teachers' instructional practices. These researchers identified four different perspectives held by teachers: traditional-based perspective, perception-based perspective, progressive-incorporation perspective, and conception-based perspective. These perspectives lie on a continuum (see Figure 1 on next page) with conception-based perspective being most aligned with reform-oriented philosophy. These perspectives were developed through a series of studies (Jin & Tzur, 2011; Simon et al., 2000; Tzur, Simon, Heinz, & Kinzel, 2001) examining different relationships between these perspectives and teachers' practices.

FIGURE 1.
Mathematics Teachers' Perspectives



Teachers who have a traditional-based perspective interpret mathematics as existing "independently from human experience" (Simon et al., 2000, p. 593). These teachers assume that their role is to tell students how to do mathematics and that students in their classroom should maintain a passive role in their learning. That is, students are expected to learn through reading textbooks and watching others, namely the teacher, solve problems. Teachers with this perspective rely mostly on directly transmitting knowledge to the student and allowing students to solve problems in class similar to the ones demonstrated by the teacher.

In contrast, those with a perception-based perspective feel that students need to see mathematics for themselves instead of being shown how to solve problems by the teacher. The teacher with this perspective views mathematics concepts as being interrelated, comprehensible, and available to any learner who is willing to discover the mathematics themselves. From this perspective, the role of the teacher is to help students discover the connections between and among mathematical concepts.

Although similar to the perception-based perspective, Jin and Tzur (2011) described the progressive-incorporation perspective as being slightly different, stating that teachers with the progressive-incorporation perspective focus on connecting ideas to students' previous knowledge and view students' knowledge as being transformed personally by the learner. This is in contrast to the perception-based perspective where connections made are not necessarily to students' previous knowledge but among different mathematical concepts.

Finally, those teachers with a conception-based perspective believe that students learn mathematics based upon their current knowledge and their past experiences. A major difference between this perspective and the previously described perspectives is the role of the teacher. In a conception-based perspective, the role of the teacher is being able to elicit, use, and make sense of student thinking as a way to guide instruction that is focused "on understanding the students' conceptions (assimilatory schemes) and determining ways to promote transformation" (Simon et al., 2000, p. 594). This perspective most closely aligns with the constructivist philosophy (Vygotsky, 1978) and also differs from the previous perspectives in that it incorporates both the learner's knowledge and experience.

Teachers' mathematical perspectives potentially can affect the instructional practices that they use in their classrooms. Therefore, to begin to understand why a teacher might implement a curriculum a particular way, one should consider the teacher's mathematical perspective and the effect that it has on what he or she views as appropriate instructional practices for the mathematics classroom.

Implementation Fidelity

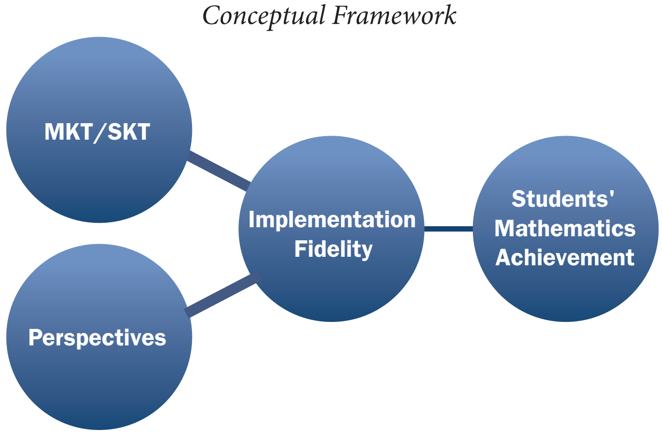
The term implementation fidelity has been used in many different disciplines with each discipline applying their own definition for the term. In this study, implementation fidelity was defined as how well the enacted curriculum aligned with the intended curriculum. Some researchers have shown that high levels of implementation fidelity are linked to high levels of student achievement (George, Hall, & Uchiyama, 2000). Others have also demonstrated that different teachers implement the same task differently (Stein, Lane, & Silver, 1996; Tarr, Chávez, Reys, & Reys, 2006) and that the same teacher has been found to implement the same curriculum differently between classes (Boaler & Staples, 2008). Given these ideas, it is important to examine this literature to better understand how teachers might alter curricula and how altering curricula can possibly affect students' achievement.

To help understand these differences in implementation, researchers have explored specific ways teachers change the implementation of specific curricula. Remillard (2005) summarized this literature and found that studies on curriculum use could be represented by three broad categories: following or subverting, interpretation, or participation with the curriculum. In the first category, teachers follow the curriculum faithfully, and Stein et al. (2007) reflected on how this often happens when teachers' philosophical beliefs align well with the curriculum philosophy. In the second category, teachers' personal beliefs and experiences shape the way they interpret and implement curricula. Reflecting on teachers' mathematical perspectives, one can see how these constructs encompass a teacher's beliefs and can potentially impact teachers' interpretation of the curriculum. Finally, the third category, teachers who participate with the curriculum, can be seen as similar to the second category previously described. However, the two are distinct in that the latter has a "focus on the activity of using or participating with the curriculum resource and on the dynamic relationship between the teacher and curriculum" (Remillard, 2005, p. 221). Overall, this research demonstrates how there are many ways that the same curriculum can be implemented.

Conceptual Framework

As illustrated in the conceptual framework found in Figure 2, a teacher's perceived MKT/SKT and mathematical perspectives (wherever they fall on the continuum shown in Figure 1) have the potential to affect teachers' implementation of a curriculum. I provide three considerations for the reader to reflect upon while considering this framework. First, for this study, MKT/SKT were combined for ease and because it was not a goal of this study to differentiate specifically between the two constructs. The literature on these constructs was provided, however, so that readers may interpret the results in light of both frameworks. Second, it is important to mention that this conceptual framework is limited in the choice of factors potentially affecting implementation fidelity. Although there are many potential factors that could affect implementation fidelity (e.g., administrative support), they were not explored explicitly in this study. Finally, given that student data was not collected in this study, the focus is on the relationship between and among MKT/SKT, perspectives, and implementation fidelity (noted by the bold connecting lines). However, I included students' mathematics achievement within the conceptual framework to display the importance of the three previous constructs. It also included so that others exploring similar ideas can think about different relationships among these four constructs and design studies around these constructs including students' mathematical achievement.

FIGURE 2.
Conceptual Framework



Methods and Methodology

Given that a description of the circumstance of implementation in the classroom was desired, Yin (2014) stated that a case-study method was appropriate. Therefore, in this section, I describe the case study in terms of the research context and participant background. I also detail the instruments used and sources of data collected. Finally, I describe the statistical unit that was implemented, the data collection and analysis procedures, and limitations and delimitations.

Research Context

This study occurred over eight days in a mathematics classroom in a rural middle school (Grades 6-8) located in the southeastern U.S. The school was part of a district that served a population of students that was 91.2% Caucasian, 5.2% Hispanic, 2.7% African American, 0.7% Asian, and 0.2% Native American/Alaskan. The total student population was 4,575 students in the 2014-2015 academic year. This district reported 57.4% economically disadvantaged students, 13.9% disabled students, and 1.3% limited English proficient students. Per results from state testing, 41.6% of the students in this district in grades 3-8 scored basic or below basic on their mathematics assessment. The study occurred within in a single sixth-grade mathematics classroom that met daily for a duration of 46 minutes. This classroom consisted of 26 students whose make-up resembled that of the district. That is, the majority of students were Caucasian and economically disadvantaged. The classroom included two Hispanic students, one of which was considered limited English proficient.

Participant

In this study, the participant, referred to as Ms. Thomas (pseudonym), was selected based upon her familiarity with reform-oriented teaching, her willingness, the willingness of her administration, and accessibility. Ms. Thomas, a certified pre-k – 6th grade teacher, had participated in a 10-day professional development session during the previous summer that was designed to help teachers implement appropriate instructional practices and had a content focus of fractions. I was able to observe Ms. Thomas within this professional development session and noted that she expressed strong interest in making her instruction align with reform-oriented philosophy.

At the onset of the study, Ms. Thomas had entered her third year of teaching, but it was her first year for teaching mathematics. Previously, she was a language arts teacher. In regards to her previous professional development experiences, Ms. Thomas had not had the opportunity to participate in any professional development specifically for statistics. She also indicated that she had taken one semester of statistics in her college career. In this way, she was similar in terms of what the research says about many of in-service teachers regarding their statistics backgrounds. Prior to implementing the statistics unit, I observed Ms. Thomas' classroom to gain an understanding of her typical instruction. During this observation, I maintained field notes as Ms. Thomas conducted her lesson. Qualitative analysis (similar to the methods described below for the overall study later) of these field notes revealed that her typical style of instruction evidenced a traditional perspective. For example, during the lesson, Ms. Thomas remained at the podium as she asked students to look at examples in the textbook. She worked a few problems at the board, and then she asked students to complete similar problems in class as she checked their work. Ms. Thomas' traditional perspective made her a suitable participant in this study as her background reflected that of many other teachers in similar contexts to Ms. Thomas as described by the literature.

Instruments and Data Sources

Data from five sources were collected: field notes, researcher journal, participant research journal, interview protocols, and a daily observation protocol. Field notes were maintained during each lesson. Immediately following each lesson, I completed the daily observation protocol (Appendix A). The daily observation protocol aligned with NCTM (2014), CCSSM (CCSSI, 2010), and GAISE (Franklin et al., 2007) documents and embodied the basic components of reform-oriented instruction. Specifically, this protocol focused on the Standards for Mathematical Practice (CCSSI, 2010), the Mathematics Teaching Practices (NCTM, 2014), and the statistical problem-solving process (Franklin et al., 2007). These features were included because they captured some of the essential practices that should occur in a reform-oriented classroom. Although it was not expected that Ms. Thomas should engage in all of these practices within one lesson, it was anticipated that she and her students engage in some of the practices daily.

I conducted interviews using a semi-structured approach. First, I interviewed Ms. Thomas prior to her implementation of the unit to gain an understanding of her background. Second, I interviewed Ms. Thomas after each daily lesson implementation. Finally, I interviewed her after the unit implementation was complete. I designed the questions to elicit potential supports and challenges to her implementation and to help her describe the implementation from her personal perspective.

Daily after each lesson implementation, Ms. Thomas and I reflected in our research journals. I was the primary data collection instrument (Creswell, 2013) as I approached the study from a subjective orientation. With a background in qualitative approaches, I used lessons learned in previous coursework and studies to maintain reflexivity throughout the study and held a non-participatory role in the classroom as I observed. It is important to note that I chose not to measure the participant's MKT/SKT with a validated instrument in lieu of her own perceived MKT/SKT. Although future studies should explore this relationship between MKT/SKT and implementation fidelity in statistics lessons, the goal of this study was to instead view fidelity and issues through the participant's perception.

Statistical Unit

By creating the statistical unit that was implemented, I had a genuine understanding of the curriculum expectations, which was important given the intent to study implementation fidelity. Although other curricula could have been used for this study, I chose to approach this by compiling rigorous tasks into a unit for Ms. Thomas for two reasons. First, I was able to ascertain if the implementation aligned with expectations as I had a key role in creating this unit and, thus, was able to note specific instances when Ms. Thomas' instruction did or did not align with curriculum expectations. Second, as most classroom instructional time is focused around mathematical tasks (National Center for Education Statistics [NCES], 2003) and the types of tasks in which students engage determine the mathematics they learn and how they learn to use it (Doyle, 1983, 1988), I wanted to be purposeful in designing a unit that focused on rich statistical tasks that specifically highlighted the mathematical goals of CCSSM (CCSSI, 2010) for sixth-grade and GAISE (Franklin et al., 2007).

To create this unit, I followed the Understanding by Design framework (Wiggins & McTighe, 2005). This framework consists of three stages: learning goals, assessments, and lesson plans. First, I identified the learning goals within the sixth-grade statistics standards in the

CCSSM (CCSSI, 2010). The two overarching goals for the unit included developing students' understanding of variability and their ability to describe and summarize distributions, the two key learning objectives for sixth-grade statistics per the CCSSM. I used these goals to create assessments that addressed the learning goals. Next, I created daily lesson plans and tasks that prepared students for the assessments. Both the daily lesson plans and assessments included the four steps of the GAISE statistical problem-solving process (Franklin et al., 2007): formulating a question; collecting data; analyzing data; and interpreting results in context. Many of the daily tasks were adapted from Browning and Channell (2003); Zbiek, Jacobbe, Wilson, and Kader (2013); and Revak and Williams (1999). To ensure that the unit engaged students in reform-based practices and aimed to develop deep conceptual understanding of statistics, a statistician, a mathematics educator, and one external reviewer who had taught statistics at the high school level for eleven years reviewed the unit. Also, the unit followed what Stein et al. (2007) referred to as a reform-based approach since the curriculum was written so that students first explored concepts and then, once they were exposed to the concept and developed an understanding, the teacher introduced vocabulary and any traditional procedures as needed.

The unit included six daily tasks to be completed on eight of the ten days and two assessments to be completed on the remaining two days. Two of the six daily tasks engaged students in the statistical problem-solving process in its entirety for both qualitative and quantitative data sets, and the remaining four daily tasks allowed students to create and analyze statistics and graphical representations for both quantitative and qualitative data sets. All of these tasks asked that students move beyond a deterministic view of the tasks (i.e., simply calculating the statistics) to a more statistical view (i.e., using multiple statistics to justify arguments based on context). Table 1 provides an overview of the intended curriculum, and Table 2 provides an overview of the enacted curriculum. Two key differences exist between these tables. First, there is a discrepancy in terms of the number of days between the two tables. The original intent was for the unit to be implemented over 10 days. However, due to unforeseen circumstances, the implementation was reduced to eight days. Second, many of the tasks were extended over multiple days per Ms. Thomas' discretion. This led to the elimination of several tasks from the unit.

Each daily lesson included a lesson goal, a list of materials and handouts needed, and a description of how one might implement the task. This description included discussion questions, anticipated student responses, anticipated student conceptions and misconceptions, and targeted ideas for certain components of the tasks (e.g., students should understand that the median is resistant to extreme values after completing this discussion and task). These components of the unit were where the Standards for Mathematical Practice, the Mathematics Teaching Practices, and the statistical problem-solving process were embedded. For

Table 1: Intended Curriculum Plan

Day	Task	Summarized Goal
1	French Fry Task	Find and interpret mean, median, mode, and range for two quantitative data sets in context
2	Answering a Statistical Question Task	Understand, collect data to answer, and represent a statistical question
3	Construct Your Own Graph Task	Find and interpret interquartile range for a quantitative data set, including a representation and description of distribution
4	I Wonder What Happens If . . . Task	Understand how different statistics affect the shape of a distribution
5	Statistical Problem-Solving Process Task	Complete the statistical problem-solving process for quantitative data set
6	Statistical Problem-Solving Process Task	Complete the statistical problem-solving process for quantitative data set
7	Categorical Data Task	Complete the statistical problem-solving process for qualitative data set
8	Categorical Data Task	Complete the statistical problem-solving process for qualitative data set
9	Unit Test	Formal assessment of previous goals
10	Oreo Performance Task	Performance assessment of previous goals

Table 2: Enacted Curriculum Plan

Day	Task	Summarized Goal
1	French Fry Task	Calculate mean, median, mode, and range for two quantitative data sets and interpret in context
2	French Fry Task	Continue from previous day
3	Answering a Statistical Question Task	Understand, collect data to answer, and represent the data answering a statistical question
4	Construct Your Own Graph Task	Calculate interquartile range for a quantitative data set, including a representation and description of distribution
5	Construct Your Own Graph Task	Continue from previous day
6	I Wonder What Happens If . . . Task	Understand how different statistics affect the shape of a distribution
7	I Wonder What Happens If . . . Task	Continue from previous day
8	Oreo Performance Task	Performance assessment of previous goals

example, each day's lesson plan included ways to elicit student thinking in the form of varying questions for students of all levels (e.g., those struggling with task or those who complete the task quickly).

Procedures

Ms. Thomas reviewed the unit two months prior to her implementation. After her review, we discussed the unit in terms of what was expected. During this discussion, I informed Ms. Thomas that she could ask me any questions about the unit. She was aware that this would be the only time that I would answer questions regarding the unit and the content to be taught. I recorded this discussion, and immediately following this conversation, I interviewed Ms. Thomas to gain an understanding of her background and perspectives of teaching and learning mathematics.

Prior to the implementation, I observed Ms. Thomas to get a sense of her typical instruction. During this observation, I followed the daily observation protocol (Appendix A) and maintained field notes. Shortly after this initial observation, Ms. Thomas began implementing the statistics unit. Each day, I videotaped her implementation, took field notes, and completed a daily observation protocol. After each implementation, I left the room and waited in an empty classroom while Ms. Thomas taught her final class of the day. During this time, I finished my daily observation protocol and wrote in my researcher journal. After her last class period, Ms. Thomas and I met for her daily interview. After this interview, Ms. Thomas responded to a participant journal prompt via email. This process was repeated daily for eight days. After the eighth day, I conducted the last interview.

Data Analysis

Following a qualitative approach, I examined the data chronologically looking for "patterns, insights, or concepts" (Yin, 2014, p. 135). After this examination, I assigned codes to these concepts based upon the conceptual framework. I then compiled these codes into larger themes based upon the literature and conceptual framework as an attempt to illuminate "the larger meaning of the data" (Creswell, 2013, p. 187). As an example of the analysis, Ms. Thomas stated one day in class, "I'm not supposed to be giving you [the students] all answers and showing you all what to do, but I'm trying to give you a good foundation to start with." Reflecting upon my conceptual framework, I assigned this statement with the code of Traditional Perspective (within the node Perspectives). While there was the potential for any perspective on the continuum in Figure 1 to be included in the codes, Ms. Thomas only provided data that aligned with the Traditional Perspective code. Codes that fell into the MKT/SKT portion of the framework consisted of Subject Matter Knowledge and PCK. Recall that the differentiation between those concepts for MKT and SKT, while important, was not explored during this study. Finally, codes that fell within the Implementation Fidelity node of the framework consisted of either Deviation (from the intended curriculum) or Alignment (with the intended curriculum). Within the codes Deviation and Alignment, I further identified sub-codes relating to challenges and supports that Ms. Thomas mentioned in regards to her implementation. I progressed through all of the data, assigning these codes and using these codes later to analyze potential reasons for Ms. Thomas' chosen implementation of the unit. Finally, I created a case study report following my chronological structure and asked Ms. Thomas to review this report, that is to provide a member check, to address construct validity (Yin, 2014).

Limitations and Delimitations

The study described had three limitations and two delimitations. The first limitation was the number of days available for the unit implementation. The original plan included 10 days of instruction for the unit. However, unforeseen school priorities arose, and the study had to be limited to eight days. Because of the time of the year, a second limitation was that Ms. Thomas was busy with many personal and work-related requirements. As a result, our interviews were often hurried and included many interruptions. The final limitation was a technical malfunction on Day Six of the implementation. The video camera failed with 20 minutes remaining in the lesson causing me to rely only on my field notes for that part of the lesson.

The selection of Ms. Thomas as the participant is considered a delimitation for this study. Given her new role as a mathematics teacher, her traditional-based perspective of teaching, and her interest in teaching with a reform philosophy, I was interested in documenting Ms. Thomas' case since she, anecdotally, reflected many other teachers in similar positions. Although interesting, this does not allow for me to generalize the results from this study. However, through thick description (Creswell, 2013) of Ms. Thomas' case, the audience can transfer the results to similar situations. The second delimitation was the use of the daily observation protocol. The instrument proved to be cumbersome, and, although it illuminated when certain portions of protocol were observed in the classroom, it did not provide much information in terms of how the portions of the protocol (e.g., the Standards for Mathematical Practice) were implemented. Reflection on video data had to be used to elaborate on how practices were addressed.

Results

As previously stated, implementation fidelity was defined as how well the intended and enacted curriculum aligned. Reflecting on Ms. Thomas' implementation, therefore, included both her deviations from the intended plan and the alignment with a reform-oriented philosophy (two of the codes described above). The results from this section are organized around sections on deviations, alignment, and challenges. The reader might benefit from knowing that the structure of the classroom was similar each day of implementation. Students were in groups of three to five, and their desks were turned to face one another to make a table on which the group could work. The table groups did not change in regards to student makeup during the length of the implementation of the unit. The study took place in Ms. Thomas' second mathematics class period of the day.

Deviations of Enacted Lesson from Intended Lesson

Across the eight days, I observed two key deviations of the enacted lesson from the intended lesson. First, on several occasions, Ms. Thomas decided to implement what she called mini-lessons as students were working on a task. For example, on Day Two when Ms. Thomas circulated the room during a task, she noticed that some students were struggling with the material. Students verbalized their confusion, and she asked the students who were confused to meet with her at the white board at the back of the classroom. With four to five students standing around her, Ms. Thomas created a data set and wrote it on the board. Then, she demonstrated the appropriate procedures for finding the statistic(s) that met the requirements of the task. In reflecting on this occurrence, Ms. Thomas stated, "I know we're not supposed to give them the answers, but some of them, if I don't show them . . . they'll never get it" (12/8/14). This practice of implementing a mini-lesson was evident on other days as well. For example, on Day Five Ms. Thomas used a mini-lesson during class when she noticed many students had issues with a certain part of the lesson. During the lesson, she stated, "Let me do it [and show you] my way" (12/11/14). She reflected on this practice during our interview, stating, "I also decided to do a little more modeling than I had in the past" (12/11/14). Ms. Thomas referred to her demonstration of how to solve the task as modeling.

Second, on some days Ms. Thomas chose to display the teacher solution sheet for the task being implemented. For example, on Day Two as students worked through a task, Ms. Thomas noticed that she was running out of class time. As the class period came to an end, Ms. Thomas displayed the teacher solution sheet on the projector with solutions to the task. She allowed students to look over the sheet and write these answers down. On this day, Ms. Thomas said to her class, "I'm not supposed to be giving you all answers and showing you all what to do, but I'm trying to give you a good foundation to start with" (12/8/14). This practice was also evident on other days, including: Day One, when she stated to the class, "Let's see what the answers would have been" (12/5/14), before displaying the teacher solution sheet; and Day Seven, when she projected the solutions for a graphical representation and asked, "What do you notice about – where is most of the data?" (12/15/14).

Alignment of Enacted Lesson to Intended Lesson

To determine how the enacted curriculum aligned with the intended curriculum, I identified evidence from the enacted lesson for each of the three components of the daily observation protocol: statistical problem-solving process, Standards for Mathematical Practice, and Mathematics Teaching Practices. First, the most addressed portion of the daily observation protocol was the Standards for Mathematical Practice section. That is, on each of the eight days, students were engaged in at least one of the eight standards (see CCSSI (2010) for a full description of all eight Standards for Mathematical Practice). Of the eight standards, students were primarily engaged in using appropriate tools strategically followed by making sense of problems and persevering in completing them. For example, during the Oreo cookie performance task on Day Eight, I observed students collecting several different types of data on a cream-filled cookie as part of the assessment. When students realized that their task required them to calculate many statistical measures for their data, several students asked if they could use their calculator to help with the calculations. On that day, some students also chose to use rulers to measure the heights of their cookies. Ms. Thomas reflected on this in her participant journal. She stated, "I feel the students enjoyed getting to decide 'how' to approach the question and how to analyze the data" (12/18/14). This practice was also evident on other days, for example, on Day Three when students also asked to use the calculator for finding the statistics of a larger data set that would have been cumbersome to do by hand.

Second, in terms of the statistical problem-solving process, on three of the eight days of implementation, the enacted lessons engaged students in three of the four steps in the process (see Franklin et al. (2007) for a full description of each level in the statistical problem-solving process). For example, on Day Three, Ms. Thomas asked the students to analyze the statistics that they had calculated and create multiple representations for the data. Ms. Thomas asked, "What do we notice looking at the histogram versus the dot plot?" (12/9/14). On Day Eight, I observed students engaged in analyzing real data. I noticed, "The students found out the name brand [cookie] is not double compared to off brand" (12/18/14). Overall, students were most engaged in the analyzing data step of the statistical problem-solving process, specifically for quantitative data.

Finally, in terms of the Mathematics Teaching Practices, Ms. Thomas demonstrated five of the practices across the eight days. Ms. Thomas was most likely to engage in eliciting and using her students' thinking. This was evident on several days when Ms. Thomas asked students for their ideas and recorded those ideas on either chart paper or the white board. For example, on Day Three Ms. Thomas asked students about the different representations that they could create for a set a data. Many of them responded, "bar graph," "bar chart," and "line graph" (12/9/14). Ms. Thomas then recorded those ideas on the board. Recognizing that no student identified histogram as a potential representation, Ms. Thomas described how to create a histogram using previously created student work during this lesson. She then asked the students to create a histogram and another representation of their choosing, many chose a dot plot, with the goal of having students reflect upon the similarities and differences between the two representations. This example demonstrated a portion of the Mathematics Teaching Practice of eliciting and using student thinking.

Other examples of her engaging in the Mathematics Teaching Practices were evident on Days One and Eight. On Day One, Ms. Thomas had two groups of students share their work for a task in which each group took a different approach to the problem. Before the second group shared, she asked of the first group, "I want you to watch to see if you catch on to the difference [in their work]" (12/18/14). This was an example of facilitating meaningful discourse in that Ms. Thomas asked the whole class to analyze the students' work. On Day Eight, Ms. Thomas asked students to "share [their] data" (12/18/14) after which she used their work to push them further in their thinking. She stated, "Is that data going to show us if the stuffing is double or not?" (12/18/14). This was an example of asking purposeful questions that required her students to justify their mathematical work.

In reflecting upon what helped her engage in these practices, Ms. Thomas referred to the unit plan. When asked specifically what about the unit plan made her engage in these practices effectively, she stated, "How [the unit is] laid out. How the lesson plan is there. How I didn't have to decide what questions to ask" (12/15/14). This sentiment came up frequently with her referring to how the unit plan allowed her to know what to expect from the students. Reflecting across the entire implementation, Ms. Thomas engaged in many of the Mathematics Teaching Practices, as well as provided her students with opportunities to engage in the statistical problem-solving process and the Standards for Mathematical Practice. Unfortunately, many of these interactions appeared to be superficial. For example, although Ms. Thomas elicited students' thinking by asking what different representations they could make for a data set, it appeared that she did so not to guide the structure of the lesson but because this was written into the lesson plan.

Barriers

During the implementation of the unit, I identified three codes within the data that Ms. Thomas used to describe challenges or barriers to her implementation. These included: a traditional-based perspective of mathematics, subject matter knowledge, and PCK. Across the eight days, Ms. Thomas revealed a traditional-based perspective of mathematics instruction that appeared as a barrier on all eight days. An example of the traditional-based perspective was observed on Day Two when students were expected to calculate statistics for a quantitative data set and then use these statistics to make sense of the distribution of the data in context. During the interview that day, I asked Ms. Thomas to reflect on her use of a mini-lesson during the lesson that was not part of the intended curriculum. She responded, "[Some of the students] still needed me to visually show them, which is why I took them to the back board, and we went over what each one of the words looks like" (12/8/14). I reflected upon this barrier in my researcher journal, describing how Ms. Thomas frequently visited students' table groups and explained or showed them how to calculate the requested statistics for the task. It seemed Ms. Thomas recognized students would not be able to meet the larger goal of the task (using the statistics to make sense of the distribution) without being told how to calculate the statistics.

In addition, subject matter knowledge appeared as a challenge on five of the eight days. An example of the subject matter knowledge barrier was evident during a lesson about creating box plots. I noticed that Ms. Thomas "thought [she] could find the number of data values in a data set with a box plot" (12/10/14). She acknowledged this lack of content knowledge in a response to a prompt regarding what helped or hindered her that day by saying, "Poor planning and content knowledge" (12/10/14). This was also evident on other days. For example, on Day Two, I reflected in my researcher journal that Ms. Thomas visited several table groups and explained, incorrectly, how to find the median for the data set. This revealed that Ms. Thomas had deficits in her MKT, specifically subject matter knowledge.

Finally, as an example of the PCK, Ms. Thomas reflected during an interview, "That is a struggle as a first time teacher of this subject – I don't know what [knowledge] they've got [*sic*]" (12/8/14). Here, Ms. Thomas specifically identified that she had a deficit in her knowledge of content and students. This was echoed in my journal, "It appeared to me that she did not know what students knew coming into her class" (12/8/14). She continued to talk about this barrier throughout the study as was evident in a later interview. Ms. Thomas stated, "I can get the answer, but I don't always feel confident [that] I'm getting the answer to the students right" (12/10/14). This quote revealed that Ms. Thomas also identified that she lacked the appropriate PCK, specifically that of knowledge of content and students, to teach the unit effectively.

Although the barriers and deviations revealed that Ms. Thomas espoused a traditional-based perspective of teaching and learning mathematics, some evidence in her engagement in the three components of the daily observation protocol demonstrated a shift in her teaching. For example, per the description of the alignment above, Ms. Thomas was able to engage herself and her students somewhat with the Standards for Mathematical Practice, Mathematical Teaching Practices, and the statistical problem-solving process components. Although this appeared to be a small shift in her perspective and much of this engagement appeared to be superficial, it demonstrated that she was beginning to move towards a conception-based perspective.

Discussion

Analysis of the results revealed new research in statistics education that aligns with previous research from mathematics education. The analysis revealed that Ms. Thomas did not completely implement the unit as intended, a finding echoed in other research focused on mathematics lessons (NCES, 2003; Stigler & Hiebert, 2004). Reflecting on the conceptual framework shown in Figure 2 and Remillard's (2005) descriptions of implementation, it appeared that her self-reported weak MKT/SKT and her traditional-based mathematical perspective influenced her implementation, described as interpretation. Although it was not expected that Ms. Thomas would implement the curriculum exactly, as Stein et al. (2007) stated that complete fidelity of implementation is impossible to achieve, there were two surprising deviations. First, Ms. Thomas used mini-lessons several times throughout the implementation. This was most likely due to her traditional-based perspective of teaching and learning mathematics given that Simon and colleagues (2000) discussed how teachers with this perspective feel it is their duty to demonstrate to students how to solve problems. Second, on a few instances, Ms. Thomas projected the teacher solution sheet to students. Ms. Thomas appeared to use this as an alternative method to having a summary discussion. As with the mini-lessons, this deviation was likely due to her traditional-based perspective for the same reason. These deviations seemed to indicate two struggles. First, it appeared that Ms. Thomas was unable to relinquish the mathematical authority in the classroom, a similar issue faced by other teachers (Wilson & Goldenberg, 1998; Wilson & Lloyd, 2000; Wood, Cobb, Yackel, 1991). Second, it seemed that Ms. Thomas was also unsure when to tell information and when to simply ask questions that might guide the students, similar to that found by Romagnano (1994).

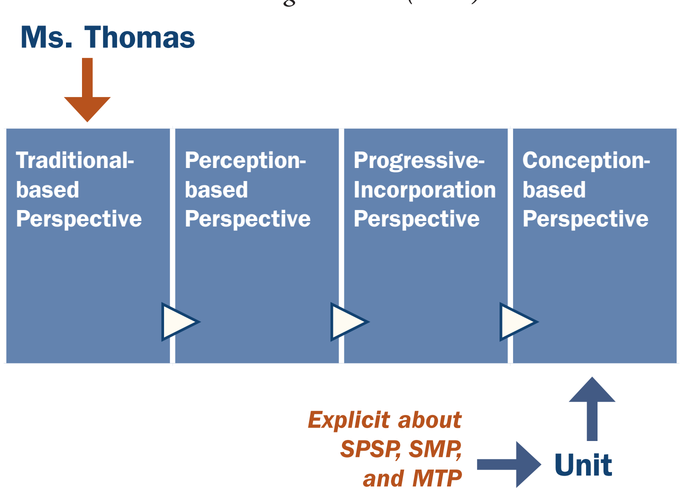
Examining the other barriers, Ms. Thomas frequently talked about challenges to her implementation that I coded as subject matter knowledge and PCK. Although Ms. Thomas and I discussed the unit's content prior to her implementation, these barriers persisted throughout her implementation. This result aligned with what several researchers have found, specifically that many teachers lack the statistical and mathematical knowledge to teach the content effectively (Groth & Bergner, 2013; Hill et al., 2005) and to be able to respond to students' thinking (Wood et al., 1991). Reflecting on the results of previous research (Copur-Gencturk, 2015; Galant, 2013), I hypothesized that because of her perceived subject matter and PCK barriers, Ms. Thomas altered her instructional practices to include the two previously described deviations (i.e., mini-lessons and showing of the teacher solution sheet), aligning with Remillard's (2005) interpretation descriptor.

Reflecting upon the mathematical perspectives described in the literature review as a continuum, Figure 3 demonstrates that continuum and how both Ms. Thomas' practice and the unit were situated within that continuum. Given that the statistical unit was created with explicit connections to reform documents, the unit was situated on the right of the continuum in the conception-based perspective cell. Reflecting on my frequent coding of Ms. Thomas' traditional-based perspective as a barrier, I situated her on the left of the continuum in the traditional-based perspective cell.

Despite the misalignment between the unit and teacher's practices, Ms. Thomas was sometimes able to engage her students and herself in many of the reform-oriented practices that were explicit within the unit (i.e., Standards for Mathematical Practice, Mathematics Teaching Practices, and the statistical problem-solving process). I hypothesized that since these practices was explicitly part of the unit, the explicitness within the unit materials supported Ms. Thomas somewhat in engaging in these practices. That is, Ms. Thomas was able to refer to the unit as a way to support her in engaging in these practices, specifically with anticipating and responding to student responses, conceptions, and misconceptions.

It is important to note that, although this unit was explicit in its details (specifically, that of how to implement), Ms. Thomas still did not implement the curriculum with complete fidelity. Also, despite Ms. Thomas engaging in many of the reform-oriented practices, reflection on the data revealed that many of these practices were implemented superficially and not for the purpose of guiding the lesson structure. Perhaps this was due to a lack of alignment

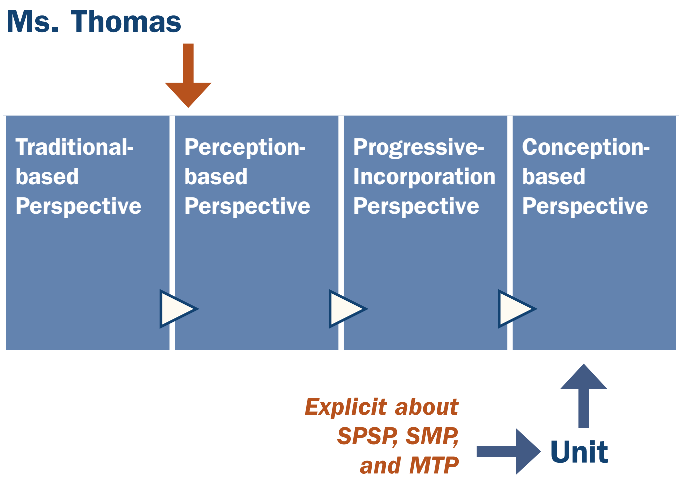
FIGURE 3.
Perspectives continuum including placement of unit and participant. The acronyms are defined as the statistical problem-solving process (SPSP), the Standards for Mathematical Practice (SMP), and the Mathematics Teaching Practices (MTP).



between the philosophy of the curriculum and Ms. Thomas' perspective of mathematics.

The results showed that at the time of the study, Ms. Thomas was making a minimal transition from traditional-based to perception-based perspective (see Figure 4). At the end of the study Ms. Thomas had not made a full transition into a perception-based perspective. Instead, similar to previous research (Cohen, 1990), Ms. Thomas seemed to embrace some aspects of reform philosophy while still maintaining some of her traditional instructional methods.

FIGURE 4.
Perspectives continuum including placement of unit and transition of participant.



Thus, in Figure 4 her name is situated almost between the first and second perspectives in the continuum. Had Ms. Thomas had more time to implement the unit or more opportunities to develop her MKT/SKT, however, a more pronounced shift in her mathematical perspective may have been observed. This study revealed that, in this case, if a teacher has a traditional-based mathematical perspective and perceived deficits in her MKT/SKT then this could affect their fidelity of implementation of reform-oriented statistics, similar to research in mathematics education.

Although this study was limited by the short length of implementation and the delimitations described previously, the results revealed teachers face similar challenges when teaching statistics at a conceptual level as they do when teaching mathematics. In the future, research in two areas could be explored. First, research needs to be conducted focusing specifically on teachers' statistical beliefs and attitudes in relation to implementation fidelity as these factors likely affect implementation fidelity. Other factors can also be explored as these suggested are not exhaustive. Second, the relationship between teachers' engagement with standards-based statistics curricula, their SKT, and students' statistics achievement needs to be explored in comparison to their implementation fidelity to determine if results are similar to that in mathematics education.

Implications

The results of this study revealed two implications for mathematics education leaders. First, Ms. Thomas' self-perceived issue with her MKT/SKT served as a potential challenge to her implementation. This result adds to the existing literature on teachers' issue with their MKT/SKT and how this affects teachers' instructional practices. Mathematics education leaders need to continue with efforts to improve teachers' MKT/SKT since research has shown that this leads to improved student achievement in the mathematics classroom. Although some of this can be done through adopting and using reform-based curricula, this study demonstrates that this is not enough to develop teachers' MKT/SKT fully, as was evident for Ms. Thomas. Mathematics education leaders cannot assume that simply adopting a reform-oriented curriculum will allow teachers to implement rigorous statistics lessons. To help teachers, leaders need to provide opportunities for teachers to engage in professional development for statistics. Recommendations for how these opportunities could be structured can be found in the SET (Franklin et al., 2015) document.

Second, Ms. Thomas changed the implementation of the curriculum based on her traditional mathematical perspective. This aligned with Remillard's (2005) interpretation descriptor and revealed that Ms. Thomas' beliefs did not align with the curriculum philosophy. Given that a reform-based philosophy undergirds both the *PSSM* (NCTM, 2000) and CCSSM (CCSSI, 2010), leaders need to consider professional development opportunities that focus on developing teachers' mathematical perspectives in a way that supports these standards (i.e., a more conception-based perspective).

Conclusion

The case of Ms. Thomas demonstrated that, when given an explicit curriculum that aligned with reform-oriented philosophy, a teacher's mathematical perspective about the teaching and learning of mathematics as well as her

MKT/SKT had the potential to affect the implementation fidelity of the curriculum. These results echo the statement by NCTM (2014) that having lessons, or by extension, a curriculum, does not guarantee "meaningful, effective, and connected lesson sequences" (p. 71). Moreover, this case aligns with what Stein et al. (2007) refer to as "substantial difference" (p. 321) between intended and enacted curricula. Overall, the findings support statements that further efforts need to be made to help teachers to teach statistics in a rigorous manner, meeting the expectations of standards-based documents. ✪

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APPENDIX A.

Date:		
Description of Classroom:		
Reform-Oriented Practice **Practices may be met in their entirety or in part. Either variation gets a YES. Not meeting any part gets a NO.**	**Was this practice met?**	**Justification**
Students were engaged in the statistical problem-solving process (Franklin et al., 2007). 1. Formulating a statistical question 2. Designing a plan for collecting useful data, implementing the data, and collecting the data. 3. Analyzing the data 4. Interpreting the results		
Students were engaged in the Standards for Mathematical Practice (CCSSI, 2010). 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		
The teacher practiced the following Mathematics Teaching Practices (NCTM, 2014). 1. Establish mathematics goals to focus learning. 2. Implement tasks that promote reasoning and problem solving. 3. Use and connect mathematical representations. 4. Facilitate meaningful mathematical discourse. 5. Pose purposeful questions. 6. Build procedural fluency from conceptual understanding. 7. Support productive struggle in learning mathematics. 8. Elicit and use evidence of student thinking.		

The teacher may not engage in all of these practices during one lesson. Make note of practices that teacher is engaged in and how this was justified during the lesson.

Leveraging Modeling with Mathematics-focused Instruction to Promote Other Standards for Mathematical Practice

Jonathan D. Bostic and Gabriel T. Matney, *Bowling Green State University*

Abstract

The Standards for Mathematical Practice (SMPs) describe mathematical proficiency in terms of behaviors and habits that every student should develop during mathematics instruction. Modeling with mathematics supports students in gaining facility with multiple representations and making sense of real-world phenomena. We investigated K-10 mathematics teaching for possible associations between mathematics teaching behaviors promoting modeling with mathematics (SMP4) and those identified by the other SMPs, using a classroom observation protocol called the Revised SMPs Look-for Protocol. Data consisted of lessons and videos of mathematics instruction from 70 K-10 mathematics teachers engaged in professional development focused on the SMPs. Results illuminated several associations between modeling with mathematics (SMP4) and other SMPs. A typical instructional case illustrates these associations and suggests potential for further opportunities. Teachers aiming to foster students' mathematical proficiency might consider instruction promoting SMP4 as a means to promote further connections to other mathematical behaviors and habits described in the SMPs.

Introduction

"School mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics" (National Council of Teachers of Mathematics, 2000, pp. 65-66).

Such opportunities likely involve modeling with mathematics, one of eight Standards for Mathematical Practice (SMPs) described in the Common Core State Standards (see Table 1; Common Core State Standards Initiative [CCSSI], 2010). This standard offers students the opportunity to connect real-life or lived experiences with mathematical problems presented in the classroom (Bostic, 2012/2013, 2015; Usiskin, 2015; Zawojewski, 2010). SMP4 also supports using multiple representations as a means to explain phenomena in everyday and mathematical terms (Bostic, 2015; CCSSI, 2010; Lesh & Zawojewski, 2007; Thomas & Bostic, 2015). In fact, SMP4 is the only standard that explicitly links classroom-based mathematics with the real world. It is uniquely positioned to foster connections among mathematical domains and traverse the division between classroom learning and everyday life. Thus, teachers should promote modeling with mathematics as a way to deepen students' mathematics knowledge (Bostic, 2015; Thomas & Bostic, 2015) and connect in- and out-of-class experiences through a mathematical lens (Matney, Jackson, & Bostic, 2013; Thomas & Bostic, 2015).

Table 1: Standards for Mathematical Practice

Standard for Mathematical Practice #	Title
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for regularity in repeated reasoning.

Note: Discussion about a specific SMP is denoted as SMP# within the manuscript.

There are instructional connections across the SMPs (Fennell, Kobett, & Wray, 2013; Kanold & Larson, 2012; Koestler, Felton, Bieda, & Otten, 2013). Some have argued that SMP1 and SMP6 are a connecting thread across the other SMPs (Fennell et al., 2013; Kanold & Larson, 2012). Engaging students in SMP1or SMP6 might be associated with fostering other SMP-like behaviors but there is no research-based evidence supporting that this will happen (Koestler et al., 2013). Conversely, SMP4 has unique features that may correlate with other SMP-like behaviors. That is, SMP4 might be a trigger mechanism that activates engagement in other SMPs because of its unique features, which include: a focus on a mathematical model; interactions between mathematical and nonmathematical (i.e., situational) referents; problems stemming from the real-world, particularly issues found in students' communities or the workplace; re-usability of mathematical models in other situations; and ideas communicated through oral and written language that make sense to the reader and problem solver (Bostic, 2015; Bostic, Matney, & Sondergeld, 2016; Floro & Bostic, in press; Thomas & Bostic, 2015). We conjectured that tasks promoting these features of SMP4 might associate with behaviors and habits found in the other seven SMPs. For instance, SMP4-focused instruction tends to foster opportunities for students to look for some underlying mathematical structure within a problem (Bleiler, Baxter, Stephens, & Barlow, 2015; Floro & Bostic, in press; Usiskin, 2015).

There is some evidence that promoting SMP4 supports other mathematical behaviors like those described in the SMPs (Floro & Bostic, in press; Thomas & Bostic, 2015); however, these studies and others often draw upon small samples (e.g., one or two teachers). There is little, if any, research-based evidence drawn from a larger sample of teachers' classroom practices exploring what SMP-related behaviors are also fostered when teachers promote SMP4 during classroom instruction.

The purpose of this study was to explore those correlational associations between SMP4 and other mathematical behaviors and habits described in the SMPs. We hypothesized that promoting SMP4 offers fruitful potential for encouraging other mathematical behaviors and habits described by the SMPs. Quantitative results and a classroom example are shared as an illustration to inform K-10 teachers' instructional practices as well as the decisions of mathematics teacher educators and professional developers. We used a mixed-methods approach to investigate possible connections between SMP4 and other SMPs and contextualize the correlations by giving instances from one teacher's classroom practice.

Context for Exploring K-10 Mathematics Instruction

Context and Participants

One hundred thirty-eight teachers located in a Midwest state volunteered to participate in one of two professional development (PD) projects. Projects met in separate locations due to geographic constraints. One project included K-5 mathematics teachers while the other was composed of grades 6-10 teachers. A shared goal of the yearlong PD projects was to foster teachers' understanding of the SMPs, particularly SMP4. An evaluation component within the projects included collecting and examining teachers' written lessons and instruction developed after experiencing the PD. Teachers were told to submit two lessons and video of them teaching one of those lessons. Instruction did not necessarily need to focus on promoting SMP4. In total, 70 grades K-10 teachers intended to promote students' mathematics proficiency through engagement in SMP4 during their videotaped lesson. Thus, these 70 teachers were a purposefully selected sample from a greater sample of PD participants. Our sample consisted of 29 grades K-3 teachers (early childhood), 35 grades 4-8 teachers (middle grades), and 6 grades 9-10 teachers. There were

16 male and 54 female teachers. On average, teachers had 13 years of teaching experience.

Instrument

Recent work by Fennell and colleagues (2013) led to a look-for protocol used by teachers, teacher educators, and mathematics supervisors. This protocol allows an observer to *look for* observable mathematics teaching behaviors that are related to the SMPs. This observation tool, used over 1,000 times in several districts, allows supervisors and teacher educators to create evidence-based records of teachers' instruction (Fennell et al., 2013). Our team revised the Fennell et al. (2013) protocol to create the *Revised SMPs Look-for Protocol* (Bostic et al., 2016). Those interested in a discussion of these revisions and the validation of the Protocol should reference Bostic et al. (2016). The revisions allow for a greater number of teacher-initiated moves to count as evidence related to an indicator. Appendix A shows the *Revised SMPs Look-for Protocol*, which includes descriptions for mathematics teaching behaviors related to the SMPs. For instance, one addition found in the revision was the phrase "and/or strategies" for indicator 1b. Content experts (i.e., mathematics teachers, supervisors, curriculum coordinators, mathematicians, and mathematics teacher educators) reviewed the *Revised SMPs Look-for Protocol* and expressed that it appropriately captured possible teacher moves indicative of promoting the SMPs.

Data Analysis

We analyzed our quantitative data in three phases. The first phase involved becoming familiar with the instruction. We read each lesson then watched the accompanying video in its entirety. The second phase was coding teachers' instruction seen in the videos using the *Revised SMPs Look-for Protocol*. A lesson received a score for each SMP based on the total number of indicators observed during the video. A score expressed the number of indicators per SMP. Thus, every lesson had eight values, one for each SMP. For example, a score of two for SMP4 meant that two indicators for SMP4 were observed on at least one occasion. Numerous instances of the same indicator for a SMP were coded the same as a single instance of an indicator for a SMP (i.e., 1). Teachers did not have the *Revised SMPs Look-for Protocol* prior to submitting their lessons and videos. Inter-rater agreement was high across coders (93%), which exceeded the minimum threshold (90%) needed to conduct quantitative analysis (James, Demaree, & Wolf, 1993). The third and final phase of our quantitative data analysis was conducting correlational analysis using these scores. Correlations such as Pearson's *r* are a measure of the strength of association between two variables (Shavelson, 1996). Statistically significant correlations for these data indicated that there was a genuine relationship between two SMPs, and there was a less than 5% likelihood that this correlation might happen by chance. All statistically significant correlations were interpreted using Cohen's (1998) guidelines: [0.01, 0.2] were considered weakly correlated, [0.21, 0.4] were moderately correlated, and [0.41,1] were strongly correlated.

We employed qualitative methods using inductive analysis (Hatch, 2002) to give meaning to the correlations. We selected Mrs. Gaston (pseudonym) from the sample of 70 teachers because her case was typical of the sample. Her case reified the numerous ways SMP4 and other SMPs appeared to be connected during classroom instruction. Inductive analysis allowed us to express salient connections that gave meaning to the quantitative results (Glaser & Strauss, 1967/2012; Hatch, 2002). Our approach to inductive analysis started with re-watching her video and reviewing her lesson. Step two was to make memos consisting of initial ideas stemming from the video and reflecting on observed indicators. Step three was to reflect on those memos and indicators as a way to synthesize them into key impressions. Step four was to search for evidence within her case to support our key impressions. Step five was to search the data for counter evidence within her case. Impressions with a paucity of counter evidence and a large set of evidence were retained. The sixth and final step was crafting clearly written impressions (themes) to share broadly that illuminate the connections between SMP4 and the other SMPs.

Results

In this section, we present descriptive statistics and correlations between teachers' promotion of SMP4 and other SMPs and then share a characterization of these correlations with descriptions from Mrs. Gaston's classroom instruction.

Descriptive Statistics

Means and standard deviations for teachers' promotion of the SMPs indicate that on average, teachers promoted numerous SMPs (see Table 2 on pg. 24). The most commonly seen SMPs were SMP3 and SMP5 whereas the least frequently observed SMPs were SMP7 and SMP8. These descriptive statistics demonstrate a picture of teachers'

Table 2: Descriptive Statistics for Teachers' Promotion of the SMPs during Instruction

	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8
Mean	1.21	1.05	1.44	1.93	1.46	0.8	0.47	0.56
SD	0.97	0.86	1.03	0.8	0.83	0.66	0.69	0.67

instruction that included numerous features of SMP-focused instruction.

Correlations

Results illuminated that teachers who promoted SMP4 tended to also foster other SMPs (see Table 3). First, there were several moderate correlations. SMP4 was moderately correlated with SMP1, SMP6, and SMP8. Moderate correlations suggested a greater positive relationship between SMP4 and other SMPs compared to weak correlations. Second, there was a weak correlation between SMP4 and SMP2 and SMP7. Weak correlations indicated some (i.e., more than none) relationship between SMP4 and another SMP. Finally, there was no statistically significant correlation between SMP4 and SMP5.

Table 3: Results from Correlational Analysis of K-10 Teachers' Instruction

Modeling SMP	All other Standards for Mathematical Practice							
	SMP1	SMP2	SMP3	SMP4	SMP5	SMP6	SMP7	SMP8
SMP4	.27*	.18*	.23*	.14	.22*	.19*	.29*	0.56

* $p < .05$

Note: SMP4 was correlated with six of the remaining seven SMPs. All correlations are Pearson *r* values.

In sum, there was a good chance that teachers in this sample who promoted SMP4 were associated with fostering mathematical behaviors and habits found in at least one and possibly up to six additional SMPs. A challenge of interpreting these correlations was expressing what this looks or sounds like in the activities of a mathematics classroom. To address that challenge, we used inductive analysis to draw out some instances that illustrated how one teacher enacted SMP4-focused instruction and encouraged promotion of other SMP-related indicators.

A Case of Classroom Instruction: Mrs. Gaston

Mrs. Gaston was a seventh-grade teacher who had taught for 18 years in a small Midwestern school district. She described her previous instruction as lecture based, focusing students on the memorization of known mathematical definitions, tricks, and processes to solve well-defined textbook and test-based problems. Mrs. Gaston's instruction seen on the video was fairly typical within the set of teachers' instruction we viewed. We purposefully selected her case (i.e., observed indicators) to share because her case was typical across the sample. Mrs. Gaston selected a ratio task that involved creating a drink mixture made from lemon concentrate and water. During the development of this lesson, Mrs. Gaston indicated that she wanted to encourage students' thinking about the situational and mathematical contexts within the topic of ratios. We describe her instruction, highlighting instances of when and how she promoted SMPs.

Nurturing classroom norms. Mrs. Gaston provided a learning space for her students to apply what they knew, comfortably make assumptions, identify important quantities, and map relationships using pictures, symbols, graphs, tables, and physical tools (SMP4) by nurturing classroom norms. Mrs. Gaston promoted students' involvement in SMP4 and used it as a lever to engage her students in SMP5. She reminded students of their usual classroom norms supporting SMP4.

> You are going to use pictures, props, tables, symbols, numbers, manipulatives and oh we're going to talk about it. You are not going to give up. If you find a way [to model and solve the task], guess what, I'm going to tell you, find another way.

Mrs. Gaston provided several tools students might use and she wanted them to strategically use whatever they needed to make sense of the problem. Thus, students considered the available tools, including representations, and selected them according to their own strategic competence (SMP5). In seeking to establish a learning environment conducive for SMP4, Mrs. Gaston focused students throughout the lesson on her desire for them to "prove it with a picture" and encouraged them to persevere in finding multiple strategies.

Mrs. Gaston drew on the SMP4-focused instruction as an opportunity to also activate students' engagement in SMP3. A classroom norm she fostered was respect for peers' ideas. She said to the class:

> You know as you participate you're going to listen to each other. You're going to give each other your attention. So when you're working in your groups don't ignore people. . . . You're going to listen by not speaking when someone else is giving their ideas. If you do not agree with someone in your group you're going to ask questions: What do you mean? What are you doing?

Mrs. Gaston reminded students to listen to one another and ask questions as a means to foster peers' model development.

Lesson launch. During the lesson launch, Mrs. Gaston offered her students an experience in a context similar to the day's task. She enacted the process of making lemonade from frozen concentrate. She asked students if they had made lemonade this way previously and most raised their hands and/or shouted, "Yeah!" Then, students proceeded to tell Mrs. Gaston how to make it. As instructed by her students, Mrs. Gaston opened three cans of frozen lemonade and dumped the contents into a large clear container. In the following dialogue, Mrs. Gaston capitalized on her instruction promoting SMP4 as a means to concomitantly engage her students in attending to precision (SMP6). She filled a separate large container with water then the following dialogue occurred.

> **S1:** Then you put the water in.
>
> **T:** Then I put the water in?
>
> S1: You need the [lemonade] container to pour it into the [large mixing] container. Because you need three of those little [lemonade] containers filled with water in it [the large mixing container].
>
> T: Ok, so you mean I can't just go like this? (Motioning to pour the whole pitcher of water in the container without measuring.)
>
> S2: Well you can.
>
> T: (pouring very little water into the container.) What if I stop now? [PAUSE]
>
> S2: You don't know what the ratio is.
>
> T: Cause what?
>
> S2: You don't know what the ratio is.
>
> T: Oh, I don't know what the ratio [emphasis added] is? The ratio of what to what?
>
> S3: The ratio of the lemonade in the container to the water in the container.
>
> T: So I need to know the ratio? Cause what if I stop right now? What would this taste like?

In this instance, Mrs. Gaston emphasized the word ratio as a means to highlight the importance of mathematical vocabulary connected to the situational context of the problem (SMP6) during this dialogue. Mrs. Gaston planned for this in her lesson; she drew attention to students' academic language use and intended to foster their precision with mathematical vocabulary, in this case with the word *ratio*. First, Mrs. Gaston started by asking the student to say it again. Then, she changed the tone of her voice to emphasize the word ratio. Later, the student mentioned ratio but did not go further to explain what two things were distinguished in the ratio relationship. Mrs. Gaston asked a question to help the student more precisely use the mathematical language of ratio within the situational context.

The class proceeded to comment on how too little water makes the lemonade strong and too much water makes it weak. They indicated that the ratio is important to make it just right. Thus, students were engaged in discussing a situational model of ratios before proceeding to mathematically model this ratio in the focal task. Later, the class identified contexts similar to this one such as making cookie dough, scrambled eggs, pumpkin pie, and cinnamon rolls. Mrs. Gaston shared a story with the class about a family member who made a pumpkin pie that tasted unusual because the individual who made the pie confused the ratios for sugar and salt hence the pie was salty rather than sweet. Then, students expressed other similar scenarios where a recipe was not followed correctly and the importance of understanding how different ratios apply to different contexts; hence, they perceived the importance of transferring ideas from this situational model to other contexts.

The lesson included numerous opportunities to engage students in SMP4 as well as an opportunity for students' engagement in SMP1. When Mrs. Gaston focused on SMP4 as she did in this lesson, she intentionally planned a lesson drawing on a relatable context that in turn allowed for more entry points into the task. This intentional focus on SMP4 brought about students' engagement in SMP1. During the lesson launch, the students reflected on their past experiences with analogous contexts and connected the launch to their own lives. This launch provided a foundation from which students could make sense of the focal task and persevere in constructing and sharing viable mathematical models to explain their thinking.

Focal task. The focal task stemmed from adapting a Connected Mathematics Project 2 task (Figure 1; Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006).

Mrs. Gaston's lesson plan indicated that she had a two-fold instructional goal for students: to answer the problem and to develop a mathematical model for judging the strength of other mixtures in different contexts. Students were given approximately 40 minutes to reflect on the problem, discuss it with a partner, and construct a brief presentation about the models for solving the problem.

Mrs. Gaston reconvened the class for presentations, which included discussing various models that demonstrated which solution had the strongest concentration of lemon flavoring. Fostering students' engagement in SMP4 during this instructional moment supported students' mathematics learning through SMP2-related behaviors. That is, behaviors indicative of SMP2 included a focus on decontextualizing and contextualizing from a mathematical problem. One group shared a bar model approach to determine the

FIGURE 1.

This was the focal task for Mrs. Gaston's instruction intending to promote modeling with mathematics.

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times. One morning, Julia and Mariah make lemonade for all the campers. They plan to make the juice by mixing water and frozen lemon juice concentrate. To find that mix that tastes best, they decide to test some mixes.

Mix A

2 cups concentrate | 3 cups cold water

Mix B

5 cups concentrate | 9 cups cold water

Mix C

1 cup concentrate | 2 cups cold water

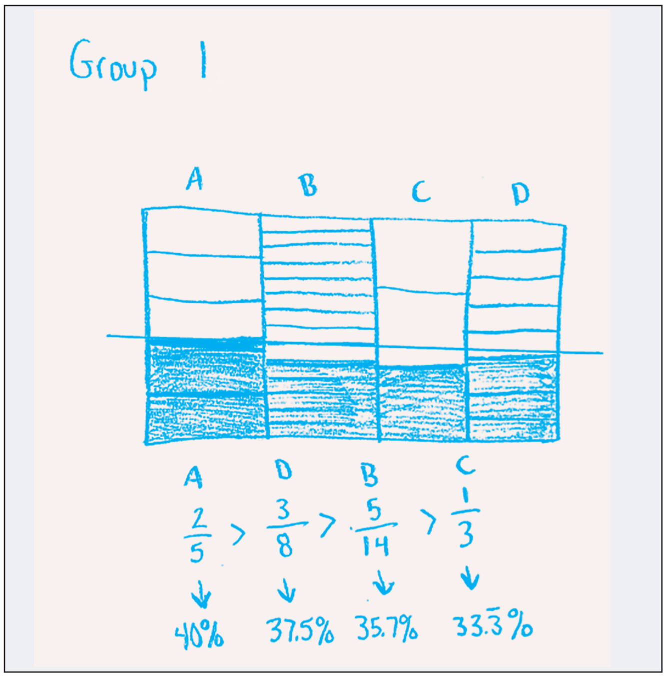
Mix D

3 cups concentrate | 5 cups cold water

Which mix did they decide tastes the most lemony?

percentage of lemon concentration that was found in the total amount of the solution (see Figure 2). Another group used a similar model (see Figure 3), and found percentages to compare how much lemon flavoring there was to how much water using circles. However, they modeled different ratios from the first group. A third model from another group involved comparing solutions using common denominators (see Figure 4).

FIGURE 2.
One group's approach for solving the focal task used a bar model representation. The horizontal line showed that mixture A was the "most lemony." The shaded region represented the concentrate and the unshaded region represented the water in the mixture. The percentages represented the ratio of concentrate:total mixture contents.



Throughout the whole-class discussion and presentations, students asked peers to explain what they meant, as seen in one interaction during the discussion of the first model (see Figure 2). We share an excerpt from the discussion, which in this case is between a peer (S4) who asked questions to one of the presenters (S5).

S4: What were the percents? And like what were the percents showing?

S5: Um, which one is greater. Because forty would be greater than thirty-seven point five, thirty-three point three three three and thirty-five percent so it's greater than all of them.

FIGURE 3.
A second group's approach used circular models to solve the focal task. The shaded region represented the amount of concentrate in the mixture. The unshaded region represented the amount of water in the mixture. The percentages represented the ratio of concentrate:water.

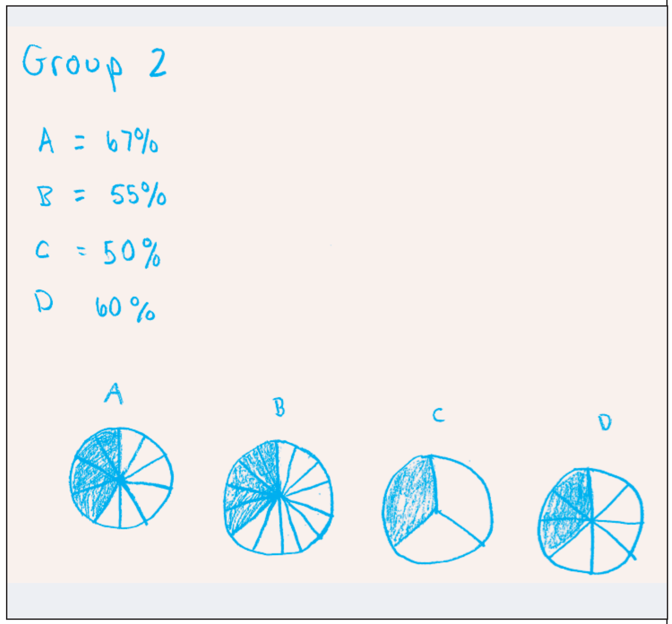
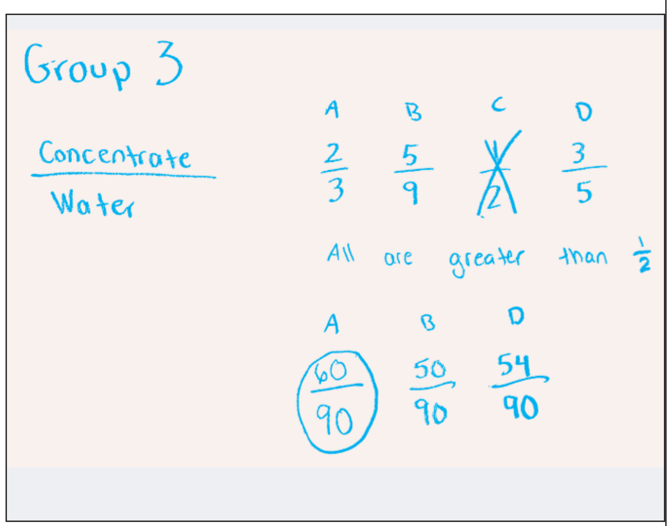


FIGURE 4.
A third group's model used common denominators for solving the focal task. Mixture C was eliminated because students immediately recognized all other fractions were bigger than ½. Fractions with common denominators represented the ratio of concentrate:water.



S4: Wouldn't you have to, um, measure something about the amount of water because the water could be like higher in all the others and could even out?

S5: Well out of one hundred percent, forty percent is greater, like one glass is a hundred percent, so this would be greater than thirty-five percent because forty percent is greater out of a hundred percent. So percent wise, it's higher.

Students sought to make sense of one another's models and justify the mixture that had the strongest lemon flavor. This prompted opportunities to discuss and use representations to make sense of quantities and their relationships as well as opportunities to decontextualize and contextualize (SMP2). The above dialogue also provides an example of a student (S4) questioning another student (S5) for the purpose of drawing out a more precise meaning of the language being used. Mrs. Gaston and her students often asked one another to tighten their language within justification statements and to precisely communicate the connection between the referents in pictures, quantities, and symbolic expressions such as the percent sign and inequalities (SMP6). For example, during Group 1's presentation of their model (see Figure 2) the following dialogue occurred.

T: Ok, I have, I have a question, Why did you, I'm looking at your picture.

S6: Yeah.

T: And then so, A) Why did you divide all of your pictures, in like differently, like A spaces are larger than B spaces in between those little lines?

S6: Because these are each, I probably should have um, made them, all have a common denominator but, they each are, they're each different fractions.

T: Ok. So then you, ok, then I like the way you lined it up like that. So, you could have had a common denominator then?

S6: Yeah, but they probably should all go together so.

In this case, Mrs. Gaston asked the presenting student (S6) to explain the dividing lines in the group's representation (see Figure 2). The question prompted the student to consider something new (common denominators) that could have made the relationship among the ratios and representation more clear. Through these kinds of classroom

Table 4: Mrs. Gaston's Case: Connecting SMP4 Focused Instruction to other SMPs

SMP 4 Teacher Indicators	Mrs. Gaston's Teaching Behaviors	SMP Look-for Protocol Indicators
A. Use mathematical models appropriate for the focus of the lesson	In launching the lesson, the teacher used a sensible real-world context that was familiar to the students and directly connected to the day's mathematical task, which could be approached through multiple strategies.	SMP 1A & SMP 1B
B. Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids)	To promote model development, the teacher nurtured norms promoting the use of manipulatives. During individual and whole class discussion, the teacher encouraged precise mathematical language as students articulated why their models were appropriate.	SMP 5A & SMP 5C SMP 6A & SMP 6C
C. Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed	To promote model development, the teacher nurtured the norm of respecting others' ideas by considering and listening to one another. She reminded students to ask questions as a means to foster peers' model development. During individual and whole class discussion, the teacher encouraged precise mathematical language in student articulation of ideas.	SMP 3A & SMP 3C SMP 6A & SMP 6C
D. Employ problems arising from everyday life, the local community, and workplace such that the solution is a model to reuse.	The teacher launched the lesson with a context connected to students' everyday lives. The planned task for students to consider involved everyday life situations in which the students must develop a mathematical model as part of the solution. The teacher leveraged these everyday life contexts and a solution model to promote students contextualizing and decontextualizing.	SMP 2A, SMP 2C1, & SMP 2C2

interactions, Mrs. Gaston and the students made moves to improve their understanding of one another's mathematical ideas and the precision of the use of mathematical language, representation, and referents (SMP6).

Mrs. Gaston aimed to promote SMP4-focused instruction, which also happened to offer opportunities for her students' to engage in SMPs 1, 2, 3, 4, 5, and 6. The case of Mrs. Gaston shows what is possible when a teacher works to promote SMP4 and gives a qualitative picture of how the correlations from the quantitative Look-for Protocol occurred in the teachers' classroom practice. Table 4 consolidates the case of Mrs. Gaston to show the connections between the codes for SMP4 and the other SMPs found during her instruction.

Implications for Mathematics Teaching

When considering this instance and several others as a teacher's first foray into instruction promoting SMP4, there are quite a few wonderful developments within the instructional scenario. One example is students developing and defending their models and justifications. A second example is the observable evidence of student engagement in multiple SMPs during the lesson. We are encouraged by these results, both the quantitative and the qualitative. As evidenced by the correlations in this study, teachers who focused their instruction on fostering SMP4 also demonstrated that their instruction facilitated opportunities for students to engage in multiple SMPs.

Although some have argued that SMP1 and SMP6 connect with the other SMPs (Fennell et al., 2013, Koestler et al., 2013), there is no guarantee that promoting SMP1 or SMP6 will always foster SMP4 much less other SMPs. However, we can conclude from our analysis that instruction by teachers in this sample who intended to promote SMP4 had a reasonable chance of also encouraging SMP1, and a slightly lesser chance for encouraging SMP6. Relatedly, K-10 teachers' instruction promoting SMP4 connected with reasoning abstractly and quantitatively (SMP2), constructing viable arguments and critiquing others' reasoning (SMP3), and looking for and expressing regularity in repeated reasoning (SMP8). Though the correlation between SMP4 and SMP5 was not statistically significant across our sample, Mrs. Gaston's instruction showed that SMP5 was not wholly absent from SMP4-focused instruction. We conclude that when teachers in our sample focused on promoting SMP4, it provided natural opportunities to foster engagement in other SMPs during mathematics instruction. These SMP connections may allow students to make sense of mathematics at a deeper level by building conceptual understanding and effectively linking mathematics learned in school with real-life experiences (Bostic, 2012/2013, 2015; Matney et al., 2013; Thomas & Bostic, 2015). In sum, our conclusion is that instruction promoting SMP4 has the propensity to support engagement in other mathematical behaviors and habits described in the SMPs. SMP4-focused instruction offers opportunities for students to engage in mathematics within tasks drawn from relevant contexts and connect ideas among various situational contexts.

Implications for Mathematics Teacher Educators and PD Providers

The ideas in this manuscript stemmed from working intensely alongside teachers to help them grow in their understanding of mathematical behaviors and habits described in the SMPs, which assisted their ability to design and enact instruction promoting the behaviors and habits. Many mathematics teacher educators are enacting PD for mathematics teachers around the SMPs with an aim to understand them and make them a part of regular instruction. An implication of our research is that fostering mathematics teacher's understanding of SMP4 and concomitantly their abilities to design SMP4-focused instruction may be fruitful for promoting other SMPs. It may be a good idea for mathematics teacher educators and PD providers to initiate mathematics teachers' thinking about the SMPs by starting with developing a deep understanding of SMP4 then following up with the other SMPs.

This manuscript also provides mathematics teacher educators and PD providers with a real-life scenario of how one teacher promoted SMP4 as well as several other SMPs. Mrs. Gaston's lesson might ignite and foster discussions about how SMP4-focused instruction leveraged other SMPs to also appear during the same lesson. Discussions, along with unpacking the correlational results, may spur thinking about possible connections between SMP4-focused instruction and other SMPs. As a reminder, these teachers' promotion of other SMPs is correlated with, not predicted or caused by, SMP4.

Further Questions: SMP4 and Predictive Validity

We aimed to illustrate correlations between SMP4-focused instruction and other SMPs in this mixed-methods study. The focus of this study was exploring correlational relationships; however, we cannot provide evidence about causal or predictive relationships. Correlations suggest the likelihood of two outcomes occurring and are often conducted before causal or predictive studies (Shavelson, 1996). Causal and predictive studies use ANOVA or regression as a means to explore whether one outcome is caused or predicted by the occurrence of another outcome (Shavelson, 1996). An experimental design could illuminate such potential causal relationships between SMP4 and other SMPs. One such design might include 30 teachers enacting the same lesson, which includes a strong SMP4 focus, to their students in their typical learning environments. The independent variable in this case might be presence of teachers' promotion of SMP4-like behaviors and dependent variable might be presence of teachers' promotion of other SMP-like behaviors. Analyses of teachers' promotion of SMP-like behaviors, using a logistic regression might illuminate any causal relationships. At this time, we cannot make any predictive statements suggesting that promoting SMP4 causes other SMPs to be promoted. We hope future research might take up this call for a causal or predictive study employing a methodology like this one described here or otherwise. Results from the present study allow us to conclude that teachers' promotion of SMP4-related behaviors is related (i.e., occurring within the lesson) to teachers' promotion of several other SMP-related behaviors. ✪

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APPENDIX A.

Revised SMPs Look-for Protocol

Place a mark in the box next to the appropriate indicator when observed.

Mathematical Practices	Teachers
1. Make sense of problems and persevere in solving them	☐ A. Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution. ☐ B. Provide opportunities for students to solve problems that have multiple solutions and/or strategies. ☐ C. Encourage students to represent their thinking while problem solving. *NOTE: Task must be a grade-level/developmentally appropriate problem. That is, a solution is not readily apparent, the solution pathway is not obvious, and more than one pathway is possible.* Comments:
2. Reason abstractly and quantitatively	☐ A. Facilitate opportunities for students to discuss representations or use representations to make sense of quantities and their relationships. ☐ B. Encourage the flexible use of properties of operations, tools, and solution strategies when solving problems. ☐ C1. Provide opportunities for students to decontextualize (abstract a situation) the mathematics within a mathematics task. ☐ C2. Provide opportunities for students to contextualize (identify referents for symbols involved) the mathematics within a mathematics task. *NOTE: Must have C1 and C2 to receive credit for indicator.* Comments:
3. Construct viable arguments and critique the reasoning of others	☐ A. Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative strategies or solution(s), and defend their ideas. ☐ B. Ask higher-order questions which encourage students to defend their ideas, consider student(s) response(s) before making code. ☐ C. Provide prompts/tasks that encourage students to think critically about the mathematics they are learning, must be related to argumentation or proving events. ☐ D. Engage students in proving events that encourage students to develop and refine mathematical arguments (including conjectures) or proofs. Comments:
4. Model with mathematics	☐ A. Use mathematical models appropriate for the focus of the lesson. ☐ B. Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids). ☐ C. Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed. ☐ D. Employ problems arising from everyday life, the local community, society, and workplace such that the solution is a model to reuse. *NOTE: Must have D to be considered a task embedded within instruction promoting modeling with mathematics.* Comments:

	Mathematical Practices	Teachers
	5. Use appropriate tools strategically	☐ A. Use appropriate physical and/or digital tools to represent, explore, and deepen student understanding. ☐ B. Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson. ☐ C. Provide access to materials, models, tools, and/or technology-based resources that assist students in making conjectures necessary for solving problems. *(Students must use the resources.)* *NOTE: Representations do NOT count as tools.* Comments:
	6. Attend to precision	☐ A. Emphasize the importance of precise communication by encouraging students to focus on clarity of the definitions, notation, and/or vocabulary used to convey their reasoning. ☐ B. Encourage accuracy and efficiency in computation and problem-based solutions, expressing numerical answers, data and/or measurements with a degree of precision appropriate for the context of the problem. ☐ C. Foster explanations and justifications using clearly articulated oral and/or written communication and grade-level appropriate conventions. Explanation or justification must go beyond Initiate-Respond-Evaluate (IRE.) Comments:
	7. Look for and make use of structure	☐ A. Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains. ☐ B. Recognize that the quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson. ☐ C. Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways e.g., 76 = (7 x 10) + 6; discussing types of quadrilaterals, etc. ☐ D. Encouraging examinations of a 'signal' and 'noise' in statistics-related tasks. Comments:
	8. Look for express regularity in repeated reasoning	☐ A. Engage students in discussion related to repeated reasoning that may occur while executing a problem-solving strategy or in a problem's solution. ☐ B. Draw attention to the prerequisite steps necessary to consider when solving a problem. ☐ C. Urge students to continually evaluate the reasonableness of their results during problem solving. Comments:

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