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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all —levels. Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
- **Professional development efforts** including how these efforts are situated in the larger context of professional development and implications for leadership practice
- **Commentaries on critical issues** in mathematics education
- **Brief reviews of books** that would be of interest to mathematics education leaders

Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We seek your reactions, questions, and connections to your work. Selected letters will be published in the journal with your permission.

Submission/Review Procedures

Submittal of manuscripts should be done electronically to the ***Journal*** editor, currently Angela Barlow, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

***Note:** Information for manuscript reviewers can be found at the back of this publication.

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS (NCSM)

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Purpose Statement

The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Angela T. Barlow, *Middle Tennessee State University*
Travis A. Olson, *University of Nevada, Las Vegas*

> *The importance of doing the right work is even more critical today as many of us have witnessed the challenges and opportunities of adopting and implementing the Common Core Standards for Mathematics (or other new state standards) and new assessments (i.e., PARCC, SBAC, …) during the past few years.* (Staley, 2017, p. 2)

In our recent newsletter, John Staley – President of the National Council of Supervisors of Mathematics – challenged us as mathematics education leaders to consider, among other questions, "What key actions and conversations are needed to make mathematics meaningful, relevant, and accessible for each and every student?" To this end, the two articles featured in this issue aim to support those actions and conversations.

With regard to actions, mathematics education leaders continue to aid teachers in re-envisioning mathematics instruction so as to better meet the learning needs of students. This re-envisioning process led Strayer and colleagues to use demonstration lessons in their professional development project for K-8 mathematics teachers. Based on the participants' feedback, however, the project team introduced two new models of demonstration lessons. In their article, "Meeting the Needs Expressed by Teachers: Adaptations of the Traditional Model for Demonstration Lessons," Strayer et al. not only describe their new models but also demonstrate how the models emerged from within their project context. In this way, their intent is to support the actions of other mathematics education leaders as they work to better meet the needs of their teachers.

In addition to actions, Staley (2017) indicated a need for conversations. At the heart of these conversations, he suggested a focus on the learning opportunities that instruction and assessment afford. Recognizing that different instructional models afford different opportunities, Wiley examines one model of the flipped classroom in her article, "Structure vs. Pedagogy: The Impact of a Flipped Classroom Model of Instruction on Fifth-Grade Mathematics Students." Through her analyses of classroom observations, student assessments, and interviews, Wiley paints a vivid picture of the flipped classroom as enacted in a group of fifth-grade classrooms as well as its influence on student achievement. In addition, she shares low- and high-achieving students' perceptions of their use of the out-of-class instructional videos. In doing so, Wiley provides implications for mathematics educations leaders that should inform conversations related to this increasingly popular model of instruction.

Mathematics education leaders play a critical role in identifying key actions and conversations that are needed within their individual contexts. We hope the ideas contained in this issue will serve to inform and enhance this process. ✪

References

Staley, J. W. (2017). Surviving leadership – Staying focused to finish the course. *NCSM Newsletter, 47*(3), 2.

Structure vs. Pedagogy: The Impact of a Flipped Classroom Model of Instruction on Fifth-Grade Mathematics Students

Bethann M. Wiley, *Winona State University*

Abstract

The Flipped Classroom model of instruction is being implemented at all levels of schooling and academic areas; yet, there is very little research regarding its effectiveness. This study attempted to expand this body of research by looking at the Flipped Classroom model as it was implemented in fifth-grade mathematics classrooms. As enacted in this study, the model involved students watching a video lecture at home and then completing traditional homework in class the next day. The participants were 112 fifth-grade students from four classrooms in a Midwestern suburban school district. Qualitative and quantitative data were collected through classroom observations, interviews, and posttests. The Mathematics Teaching Practices were used as a framework to analyze the classroom instruction. Further, research on students' conceptual understanding of decimals and fractions formed the basis for understanding student thinking during interviews. The data suggested that the Flipped Classroom model, as enacted in this study, strongly supported the use of rules and procedures, not always accurately, to the detriment of developing conceptual understanding. Of equal concern was that low-achieving students had less access to the videos at home and more frequently found them frustrating or confusing. Implications for mathematics education leaders are provided.

Introduction

"All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence" (National Council of Teachers of Mathematics [NCTM], 2000, p. 5). This statement in conjunction with the prevailing achievement gap in mathematics (Boykin & Noguera, 2011) has given rise to innovations and research on teaching and learning mathematics in an effort to truly provide "high quality mathematics instruction for all students" (NCTM, 2000, p. 3). One of the many innovations that has become increasingly popular, the Flipped Classroom model of instruction, has now made its way from predominantly post-secondary classrooms into middle school and elementary classrooms (Bishop & Verleger, 2013; Hamden, McKnight, McKnight, & Arfstrom, 2013; Yarbo, Arfstrom, McKnight, & McKnight, 2014). This gives rise to two important questions. First, how does this model of instruction impact elementary-age students and their conceptual understanding and achievement in mathematics? Which then leads one to ask: based on these findings, how do we as mathematics education leaders continue to support high quality mathematics instruction when this model is implemented in elementary and middle school mathematics classrooms in order to promote student conceptual understanding and increased levels of achievement for all students?

The first question can be partially answered by this research study, which sought to examine the Flipped Classroom model of instruction as it was enacted in four fifth-grade mathematics classrooms. Previous research in the area of the Flipped Classroom model has been done at the secondary and post-secondary levels and typically in the areas of science and mathematics with the instructor serving the dual role as the researcher. Achievement on final exams and measurements of attitude based on course reviews have served as the major pieces of evaluation data in most of these studies (Bishop & Verleger, 2013; Hamden et al., 2013; Yarbo et al., 2014). Based on the current body of research, however, there is a significant need for research on the Flipped Classroom model at the elementary level specifically in mathematics with attention to teaching practices and the learning outcomes. Therefore, the purpose of this study was to examine how the Flipped Classroom model of instruction impacted fifth-grade students' achievement in mathematics with a particular focus on conceptual understanding versus procedural understanding. This study also examined teacher practices within the Flipped Classroom model enacted in the classrooms in this study and their alignment or misalignment to the Mathematics Teaching Practices (NCTM, 2014). Specifically, this study addressed the following questions intended to examine both the use of effective teaching practices and student achievement.

1. To what extent does the observed model of Flipped Classroom instruction align with the Mathematics Teaching Practices for high quality mathematics instruction in four fifth-grade classrooms? The specific practices addressed were:
   a. Implement tasks that promote reasoning and problem solving;
   b. Use and connect mathematical representations;
   c. Facilitate meaningful mathematical discourse;
   d. Build procedural fluency from conceptual understanding; and
   e. Elicit and use evidence of student thinking.

2. How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction in this study?
   a. Do the students meet the State Standards for decimal and fraction concepts as measured by the curriculum post-unit tests?
   b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?
   c. To what extent are there differences between high-achieving and low-achieving students' conceptual understanding and achievement in the Flipped Classroom model?

The significance of this study was its ability to inform mathematics education leaders with regard to areas to be addressed in professional development related to the use of class time in a flipped classroom model and issues of equity when enacting a flipped classroom model.

The Flipped Classroom Model

The definition of *Flipped Learning* or the *Flipped Classroom* used in this study was developed by members of the Flipped Learning Network (FLN, 2014) and stated on their website. It defines Flipped Learning as:

> A pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter. (FLN, 2014, "Definition of Flipped Learning")

In line with this definition, the students often watch a video lecture at home for homework and then work on problems or activities related to the video in class the next day. This can be enacted in a variety of ways with the most traditional model being that the students complete the typical pencil-and-paper homework in class (FLN, 2014; Hamden et al., 2013; Strayer, 2012). The teacher is then present to assist students with these practice problems.

This traditional model of the Flipped Classroom was the model observed in the classrooms in this study. The students watched a video each night made by district teachers and based on a lesson in the curriculum. On the next day in class, students worked on the corresponding lesson pages in a workbook. The idea of using the video instruction as homework made class time available to offer high quality, interactive, mathematical experiences to all students. Further, this model allowed the teacher opportunities to engage and interact with students and mathematics in significant ways that a traditional lecture model would not.

Conceptual Framework

Mathematics Teaching Practices

The eight Mathematics Teaching Practices, detailed in *Principles to Actions: Ensuring Mathematical Success for All*

(NCTM, 2014) "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9). These eight Mathematics Teaching Practices are:

1. Establish mathematics goals to focus learning;
2. Implement tasks that promote reasoning and problem solving;
3. Use and connect mathematical representations;
4. Facilitate meaningful mathematical discourse;
5. Pose purposeful questions;
6. Build procedural fluency from conceptual understanding;
7. Support productive struggle in learning mathematics; and
8. Elicit and use evidence of student thinking. (p. 10)

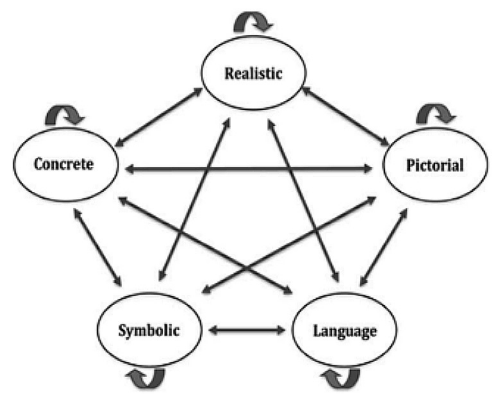
Teacher and student actions are outlined in this document to guide the development of these high-leverage practices and support the development of conceptual understanding of mathematics that students need to acquire. It was these practices, in conjunction with research on conceptual understanding specifically in the areas of fractions and decimals, which created the foundation for the conceptual framework of this study.

Conceptual Understanding

The idea of meaningful mathematics is generally connected to the work of Brownell (1935) who wrote extensively on the importance of teaching for understanding or meaning. Although a balance of meaning and skill is needed to be successful in mathematics (Brownell, 1956), what it means to truly understand needs to be defined. Skemp (1976) defined two types of understanding: relational understanding and instrumental understanding. Relational understanding involves knowing the why behind what one is doing whereas instrumental understanding involves knowing the rules. Relational understanding has been emphasized in curriculum documents (e.g., Common Core State Standards Initiative, 2010; NCTM, 2000) so that procedural or instrumental understanding is developed with accuracy and purpose.

Understanding relationships in mathematics comes from creating and internalizing mental models and making connections among these mental representations (Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007). "Understanding occurs as representations get connected into increasingly structured and cohesive networks" (Hiebert & Carpenter, 1992, p. 69). These mental models or representations are created over time and through experiences. The Lesh Translation Model (Lesh, Post, & Behr, 1987) demonstrates the types of representations and translations that students must experience in order to support the development of conceptual understanding (see Figure 1). For example, when learning about the relative size of fractions, students can use fraction circles or fold paper strips to see them concretely. From there, students may draw pictures, describe them to their classmates, and finally record various equivalent fractions symbolically.

FIGURE 1.
Lesh Translation Model
Adapted from "Representations and Translations Among Representations in Mathematics Learning and Problem Solving," by R. Lesh, T. Post and M. Behr, 1987, In C. Janvier (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics,* pp. 33-40. ©1987 by Routledge. Reprinted with permission.



Research explaining what it means to conceptually understand decimals and fractions, the mathematical focus of this study, includes the use of mental models and translations between models (e.g., Behr, Wachsmuth, Post, & Lesh, 1984; Cramer, Behr, Post, & Lesh, 1997; Cramer, Monson, Wyberg, Leavitt, & Whitney, 2009; Cramer, Post, & delMas, 2002; Hiebert & Wearne, 1986; Hiebert, Wearne, & Tabor, 1991; Roche & Clark, 2004). Researchers are concerned that in order to understand the relative size of fractions and decimals, as well as to compare, order, and compute accurately with fractions and decimals, students need to have many experiences with a variety of representations (Cramer et al., 2009; Hiebert & Wearne, 1986; Hiebert et al., 1991; Roche & Clark, 2004). In addition, connections among these representations are needed in order to develop a deep understanding of fractions and decimals. This research has also suggested that students struggle with interpreting symbolic representations of fractions and decimals. Much of students' difficulties result from their tendencies to employ whole number thinking to a variety of situations, which leads to inaccurate interpretations when comparing, ordering, and estimating with fractions and decimals (Cramer et al., 2009; Hiebert & Wearne, 1986; Hiebert et al., 1991; Roche & Clark, 2004).

Methods

This study was designed to examine what teaching practices existed in four elementary classrooms using a Flipped Classroom instructional model and how these practices affected the conceptual understanding and achievement in mathematics of the students in these classrooms. This study took place during two fifth-grade Math Expressions (Fuson, 2011) curriculum units of instruction (decimals and fractions), which occurred over eight weeks of time in four classrooms (117 students). All four classroom teachers used the Flipped Classroom model. The classroom teachers taught all of the lessons and administered all assessments that included the curriculum posttests. The posttests covered the mathematics content for each unit and were developed by the curriculum authors. This study was unique to the current body of research on the Flipped Classroom in that the majority of the other published work places the researcher in the role of the teacher. In contrast, the researcher in this study was an outside observer.

The context for this study was a suburban school district outside of a large Midwestern metropolitan area. At the time of the study, the district was in its fourth year of Flipped Classroom mathematics instruction at the fourth- and fifth-grade levels. The two schools featured in this study had relatively different demographics from each other although they were in the same district. Because the Flipped Classroom model was used throughout the district (10 elementary schools), the varying demographics of the selected schools allowed for a broader understanding of the impact this model of instruction had on students. Three classrooms were studied at Southside Elementary (pseudonym) because each fifth-grade teacher taught mathematics to his/her own students. One classroom was studied at Central Elementary (pseudonym) because this teacher taught mathematics to all of the students in this grade level. These four classrooms were also chosen based on the teachers' experiences with the Flipped Classroom model and their willingness to participate in the study. The demographics of the students in the study from the two schools as well as the district are shown in Table 1.

In order to document the actions of the teachers and students, classroom observations recorded as field notes were completed during 32 class periods. These observations were guided by the Teacher Actions and Student Actions identified for five of the eight Mathematics Teaching Practices in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). These five practices (see Research Question 1) were chosen because they were observable during the class periods. In considering the remaining three practices, two were considered difficult to observe. Although the third practice of *Pose Purposeful Questions* was observable, the researcher felt this practice could generate enough data to be a study on its own and would, therefore, distract from the purpose of this study. Therefore, this practice was excluded. In connection to

Table 1: Demographics of School Enrollment

Subgroup	School District N = 8,800	Southside Elementary N = 88	Central Elementary N = 29
Amer. Indian/Alaskan	.7%	0.8%	0.5%
Asian/Pacific Islander	5.2%	9.5%	7.2%
Hispanic	3.4%	9.5%	1%
Black, not of Hispanic origin	4.4%	13.8%	1.5%
White, not of Hispanic origin	86.2%	66.3%	89.7%
ELL	2.0%	13%	1.7%
Special Ed.	13.9%	13.6%	14.3%
Free/Reduced Lunch	16.5%	30.3%	5.2%

observing teacher and student actions through this lens, the elements needed for conceptual understanding, such as multiple representations of fractions and decimals, were noted whenever possible.

The field notes were coded first by activity and then by teaching practices and student actions observed. These actions were then matched, where possible, to the selected Mathematics Teaching Practices (NCTM, 2014). Themes developed that illustrated the routines and practices that were typical in the classrooms. These themes emerged as routine practices in each classroom and then across the four classrooms.

Students completed posttests after the conclusion of each unit. Additionally, 20 students participated in student interviews at the end of each unit (40 students total). These interviews included specific questions about both the students' experiences in the Flipped Classroom and their conceptual understandings of the content from each unit. The students were selected by their teacher and were identified as either high achieving or low achieving based on their test scores and the teacher's knowledge of the student. This was done purposefully to identify possible differences in thinking patterns between the two groups of students as well as how the Flipped Classroom model may or may not have impacted students differently. Questions related to the Flipped Classroom experience included students' opinions of the videos, how often they watched the videos, and their access to adequate internet devices at home. Specific questions involving decimals and fractions were asked so that students could demonstrate their conceptual understandings of comparing, ordering, estimating, and computing with decimals and fractions by explaining their thinking and their use of procedures.

This pragmatic approach of combining both qualitative and quantitative data to answer the research questions allowed for rich descriptions to be developed of what was taking place in the Flipped Classrooms in this study. This approach "attempts to provide evidence that meets the epistemological standard of what John Dewey called warranted assertability" (Johnson & Christensen, 2012, p. 432); that is, what can be a justified belief versus an opinion. The data generated from the themes found in the classroom observations and student interviews was put in concert with quantifiable data such as the frequency of various types of classroom activities and unit test scores to establish a more complete picture of these classrooms using the Flipped Classroom model of instruction and the resulting impact on students' mathematical understandings and achievement.

The results of the study follow in the next sections along with conclusions and recommendations. These conclusions and recommendations are based on the data collected in this study and supported by the research behind its conceptual framework and the research on conceptual understanding of decimals and fractions. However, this study has several limitations. First, the literature on the Flipped Classroom suggests that there are many ways that the model can be enacted. This study only observed one such model; therefore, other versions of the Flipped Classroom may offer different outcomes or results. Second, every student in each classroom was not interviewed so there may be perspectives from the average student not represented in these findings. Finally, the duration of the study was limited to approximately eight weeks of instruction and not every lesson in every classroom during those eight weeks was observed. It would be possible that over a longer period of time, different observations could lead to additional supportive or conflicting findings.

Findings Related to Classroom Activities and Teaching Practices

The classroom observations were conducted over the span of approximately eight weeks during two units of study: Unit 3 – Decimals and Unit 5 – Fractions. The classroom teachers used the district adopted Math Expressions (Fuson, 2011) curriculum for the majority of the students and an alternative sixth-grade textbook for those who passed the unit pretest with a score of 90% or better. In this section, results of the analyses from classroom observations are presented, followed by the alignment of these instructional practices with the selected Mathematics Teaching Practices.

Classroom Observations

Two variations of instructional models were observed (see Table 2). Most lessons began with warm-up problems and then a mini-lecture, which typically lasted 5 – 10 minutes and was based on the previous night's video. The rest of the class period was devoted to independent work time in the student workbooks (see Tables 2 and 3). In the homework videos, the teacher demonstrated the steps in

Table 2: Two Types of Observed Instructional Models

Instructional Model A	Instructional Model B
Students begin the class period with a warm-up or review problems.	Students work on workbook pages individually or with a partner (informal).
Teacher gives a 5 – 10 minute lecture based on the video from the previous night.	Teacher pulls a small group of students together for a short mini-lesson based on need.
Students work on workbook pages individually or with a partner (informal).	Teacher circulates the room assisting individual students.
Teacher circulates the room assisting individual students.	

Table 3: Classroom Activity Descriptions

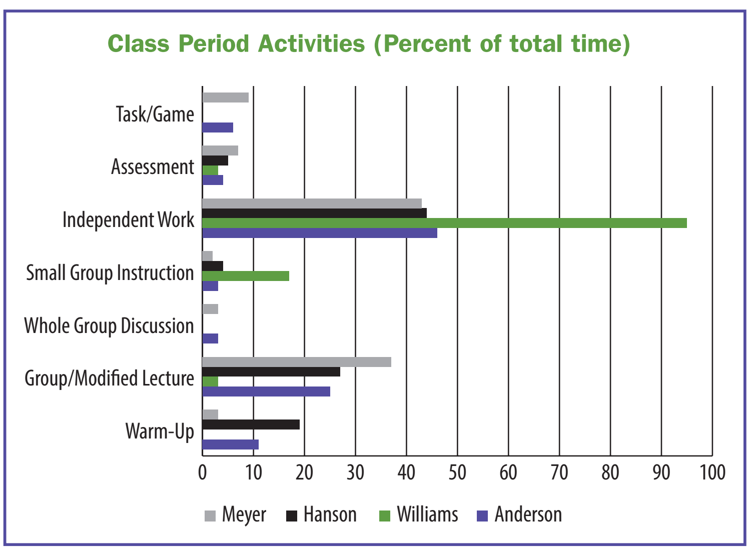
Activity	Activity Description
Task/Game	A whole class activity such as a problem-solving task, a game, or skills practice on a computer
Assessment	A quick quiz from the curriculum
Independent Work	The time that students are working out of their workbook or textbook
Small Group Instruction	A small group session of 3-8 purposefully selected students with the teacher to review specific content
Whole Group Discussion	A whole class session during which students are sharing their strategies, offering new strategies, and asking questions of each other and the teacher – conversation like
Whole Group/ Modified Lecture	A whole class session in which the teacher demonstrates a procedure and sometimes asks procedural questions in an IRE (initiate, respond, evaluate) type dialogue (e.g., "What is 3 x 4?" during the procedure to make common denominators)
Warm-up	Either a commercially made worksheet with one problem from each mathematics strand or several practice problems on the Smart Board connected to the video from the previous night

a procedure and then modeled several practice problems. In the mini-lectures, the teacher did the same thing using sample problems, put on the Smartboard, based on the problems and procedures in the video. Afterwards, students worked with a partner or by themselves on problems in their workbook, and the teacher circulated the room assisting individuals as needed. At times, small groups were pulled to work on a specific skill based on quiz scores or common student questions that had occurred in a previous class period. It appeared, and was also shared in teacher interviews, that the teachers depended on the video as the main vehicle to deliver the instruction.

Within both Instructional Models A and B (see Table 2), a variety of activities, such as whole group lecture, small group instruction and other instructional practices, took place. These activities are described in detail in Table 3. Based on the field notes, the duration of these activities were calculated (see Figure 2). The total percentage for each classroom exceeded 100% because some activities were going on simultaneously in the classroom such as small group instruction and independent work time.

FIGURE 2.

Classroom activities. (Teacher names are pseudonyms)



Alignment with Mathematics Teaching Practices

Overall, there was a weak alignment between the observed actions in the classrooms and the suggested NCTM actions. In general, the observed teacher and student actions rarely matched those identified with the Mathematics Teaching Practices (NCTM, 2014) as evidence of the practice occurring. In the sections that follow, results related to each of the selected Mathematics Teaching Practices will be presented.

Implement tasks that promote reasoning and problem solving. In this study, this practice referred to the teacher-selected work that the students completed in their workbook or notebook. The teacher chose this work from the textbook, as this was the main source for the student tasks. The selected problems aligned with the procedures taught in the video and reviewed in class.

The suggested actions for this practice (NCTM, 2014) call for engaging problems with multiple entry points. The featured problems were procedural-type questions used to practice what the students observed on the video and in class. Although there might have been multiple entry points, a variety of strategies were not observed being discussed and, therefore, were not likely used by the students. In addition, it was difficult to assess the types of reasoning and problem solving that the students used because this was not typically discussed in relation to their independent work. In general, the types of tasks recommended by the NCTM and the subsequent teacher and student actions were not typically observed in these classrooms.

Use and connect mathematical representations. The mathematical representations featured during instruction were all in pictorial form and appeared only when present in the curriculum materials. Typically, this occurred in the first few lessons of each unit. In addition, there was a fraction bar poster in each classroom that was occasionally referenced by the teacher. There was no evidence that any students used the posters as a tool. Further, there were no observations of connections being made among any of the pictorial representations. In part, this could be because of the types of conversation that were observed in these classrooms.

Facilitate meaningful mathematical discourse. When considering the practice of *Facilitate Meaningful Mathematical Discourse*, the majority of the dialogue heard involved the steps in procedures with short student responses. Students were typically asked to contribute the correct answer to the next step in the procedure. Alternatively, students occasionally shared how they answered a question, but this generally involved the steps used rather than the reasoning behind the steps. The *turn to your neighbor* protocol was frequently observed, although what was shared was a single answer or procedure to solve the problem. Students appeared comfortable sharing their ideas both with their partner and with the whole class and in several instances were observed modeling a procedure in front of the class in the role of the teacher. The observed sharing was generally focused on the steps of a procedure and the answer to the problem. An example of the type of discourse most frequently observed follows.

T: Who can tell me what an equivalent fraction is?

S1: Umm, I'm guessing but two fractions with the same denominator?

T: (calls on another student)

S2: Two fractions worth the same amount.

T: (Writes 1/2 and 3/6 on the board) These two fractions are equal – they show the same amount. Now we need to find the multiplier – the factor that we are going to multiply both the numerator and the denominator by to get the equivalent fraction. (Teacher writes a small x3 next to the numerator and denominator of 1/2)

T: (Writes 5/6 = 10/12 on the board) What do you multiply each number in 5/6 by to get 10/12?

S: (Chorally) 2

T: So if you have 15/18 = 5/6 (writes this on the board) what is the divisor?

S3: 3

* Teacher continues with two more examples this time having the students do this in their notebooks and then check with their neighbors about the multipliers. After a few minutes the teacher calls on a couple of students to give the answers – she writes the answers in on the board. (From Lesson 5.12 – Equivalent fractions)

This dialogue was typical of the type of discourse that occurred in these classrooms. That is, the teacher told the students what was needed to solve the problem and demonstrated how to solve the problem. Discussion about why the procedure made sense or how it related to the concept was generally not part of the discourse.

Build procedural fluency from conceptual understanding. Building procedural fluency was observed in all classrooms; however, *Building Procedural Fluency from Conceptual Understanding* was generally not observed. The emphasis was clearly on learning rules or procedures and then practicing these procedures. A great deal of class time was devoted to independent student practice, which stemmed from the instruction in the video and the mini-lecture at the beginning of each class period. Because of the limited use of multiple representations and connections through meaningful discourse in the classrooms, it was difficult to ascertain what level of conceptual understanding the students were using to do the work, compared to memorized rules and procedures.

Elicit and use evidence of student thinking. Two main actions were linked to the practice *Elicit and Use Evidence of Student Thinking*. First, in all classrooms, the most common action observed was that teachers spent a great deal of time talking with students individually. Usually this was driven by the student asking a question specific to a problem that he or she was working on in the workbook. Based on the question, the teacher gained an idea of what was likely misunderstood or confusing to the student. The second action occurred when, in some cases, the teacher pulled small groups of students together who needed similar support based on these individual conversations or previous quiz results from an earlier lesson. However, beyond talking with individual students, instructional decisions on the pacing or order of lessons appeared to be dictated by the curriculum. Every day the video for the next lesson was posted as homework and the in-class work the next day was the lesson workbook pages that went with it. The exception to this was the students working in the sixth-grade textbook who worked at their own pace so they could move ahead if they completed the work. Occasionally, the teacher announced that if a student had finished their assigned Math Expressions (Fuson, 2011) workbook pages they could go on to the next lesson as well or do some other worksheets that may or may not be more challenging. In general, eliciting student thinking centered on student questions or needs based on their ability to complete the questions in their workbooks or on quizzes accurately. Using evidence of student thinking was limited to pulling groups together or allowing students to work in the alternate textbook and work ahead.

Findings Related to Achievement and Conceptual vs. Procedural Understanding

Posttest Analyses

Each unit culminated with a posttest designed by the Math Expressions curriculum. The tests, as well as the lessons in each unit, were aligned to the state standards for fifth grade in the areas of decimals and fractions. All students in the classrooms in this study took the same posttest. This study used the score of 80% or greater as the cutoff to likely meet the state standards. This was a practical decision in that anything less than 80% clearly showed some understanding of the topic; however, misconceptions or errors were taking place which could limit the student's ability to meet the standards in that area at this time. The Unit 3 – Decimal test had a mean score of 91.45% with 94% of the students receiving a score of 80% or higher. The Unit 5 – Fraction test had a mean score of 81.31% with 63.4% of the students receiving a score of 80% or higher (see Tables 4 and 5). Possible insights regarding the difference in student achievement between Unit 3 and Unit 5 could be gained from the analysis of procedural versus conceptual understandings found during the student interviews after each unit. These results follow in the next section.

Table 4: Posttest Achievement Scores

Posttests	N	Mean (%)	SD	Min	Max
Unit 3 Posttest – Decimals	112	91.45	6.91	71.00	100.00
Unit 5 Posttest – Fractions	112	81.31	15.86	37.50	100.00

Table 5: Posttest Scores by Percentage Levels

Score	Unit 3 Posttest N = 112	Unit 5 Posttest N = 112
90 - 100%	75	48
80 – 89.9%	30	23
70 – 79.9%	7	19
60 – 69.9%		9
50 – 59.9%		7
40 – 49.9%		2
30 – 39.9%		4

Interview Analyses

The interviews revealed more detailed information as to how the students were actually thinking about decimals and fractions. During the decimal interviews, the use of whole number thinking was observed across the group of students, both high achieving and low achieving (see Tables 6 and 7). This type of procedural thinking (e.g., "0.7 is greater than 0.4 because seven is more than four" or "add zeros and line up the decimals") typically enabled the students to produce the correct answers while not necessarily understanding what they were doing. Only one student referred to a mental image of a grid and bar to explain how he got his answer.

Both high-achieving and low-achieving students struggled with the two estimating questions because their whole number thinking became an unreliable strategy. For example, the students were asked to think about the number 0.57. Then, they were asked if they were to take away 0.009, would they be left with a little more than a half (0.5) or a little less than a half (0.5). The relative size of the decimal, in the case of 0.009, was not generally thought of as being very small and, therefore, would cause little change to the original number of 0.57. Most students who correctly answered this question provided a procedural explanation, such as, "I imagined doing the problem in my head. I added a zero behind the 0.57 and then lined up the decimals." The purpose of this question, though, was to determine whether a student could use the relative size of a decimal number to make a correct estimation instead of using a procedure to get an answer. The unit posttest did not have any estimating questions on it; therefore, the

Table 6: Decimal Interview Responses by Type from Low-achieving Students

Low-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
0.7 or 0.4 Which is larger?	9	1	1	9
0.103 or 0.13 Which is larger?	5	5	1	9
Put these decimals in order from least to greatest: 0.245, 0.025, 0.249, 0.3	5	5	0	10
Estimate 0.37 + 0.4	1	9	0	10
Picture 0.57. If you took 0.009 away, would the amount left be more than a half or less than a half?	2	8	0	10
Solve 0.375 + 2.5	9	1	0	10
Solve 4.85 – 0.437	8	2	0	10

Table 7: Decimal Interview Responses by Type from High-achieving Students

High-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
0.7 or 0.4 Which is larger?	10	0	1	9
0.103 or 0.13 Which is larger?	10	0	0	9
Put these decimals in order from least to greatest: 0.245, 0.025, 0.249, 0.3	10		0	10
Estimate 0.37 + 0.4	8	2	0	10
Picture 0.57. If you took 0.009 away, would the amount left be more than a half or less than a half?	7	3	4	6
Solve 0.375 + 2.5	10	0	0	10
Solve 4.85 – 0.437	10	0	0	10

Table 8: Fraction Interview Responses by Type from Low-achieving Students

Low-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
Put these fractions in order; 1/5, 1/3, 1/4	6	4	6	4
Which fraction is larger 4/5 or 11/12?	0	10	4	6
Which fraction is smaller 1/20 or 1/17?	5	5	7	3
Are these fractions equal or is one less, 5/12 or ¾?	7	3	3	7
Are these fractions equal or is one less, 6/4 or 6/5?	6	4	2	8
2/5 + ¾ = 5/9 Do you agree?	9	1	0	10
Estimate: 7/8 + 12/13	1	9	1	9
Solve: 2 1/5 + 1 ¾ =	0	10	0	10
Solve: 4 1/8 – 2 2/4 =	0	10	0	10

Table 9: Fraction Interview Responses by Type from High-achieving Students

High-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
Put these fractions in order; 1/5, 1/3, 1/4	10	0	8	2
Which fraction is larger 4/5 or 11/12?	8	2	4	6
Which fraction is smaller 1/20 or 1/17?	10	0	7	3
Are these fractions equal or is one less, 5/12 or ¾?	9	1	1	9
Are these fractions equal or is one less, 6/4 or 6/5?	10	0	4	6
2/5 + ¾ = 5/9 Do you agree?	10	0	2	8
Estimate: 7/8 + 11/12	7	3	7	3
Solve: 2 1/5 + 1 ¾ =	10	0	0	10
Solve: 4 1/8 – 2 2/4 =	10	0	1	9

use of whole number thinking and following rules likely allowed many students to provide correct answers regardless of whether they had a conceptual understanding of the relative size of the decimal number.

The interviews after the fraction unit test showed more use of mental images or pictorial representations to explain some answers, such as working with unit fractions; however, they were not used consistently or to support estimation with fractions (see Tables 8 and 9). Further, all of the students interviewed could state the need for making common denominators prior to adding or subtracting fractions; however, very few were able to explain why they should do that and only 10 of the 20 students could do it accurately. Most students could explain how the denominator relates to the size of a piece of pizza or a candy bar when looking at unit fractions or fractions with a common numerator. Some students described this while others drew a simple picture. However, this same type of thinking tended to not be used when students were asked to compare fractions with unlike numerators. For example, when asked, "Which is greater 4/5 or 11/12?" common responses included, "They are equal because they are both one piece away from a whole," or, "The answer is 11/12 because the numbers are bigger." Regardless of the type of question, the students typically tried to find the common denominators before comparing, estimating, or computing with fractions. This frequently resulted in the wrong answer or a correct answer based on a procedure versus any demonstration of the conceptual understanding of the relative size or equivalence of a fraction.

The inconsistent demonstration of conceptual understanding and consistent, but frequently inaccurate, use of procedures potentially contributed to the wider range of test scores on the fraction unit test as well as the smaller number of students receiving a score of 80% or greater compared to the decimal posttest. Based on the student interviews, it would appear that a limited number of students had developed a conceptual understanding of fractions.

Findings Related to Overall Classroom Experience

All 40 students (20 high achieving and 20 low achieving) interviewed were asked the same five questions about their feelings toward mathematics and specific aspects of the Flipped Classroom model. Many students liked mathematics to some degree. In addition, they liked working with friends and having a video for homework instead of pencil-and-paper homework. The differences emerged, however, when asked specifically about the videos and their home computer and internet access (see Table 10). The high-achieving students generally liked how the videos told the student what to do. In contrast, the low-achieving students frequently reported the videos to be confusing. Many of these students also reported frustration with not being able to ask their teacher a question during the video and typically did not re-watch a video as often as the high-achieving students. This difference in re-watching the videos could be linked to the fact that some of the low-achieving students had to watch the videos at school because they did not have computer access at home. A few shared that they did not like to miss class to watch the video therefore re-watching the video could make this a worse situation. In general, the high-achieving students reported the use of multiple home devices to watch the videos and good internet connections. Alternatively, the low-achieving students typically had one device at home with mixed comments on their internet connections. The interview data suggested that there were discrepancies in access to computers and the internet as well as in experiences with the videos between low-achieving and high-achieving students.

Discussion

The definition of the Flipped Classroom used in this study began with the language, "a pedagogical approach" (FLN, 2014, "Definition of Flipped Learning"). This implies that what the teacher does within the model is critical to the success or failure of the model and that of the students. Further, the definition described a classroom that is a "dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (FLN, 2014, "Definition of Flipped Learning"). These ideas would

Table 10: Student Interview Responses Regarding Video and Technology Access

	Low-achieving Students	High-achieving Students
Positive Feedback	• Videos are helpful • Liked video homework better than workbook homework	• Liked video homework • Videos tell you how to do it
Negative Feedback	• Videos are too long • Videos are confusing and go too fast • Prefers lesson in class so you can ask questions • Misses having a teacher • Didn't like missing class to watch the video	• Videos are boring
Re-watching Videos	• 8 out of 20 had re-watched a video	• 11 out of 20 had re-watched a video
Computer Access	• Most have only one device in their home to watch the videos • 6 out of 20 students reported that they do not have internet access at home • About half reported a slow connection	• Most have multiple devices to watch the videos • Most report that they have a good internet connection

appear to align with the expectations for high quality mathematics instruction for all students (NCTM, 2014). The teacher is responsible for intentionally and purposefully selecting engaging tasks with multiple entry points, offering many experiences with multiple representations, making connections among the representations, and then making instructional decisions based on elicited student thinking. The purpose in these actions, based on research, supports the deep learning of mathematics both conceptually and procedurally. The qualitative data in this study, however, suggested that the observed Flipped Classroom model supported the teaching of rules and procedures and did not necessarily align with the expectations that support deep learning.

During the student interviews, the use of rules or procedures dominated the processes used by the students, although not always accurately. When merging qualitative findings with the quantitative posttest data, it suggested that students were able to demonstrate their ability to meet the state standards more frequently in the area of decimals when taking a test based on the use of procedures. Conversely, when the students were less able to utilize the procedures and had limited conceptual understandings, they did not perform as well, as in the case of the posttest on fractions in which fewer students were likely to meet the state standards at that time. Further, the data from the student interviews suggested that lower-achieving students tended to be more frustrated by the videos, did not re-watch the videos as often, and had more access issues to computers and the internet compared to their high-achieving classmates.

Research-based practices were generally not employed in the Flipped Classroom model examined in this study. Further, the FLN description, stated at the beginning of this section, did not seem to describe the classrooms observed. Teacher beliefs about teaching and learning mathematics can be productive or unproductive (NCTM, 2014) and greatly influence what happens in the classroom. The importance of doing this study from an outside observer perspective brought these conflicts to light.

Of equal concern were the issues surrounding the differences between high-achieving students and low-achieving students in regards to their reactions to the videos and their access to computers at home along with the internet connection. From an adult perspective, including secondary and post-secondary students in other studies (Bishop & Verleger, 2013; Hamden et al., 2013; Yarbo et al., 2014), the opportunity to be able to watch a video repeatedly is very appealing when working with challenging material. Elementary-age students in this study, however, did not appear to share this same thought. Likely due to computer access issues, this may be especially true for those low-achieving students in need of the most support mathematically. The use of videos at home may be supporting the disparity in achievement between high-achieving and low-achieving students in this study instead of being a useful tool for learning, as perceived by adults. As the NCTM Equity Principle states, "Access to technology must not become yet another dimension of educational inequity" (NCTM, 2000, p. 14).

Recommendations

The idea of flipping the classroom has become very popular across all levels of education and many content areas (FLN, 2014). This study demonstrated that teachers who choose to implement this model in their classrooms need to be very intentional with their pedagogy within this model just as they would within the standard classroom model. The idea or structure of flipping the classroom does not necessarily support students any more than the traditional classroom model. The intentional use of effective practices is one critical element to the success of the students. Based on this research, three recommendations are offered.

First, teachers utilizing a Flipped Classroom model need the opportunity to explore how to use the classroom time that is freed from lecture and turn it into productive activity that supports the significant understanding of mathematics. Mathematics education leaders cannot assume that because a teacher has adopted a new instructional model in his/her classroom that the instruction in the classroom will change or that students will automatically benefit.

Second, alternative methods to support students' access to the videos in a Flipped Classroom model need to be developed. Using other class time during the school day is not an equitable approach to solving this problem. Communication with families, while potentially challenging, could play an important role in working to resolve this issue.

Third, support is needed for teachers who want to implement a Flipped Classroom model that is consistent with the Mathematics Teaching Principles (NCTM, 2014). Collaborative planning or coaching that focuses on using the newly available classroom time for encouraging productive discussions and engaging students in high quality tasks is essential. A possible model could be the Four-Phase Process designed by Strayer, Hart, and Bleiler-Baxter (2016), which involves using the homework video as a jumpstart to the in-class lesson. Students come to class having had the opportunity to think about a problem ahead of time, based on some background information, and then use the class time to engage in rich discussion and problem-solving activities to learn the mathematics content. This is a new space in professional development that would be valuable for teachers interested in using a Flipped Classroom model at all levels.

Concluding Remarks

Implementing the Mathematics Teaching Practices (NCTM, 2014), changing pedagogy, and creating a new learning structure or environment are very complex tasks that teachers are undertaking. The intent is to provide students with excellent instruction so that all students have the opportunity to succeed. This study used the research behind the Mathematics Teaching Practices and conceptual understanding of fractions and decimals to examine the Flipped Classroom model. In doing so, it offers insight into a very popular, yet minimally researched, instructional model. The results of this study highlight the importance of considering how changes in class structure influence not only student learning but also the learning experiences of specific groups of students. The Flipped Classroom model will continue to be implemented across the United States; therefore, it is critically important to continue to support the development of research-based teaching practices as well as encourage an acute awareness of newly created issues of equity based on the use of technology. Research-based practices that support high-quality mathematics instruction for all students as well as equitable learning environments are necessary regardless of the teaching model, if we are going to close the achievement gap. ✪

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Meeting the Needs Expressed by Teachers: Adaptations of the Traditional Model for Demonstration Lessons

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Abstract

Demonstration lessons are one means for providing teachers with opportunities to reflect on instruction. Although different models for demonstration lessons are described in the literature, the Implementing Mathematical Practices And Content into Teaching Project, or Project IMPACT, developed two additional models of demonstration lessons in response to the expressed needs of project participants. In this article, we introduce these two models with the goal of supporting mathematics education leaders in enacting these models, or further adapting them, in their own work. Further, we aim to demonstrate how these models were developed in response to project participants' needs.

Introduction

With increased expectations regarding mathematics learning (e.g, Common Core State Standards Initiative [CCSSI], 2010), there is a strong need to support teachers as they "envision and implement classrooms in which students are effectively engaged in learning mathematics and understand the instructional decisions that they need to make in order to create this environment" (National Council of Supervisors of Mathematics [NCSM], 2014, p. 1). This type of mathematics teaching is complex (National Council of Teachers of Mathematics [NCTM], 2014) and requires that mathematics education leaders "model effective instructional strategies" (NCSM, 2014, p. 16) as a way to encourage teachers to professionally reflect on instruction (NCSM, 2014). One way to provide teachers with the opportunity for such reflection is through the use of demonstration lessons (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010).

Demonstration lessons represent one type of public teaching[1], where an instructor conducts a lesson with students and

[1] We recognize that there are many ways to use public teaching for professional development, such as lesson study and model lessons. In this paper we focus solely on demonstration lessons.

invites other teachers and colleagues into the classroom to observe and reflect upon that lesson (Loucks-Horsley et al., 2010). The traditional demonstration lesson model combines briefing (prior to the lesson) and debriefing (after the lesson) discussions with a lesson observation to provide rich opportunities for teacher learning and is recommended as an effective tool to facilitate professional development activities for teachers (Conference Board of the Mathematical Sciences, 2012). There are many purposes for which demonstration lessons can be employed, therefore different recommendations exist regarding how to conduct demonstration lessons (Casey, 2011). In our professional development project, however, the purpose and design of demonstration lessons emerged in direct response to the needs of participating teachers. In this article, we discuss different models of demonstration lessons for professional development in the literature and then share our refinements of demonstration lessons to better meet the professional learning needs of project participants. In doing so, our intent is two-fold. First, we aim to introduce two new models of demonstration lessons and support the reader's understanding of these models so that they may be employed in other settings. Second, we seek to demonstrate how these models of demonstration lessons emerged based on the project goals and participants' needs.

Demonstration Lessons in the Literature

Most professional development providers who use demonstration lessons do so to create a space for teachers to critically reflect upon the practice of teaching by observing the overall classroom environment and the teacher's actions during a lesson (Clarke et al., 2013). To accomplish this, demonstration lessons are generally structured to include: a briefing that focuses observing teachers' attention on selected mathematical or pedagogical features of the lesson; the observation of the demonstration lesson where observers record notes on the features of interest; and a debriefing where the observing teachers' observations are discussed along with implications for future instruction (Clarke et al., 2013; Loucks-Horsley et al., 2010). This structure allows a demonstration lesson to be conducted in one sitting.

When reviewing the literature, we noticed specific reasons for which demonstration lessons were implemented. The most common purpose was to cast a vision for what mathematics instruction that focuses on student thinking can look like and to invite observing teachers to consider how they might change their practice to align with this approach. We refer to this model of demonstration lessons as *exemplar demonstration lessons*. For example, Clarke and colleagues (2013) used exemplar demonstration lessons in a large (over 650 teachers), multi-year professional development project aimed at supporting teachers as they transitioned to incorporating reform-oriented teaching practices in their classrooms. Throughout the project, teachers attended one or more demonstration lessons. During a demonstration lesson briefing, teachers were given the freedom to choose their own focus areas for the observation with regard to both teaching and student learning. As teachers observed the demonstration lesson, they recorded what they noticed on an observation form that encouraged observing teachers to consider the connections between teacher actions and student responses. During the debriefing, teachers reported what they had observed. After the debriefing, teachers reflected on the experience and shared anything that occurred that they believed would contribute to a change in their own teaching practices. Teachers were also asked to describe any intended changes in their practices. In general, observing teachers often initially struggle to focus on anything other than the teacher during a demonstration lesson. However, as a result of this work, Clarke and colleagues concluded that their structure for a demonstration lesson resulted in observing teachers having a greater focus on both student thinking and teacher actions. Also, many of the observing teachers in this study intended to change their practice to include greater opportunities for students to articulate their thinking and to increase their use of hands-on resources to support student thinking. These results are typical of successful uses of exemplar demonstration lessons.

Other professional development projects have conducted exemplar demonstration lessons with the added step of having the observing teachers return to their classrooms and teach the exact same lesson with their own students. We call this model *replicated demonstration lessons*. In one such study, Herbert, Vale, Bragg, Loong, and Widjaja (2015) chose teachers' ability to notice and attend to students' mathematical reasoning as their focus. The briefing prepared participating teachers for this focus. Then teachers observed a demonstration lesson and, during the debriefing, discussed what they noticed about students' mathematical reasoning throughout the lesson. Next, each teacher taught the exact same lesson in his/her own classroom. After the lesson replication, the researchers interviewed teachers to gain insight into their developing abilities to notice and respond to students' mathematical reasoning. Later in the project, the teachers participated in another replicated demonstration lesson. They observed a second demonstration lesson, debriefed with other teachers, taught the exact second lesson with their own students, and participated in a second interview. Herbert and colleagues analyzed data collected throughout the study and classified the various ways in which teachers perceived what constitutes mathematical reasoning, which included: thinking; communicating thinking; problem solving; validating thinking; forming conjectures; using logical arguments for validating conjectures; and connecting aspects of mathematics. During this study, the replicated demonstration lesson model provided teachers with multiple vantage points from which to notice student mathematical reasoning during a lesson: an outsider's view as observer and an insider's view as the teacher of the lesson.

Both exemplar demonstration lessons and replicated demonstration lessons engage teachers in meaningful reflection on instructional practices. The lessons observed in exemplar demonstration lessons aim to provide a vision of the type of mathematics instruction needed to engage students in learning meaningful mathematics. This vision is extended to include implementation within the classroom in replicated demonstration lessons. During our professional development project, we wondered if other models of demonstration lessons might be useful for moving teachers beyond envisioning and implementing effective mathematics instruction towards understanding the instructional decisions made in this regard. Therefore, the following section presents an overall description of our project followed by descriptions of our models for demonstration lessons.

Demonstration Lessons in Project IMPACT

The Implementing Mathematical Practices And Content into Teaching Project, or Project IMPACT, is an ongoing professional development effort that serves over 150 K-8 mathematics teachers, primarily drawn from five partner districts. The project seeks to promote teacher growth in four critical areas: building mathematical knowledge and employing it in the work of teaching; utilizing student thinking during instruction; developing productive habits of mind; and building collegial relationships to support continued learning (NCTM, 2010). The work of this five-year project has entailed classroom observations of a sample of teachers, two-week intensive summer institutes that incorporate immersion and practice-based experiences (Loucks-Horsley et al., 2010), fall and spring sessions that continue to provide immersion and practice-based experiences during the school year, and multiple fall and spring demonstration lessons.

During Project IMPACT, demonstration lessons have been key to supporting participants' continuous professional learning. So that participants can observe the demonstration lessons live, the project pays for substitute teachers and participant mileage. Participants travel to a selected school where the demonstration lesson is conducted in a large room, such as a library or gym. Between 30 and 60 participants participate in any given IMPACT demonstration lesson. Including the briefing and debriefing sessions, one demonstration lesson is typically completed during a three-hour block of the school day.

From the beginning, our broad goal has been to use demonstration lessons to help participating teachers move from a practitioner's stance to professional development (Farmer, Gerretson, & Lassak, 2003), which focuses on taking ideas from professional development and using them with little modification in the participant's own classroom, toward an inquiry stance, which focuses on using professional development as an opportunity to investigate the teaching process. To help participants embrace an inquiry stance, we knew we needed to seek to impact participants' knowledge and beliefs, which have been shown to influence their instructional practices (Ernest, 1989).

As the IMPACT team planned, implemented, studied, and revised the work of the project, we developed different models of demonstration lessons in response to the needs of participating teachers. In the paragraphs that follow, we describe how the team has used demonstration lessons to support participants' professional growth during the project.

Initial Demonstration Lessons

At the onset of the project, IMPACT staff conducted classroom observations in pairs of a subset of teachers that represented approximately 25% of the participants and was drawn from its two primary partner districts. The Reformed Teaching Observation Protocol (Arizona Board of Regents, 2002) was utilized during these observations. In utilizing this protocol, each observer developed a written record of the lesson that included statements and questions offered by students and teachers as well as pictures of

FIGURE 1.
Exemplar demonstration lesson structure.

Exemplar Demonstration Lesson

Briefing	Lesson	Debrief
• Consider lesson plan and mathematical goals • Determine observation focus	• Observe according to area of focus • Attend to both teacher and student actions	• Share significant observations • Relect upon individual teaching practice

FIGURE 2.
Summary of the Acrobat Task (Burns, 1996).

In round 1 of a tug-of-war, four acrobats tied with five grandmas.

In round 2 of a tug-of-war, one dog tied with two grandmas and an acrobat.

In round 3 of a tug-of-war, three grandmas and the dog are pulling against four acrobats. Who will win?

student-generated artifacts from the lesson. An analysis of these written records revealed two common instructional features that were grounded in participants' knowledge and beliefs. First, participants did not scaffold student engagement in the problem-solving process. Therefore, students did not engage in productive struggle. Second, participants demonstrated mathematical procedures first and then asked students to apply these procedures to solve problems. This pattern of instructional practice did not support deep learning of mathematics (NCTM, 2014).

In response to these observations, the IMPACT team planned an initial round of exemplar demonstration lessons to model teaching through problem solving. The goals of these demonstration lessons were: first, to set a vision of effective mathematics instruction; and second, to gain traction with teachers and inspire them to change their practice to align with the vision. In this way, these initial demonstration lessons aligned with the literature, both in terms of purpose and design (Loucks-Horsley et al., 2010). The vision for instruction in the demonstration lessons was grounded in the Standards for Mathematical Practice (CCSSI, 2010) and the Mathematics Teaching Practices (NCTM, 2014). Figure 1 provides a visual of these exemplar demonstration lessons with the briefing and debriefing occurring immediately prior to and after the demonstration lesson, respectively.

As an example, the initial demonstration lesson in Project IMPACT featured the Acrobat Task (Burns, 1996), which is summarized in Figure 2. Note that the original task uses pictures to communicate what happens in each of the tug-of-war rounds. During the briefing, participants reviewed this task and expressed concern. They imagined that the IMPACT instructor would give students the task and ask them to work independently for 10 minutes before sharing their thinking with others. Participants communicated that students' unfamiliarity with such a task would lead to an inability to successfully find an entry point to solving the problem. These concerns disappeared, however, once the lesson plan was distributed and participants gained insight into the scaffolding that was provided to support students' engagement in the problem. Figure 3 (next page) provides the opening portion of the lesson plan, with the intended scaffolding represented in the bolded statements. The enacted lesson demonstrated this scaffolding, which led to students successfully engaging in problem solving and producing their solutions. In this way, the lesson provided a vision for how to support students' engagement in productive struggle along with a vision for instruction that did not follow the common teaching practice of demonstrating procedures to be applied by students.

During subsequent project activities, some participants shared that they had successfully implemented the demonstration lessons in their own classrooms. In this way, although the IMPACT team did not intend for these initial demonstration lessons to be replication demonstration lessons, some participants sought to make initial changes to their practice by replicating the demonstration lessons in their own classrooms.

Day Two Demonstration Lessons

During project IMPACT's second year, the team continued to implement exemplar demonstration lessons. At the debriefing sessions, participants were encouraged to consider ways in which they might change their own practice to align with research-based instructional practices. As participants shared during these sessions, though, it became clear that demonstration lessons exposed students' mathematical

FIGURE 3.
Opening portion of the lesson plan for the Acrobat Task (Burns, 1996) with scaffolding aspects in bold.

Warm-up (5 minutes)

Display the following question on the document camera.

What do you know about the game of Tug of War?

Allow 30 seconds for independent think time, 30 seconds of pair time, and 3 minutes of share out time. Utilize index cards to call on groups to share out. At this time, try to bring out strength as the key factor in winning a tug of war.

Acrobat, Grandmas, and Ivan Task

Understanding the Problem (15 minutes)

Display the initial problem sheet. Introduce the people who will be featured in the problem.

Display the Round 1 picture and context on the document camera. Read Round 1 aloud.

Think-pair-share: **Based on this information, what is something that we know about the grandmas and acrobats?** As students share their ideas, record these on a piece of chart paper with "Round 1" as the heading.

Display the Round 2 picture and context on the document camera. Read Round 2 aloud.

Think-pair-share: **Based on this information, what is something that we know about the grandmas, acrobats, and Ivan (the dog)?** As students share their ideas, record these on a piece of chart paper with "Round 2" as the heading.

misunderstandings yet left little time to resolve them in a single lesson. This was problematic for participants, and they would often ask during debriefing discussions, "What would you do the next day?" In response, we structured *day two demonstration lessons* for the third year of the project to answer this question. A general description of this model is provided in the next section, followed by an example taken from Project IMPACT.

Description. *Day two demonstration lessons* aim to help teachers develop lessons that build from one day to the next based on students' thinking. In this model (see Figure 4), an instructor first teaches a lesson to a teacher's class. The lesson is video recorded. On the next day, a group of teachers gather for the day two demonstration lesson. In this setting, the briefing involves the group examining the lesson plan for the previous day's lesson along with edited video of the lesson. This discussion includes reviewing the original day two lesson and any modifications to the lesson that resulted from students' conceptions and misconceptions that surfaced during the previous lesson. Then, teachers observe the day two demonstration lesson, followed by a debriefing that considers implications for future instruction. In addition, teachers discuss ways in which they might design instructional experiences based on the understandings and misunderstandings of students in their own classrooms.

Example. To describe how the day two demonstration lesson process unfolded in Project IMPACT, we share an example that is based on the L Problem (Watanabe, 2008). The L Problem (see Figure 5 on next page) is intended to engage students in finding the area of a non-typical shape, with a goal of developing strategies that can later be utilized to generate area formulae for shapes (e.g., parallelograms, triangles, trapezoids). An IMPACT instructor taught this lesson in one participant's classroom, while other staff members observed and video recorded the lesson. During this lesson, many students decomposed the shape into rectangles, found the area of each rectangle, and then incorrectly multiplied the different subareas to find the total area.

FIGURE 4.
Day two demonstration lesson structure.

Day Two Demonstration Lesson

Day One
- Only project staff are present
- Project staff teaches and records a lesson with students

Day Two

Briefing
- Teachers observe edited video of day one lesson
- Teachers consider lesson plan, goals, and observation focus

Lesson and Debrief
- Constructed in a manner consistent with an exemplar demonstration lesson

FIGURE 5.
The L Problem (Watanabe, 2008).
Note that the figure should be drawn to scale.

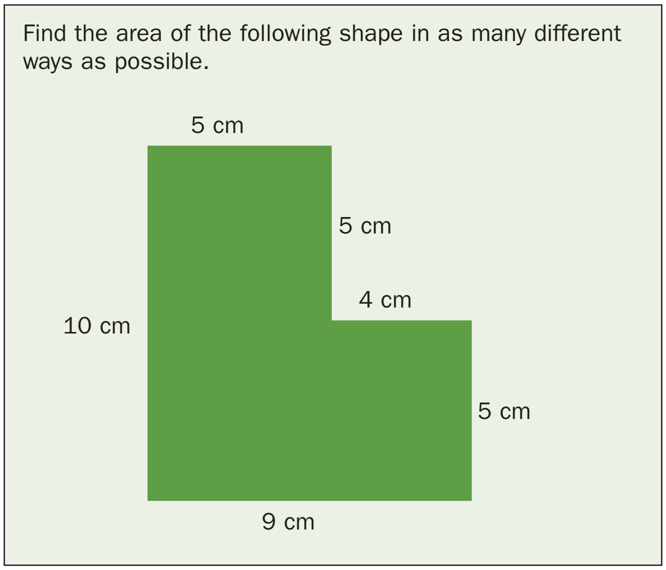


FIGURE 6a.
The F-Shape as distributed to students in the day two demonstration lesson. Note that the figure was drawn to scale and students had rulers with which to work.

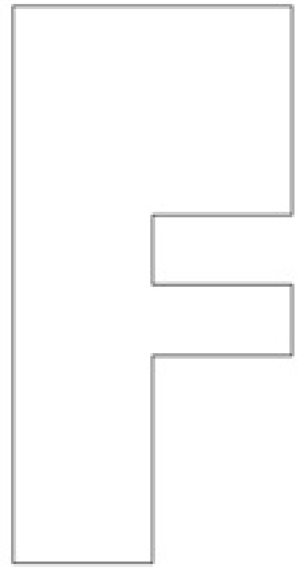
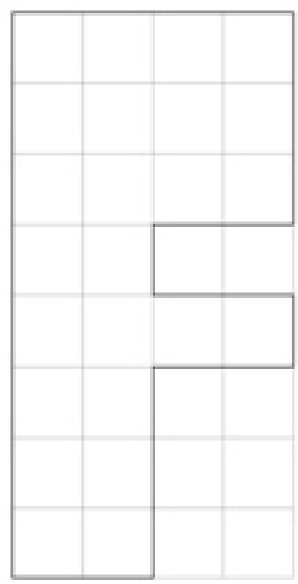


FIGURE 6b.
The F-Shape with an inch-grid superimposed on it.



During the briefing of the day two demonstration lesson the next day, participants examined the lesson plan for the L Problem along with the edited video of the lesson. Participants then reviewed the student work from this lesson to better understand the students' views of area, which appeared to be limited to length times width and relied on multiplication as the operation without justification as to why this might (or might not) be appropriate. Next, IMPACT staff described the need for students to consider counting squares as a means for thinking differently about area and for verifying and/or making sense of solutions. To accomplish this, the lesson instructor explained that the upcoming lesson would include asking students to hold centimeter grid paper behind the L-shape in order to count and determine the area. Then, students would be directed to find the area of the F-shape (see Figure 6a), which provided an opportunity to apply this square counting strategy or other strategies. In working with the F-shape, students would be given one-inch graph paper and rulers. Figure 6b shows what the F-shape looks like when replicated onto graph paper.

During the day two demonstration lesson, the participants observed how the lesson built from the previous day's work, as students counted the centimeter squares to determine the area of the L-shape and compared this solution and process to their ideas from the previous day's lesson. Once students realized that the subareas should be added rather than multiplied, they were better prepared to successfully find the area of the F-shape. In doing so, students either replicated the F-shape onto graph paper and counted the squares or decomposed the F-shape into rectangles and added the areas of these subregions. After the lesson, participants' discussions during the debriefing centered on the notion of using students' mathematical reasoning to build from one lesson to the next.

Double Demonstration Lessons

As the IMPACT team planned the work for year four of the project, we considered the professional growth of the participants. During the day two demonstration lessons, we noticed that some of the participants were still operating with a practitioner's stance toward professional development (Farmer et al., 2003). That is, they were focusing on specific ideas that could be taken from the demonstration lessons and used with little modification in their classrooms. However, we also observed other participants adopting an inquiry stance during the debriefing. These participants focused on how they could use the demonstration lesson to investigate the teaching process, suggested changes to the lesson, and hypothesized how those changes might influence the lesson outcomes. In fact, one participant, who was clearly demonstrating an inquiry stance, stated, "I wish we could teach this lesson again to see how our suggestions will impact the lesson." In response, we introduced *double demonstration lessons* during the fourth year with the goal of supporting all participants in adopting an inquiry stance. A general description of this model is provided in the next section, followed by an example taken from Project IMPACT.

Description. Double demonstration lessons (see Figure 7) incorporate two rounds of the briefing, observation, and debriefing cycle in a single day. In the first briefing, teachers review the lesson plan for the demonstration lesson. As teachers reflect on the lesson plan, they discuss: what they hope to observe during the lesson with regard to teacher and student actions; how they will know it if they see it; how they will record their observations; what student misconceptions they might observe; and what portions of the lesson plan currently concern them. This section of the briefing is intended to help teachers see the demonstration lesson as an inquiry process in which they can learn about the lesson in order to improve the lesson for student learning.

FIGURE 7.
Double demonstration lessons.

Double Demonstration Lesson

First Lesson	Second Lesson
• During the briefing, teachers consider the lesson plan and determine anticipated student responses	• During the briefing, lesson modifications are explained to the whole group
• Teachers observe the lesson	• Teachers observe the lesson
• During the debrief, teachers propose and defend improvements to the lesson	• During the debrief, teachers reflect on how the lesson changes affected lesson outcomes

After the briefing, an instructor teaches the lesson to a class of students. During the first debriefing, teachers reflect on areas for improvement in the lesson with regard to student engagement in the task, content, and the mathematical practices. Teachers decide on recommended revisions for the lesson and present these with justifications. Then, depending on the size of the group, a subset of teachers determines the final revisions for the lesson. With these revisions in hand, the same instructor teaches the modified lesson to a second class of students. Finally, during the debriefing of the second demonstration lesson, teachers reflect on how the changes from the first to second lesson affected lesson outcomes.

Example. To describe how the double demonstration lesson process unfolded in Project IMPACT, we share an example that is based on the Sharing Chocolate Task (Enns, 2014). This problem features a group of four students sharing three chocolate bars equally and a group of eight students sharing six chocolate bars equally. The problem asks students to consider how much chocolate students in each group receive as well as which group of students will receive more chocolate. Although there are several potential mathematical goals for which this problem could be used, the IMPACT instructor chose to use the problem as a means to help students understand that the fraction of chocolate received by each person can be represented by the number of chocolate bars divided by the number of people. The IMPACT instructor wrote the lesson plan to match the description of its enactment found in the article by Enns (2014).

Time in the initial briefing was spent acquainting the participants with the problem and its accompanying lesson plan. IMPACT staff also made participants aware of the double demonstration goal: to watch the first lesson with an eye on modifications that could be made to the lesson that would influence student learning. After observing the demonstration lesson, the first debriefing involved participants in small groups discussing their observations and developing suggested modifications for the lessons with justifications. Then, each group presented their ideas to the

Table 1: Summary of Demonstration Lessons Used in Project IMPACT

Model	Motivation	Intended Impact
Exemplar Demonstration Lesson[s]	• Participants did not scaffold students' engagement in problem solving. • Participants demonstrated procedures to be duplicated by students.	• To provide participants with instructional strategies for scaffolding towards problem solving. • To set a new vision for what effective mathematics instruction might look like.
Day Two Demonstration Lesson	• Participants expressed uncertainty regarding how to follow up a lesson that exposed students' mathematical misunderstandings/ shortcomings.	• To support participants' understandings of designing lessons that build on students' mathematical reasoning.
Double Demonstration Lesson	• Participants wondered how their suggestions for lesson modifications would influence the learning outcomes.	• To provide all participants' with the opportunity to engage in practices associated with an inquiry stance towards teaching.

[a] Although Project IMPACT chose not to use replicated demonstration lessons, many participants elected to utilize the exemplary demonstration lessons as if they were replicated demonstration lessons.

whole group. Because our group was large (60 participants), a subset of participants (i.e., those that had been with the project since its inception) were tasked with making the final decisions regarding lesson modifications. Then, the IMPACT instructor taught the modified lesson, which was followed by a second debriefing that focused on evaluating the impact of the lesson modifications.

As participants reflected during the second debrief, several noted that small changes led to significant influences on students' mathematical understandings. Other participants stated that seeing the enactment of the lesson modifications caused them to rethink some of the instructional assumptions that led to the suggested modifications. In this way, double demonstration lessons provided participants with an opportunity to adopt an inquiry stance and to recognize that the act of teaching is an opportunity for their own personal professional learning to occur.

Summary of Demonstration Lesson Models

As we reflect on the models of demonstration lessons, both from the literature and from our own work, we recognize that the use/development of each model was motivated by different circumstances and with different intentions based on project goals and participants' needs. Table 1 summarizes the models as they were utilized in Project IMPACT.

Conclusion

With increased expectations regarding the mathematics that students are to learn (e.g., CCSSI, 2010), there exists the need to support mathematics teachers in understanding the instructional decisions that will lead to deep mathematical learning (NCSM, 2014). Although Project IMPACT utilizes a variety of professional development activities, feedback from participants has indicated that demonstration lessons hold the most potential for supporting teachers' reflection on instructional practices. In this paper, we have not only described two models for demonstration lessons that have emerged from our work but also the circumstances that led to their development. In doing so, it is our hope that other mathematics education leaders might utilize these new models, should they fit within the circumstances of their work, and contemplate other models developed in response to the needs of their teachers.

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