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Call for Manuscripts

The editors of the *NCSM Journal of Mathematics Education Leadership* are interested in manuscripts addressing issues of leadership in mathematics education and reflecting a broad spectrum of formal and informal leadership at all levels.

Categories for submittal include:

- **Key topics** in leadership and leadership development
- **Case studies** of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
- **Reflections** on what it means to be a mathematics education leader and what it means to strengthen one's leadership practice
- **Research reports** with implications for mathematics education leaders
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- **Commentaries on critical issues** in mathematics education
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Other categories that support the mission of the journal will also be considered. Currently, the editors are particularly interested in manuscripts that address the leadership work of mathematics coaches and mathematics specialists.

We also invite readers to submit letters to the editor regarding any of the articles published in the journal. We seek your reactions, questions, and connections to your work. Selected letters will be published in the journal with your permission.

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Submittal of manuscripts should be done electronically to the *Journal* editor, currently Angela Barlow, at ncsmJMEL@mathedleadership.org. Submission should include (1) one Word file with the body of the manuscript without any author identification and (2) a second Word file with author information as you would like it to appear in the journal. Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.*

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Purpose Statement

The *NCSM Journal of Mathematics Education Leadership* is published at least twice yearly, in the spring and fall. Its purpose is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

Comments from the Editors

Angela T. Barlow, *University of Central Arkansas*
Carolyn Briles, *Loudoun County Public Schools/Riverside High School*

Because teachers and their teaching matter, improving their knowledge and instructional practice through professional development (PD) has become increasingly important for districts, schools, and, of course, teachers themselves. (Sztajn, Borko, & Smith, 2017, p. 793)

In reflecting on this quote taken from the recently published *Compendium for Research in Mathematics Education* (Cai, 2017), three key ideas resonated with us. First, teaching matters. Teaching that supports the mathematical achievement of all students is complex (National Council of Teachers of Mathematics [NCTM], 2014), as it includes instructional practices like those outlined in the Mathematics Teaching Practices. Second, teachers matter. Enactment of the Mathematics Teaching Practices requires knowledgeable teachers who are provided with meaningful feedback as they engage in these instructional practices. Third, professional development matters. Professional development that meaningfully engages teachers is paramount, understanding that “practice is the premier space for teacher learning” (Sztajn et al., 2017, p. 794). With these ideas in mind, this issue of *JMEL* offers two articles for readers’ consideration.

In the first article, “An Examination of the Nature of Post-Observation Feedback Provided to Middle School Mathematics Teachers,” Trinter and Carlson-Jaquez report a study in which they examined the feedback provided by evaluative and non-evaluative administrators with differing formal mathematics education experiences. Motivated

by the call in NCTM’s (2014) *Principles to Actions*, the authors describe the processes used by the administrators to develop and provide feedback, attending to the differences in the content-focused feedback. Both the teachers’ and administrators’ perspectives are shared. Implications for leaders in mathematics education describe how to support the processes of providing meaningful feedback to mathematics teachers.

In the second article, “Math Labs: Teachers, Teacher Educators, and School Leaders Learning Together With and From Their Own Students,” Kazemi and colleagues describe a professional development structure, referred to as Math Labs, that engages teachers in implementing and reflecting on lessons. Unique to this professional development structure is the collaborative implementation of lessons that includes Teacher Time Outs (Gibbons, Kazemi, Hintz, & Hartmann, 2017) to support the exploratory nature of the classroom visits along with instructional activities that may be implemented over the course of a school year. In addition to describing these features, the authors provide participants’ perspectives of Math Labs along with evidence of the influence of their participation on student achievement.

In both articles, the authors communicate the importance of teaching, teachers, and professional development. With their emphases on feedback and teacher learning in practice, we hope these articles will support readers in reflecting on ways to enhance their own work with teachers. ★

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An Examination of the Nature of Post-Observation Feedback Provided to Middle School Mathematics Teachers

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Abstract

The feedback mathematics teachers receive following an administrator's observation of instruction is a critical component of the teachers' professional development. This study examined the nature of feedback middle school mathematics teachers received from administrators who had differing formal mathematics education or experiences. Data included teacher evaluations, classroom artifacts, classroom observation field notes, and interviews with teachers and administrators. Within the framework of leadership content knowledge and complexity leadership theory, three major themes emerged with regard to how different mathematical backgrounds and/or evaluative roles of observers influenced their feedback. These themes focused on the form of the feedback (written and oral), the process for developing feedback (inductive and deductive), and the nature of feedback (content or pedagogical focus). The findings from this study most notably pointed to the difference in the nature of feedback to middle grades mathematics teachers from observers who had formal mathematics education or experience and those who had different subject backgrounds. The findings also provide evidence to support several implications for mathematics education leaders, which are discussed.

Introduction

Evaluation of teaching is a common practice with school districts nationwide investing significant time and resources into developing teacher evaluation instruments and protocols that assist administrators in documenting teacher effectiveness to meet federal policies (U.S. Dept. of Education, 2012). Although the passing of the Every Student Succeeds Act (2015) removed some of the restrictions on teacher evaluation, accountability is still a top priority in school districts. Evaluation models take many forms and researchers recommend models that include multiple methods of data collection (Milanowski, 2011). For example, Rockoff and Speroni (2011) found evidence to support that first-year teachers, who received subjective evaluations by trained mentors, produced greater gains in student achievement with future students but recommended both subjective evaluations by trained professionals and objective performance data to identify weaknesses in instruction. Similarly, Darling-Hammond, Amrein-Beardsley, Haertel, and Rothstein (2012) reported that effective systems utilized trained evaluators, provided frequent evaluation and feedback, and integrated measures (e.g. observations, videos, artifacts) that linked teachers' actions to outcomes.

These models are not only designed for evaluation purposes but also play a significant role in the professional growth and careers of many educators. Typically, evaluators are expected to provide feedback to multiple teachers in many content domains with the goal of improving

instructional practice. A critical component in a teacher's professional development is the feedback evaluators provide to teachers following the administrator's observation of instruction (Scheeler, Ruhl, & McAfee, 2004). Despite the importance of this post-observation feedback and the recommendations for specificity in accountability measures, there is little research pertaining to the ways in which administrators attend to subject-specific details in evaluation and instructional improvement (Lochmiller, 2016).

In concert with the national stir about teacher accountability is the nation's continued focus on students' mathematics achievement with federal initiatives seeking to increase the number of highly qualified STEM teachers, federally funded professional development programs, and partnerships between education and industry. In an effort to support the development of mathematics teachers, the National Council of Teachers of Mathematics (NCTM, 2014) recommended that leaders and policymakers empower teachers to create effective classrooms and learning environments by aligning accountability measures with mathematics teaching practices. These practices focus on clear mathematical learning goals, reasoning, problem solving, connecting representations, discourse, prior knowledge, conceptual understanding, and productive struggle. Considering the importance of observation and feedback to teachers' professional growth, coupled with the NCTM's recommendation for content-focused accountability measures, it is plausible that observers' content knowledge may influence the type of feedback provided and, hence, play a role in the professional growth of teachers.

Purpose of the Study

In this study, we examined the nature of feedback middle school mathematics teachers received from administrators who had differing formal mathematics education or experiences. We were particularly interested in teachers' perceptions of the feedback they received, in administrators' perceptions of the feedback they provided, and in comparing administrators' perceptions to the written feedback that teachers received. This exploratory study included 10 participants from three different schools and districts. We collected several forms of data including teacher evaluations, classroom artifacts, classroom observation field notes, and approximately 4 hours of interviews. We begin this paper by providing a review of literature relevant to teacher feedback and include a specific focus on feedback provided to mathematics teachers. Following this review,

we include the theoretical perspectives that framed the study, our study findings, and the ensuing discussion and implications for mathematics teacher development. The research questions explored were:

- 1) In what ways does post-observation feedback differ among observers with different mathematical backgrounds and evaluative roles?
- 2) How does the mathematical background of the observer shape his or her use of the school district's teacher evaluation system observation instrument?

Relevant Literature

The Nature and Benefits of Observation and Feedback to Teachers

A commonly employed method for promoting dialogue between evaluators and teachers and one that is included in recommended evaluation models (Darling-Hammond & Snyder, 2000; Moss et al., 2004) is observation and feedback from administrators, with feedback being a critical component to this cycle (Darling-Hammond et al., 2012; Scheeler et al., 2004). For the purposes of our study, we used Hattie and Timperley's (2007) conceptualization of feedback as "information provided by an agent regarding aspects of one's performance or understanding" (p. 81). We interpreted performance to mean teachers' instructional choices and understanding to mean teachers' understanding of content and pedagogy that influenced instruction. A review of the literature on feedback to teachers conducted by Scheeler and colleagues (2004) found 208 articles that were published on feedback to teachers between 1970-2004; however, only 4% of those articles focused on in-service teachers, and the rest focused on pre-service teachers. They concluded, "Feedback is better than no feedback, immediate feedback is better than delayed feedback, and feedback that is immediate, specific, positive, and corrective holds the most promise for bringing about lasting change in teaching behavior" (p. 405). Although it is documented that providing feedback is important, the type of feedback provided to teachers is also important, as it plays a role in the feedback's efficacy (Cherasaro, Brodersen, Reale, & Yanoski, 2016).

Characteristics and teacher perceptions of effective feedback. Research-based characteristics of effective feedback for teachers often center on the specificity of the feedback. For example, assessment research indicates that feedback is most effective when it communicates the current level

of achievement in relation to specified goals and provides steps to attaining these goals (McMillan, 2011). Similarly, Milanowski (2011) purported that teacher evaluations should include specific and clear feedback so that teachers have the opportunity to use the results to improve their practice. Furthermore, quality feedback can be described as timely, specific, and frequent (Northcraft, Schmidt, & Ashford, 2011; Price, Handley, Millar, & O'Donovan, 2010).

In Ovando's (2005) study, which examined the experiences of teachers and administrators during their observation and feedback cycles, teachers noted the importance of specificity in written feedback, and they appreciated face-to-face conversations about the observation and the written feedback. These teachers believed that effective feedback included post-observation conferences between the administrator and teacher that focused on the strengths of the instruction, were based on observable actions, and resulted in professional development goals for the teacher. In this study, administrators reported that in order to provide this level of specificity in written feedback, they needed to develop a knowledge of quality instruction, scripting skills, and appropriate professional language during their graduate studies. Considering the personal nature of the observer-feedback evaluation cycle, these experiences and perceptions of teachers and observers are noteworthy.

Other studies that have looked at teacher and administrator perceptions and experiences emphasized: the need for multiple observers; specific, written feedback coupled with dialogue; and adequate time for the full cycle to be effectively employed (Collins, 2004; Ovando & Ramirez, 2006). In one qualitative study, teachers and administrators had different perceptions of the nature of the given feedback following teacher observations (Collins, 2004). Teachers in this study believed that when instruction was satisfactory, they received no feedback from administrators. This was problematic for teachers as they expressed a need for feedback, regardless of the nature of instruction. Collins recommended addressing teachers' concerns by including supplemental observers such as department heads and senior teachers who had subject expertise and believed that their inclusion would result in a more comprehensive evaluation.

Further, not having multiple observers can lead to subjectivity in evaluation as recognized by Sartain, Stoelinga, and Brown (2011). In their study, the researchers examined the effectiveness of a teacher evaluation framework

that employed the observation/feedback model between administrators and teachers. To do this, the teacher was evaluated by both an administrator and a researcher. When reporting on the higher end of the scale (proficient or distinguished instruction), there were significant discrepancies between the observation ratings. Administrators were more likely than the researcher to rate a teacher as distinguished. In this same study, administrators were more likely to ask low-end questions that did not invoke reflective conversation than high-end questions that sparked deeper discussion about the instruction. The researchers concluded these forms of in-depth conversations were needed to improve teaching practice.

Subject-specific feedback. With the current focus in mathematics education on process standards, student mathematical dialogue, justification, and modeling (Common Core State Standards Initiative, 2010; NCTM, 2000, 2014), it is critical that administrators not only direct their attention to pedagogical and behavioral concerns in instruction but also value subject matter in both the content and the practice of disciplines (Nelson & Sassi, 2000). As noted by the NCTM (1989) in the *Professional Standards for Teachers of Mathematics*, central to the process of evaluation is the inclusion of multiple observations from more than a single observer. The teacher's role in these interactions is that of a reflective practitioner who provides information to the observer about his or her goals and self-analysis of teaching. The post-observation dialogue should be a means for developing a professional development plan focused on improving instruction.

Recently, the NCTM (2014) extended this work by outlining specific teacher and stakeholder actions in the publication, *Principles to Actions: Ensuring Mathematical Success for All*, which, they contend, should ensure students' success in mathematics. In particular, this document recommended that leaders and policymakers provide supports for ensuring student access to high-level mathematics education by aligning accountability measures with the Mathematics Teaching Practices and classroom observations that focus on these practices. These recommendations for teacher evaluation aligned with the research literature, which endorsed evaluation models that included multiple data collection sources.

Despite the NCTM's recommendation for mathematics-specific dialogue and evidence of content mastery, very few studies have taken a look at subject-specific observation

and feedback (Lochmiller, 2016; McDonald, 2008; Nelson & Sassi, 2000). Nelson and Sassi (2000) examined the nature of administrators' observations of a video-recorded fifth-grade mathematics lesson. The researchers reported that during the first observation, administrators noted the structural features of the lesson including "orderliness, good classroom management, understandable and well-executed structural components to the lesson and teacher behaviors such as wait time and gender equities" (p. 565). Following eight months of a professional development seminar for administrators focused on observation and supervision of elementary mathematics, the administrators viewed the video for the second time and noted subject-specific features of the lesson such as students' mathematical discourse. The observations shifted from teacher action and surface features of instruction to the development of mathematical ideas.

More recently, Lochmiller (2016) interviewed 51 participants, including 20 mathematics teachers, 19 science teachers, and 12 administrators, and examined these participants' perceptions of feedback that they received or provided. Findings indicated that the mathematics and science teachers perceived the feedback that they received as being general in nature and did not address content-specific instructional matters. Administrators used their past teaching experiences to help frame their feedback to teachers across content areas. Recognizing this tendency, Steele, Johnson, Otten, Herbel-Eisenmann, and Carver (2015) used Stein and Nelson's (2003) leadership content knowledge as a framework for designing professional development aimed at increasing principals' algebra content knowledge. The researchers found that the professional development experience changed principals' understandings and perspectives of algebra, which, in turn, influenced their leadership practice. These principals expressed that, prior to engaging in this content-specific professional development, observations were short in duration and feedback consisted of broad brushstroke statements. With the change in their content knowledge, the principals stated that future observations would focus on specific details of mathematical thinking. The researchers in this study did not follow principals into classrooms to document their evaluation practices, which was one way we intended to contribute to the knowledge surrounding teacher feedback in the current study.

Another study supporting the need for examining content-focused feedback was conducted by Cherasarso et

al. (2016). The researchers used correlational analysis to explore teacher perceptions of the feedback they received and found that their perceptions were related to four characteristics with one of the most critical being the credibility of the evaluator. The researchers suggested that credibility was linked to the evaluator's knowledge of the subject being evaluated. The current study addressed the importance of evaluator credibility by examining the feedback provided by observers with mathematical backgrounds and those without.

Summary

Considering the importance of feedback for improving instruction, we sought to extend the literature centered on discipline-specific leadership and teacher development by examining the nature of feedback provided to teachers by observers with different content backgrounds. We were particularly interested in feedback for middle grades mathematics teachers because teachers in this grade band are challenged with building on elementary mathematics content in preparing students for high school content with many middle grades teachers responsible for teaching higher-level mathematics courses. Hence, teachers in these grades feel pressure from both the grade bands below and above them and effective feedback may support their instructional practice.

Feedback literature shows the importance of multiple perspectives, specificity, and timeliness (Collins, 2004; Northcraft et al., 2011; Ovando & Ramirez, 2006; Price et al., 2010) with little research centered on content-focused feedback (Lochmiller, 2016; McDonald, 2008; Nelson & Sassi, 2000). Our study examined the content, development process, and form of post-observation feedback provided to teachers. This study is unique in that the observations were conducted by both an observer with mathematical content background and an observer with a different subject matter expertise. These different perspectives allowed us to compare the types of feedback received by middle grades mathematics teachers from each observer while also considering teachers' and observers' perspectives about this feedback.

Theoretical Framework

We drew from two theoretical perspectives in the design and analysis of this study: leadership content knowledge (Stein & Nelson, 2003) and complexity leadership theory (Uhi-Bien, Marion, & McKelvey, 2007). Leadership content

knowledge contends that the subject matter knowledge of an administrator plays a role in his or her leadership functions. Stein and Nelson defined leadership content knowledge as “the knowledge of subjects and how students learn them that is used by administrators when they function as instructional leaders” (p. 445). At the school level, this form of knowledge may play a role in an administrator’s feedback about lessons or instruction. Stein and Nelson explained that leadership content knowledge is at the cross-road between subject matter knowledge and leadership practice and stated, “Without knowledge that connects subject matter, learning, and teaching to acts of leadership, leadership floats disconnected from the very processes it is designed to govern” (p. 446). This intersection between subject matter and leadership framed our study.

Recognizing that school administrators cannot become experts in all content areas within one school, Stein and Nelson (2003) recommended a *distributed approach* to leadership that employed *postholing* to support disparities in leaders’ subject matter knowledge. A distributed approach acknowledges that schools are complex entities with many resources for supporting leaders in increasing their subject matter knowledge. In mathematics, these resources may include mathematics specialists, teachers, curriculum coordinators, or tangible materials such as curricula, standards, or observation protocols. Leaders should draw from these available resources for building their own capacity in a subject area. Postholing refers to the process of learning a slice of one subject at a very deep level. In this way, administrators gain an understanding of how the subject is constructed, what conceptual meaning looks like in that subject, and how students come to understand the content. Administrators should have a firm understanding of one discipline. With regard to knowledge of other disciplines for which they are responsible, Stein and Nelson recommend postholing when providing instructional leadership.

In addition to the theory of leadership content knowledge, we drew from complexity leadership theory (Uhi-Bien et al., 2007) in analyzing the data for this study, because school leadership is a multifaceted arena, and our administrative participants held different evaluative roles within this space. This theory purports that there are three ways in which leadership manifests itself in a knowledge building environment: administrative leadership, adaptive leadership, and enabling leadership. A knowledge building environment positions organizations as “complex adaptive

systems that enable continuous creation and capture of knowledge” (p. 301) and we believe that schools fit this description. Administrative leadership acknowledges the bureaucracy inherent in managerial leadership. This form of leadership takes a top-down, authoritative approach, which allows for assertive decision making. Within the realm of complexity leadership theory, administrative leadership considers the organization’s need for adaptability and creativity as this consideration benefits the decision-making process and outcome. One example of this form of leadership in schools is the administrative use of an observation protocol. The administrator has the authority to use this protocol in an evaluative way. Adaptability and creativity come into play when the administrator adapts the protocol or the process for using it in a way that honors the teacher’s professional goals. In contrast, adaptive leadership considers the fluidity and interactive nature of leadership that “produces adaptive outcomes in a social system” (p. 306). Adaptive leadership promotes change in an organization and does not result from one individual or entity but rather dynamic interactions between people and ideas initiated by a problem or struggle. Finally, enabling leadership assists in the emergence of adaptive leadership by providing resources, structures, systems or facilitating dynamics that catalyze adaptive leadership. “Catalyzing refers to activities that bring together the enabling conditions (mechanisms and contexts) necessary for adaptive leadership to emerge” (p. 309). Enabling leadership promotes interdependency, and complexity leadership theory posits that leadership exists in, and is a function of, interaction.

For the purposes of this project, we saw complexity leadership theory as providing a framework for the interactions among observers who held different roles in the school system (e.g., central office administration, principal and assistant principals, and mathematics specialists) and their interactions with veteran and novice teachers. Additionally, complexity leadership theory attends to the dynamics between these stakeholders and their material resources such as observation tools and curriculum and considers how these relationships and interactions fit into the larger school system. These dynamics may play a role in the nature of feedback that teachers receive.

We felt that leadership content knowledge and complexity leadership theory complemented one another in that both theories rely on interactions between leaders and personnel on multiple levels. We purported that leadership content

knowledge had the potential to promote enabling and adaptive leadership while administrative leadership would also be influential.

Method

We conducted a multi-case qualitative study using data collected from three middle schools located in three different school districts in a mid-Atlantic state. Our analysis was grounded in the theoretical frameworks (i.e., leadership content knowledge and complexity leadership theory), and as such we chose a multiple case design and employed replication logic as means for increasing external validity of the study (Yin, 2003). Because of the similarities among the experiences of the four teacher-observer groups, findings are reported as a cross-case analysis (Yin, 2003). In this way, single cases are not presented separately; they are threaded among the four themes, which frame the findings.

Site and Participant Selections

The study topic, middle grades mathematics teacher feedback, was initiated by a group of superintendents who were members of an educational research consortium. Members of this consortium included faculty from one university and school personnel from several school districts who were all located within the same metropolitan area. As part of this consortium, school district personnel worked in partnership with university-based researchers on investigating problems of practice within their districts. In doing so, researchers and school-based personnel formed a study team and worked collaboratively in identifying the study questions and designing the project. The school-based personnel each worked with the university researchers to identify and recruit participants from their school districts. The study design called for three-person teams consisting of two administrators, who had different levels of formal mathematics education or experience, and one teacher. These teams allowed for the teacher to be observed by each administrator (on separate occasions) and given feedback about his or her instruction. Four teams of teacher-administrators volunteered to participate in the study. Three of these teams included two administrators with contrasting mathematics backgrounds and one teacher. The fourth team included one teacher and one administrator who did not have a formal mathematics background (See Table 1). These participants were employed in three different schools and districts. The

administrators completed surveys detailing their level of mathematics education/experience prior to the start of the study.

As noted in Table 1, the administrators held different roles within each of their school districts and these roles played a part in our analysis. Because Jennifer Garcia was employed as a mathematics specialist with Madison school district, she did not formally evaluate teachers. Therefore, Ms. Garcia will be referred to as a non-evaluative observer. The other five administrators all held evaluative roles in their districts and will be referred to as evaluative observers. When referring to the whole group of administrators (evaluative and non-evaluative), we will use the term observers. In the interest of protecting participants' identities and also to maintain the multi-case reporting in the aggregate, the pseudonyms used in Table 1 will not be used when reporting the data.

Data Collection Procedures and Sources

Data collection was conducted at the participating middle schools both in the classroom and during post-observation meetings in locations chosen by the teachers and observers such as offices or the school library. Data sources included: field notes taken during observations of teacher instruction; teacher evaluation and/or post-observation written documents; teacher lesson plans or other classroom artifacts; semi-structured interviews conducted individually with teachers and observers; mathematical background surveys completed by observers; and teacher evaluation protocols for each participating district.

Teachers were each observed on two occasions, one time by both the first author and an observer with a mathematics background and a second time with the first author and an observer without a mathematics background. Researchers recorded detailed notes during these observations. Observations were unannounced but the teacher was aware that a researcher and an observer would be watching his or her instruction within a two-week timeframe. Following instruction and the observer/teacher post-observation conference, the first author interviewed each observer and teacher separately. These interviews averaged 30 minutes each, were audio recorded, and were transcribed by the researchers. All but one observer, Margaret Dade, fully participated in this observation and interview process. Upon initiation of this study, Ms. Dade had already observed the participating teacher and engaged in a post-observation conference. Hence, the teacher shared

Table 1: Participants

School District	Teacher	Observer
Madison # of students: 19,400 Free/reduced meals: 17.17% Student demographics: White: 83.31% Black: 9.60% Hispanic: 2.83% Other: 4.24%	Mary Thomas # of years teaching: 11 years	Beth Smith Role: Assistant Principal 13 years science teacher, history & language arts 2 years; liberal studies/elementary education undergraduate; administration masters <hr/> Jennifer Garcia Role: Mathematics Specialist Master's Degree in Education with focus on mathematics; 23 years teaching mathematics; mathematics undergraduate
Madison Same as above	Lisa Niles # of years teaching: 3 years	Beth Smith See above. <hr/> Kate Rand Role: Central Office Administrator Master's Degree in Supervision with mathematics endorse- ment; 10 years mathematics teaching; elementary and middle education undergraduate with high school endorsement
Washington # of students: 2,451 Free/reduced meals: 28% Student demographics: White: 71.2% Black: 23.16% Hispanic: 3.7% Other: 1.2%	Rob Russo # of years teaching: 16 years	Carol Jones Role: Principal Counselor, Secondary Educational Leadership certificate*
Jefferson # of students: 58,000 Free/reduced meals: 36% Student demographics: White: 54.31% Black: 26% Hispanic: 11.54% Other: 7.87%	Ann Mayer # of years teaching: 1 year	June Flowers Role: Assistant Principal 7 years mathematics teacher; liberal studies undergraduate; k-8 mathematics and administration masters <hr/> Margaret Dade Role: Principal Special Education*

*Ms. Jones and Ms. Dade did not complete the background survey. The information about the backgrounds of these participants in Table 1 is taken from our interview data and is not necessarily a complete description of each of their academic or professional experience but we were informed that neither of them had formal mathematics education or professional experience. All names are pseudonyms.

her perception of this conference and provided the written feedback she received. Teachers also shared the written evaluations they received from the observers during their post-observation conference and some provided lesson plans or other classroom artifacts.

Data Analysis

We used qualitative methods to analyze the data in this descriptive, exploratory study, employing three phases of analysis. In phase one, we analyzed the teacher and observer interviews using open and axial coding (Strauss

& Corbin, 1990). Although we were not seeking to develop theory, we felt that open coding was most appropriate given the limited research done on this topic. Given that our theoretical framework influenced the design of the study, we gathered data which also adhered to these guiding principles. Specifically, with regard to complexity leadership theory, we considered the many roles evaluative observers played that impacted their observations and feedback in terms of the administrative, adaptive, and enabling leadership. When coding, we recognized that, administratively, evaluative observers were required to use protocols that influenced the type of written feedback they provided. Similarly, we considered the ways in which observers interacted with teachers and provided feedback with potential for enabling change in this process. We also maintained that leadership content knowledge manifested itself in different ways depending on the leader's role, responsibilities (e.g., evaluative, non-evaluative), and depth of content knowledge. During coding, we explored these variations in leadership content knowledge and also considered the overlap between leadership content knowledge and complexity leadership theory. Hence, our open codes included many references to the nature, development, and perceptions of feedback through the lens of the constructs surrounding complexity leadership theory and leadership content knowledge. We then grouped these open codes into sensible themes during the axial coding phase. In this way, we came up with specific themes such as *inductive and deductive approaches to developing feedback*. Finally, these themes were analyzed against the written feedback teachers received in their post-observation conferences along with our observation notes in search of confirming or disconfirming evidence.

To increase the internal validity of the study, we employed a peer-debriefing process (Creswell & Miller, 2000; Lincoln & Guba, 1985). The first author conducted the primary data analysis and the second author acted as a peer debriefer. Although the second author had been involved in the study from the outset, she was primarily involved in developing the literature review and transcribing interviews, which allowed her to maintain a more objective role. Her professional and academic background was in educational psychology, which positioned her well to counter the first author's bias as a mathematics educator. The focus of the peer debriefing process was to carefully look for overemphasized points, underemphasized points, vague descriptions, general errors in the data, and biases or assumptions. To do this, the peer debriefer read the

findings and compared these to the raw data. Based on this analysis and her accompanying report, the first author made modifications to the findings such as including more descriptive terminology for underemphasized points.

Findings

The design of this study involved a descriptive analysis of the differences in post-observation feedback provided to teachers, the teachers' and observers' perceptions of the feedback, and the alignment with the employed observation protocol. Feedback took both oral and written forms, with the written feedback from observers being influenced by the evaluative roles and school district protocols. Within the framework of leadership content knowledge and complexity leadership theory, three major themes emerged with regard to how different mathematical backgrounds and/or evaluative roles of observers influenced their feedback: the form of the feedback (written and oral); the process for developing feedback (inductive and deductive); and the nature of feedback (content or pedagogical focus).

In this section, we begin by providing an overview of the two forms of feedback teachers received (i.e., oral and written), and the participants' perceptions of each of these forms of feedback. Next, we describe the differences in the approach observers, with different mathematical backgrounds, took to documenting observations and follow this with an exploration of the contrast between the natures of feedback produced from these observations. We conclude with a description and analysis of teachers' perspectives of the alignment between their evaluations and the mathematical learning goals of the observed lessons.

Forms of Feedback

The evaluative role of the observer seemed to influence the type of feedback provided (oral and/or written) to the teachers in that the non-evaluative observer focused more on oral communication with some written narrative, and the five evaluative observers prioritized written feedback and contextualized this with some level of oral discussion. We begin by describing the written and oral feedback the teachers received and participants' perspectives on the importance of these forms of feedback.

Written feedback. All observers commented on the importance of written feedback for teachers' reference and reflection, and our analysis revealed that the evaluation

FIGURE 1.
Pre-designed Observation Protocol.

Evidence: STANDARD 1 - Professional Knowledge

STANDARD 2 – Instructional Planning

- ☐ 2.2 Plans time realistically for pacing, content mastery, and transitions.
- ☐ 2.3 Plans for differentiated instruction.
- ☐ 2.4 Aligns lesson objectives to the school's curriculum and student learning needs.
- ☐ 2.8 Uses comprehensive materials and resources to support student learning.

protocol, required by the districts, influenced the nature of the written feedback. Five of the six observers held evaluative roles and one held a non-evaluative role. As part of the teacher evaluation process, school districts required the five administrators to submit pre-designed observation protocols, aligned with the state standards for the professional practice of teachers. These protocols included space for observers to include narrative descriptions of the observation; identify observed professional standards by checking boxes associated with each standard and sub-standard; and in one case, rate the level of observed implementation of professional standards on Likert scales.

Analysis of the written documentation revealed that the design of the protocols influenced the amount and nature of the feedback provided to teachers. Many evaluative observers, who used these protocols, included verbatim, scripted documentation of the interaction between teachers and students. For example, one evaluative observer scripted the teacher's actions and dialogue to document the instructional delivery standard: "With an orange marker, Ms. (teacher name) wrote a top, bottom chart on several students' papers. . . Student asks, 'Can I use a calculator?' You can do it first without a calculator, then check with a calculator." Scripting was consistent in all of the five evaluative observers' written documents.

In addition to scripting, evaluative observers checked boxes indicating that a teacher met various parts of each standard. One of the district's protocols included a pre-designed narrative describing teachers' attainment of each standard, as illustrated in Figure 1. All teachers in this district, regardless of discipline, received the same narrative feedback for each standard and custom feedback at the end of the protocol in the form of overall comments.

In all three districts, evaluative observers provided some level of personal, written feedback to the teachers, apart from the scripted documentation of the lesson or the pre-determined text. These narratives ranged from three sentences to two paragraphs in length. The following is an entry from one of these evaluation summaries.

Mrs. [Teacher] is a valuable member of the [school] staff and the math department. She led a school improvement standard committee and was instrumental in planning activities that added to the positive climate this year. Her efforts to work with the team on grade level and with our math coach are commendable. Her warm and friendly demeanor, coupled with her professional knowledge make her an outstanding instructor. Mrs. [Teacher] has played a very active role in our school both in and out of the classroom.

The above quote illustrated feedback that was general in nature and did not directly address the instruction. This feedback did not align with our chosen feedback definition, which indicated the specific teacher's instructional choices: "information provided by an agent regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007, p. 81).

The checked boxes, scripting, and general language used in the above examples provided the teacher with information regarding his or her performance, albeit at a very general level. Due to the lack of specificity, none of these forms of written dialogue attended to the individual teacher's understanding. The pre-designed protocol seemed to influence the level of detail pertaining to the uniqueness of the teacher's instruction, which directly impacted the narrative's alignment with the feedback definition. Devoid

of oral feedback that contextualized the protocol, these written documents did not represent our working definition of feedback because of the lack of specificity.

The mathematics specialists' non-evaluative role excused her from submitting a pre-designed district protocol for the teacher's evaluation file. She developed written feedback in the form of open notes during the post-observation conference with the teacher. The non-evaluative observer commented that she was interested in maintaining a non-evaluative relationship with the teacher and did not script observations. Instead, she focused on building teacher capacity for reflective thinking as evidenced by her statement:

So when I meet with them I try to look at more – get them reflective thinking about what they did, how it worked, what could we do differently, and I think the angle is here is what these students need to know.

This statement illustrated the non-evaluative observer's interest in crafting a conversation centered on the unique needs of the teacher's students and helping the teacher reflect on his or her practice. The written documents from her post-observation conference included four quadrants titled (1) what's working, (2) focus-concerns-challenges, (3) teacher's next steps, and (4) coach's next steps. The non-evaluative observer and teacher collaboratively responded to each of these quadrants during the post-observation conference. Examples of this feedback included: "Couple of the numbers were too hard (changed the order in subsequent blocks)," and "Continue to develop activities to engage students." This feedback was noteworthy because the observation protocol (or absence of a protocol) influenced the form of the observer's written feedback. The non-evaluative observer took an inductive approach to observations and was not required to link her written observations to a set of pre-determined standards. As a result, the written feedback aligned with our aforementioned definition in that the non-evaluative observer provided information specific to the teacher's instruction. This differed from scripted or Likert-style protocols in that these written, standardized protocols documented teacher actions but did not speak directly to the teacher in a personalized way. These forms required a follow-up conversation to situate the narrative and, hence, the written documentation was not feedback but instead evidence to support the oral conversation.

Oral feedback. Both teachers and observers noted the importance of oral post-observation feedback. Teachers expressed that engaging in discussion about the complex interactions occurring in the classroom was more beneficial than only reading written feedback. For example, one teacher commented, "For me what I take out of it is what I hear from them." Another teacher explained, "The one-on-one conversation is more effective than this [written feedback] because I can sit here and read this but . . . I take so much more from talking to someone than just reading through it." Another teacher described the importance of conversation because of the emotional and physical characteristics embedded in communication. Similarly, observers felt that oral communication provided an opportunity for contextualizing the feedback and several of the observers credited this conversational feedback with teacher understanding. One observer stated, "The oral piece is what helps teachers understand what you can't always say because you're limited to a document or a form." These quotes showed that both teachers and observers valued the opportunity to have an oral discussion of the written feedback. These statements implied that feedback provided only in written form may not include enough information.

Oral feedback also provided an opportunity to solve problems of practice. "I think that your problem-solving piece comes out of the oral discussion with teachers if there is a problem. It doesn't come out of the written piece usually." This statement aligned with the idea that oral conversations provided more context than written feedback and allowed teachers and observers to address issues. Even in situations when the observation protocol did not require a post-observation debrief, the evaluative observers commented on the importance of finding the time to discuss written feedback.

For informals and formals, I tend to still schedule that conversation especially if it's not someone who necessarily knows me, because I think you begin with the conversation but you try to capture what you said in written form for people to go back and reflect on.

As noted, teachers and observers valued the oral communication surrounding observations. The literature showed the importance of feedback to teachers (Scheeler et al., 2004), and in our study, observers also held such strong feelings toward oral feedback that they sought this discussion out even when it was not required, as illustrated in the above quote.

Summary. Although observers' mathematical backgrounds did not influence their choice of including written or oral feedback in their post-observation communication, the evaluative observers' roles required them to use a pre-designed protocol. This protocol illustrated a form of administrative leadership (Uhi-Bien et al., 2007), while the non-evaluative observer's freedom in selecting the focus of her feedback was not influenced by an administrative provision. The use of a pre-designed protocol seemed to limit the specificity of the feedback, and hence, observers and teachers expressed appreciation for engaging in oral dialogue in an effort to contextualize the written documents. The non-evaluative observer's freedom to develop her own narrative supported her in choosing the focus for the observation and ensuing discussion.

Processes for Developing Feedback

While observers' evaluative roles influenced the form of written feedback that they produced in terms of aligning observations to professional standards or providing open notes, their mathematical backgrounds also seemed to influence the approach each took when engaging in their observations and developing feedback. Observers with mathematical backgrounds took inductive approaches and observers with non-mathematical backgrounds took deductive approaches to preparing feedback. As noted, evaluative observers were required to submit a post-observation protocol for the teacher's professional file, but they were not required to use these forms when developing feedback. We found that the evaluative observers who did not have mathematical backgrounds chose to use the post-observation protocol as a guideline while observing teacher instruction and those with mathematical backgrounds did not.

Inductive approach. When asked to describe the process that they used to prepare feedback to teachers, all three of the observers with mathematical backgrounds described an inductive approach to documenting observations and preparing feedback, taking extensive notes while observing the teachers' instruction. These accounts were supported by the first author's observations in the classrooms. This process for documenting teachers' discourse and actions did not begin with a pre-determined list of standards or look-fors; instead, each observer used his or her expertise, experience, and teacher's personal growth goals to decide where to focus her attention. One observer described her process in this way: "I type everything, I just take notes the whole time and then I ask question within my notes."

Another observer began her observation by documenting student engagement and student-teacher mathematical discourse on a student roster then summarized those notes for use in the post-observation conference. Similarly, the third observer listened for student discourse and documented circulation and student activity on a seating chart. Her focus was on patterns that emerged during instruction. This observer described her process (in part) in the following way:

Initially, I try to take in everything and see if I see a pattern emerge. You'll see lots of different data points in my notes. I tend to capture a lot of questions that teachers are asking so I'm really looking at that level of questioning and engagement. I look for what students are saying . . . so that's my entry point of conversation for her, is to kind of present what I've collected in my observation and for the teacher to really have a point of analysis with it before I make a judgment or suggestion. . . . I'll also take that and put it into the seven standards.

In addition to aligning the narrative with the professional standards on the district's evaluation protocol, the observer also included a copy of her observation notes to the teacher (Figure 2, next page). All three observers with mathematics backgrounds commented that their open notes drove their post-observation conferences, and two of the three provided these notes to the teacher for the purpose of teacher reflection.

The above quotes represented a selection of observers' descriptions for inductively developed feedback. These quotes showed the observers' focus on interactions within the classroom, which is different from a focus on matching a list of standards with the classroom interaction. These observations were then used to drive the post-conference between observer and teacher.

Deductive approach. The two evaluative observers with non-mathematical backgrounds who were interviewed for this study stated that they used their district's evaluation protocols when developing notes for teacher feedback. These statements were supported by the first author's observations of these classroom visits. One of these evaluative observers explained that she looked for an engaged classroom environment, with an agenda, standards, and objectives posted on the board, and a variety of other management strategies such as bell ringers and transition time. The evaluative observer used the district's electronic

FIGURE 2.

Inductively generated observation notes.

Teacher	Student
• J What does Quotient tell us?	None (individual questions to teacher during ind. work?)
• J Does this look similar to what the difference?	
— C How would I write 3 times a number?	
— C Do I have to have the (?)?	
— J What do you think? Does that match this?	
• C What does the equation say?	
• C What part makes it an equation?	
— J What symbol does an equation have that an expression doesn't?	
— C What is the relationship? Key word?	
• C What do I need? (=)	
— Do you agree?	
— What's the goal?	
• Why did she bring in $\frac{1}{2}$?	
• J What property does this represent?	
— C Can you show me how to solve it?	
• C What is who is at the board?	
— C Why are you doing that?	
— C What do you need to do now?	
• M Is there a way you could $\sqrt{100}$?	
— C Can you tell me why?	
— C Why?	<p>1 - responded to question or spoke B - worked problem on board □ - no oral participation in class discussion</p> <p>Student Problem Solving</p> <p>40 minutes • 5 problems - one student works on whiteboard modeled class observes</p> <p>• 4 problems on worksheet - students completed independently; then worked by one student on whiteboard</p> <p>• HW (4 problems)</p>
— C Why?	
— C Why did you draw red lines	
— J Does it matter what side the "x" is on?	

evaluation protocol to rate the level (i.e., exemplary, proficient, developing, or unacceptable) for which she observed the standards that aligned with her observations. For example, Standard 3.5 stated, "Uses a variety of effective instructional strategies and resources," and the evaluative observer checked a box for exemplary. She also provided a narrative summary at the end of the document.

The other evaluative observer also used an online system, but the teacher received the written feedback in hard copy form during the post-observation conference. This evaluative observer used the district evaluation protocol to structure her note taking during the observation and then re-organized her notes prior to discussing them with the teacher. Her process is illustrated in the following quote.

While I'm watching the class I have this form on my computer, and taking notes, and I really do go back and forth between the seven standards. So a lot of what I saw today in the lesson. . . . I documented under instructional delivery because that's usually what you see the most of, you're giving feedback on the delivery. But then as I go back to it later in my office. . . . I'll dis-

sect it a little bit more and figure out where would then our district form fit.

These statements represented the ways observers without mathematics backgrounds approached observations and feedback generation. The pre-designed form, containing the seven professional standards, provided a focus for observation.

Teachers' perspectives. Three of the four participating teachers received feedback from both an observer with a mathematical background and an observer with a non-mathematical background, which enabled teachers to make comparisons across the two different sets of feedback. There were notable differences between teachers' perceptions of the feedback they received in cases when the inductive, open notes were shared and cases where teachers only received the formal observation protocol. All teachers appreciated feedback, regardless of the format, but the two teachers who were given the open notes were particularly impressed by the specificity and comprehensiveness of the narrative. These teachers viewed the purpose of feedback as a means for improving instruction

and the open narrative as an added support to the formal observation protocol.

In comparing the inductively versus deductively designed feedback, one teacher indicated that both were valuable and the open notes coupled with oral conversation provided specific information about instruction that supported the formal observation protocol. Below is her description of the feedback from the two observers.

[Math specialist] is a math person and so she can come in and give me very specifics of things that she thought I should touch on. . . . She would say, “Hey you really need to emphasize this a little bit more,” and [administrator] is going to give me the nuts and bolts, making sure everything stays together. [Math specialist] gets to dig a little bit deeper into the math end.

This teacher appreciated the specificity of the inductively developed feedback and held a particular appreciation for the content focus of the feedback.

The other two teachers in this study received the formal observation protocol from their evaluative observers and did not see any open notes from these observations. One of these evaluative observers prepared her observation using an inductive approach, but these notes were not shared with the teacher in written form and were only summarized into the standards of the formal protocol. The teacher in this situation described the purpose of the feedback as documenting evidence of her work for her accountability and “keeping everyone on task and in the right direction.” She commented that her mathematics-focused evaluative observer (who developed inductive open notes but did not share these with the teacher) provided valuable suggestions specific to improving her mathematics instruction, and she noted that the difficulty lay in that these were not written down. The teacher described her conversation with this evaluative observer as discussing the formal observation protocol bullet points and “on top of that she’ll tell me, well here’s how we work on that because she’s had experience in the classroom teaching math.” This case illustrated the teacher’s perception of the value in mathematics-specific feedback.

Similarly, the other teacher who did not receive this form of inductive feedback indicated a preference for mathematics-specific suggestions, which were not present in the formal observation protocol. While this teacher (and

all teachers in this study) highly valued the feedback they received, it was noted that immediate changes to classroom instruction often resulted from pedagogical, content-specific suggestions.

If it is an administrator talking about the layout of the classroom or student engagement that feedback might take longer to implement but when a colleague comes in and says, “Well you are using the slide and divide method for factoring and we really want you to use grouping,” that is an immediate change I can make.

This teacher compared the ease of changing a specific strategy for solving a mathematical task to the difficulty in changing a broad classroom culture such as student engagement. Again, this statement supported the importance of concrete feedback.

The data indicated a relationship between mathematics content knowledge, inductively developed feedback, and the level of specificity of the feedback. These characteristics played a role in teachers’ perceptions of the purpose of feedback. Teachers perceived instructional improvement as the goal of feedback developed inductively and “keeping everyone on task” as the goal of deductively developed feedback. It seemed that the comprehensiveness of inductively developed feedback provided teachers with information centered on their instruction as per Hattie and Timperley’s (2007) feedback definition. Indeed, it is not surprising that teachers perceived instructional improvement as the goal of this form of feedback.

Summary. As shown in Figure 1, evaluative observers using a deductive approach relied on the pre-designed district protocol as a framework for the criteria in an observation. Observers who used the inductive approach explained that they were looking at classroom discourse, development of mathematics, student engagement, and activity. The standards listed in the pre-designed protocol seemed to support observers with non-mathematical backgrounds by providing a focus for their observations. Alternatively, observers with mathematical backgrounds chose not to use any observation protocol and relied on their content expertise to drive their observation.

Feedback Focused on Content and Pedagogy

All participants in this study believed that the most effective form of feedback came in specific and concrete suggestions for improving instruction. Moreover, all

four of the teachers observed a difference in the nature of feedback, and hence, the kinds of suggestions, they received from observers who had formal mathematical backgrounds and those who did not. Analysis of the written documents supported teachers' perceptions of these differences with observers who had mathematics backgrounds providing more mathematics-focused feedback and observers with backgrounds from non-mathematics disciplines focusing more on pedagogy and classroom management. Observers recognized challenges in providing feedback outside of their content areas and provided examples of the ways in which they addressed these challenges in order to provide effective support for their teacher colleagues.

Teachers' perspectives. Teachers perceived the nature of feedback from observers with mathematics backgrounds as focused on the development of the mathematics in the lesson and the feedback from administrators with non-mathematics backgrounds as centered on general pedagogy and classroom management. When asked about the difference in the nature of feedback from each observer, teachers noted the ability of observers with mathematics backgrounds to (1) provide guidance on vertical alignment of content, (2) suggest mathematics-specific pedagogy, and (3) give recommendations for how to increase the level of questioning. When asked to describe differences in feedback between observers with mathematics backgrounds and those with different disciplinary knowledge, one teacher stated:

Certainly, someone with a math background would be able to look at my lessons, pick them apart, more so than someone without a math background. Um, because they are going to be willing to ask questions like, "Why do you use this method of factoring versus another" or "Why is it that you teach laws of exponents before you teach some other topic?" So they could ask more pointed questions. And I also think that person if they are evaluating the vertical team, going from the algebra to the geometry to the algebra II to the pre-calculus, they probably could give feedback along the lines of what you are doing is setting students up for the next level.

This quote detailed the teacher's belief that observers with mathematical backgrounds used their specialized knowledge to promote reflective thinking centered on instructional choices in a different way than an observer

without a mathematical background. Moreover, this teacher suggested that subject matter knowledge has the potential for supporting the teacher's vertical articulation of the content.

Similarly, teachers perceived the feedback they received from observers with non-mathematics backgrounds as focusing on instructional strategies and classroom management. For example, one teacher speculated, "I think [administrator with non-math background] will talk to me about how everything ties together. Like the professional knowledge, the classroom behavior and demeanor, my management system." Another teacher described the type of feedback she received from her evaluative observer as focusing on discipline and classroom management with suggestions such as using popsicle sticks for selecting students, working on transitions, and other classroom management tricks. A third teacher stated that these observations focused on a broad spectrum of topics such as scaffolding for students with individual education plans, behavior, a little bit of content knowledge but "less focused on specific math content rather than more so everything overall." The fourth teacher noted that the evaluative observer with non-mathematical background "is going to look at classroom environment or classroom engagement or those types of things." As shown in these statements, teachers recognized the feedback from observers without mathematical backgrounds as focused on student behavior and classroom management with little to no content focus.

Our analysis of the written observation protocols supported these perspectives. Specifically, observers with mathematics backgrounds focused on the development of the mathematics. For example, one observer documented, "Students were asked questions that required them to draw on prior knowledge and connect new learning to prior learning. Examples: What does the quotient tell us? What property does this represent? Does this look similar?" This feedback recognized the importance of utilizing prior knowledge, which aligned with the NCTM (2014) evaluation recommendations. Another observer noted, "Students were able to readily manipulate algebra tiles (1, -1, and $-x$) which indicated that the use of modeling and the use of algebra tiles has been part of instruction to develop conceptual understanding." This statement highlighted the importance of modeling mathematics for developing conceptual understanding, which is a tenet of mathematics education.

The inductively developed feedback included more specific details surrounding mathematics content than deductively developed feedback. The process for developing this feedback may have influenced the specificity of the information provided to the teacher in that observers were not limited to a pre-designed list of standards. As noted earlier, much of the written narrative from evaluative observers with non-mathematical backgrounds was in the form of scripting. Evaluative observers also made broad, general statements such as:

You consistently incorporated 21st century skills in your delivery. It is evident that students enjoy your class, understand the content and are able to apply what they know. The learning environment you have created challenges students and is actively engaging.

The protocols used in deductively developed feedback included general statements about instruction. While these may have aided evaluative observers in providing feedback, they also seemed to impede the specificity of the written suggestions provided for instructional improvement. Arguably, the use of the protocols may have inadvertently impacted teachers' perceptions of the goals of the observation-feedback cycle.

Observers' perspectives. All observers indicated that there was a difference in the level of discipline-specific feedback they provided when they were conducting observations outside of their content area than when they were observing in their own field. They noted the value in collaborating with administrators or teachers in each discipline to support their understanding of these fields and, in turn, use this understanding to provide content focused feedback. This interdependent practice aligned with Stein and Nelson's (2003) definition for distributed leadership and these interactions enabled adaptive leadership (Uhi-Bien et al., 2007) in that, together, stakeholders worked toward solving problems of practice. When asked to describe the kinds of difficulties they faced in providing effective feedback, one evaluative observer commented:

My personal difficulties lie in that I don't have a math background, so in order to talk about math with someone who has been teaching Geometry and Algebra 1 for years, I have to do a lot of thinking about math that, it's not part of my background, it's not innate to me.

This statement reflected the challenge felt by this observer in developing feedback outside of her content area. This disconnect between content preparation and the observation cycle presented itself in all content areas, as some observers with mathematics backgrounds recognized their difficulties in conducting observations and providing effective feedback in other disciplines. One of the observers described her challenges and strategies for overcoming these difficulties when observing in oral language classes.

For me probably one of my most challenging was oral languages. I had no background in it. . . . I couldn't give them content feedback. . . . That's when I called a lead teacher specialist from central offices and. . . we would do the observation in tandem but then often talked about it together so that lead teacher specialist in world language would say. . . look for these types of things. . . . There are usually resources within your district to help you if you've been given a department which you really it's not your background to try to build that capacity to give me full feedback. . . . Instructional strategies, student engagement, that would be more where I probably would end up giving more feedback.

This observer noted his/her inability to provide content-focused feedback to a teacher and the value in taking advantage of district resources and other colleagues for support. Similar to teachers' perceptions of feedback from observers with different content backgrounds, this observer commented that, without the support of using other resources, he/she was limited to providing feedback about student engagement or instructional strategies.

These observers referenced their abilities to provide guidance about general practices with regard to classroom management and instructional strategies. They translated their expertise and experiences in other subjects to other classrooms. One observer without a mathematical background noted deep understanding in at least one discipline or teaching practice and described drawing from this understanding and collaborating with a mathematics peer to build a knowledge base for providing feedback to mathematics teachers (Stein & Nelson, 2003).

I work with my math, we have a math coach who is part time with us and part time at the high school. So if I see something I will say, "Coach, tell me about this, I saw this in a math class, why were they doing that?" And she will say, "I don't know, that's crazy," or she will

say, "That makes perfect sense to me, let me tell you why they are doing that." And so if I see something mathematical that I don't get she'll help me. I guess I have to say I rely on her a lot.

This statement shows the evaluative observer's use of a peer as a resource for helping the observer better understand the mathematical content which, in turn, aids the development of content-focused feedback. All observers indicated difficulty in providing feedback outside of their content area (mathematics or otherwise) and used their expertise in instructional strategies and classroom management to parlay this challenge into beneficial feedback for the teachers. These observers recognized that, when working with teachers from different disciplines than their own, they needed to draw from another colleagues' expertise. Leaving the content expertise to a peer allowed them to focus on pedagogical and behavioral aspects of instruction. This interaction between observers and available content-focused resources illustrated a form of adaptive and enabling leadership (Uhi-Bien et al., 2007). Adaptive leadership takes advantage of dynamic interactions when seeking to make a change in an organization or structure and enabling leadership provides the resources or tools to enable this change to occur. In this way, the evaluative observers were enabled by their content area colleagues to adapt their leadership knowledge in a way that enabled this observer to provide content-focused feedback to the teacher.

Teachers' feedback preferences. The teachers appreciated both content-specific and general pedagogical and management feedback for different reasons. They were particularly enthused about receiving suggestions or commendations that focused on the mathematics content and recognized this as critical to their instruction. Teachers noted that observers with mathematics backgrounds provided specific suggestions for improving instruction, and these observers also acknowledged the positive aspects of the mathematics in the instruction. One teacher noted:

She definitely has a math background and she touched on a lot of the content. . . . I feel like maybe there is a little bit more recognition of my content knowledge by somebody who has a math background recognizing the way that I'm saying things, how I'm saying things, how I'm scaffolding things. Maybe being able to recognize the thought that I put into how I conduct my lessons based on the math content. So maybe it's, the

feedback is not any less significant by those who aren't math-content related but there are certain things that are capitalized on and are more noticed by those people than I guess the ones who are not.

Similarly, a different teacher offered:

I do like it when someone who does have a math background so that they can either share specific examples of how they taught that or how they would teach that or just recognizing how I'm teaching the content. So I do, I think I do prefer someone with a math background and I do think that sometimes things are noticed more by the math people, like whether it's an assessment or a warm-up, noticing that was a really great math content that you pulled in there and how you pulled that in.

These two quotes represent a larger group of statements by all of the teachers interviewed. The teachers all noted their affinity for receiving content-focused feedback. While teachers appreciated math-specific feedback, they also saw great value in pedagogical and behavioral feedback. Teachers noted that the evaluative observers detected different aspects of the mathematics instruction than they would have recognized themselves, and they broadened the teachers' understanding of instructional strategies. There was a consensus among evaluative observers and teachers that feedback took on a different focus based on the lens of the observer. Evaluative observers used strategies to help them provide the most useful feedback they deemed possible, while teachers appreciated receiving both forms of feedback and made use of these forms for different purposes.

Discussion

This exploratory study looked at the nature of feedback provided to middle school mathematics teachers from observers with differing content expertise. The findings extended the research literature pertaining to discipline-specific feedback, particularly in mathematics education (Nelson & Sassi, 2000). Through the lens of leadership content knowledge and leadership complexity theory, we found that observers used their subject-specific knowledge and past experiences to develop feedback for teachers, which supported current research (Lochmiller, 2016; Steele et al., 2015), and their evaluative roles required them to use a pre-designed observation protocol, which influenced

the depth and form of written feedback that teachers received.

In this study, we recognized forms of administrative, enabling, and adaptive leadership (Uhi-Bien et al., 2007) in the ways that observers approached the task of providing feedback. In particular, evaluative observers were required to submit a pre-designed protocol, which influenced the feedback that they provided to teachers and illustrated a form of administrative leadership. Concurrently, some observers provided teachers with comprehensive, specific feedback in both written and oral form, which enabled teachers to reflect on their instruction, and, in some cases, held potential for adaptive leadership. Uhi-Bien et al. (2007) defined adaptive leadership as “a collaborative change movement that emerges nonlinearly from interactive exchanges” (p. 306). Indeed, we observed these interactive exchanges between teachers and observers and recognized the potential for adaptive change resulting in the form of instruction from these exchanges.

We speculated that the difference in the inductive and deductive approaches that observers took to documenting classroom activity was, at some level, attributed to the observers’ expertise in the content area. As the pre-designed observation protocols did not include discipline-specific standards, it is possible that the observers with mathematical backgrounds were interested in capturing the development of the mathematics and student learning of the mathematics, knowing that this could later be translated into more generic terms for the purposes of the evaluation protocol, hence, an inductive approach. In the same vein, administrators for whom mathematics was not their formal discipline may have used the evaluation protocol as a framework to direct their (deductive) observations because they were interested in observing non-discipline-specific instructional strategies. These assertions are merely speculation as our interviews did not include questions pertaining to why observers chose inductive or deductive approaches. The process for developing feedback is an area that holds potential for further investigation.

Our findings supported research citing teachers’ preferences for specific, concrete feedback (Northcraft et al., 2011; Price et al., 2010). All teachers in our study expressed a need for specific feedback that informed their instruction. Along these lines, teachers noted their appreciation for content-specific feedback indicating its immediate impact on instruction. Even in cases when the school district

did not require oral feedback, administrators engaged in post-observation discussions with the teachers, noting their importance. This finding aligned with Collins’ (2004) study, which recognized the value in providing feedback regardless of the level of student achievement or teacher instructional capacity.

The findings from this study most notably pointed to the difference in the nature of feedback to middle grades mathematics teachers from observers who had formal mathematics education or experience and those who had different subject backgrounds. Similar to Nelson and Sassi’s (2000) findings, observers in this study with mathematics education or experience focused on the development of mathematical ideas while the other observers looked at structure or management aspects of the lesson. Furthermore, the feedback provided by the mathematics-focused observers included content-focused pedagogy, which indicated that content and pedagogy are intertwined and unique to each discipline (Nelson & Sassi, 2000).

Stein and Nelson (2003) purported that administrators do not need to be experts in all subject areas, rather a distributed approach to leadership yields a solution for leading teachers in various disciplines. In each of our cases, observers called upon their subject matter expertise, whether that was in mathematics, special education, counseling, or science, to provide instructive feedback to teachers. Evaluative observers conducting observations outside of their content expertise (both mathematical and other subject areas) described using mathematical resources, such as colleagues, to help them develop and provide effective, content-focused feedback, which exemplified a distributed approach to leadership. Their use of outside resources held the potential for developing observers’ understanding of the content. This approach coupled with deep content knowledge in their respective fields positioned observers well for postholing (Stein & Nelson, 2003). Although we did not observe postholing in our study, we did recognize the ways in which observers were employing facets of this process. Alternatively, observers with mathematical backgrounds provided detailed feedback focused on content while also integrating pedagogy.

Feedback has implications for teacher development and, in turn, student achievement; as such, it is critical that teachers receive productive oral and written feedback from their observers. This study was limited in that we were unable to observe the post-observation meetings between teachers

and observers. We were not granted access to these conversations because school district research office personnel felt that including an observer in these meetings would be invasive to the teacher. This form of data would increase the robustness of a study like this one and complete the picture for the full spectrum of feedback that teachers received. Hence, this study provided evidence that further research is needed in examining the nature of feedback that is provided to teachers from observers with different content backgrounds and administrative roles. Despite this limitation, the findings indicated teachers' preference for specific, content-focused feedback, which calls for a closer look at the type of subject-specific preparation observers received, similar to that proposed by Steele et al. (2015).

Implications for Mathematics Education Leaders

Our findings have several implications for mathematics education leaders centered on the form, the nature, and the generation of post-observation feedback to teachers. In our study, teachers considered feedback most effective when it was provided in both written and oral forms. Conversations surrounding the written documents presented teachers with opportunities to ask questions and discuss the observations. Additionally, this dialogue allowed leaders to contextualize the written feedback. Hence, mathematics education leaders should be purposeful about dedicating time to discuss observations with teachers, and these conversations should be crafted to meet the unique needs of the teacher and includes specific, concrete feedback for instruction.

Content-focused feedback provided teachers with specific, concrete suggestions for improving their practice. In cases where observers did not have the mathematical background to provide this form of feedback, they described their reliance on a colleague with subject matter expertise to support their thinking about mathematics instruction. Considering the importance of specific, content-focused feedback for teachers' professional growth, mathematics education leaders should seek opportunities to support and collaborate with observers from non-mathematical

backgrounds. Additionally, mathematics education leaders should advocate for observations to include multiple observers that include both observers with mathematical backgrounds and those from other disciplines.

Understandably, many school districts require leaders to utilize an observation protocol standardizing observations across disciplines. In these cases, teachers in our study particularly appreciated receiving written observation notes, generated inductively, in conjunction with the observation protocol. Mathematics education leaders should consider developing written feedback that goes beyond selecting proficiency ratings or scripting teacher and student dialogue. The written documentation should include content-focused feedback, that is centered on a specific teacher's instructional decision making and is supported by observation data.

Conclusion

In the current era of teacher accountability, a school district's evaluation model has the potential for influencing the effectiveness of the feedback teachers receive. Feedback is a critical component in instructional improvement and, hence, an important consideration when crafting plans for teachers' evaluation and professional development (Cherasaro et al., 2016). Research literature points to the need for increased attention toward content-specific feedback for mathematics teachers (Lochmiller, 2016). The current study addressed this need and expanded the literature by including the written feedback and perspectives about this feedback from observers in evaluative and non-evaluative roles, some with mathematics backgrounds and others with different subject matter expertise. Findings indicated differences in the form, development process, and content specificity between evaluators with mathematics backgrounds and those without mathematics experience or training. These differences seemed to impact the specificity of the feedback, which held the potential for influencing its efficacy. In conclusion, this study supports the current body of research surrounding the importance of specific, timely feedback crafted to meet the individual instructional needs of the teacher receiving the feedback. ★

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Math Labs: Teachers, Teacher Educators, and School Leaders Learning Together With and From Their Own Students

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Abstract

This article describes a structure for embedding professional development within a school day, which we call Math Labs. It enables teachers to come together, with the guidance of a teacher educator, to engage in collective inquiry into the teaching and learning of mathematics with time to experiment with new ideas with their own students. We explain the design principles, reflecting our commitments to equity and social justice, that motivate what occurs during a typical Math Lab. When Math Labs become an integral part of the school's culture, they allow teachers and school leaders to negotiate (1) how they position and empower students; (2) what opportunities they give students to learn rich mathematics; and (3) what shared professional values guide their inquiry into students' mathematical learning.

Introduction

We begin this article with a vignette to provide an image of what it might look like for teachers to work with their colleagues on complex aspects of mathematics teaching. The vignette features a professional learning structure called

Math Lab, which is designed to support teachers to build their understandings and skills as teachers who center mathematics instruction on students' ideas, empowering students to meaningfully engage in learning mathematics.

* * * * *

A group of third-grade teachers has planned a lesson together with their math coach to support students in making use of doubling when solving related multiplication problems. They are eager to see how their plans play out in a classroom visit and curious about what students will do. They have just posed 3×7 to one of their classes and plan to pose 6×7 next before moving on to a new set of problems with a doubling relationship (2×6 ; 4×6 ; and 8×6).

Savion, a third-grade student, is sharing how he thought about the problem 3×7 : "I did seven and seven and seven, and got 21."

Mrs. Brown makes a "T" with her hands and says, "Teacher time out." She looks around the room at her third-grade teaching colleagues who are sitting among the students and asks, "How should I represent Savion's strategy? I'm not sure if we should use a number line or an array. We talked about both possibilities in our planning."

One of her colleagues suggests, "Let's ask him how he thought about it." Mrs. Brown smiles agreeing with this

suggestion, looks at Savion and asks, “Savion, how were you picturing the sevens?” Savion again says, “I saw seven, seven, seven,” and moves his hand horizontally each time he says seven, one seven below the next, as if they were stacked, moving it down a bit for each subsequent group of seven. Mrs. Brown revoices Savion’s idea, saying, “It sounds like you were thinking of three rows of seven.” He nods. She turns her attention back to her colleagues and says, “In our planning, we talked about moving away from directly modeling each item. I’m thinking I’ll record this with the number 7 instead of dots. Does that sound okay?”

After seeing her colleagues nod in agreement, she draws the following:

7
7
7

She says, “Savion, does this match how you were thinking about it?” Savion nods his head and then Mr. Sampson signals a teacher time out and says, “One of our goals was to help students make sense of each other’s ideas. What if we asked a question about Savion’s strategy to the whole group? Maybe, ‘How does Savion’s picture help us figure out three times seven?’” Mr. Chen chimes in, “Or, we could ask, where do we see the three from three times seven in Savion’s picture?”

After hearing from two students who suggested ways of using Savion’s picture to solve the problem, Mrs. Brown returns to Savion and invites him to share how he thought about it. Mr. Sampson then trades places with Mrs. Brown and, as planned, takes over the facilitation of the lesson for the next part of the number string. He says, “Okay, I’m going to write the next problem in our string. I want you to ask yourself, ‘Hmmm . . . can 3×7 help me think about this next one?’” He writes 6×7 on the chart paper and says, “Let’s make sure everyone has some quiet think time. Show me a quiet thumb on your chest when you have an idea about 6×7 .”

As the lesson unfolds, Mrs. Brown, Mr. Chen, Mr. Sampson, and their colleagues work together to orchestrate a mathematical discussion, eliciting and responding to students’ thinking. In this lesson, they use a particular

instructional routine (i.e., number strings – posing related computation problems) as a focal point to learn more about their students’ ideas (Fosnot & Dolk, 2001). They also want to develop shared practices that support students in developing identities as capable mathematicians and create classroom environments where students believe that who they are and how they think matter. Their students love having their ideas listened to by so many adults on these Math Lab days. The students see their teachers working hard at listening to and representing their ideas and strategies. The driving purposes of the teachers’ collaboration enable them to move forward on their commitments to developing equitable learning environments.

* * * * *

What are these teachers doing and how does this work benefit them and their students? Embedded within the school day, educators work together during Math Labs to plan, enact, and reflect on the work of teaching. A unique feature of Math Labs is to allow teachers the opportunity to immediately try out new ideas with their own students and reflect with their colleagues. Although Math Labs are facilitated by a teacher educator, such as a building or district coach or a university-based teacher educator, the facilitator is not an expert who is there to demonstrate “how to do it right.” The structure organizes teachers’ workplace interactions by giving them opportunities to engage in collective and ongoing inquiry into the teaching and learning of mathematics. Our goal in this article is to describe the Math Lab structure, the design principles that underlie the structure, and the potential of the structure to support learning and school improvement. The descriptions are based on our collective experiences facilitating Math Labs over the last decade. We conclude this article with data that convey the role Math Labs can play in school improvement.

Supporting Educators’ Learning Together in Practice

Our goals for student learning in mathematics are complex, demanding, and even aspirational. Researchers in mathematics education have argued for a range of learning goals for students that attends to both procedural and conceptual fluency, engagement with disciplinary ways of knowing, and the cultivation of positive identities and agency with respect to using mathematics critically and meaningfully (Aguirre, Mayfield-Ingram, & Martin, 2013;

Gutiérrez, 2012; Gutstein, & Peterson, 2013; National Research Council, 2001; Turner et al., 2012). Goals for student learning have implications for what mathematics teachers need to know and be able to do, including cultivating learning environments in which students can do substantive mathematics and where students are treated as sense-makers and empowered to use mathematics in culturally meaningful ways.

Teaching mathematics is complex. It requires continual learning about the subject matter itself as well as how to make learning relevant and meaningful for particular students in particular contexts. Teachers must be adept at moment-to-moment decision making, in order to engage students in rich discussions of mathematical content (O'Connor & Snow, 2018). This type of instruction envisions that teachers orient students to each other's ideas and to the mathematical goal and position students competently (Kazemi, Franke, & Lampert, 2009). Teachers attend carefully not only to the way their students are making sense of mathematics but also to the way the students are relating to one another socially and mathematically. In addition, they focus on how students are able to bring their whole selves to the school. This means, for example, continually learning about the ways students see themselves with respect to race, culture, and gender. Teachers play significant roles in creating classroom learning environments that are inclusive, intellectually rigorous, and socioemotionally supportive (e.g., Aguirre et al., 2013; Franke, Kazemi, & Battey, 2007; Gutiérrez, 2012; Hunter & Anthony, 2011).

For us, this vision of mathematics teaching and learning pushes back against school structures and policies, often shaped by race, class, language, and/or gender, that have typically sorted and labeled children as being capable or not. We aim to create schools where children and adults are known and cared for and where they feel connected and invested (Martin, 2012). In order to support these communities and teachers in developing these types of instructional practices, we believe that schools need to be organized in ways that cultivate supports for teaching that aim to engage all kinds of learners successfully in complex mathematical learning—including adults and children.

The work described in this article is part of advancing an equity and social justice agenda, where both teachers' and students' experiences and knowledge are not seen as deficits but recognized as assets (Aguirre et al., 2013; Bartell et al., 2017; Turner et al. 2012).¹

Over the past 10 years, we have worked alongside instructional coaches and teachers to design Math Labs and use them as a resource in creating thriving school communities. Math Labs are intended to support individual, group, and system learning in order to generate practices that continually renew and transform schools in ways that support the shared aims described above (Boreham & Morgan, 2004).² Therefore, Math Labs are most powerful when they include collaboration among classroom teachers, specialists teachers (e.g., ELL or SPED), mathematics coaches, and principals. At their best, Math Labs become an integral part of the school's fabric, allowing teachers and school leaders to negotiate (1) how they want to position students; (2) what opportunities they want to give students to learn particular mathematical content; and (3) the means by which to develop shared professional values through which they can discuss students' mathematical learning. Elsewhere we discuss how important the principal and coach are in embedding Math Labs in a broader view of supporting teachers' work lives and professional interactions (Gibbons, Kazemi, & Fox, 2017; Gibbons, Kazemi, & Lewis, 2017).

Designing Math Labs: Principles for Teaching and Learning to Teach

The Math Lab design is informed by a set of principles about both teaching (Figure 1) and learning to teach (Figure 2). These principles build on work that was concerned with supporting preservice teachers' learning in mathematics in which several authors were engaged (see Lampert et al., 2013) and have been further refined through collaboration with colleagues (Dutro & Cartun, 2016). The principles shape both the focus and structure of teacher learning during Math Labs. They are living principles in that they are refined and changed as communities learn. These principles convey that taking risks, being critical

¹ Although beyond the scope of this article, educators we have worked with have also used the Math Lab structure for teachers and families to engage in dialogue about the goals and processes of classroom instruction.

² Math Labs share many features with other professional development structures through which teachers inquire about their practice and get critical feedback, such as Lesson Study (Fernandez, 2002; Lewis, Perry, & Murata, 2006), video clubs (van Es & Sherin, 2010), cognitive coaching (West & Staub, 2003), and studio days (Teachers Development Group, 2010).

FIGURE 1.
Principles for teaching.

1. Teaching is both intellectual work and a craft. Deep knowledge of content and pedagogy, creativity, and passion fuel both learning and teaching.
2. Teachers must position students as sense-makers and knowledge-generators, who desire to invest and succeed in school. This involves building relationships with children, their families, and communities, as well as valuing their perspectives and attending to their thinking, curiosities, and capabilities.
3. Teachers must design equitable learning environments in which all children are engaged in robust and consequential learning.
4. Learning is a process of inquiry for both teachers and students. Teaching includes becoming a student of your students. Teachers must draw on multiple sources to deepen understanding of students as mathematics learners and how to support them to develop their mathematical knowledge and identities.
5. Teaching for equity involves analysis of language and positioning at three levels: individual, institutional, and societal. Our work together involves making the structures surrounding teaching and learning visible, thinking about how those structures impact individuals and groups, and working together toward action and advocacy.

about the impact of school structures and policies on students' experiences, and attending carefully to students are important in the process of growing as educators. They convey that inquiry is fundamental to learning and, at the same time, that communities develop when educators come together around shared experience.

Math Lab Structure

As we have come to know them, Math Labs involve small teams of teachers in full- or half-day, job-embedded experiences multiple times throughout the school year. Typically teachers in the same grade level come together for the experience, which is led by a mathematics coach. In our work, we have found it important for the principal to participate as a lead learner (Gibbons, Kazemi, & Lewis, 2017). Math Labs take place during the school day so that teachers and leaders can learn from students. For some schools, they have secured substitute guest teachers to be in the regular classroom teachers' rooms for the whole day. Other schools have found creative ways to cover classes for certain periods of time so that teachers can work together.

To support learning from the classroom experience, the work in a Math Lab is organized around a *learning cycle* with four phases: learning together, co-planning a lesson, enacting the lesson together, and debriefing together (McDonald, Kazemi, & Kavanagh, 2013). At the center of this learning cycle is an instructional activity that

FIGURE 2.
Principles for learning to teach.

1. Teaching is intellectual work and requires specialized knowledge.
2. Teaching is something that can be learned.
3. Learning to do something requires repeated opportunities to practice.
4. There is value in making teaching public.
5. We all bring our histories forward. Our own learning experiences and identities shape what we know and do. Our developing identities as mathematics teachers matter to our work with children.

provides the practical means for focusing student and teacher learning. The third phase of the Math Lab (i.e., enacting a lesson together) is what sets Math Labs apart from many other approaches to professional development. During these classroom enactments, teachers experiment together with new teaching practices and learn together about students' mathematical thinking. Like Lesson Study (Fernandez, 2002), teachers spend time together in classrooms. However, in a Math Lab teachers work together to experiment with instruction during *both* planning and the classroom enactment by collectively discussing instructional decisions in the moment (e.g., Gibbons, Kazemi,

Hintz, & Hartmann, 2017). In what follows, we describe the norms required in this setting followed by each phase of a Math Lab, attending to what teachers do and the role that the facilitator might play in supporting them to collectively learn in and from practice.

Setting Norms for Experimenting with Practice Collectively

Asking teachers to experiment with new teaching practices collectively, with each other's students, can lead teachers to feel vulnerable. Teacher Educators must confront the challenge of designing learning spaces that are not typical in the current culture of U.S. schools, where teachers often work in isolation from one another. We have found that setting norms at the first Math Lab and continuing to revisit them over time is vital. Often facilitators will offer a provisional set of norms to the group of educators and ask for their reactions. Some example norms that we have seen facilitators use are provided in Figure 3. Facilitators will ask educators to read them and discuss with their colleagues what they might change, remove, or add to the list. They ask educators to reflect on which norms resonate with them and why.

At the beginning of each Math Lab thereafter, facilitators typically revisit norms with the educators. Sometimes, they ask educators to focus on a particular norm for that day's experience. We cannot overstate how important it is to attend to norms with educators. This allows facilitators to productively respond to the vulnerability that educators might feel around discussing and engaging in professional learning in new ways. In the following sections, we will also refer to other norms, specific to each phase, that we have found useful in enacting Math Labs.

Phase 1: Unpacking New Learning

A Math Lab begins with opportunities for collective learning about mathematics, student learning, and pedagogy. The intent of this phase is to support teachers' knowledge of mathematics, students' thinking, and pedagogy that supports listening and responding to students' thinking. For the sake of explaining what typically takes place in Phase 1, we tease apart each of these domains; however, they are often developed concurrently. What materials and ideas are explored during this phase depends on the overall plan for the team of teachers. For example, the teachers could be focusing on the teaching of a particular content domain or mathematical practice (e.g., properties of multiplication as depicted in the opening vignette or making

FIGURE 3.

Sample norms used by facilitators.

<ul style="list-style-type: none"> • Be willing to take risks with new ideas.
<ul style="list-style-type: none"> • Listen actively and generously.
<ul style="list-style-type: none"> • Build on others ideas and invite others to participate.
<ul style="list-style-type: none"> • Give each other time to think and process ideas.
<ul style="list-style-type: none"> • Be open to sharing ideas in progress and revise your thinking.
<ul style="list-style-type: none"> • Use specific language to describe what you see students doing, rather than labeling students. Avoid labels such as "low" and "high."

use of structure). They could be working on integrating knowledge about supporting multilingual students with the teaching of mathematics or how to develop norms for advancing the rigor of classroom conversations while attending to how children use arrays to solve multiplication problems. The facilitator needs to consider a goal for teacher learning so that the Math Lab experience is coherent and reflects some intentionality about teachers' development.

Knowledge of mathematics. An important activity that takes place during this phase is engaging in mathematics content. The aim of engaging in mathematics is to challenge educators' specialized mathematical knowledge, which comprises the content knowledge and pedagogical skills required for effective teaching (Ball, Thames, & Phelps, 2008; Suzuka et al., 2010). To do so, the facilitator engages the group in rich explorations of mathematics, where the educators are placed in the role of students. This allows them to develop a stance of inquiry and cultivate a disposition that examines ideas. Further, engaging in the mathematical content will prepare them for Phase 2 in which they will anticipate students' strategies and consider how they might respond to those strategies. Educators may also examine content standards and discuss how the standards build on each other over time.

Knowledge of student thinking. In order to learn about how students' mathematical reasoning develops over time, facilitators engage teachers in reading articles from publications such as NCTM's *Teaching Children Mathematics* or

books such as *Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson, 2014) or *Extending Children's Mathematics* (Empson & Levi, 2011). The facilitator leads discussions about what teachers learned in the readings about how students develop particular mathematical ideas and what tasks supported them to do so. Facilitators can also select video of students being interviewed by a teacher or engaging in mathematical discussions as a class. The educators can be asked to notice how students' ideas are developing, what language and tools they are using, and/or how their ideas are being taken up by the teacher or other students. This allows the educators to consider what they have just read against what they see students engaging in as they solve or discuss a particular task. Facilitators also support educators' understanding of student learning by engaging them in examining student work. Through examining student work, educators can learn how students' understanding of particular disciplinary ideas develop and also support educators in coming to appreciate the range of their students' ideas. In Phase 2, the educators can later consider how to build on those ideas during instruction.

Knowledge of pedagogy. Phase 1 provides the initial grounding for teachers to develop further understanding of a particular aspect of teaching and instructional decision making. Educators can raise questions about how teachers' actions might be consequential for students' experiences in the classrooms. Further, they may press each other's understandings of commitments around equity, and how these commitments come to life in the tasks on which they work with students and the way they conduct instructional conversations. Educators can consider the concept of voice, for example, and how students are positioned in the classroom and with what outcomes. They might then focus on one or two *talk* moves (e.g., revoicing or reasoning; Chapin, O'Connor, & Anderson, 2013) or moves that support linguistic development for English learners in order to consider how to use them during instruction to elicit and respond to students' ideas and to position students as capable and valued contributors to the classroom. To develop shared images of these practices, the facilitator might engage teachers in discussing classroom video, commercially or locally produced, or reading a case that gives a representation of a particular idea. The facilitator leads a discussion that highlights the focal ideas and supports educators to make sense of them in relation to goals for their own students.

Within Math Labs, we typically use routine Instructional Activities that provide a common focal point for teachers and instructional leaders to work together on teaching practices, student thinking, and mathematical content (Kelemanik, Lucenta, & Creighton, 2016; Lampert & Graziani, 2009). The Instructional Activity itself may be new to teachers, and facilitators choose to focus on supporting teachers to develop an understanding of the general contours of the activity and the mathematical opportunities generated by the activity. For example, if teachers were new to the number strings activity described in the opening vignette, the facilitator might engage teachers in this Instructional Activity as learners or the group might watch video of an example of a class engaging in the strings activity. The purpose of this kind of engagement is for teachers to experience the activity and begin to unpack the purpose of the activity in relation to its constituent parts (Grossman et al., 2009).

Facilitation of Phase 1. For each activity that a facilitator may choose to engage educators in during Phase 1, there are important norms to be established. The norms may vary, depending on the activity. For example, when doing mathematics, facilitators can share that they will press educators for an explanation about their work. Further, they can explain that mathematical errors can help examine further questions and ideas about mathematics. When viewing video, facilitators can encourage educators to be empathic with their observations of the classroom and in what the teachers and students are engaged and use their noticings to ask questions about their own classrooms. When analyzing student work, facilitators can encourage educators to focus on what students seem to be working on and thinking about. They can raise questions about what they want to understand further about the student but avoid evaluating the student.

Although we cannot detail facilitation practices around each activity, some literature exists to guide facilitators. For example, Little, Gearhart, Curry, and Kafka (2003) have identified a number of facilitation practices around the activity of examining student work. Elliott and colleagues (2009) and Borko, Koellner, and Jacobs (2011) examined facilitation practices that supported educators to engage in mathematics. van Es and colleagues (2014) have identified facilitation moves that supported educators to collectively examine and analyze video. White, Crespo, and Civil (2016) offer a number of cases for educators to consider

how to engage in conversations regarding equity and justice in the context of teaching mathematics.

Phase 2: Co-Planning

Next, the group takes their new learning from Phase 1 and incorporates it into a process of collaboratively planning an instructional activity that they will soon enact together in a classroom. Selecting which task to do in the classroom with the group is often one of the most challenging aspects of planning Math Labs. Facilitators have to keep *both* teacher and student learning goals in mind when considering what they want the group of educators to try out together. Although the facilitator typically decides ahead of time the Instructional Activity in which the group and students will engage, the facilitator encourages the group to make the final decisions about the specific quantities, contextual features, and mathematical ideas they want to explore.

It is important that the group develops shared ownership of the lesson—it is not one teacher’s lesson that is going to be modeled for others to observe. To plan for student learning, the group works together to first consider what they want to learn about students’ mathematical thinking and then come to a consensus about a goal that might be productive for the students themselves. During this planning phase, the objective is not to plan a scripted, rigid lesson. Instead, lessons are planned with the intention for teachers to alter the flow of the lesson as they make sense of students’ responses. The group works together to anticipate student thinking, and against this, they consider the affordances and constraints of particular instructional moves and representations they might use. The group brainstorms particular questions to uncover or press on student thinking. In some cases, this can include rehearsing some or all of the lesson (Lampert et al., 2013). For example, if the group is grappling with specific representations for a word problem, they may practice creating the representation during this phase in order to “work out the kinks” ahead of time. Alternatively, they may identify investigative questions they have and plan two different ways of representing—one during the first enactment and one during the second enactment.

Typically the group will spend 20-30 minutes preparing for their classroom visit. The group discusses not just what they will try but also to what they will attend. For example, while teachers might be planning an instructional activity that involves students using doubling as a strategy to multiply two numbers, they are also keeping in mind

that as they try out the activity their goal will be to explore student understanding of and reasoning about how multiplication as an operation behaves. It is important for the plan to remain flexible enough that teachers can adapt instruction during the classroom visit in response to student thinking. To this end, teachers also make plans about how they will collaborate during the classroom visit. One or more teachers volunteer to take the lead on instruction during the visit, but the goal is for the visit to be collaborative. Teachers spend some time during Phase 2 establishing norms for their collaboration in the classroom. For example, teachers might agree on a way to chime in during instruction (see further discussion in Phase 3). They might also identify particular moments in the lesson about which they are curious or unsure to focus their noticing in the classroom visit. By the end of the planning time, teachers should have the basic flow of a plan written out and be ready to learn from how the students engage in that plan.

Phase 3: Co-enactment

With their co-planned lesson in hand, the group enters a classroom to try out the lesson. Classroom visits take place in the classroom of one or more participating teachers. The facilitator has negotiated which classroom(s) the group will visit and coordinated schedules with those teachers ahead of time. Although bringing the lesson to life is the responsibility of the group, the group determines who will take the lead for some or all of the lesson. Teachers typically do not take the lead in teaching when visiting their own classrooms. We have found this norm to be powerful for establishing a culture of risk-taking, because it helps reinforce the idea that the classroom visit is experimental in nature. When working with their colleagues’ students, teachers seem more open to asking for input. This norm also provides a unique opportunity for the classroom teacher to be with her students and learn deeply about students’ thinking without also being responsible for facilitating instruction.

When the educators enter the classroom, the group sits with and among the students in preparation for listening to and noticing carefully what the students say and do. The coach or principal starts by framing the visit for students by emphasizing that the teachers have spent the day learning together and that they are there to try out something new and to learn from the students. We have found this framing to be powerful for both teachers and students. Teachers are reminded that they are there as learners and no one is expected to model a perfect lesson. In addition,

students are positioned as having important ideas that help their teachers learn.

Although there is a lead teacher who facilitates the lesson, a key element of classroom visits is that all of the educators in the group will collaborate *during instruction* to make decisions that are responsive to student thinking. Decision making is shared in the moment (as opposed to reflecting on and discussing decisions *after* the lesson is over) through an important routine called teacher time out (see Gibbons, Kazemi, Hintz, & Hartmann, 2017). By providing opportunities to pause within the lesson to think aloud, share decision making with one another, and determine where to steer instruction, teacher time outs help shift the focus of the classroom visit from one of judgment and evaluation to one of collective consideration about teaching and learning (Hiebert & Morris, 2012). For example, in the opening vignette, after Savion shared his strategy, Mrs. Brown (the lead teacher) paused to get a quick opinion from her colleagues about whether she should use a number line or an array to record his thinking. Later in the vignette, Mr. Chen (who was sitting among the students) initiated a teacher time out and suggested a question that could be asked in order to work towards the group's instructional goal.

There are often two classroom visits in one lab day so that multiple teachers can lead the Instructional Activity, and so the group has an opportunity to revise their plan based on what they learn in the first classroom visit and try out their revisions right away. It is the exploratory nature of the classroom visits, the dual focus on teacher and student learning, and the flexible use of classroom time that distinguishes our model from other professional development designs.

Phase 4: Debrief

Following the classroom visit(s), the teacher educator facilitates a debriefing conversation that focuses on what the team learned about students' thinking in relation to content, what this new learning means for instructional practice, and the implications for teachers' own classrooms. This often takes the form of a discussion that begins with prompts from the facilitator, but it can also include viewing video of the classroom visit(s) in order to revisit specific moments. We have found that it is important to start this debrief by asking what students seem to know or understand in relation to the content or instructional goal for the lesson. By starting the conversation with students'

strengths, as opposed to limitations of students' understanding, teachers have an opportunity to develop their visions of students' mathematical capabilities (Jackson, Gibbons, & Sharpe, 2017) and consider how instruction can be responsive and build upon students' assets. As the group discusses what they noticed and wondered about students' thinking, the coach also encourages the group to consider instructional moves--both those that were made during the classroom visit as well as those that they could use in the future based on what they experienced in the classroom. The debrief typically ends with teachers making commitments about what they will try in their own classroom and when they will try it. By identifying when they are going to try something in their classrooms, both coaches and principals have the opportunity to ask, "Can I come try it with you?" or "How can I support you?" These common commitments provide opportunities for teachers to continue to learn about teaching mathematics between labs as they try common instructional activities and practices and share their experiences with their teammates, coaches, and principals. They can learn from one another about how these plans play out in their various classrooms.

Impact of Math Labs

Math Labs adhere to many recommendations of good professional development but as our previous description highlighted they have several unique features: (1) lessons worked on in Math Labs are typically activities that teachers can use routinely across the school year and across grades; (2) plans for teaching these activities are collectively created and owned by the participating mathematics teachers; (3) the teacher educator attends to teacher learning goals and serves not as the sole authority but instead as someone who invites teachers to experiment in planning and enacting lessons; and (4) during enactments, teachers make their decision making public with one another and can collectively steer the lesson through the use of teacher time outs as they make sense of student thinking. When Math Labs are used intentionally within schools, these unique features positively impact teachers' learning and growth. As evidence of this impact, in the following sections we share the voices of educators who have participated in Math Labs, relying on interviews conducted with teachers and school leaders. The educators were asked to describe their experiences with Math Labs and how they influenced their understanding of teaching mathematics and instructional leadership as well as their relationships with their colleagues. In addition to teacher learning, we

have also seen significant growth in children's learning. We conclude this section with a description of this evidence.

Teachers' Experience and Learning

When asked to reflect on their experiences in Math Labs, participants' responses fell within four broad categories: the focus on Instructional Activities, multiple enactments of Instructional Activities, shifts in perspectives of student capabilities, and collaborative enactments. Each of these categories will be described in the following sections.

The focus on Instructional Activities. When asked about their participation in Math Labs, teachers reflected on the power of centering the experience around Instructional Activities. Teachers shared how these activities provide different kinds of experiences for their students because they: (a) have multiple entry points often with many ways for students to be correct or to be successful; (b) afford many opportunities for students' voices to be heard; and (c) offer students the opportunity to engage in rich mathematical practices beyond recalling and practicing procedures. The use of Instructional Activities encouraged teachers' experimentation and practice with similar routines multiple times and with different content objectives. Teachers noted that as a result of this practice they felt more prepared to try things on their own in their classrooms. In particular, having opportunities to plan for and try particular kinds of conversations associated with each Instructional Activity helped teachers feel more prepared to do the same on their own. As one teacher explained, "I think a lot of people would have tried an [Instructional Activity] but I don't know that it would have been implemented in the way that it's intended without the professional development or the space to try it and see what it looks like."

In addition, teachers who experienced Math Labs across grade levels in a school reported that the focus on Instructional Activities supports a feeling of coherence across the school. One kindergarten teacher described a moment when she realized how Instructional Activities had changed the kinds of conversations she could have with colleagues.

We spent one of our staff meetings . . . talking about what was going well. And I was sharing about an Instructional Activity in my classroom that had been going well, and then one of the fifth-grade teachers – she was sharing about the same Instructional Activity but happening in fifth grade . . . And we were able to

have a conversation about the same activity at different ends of the spectrum . . . It was really cool because I couldn't have had that conversation [before Math Labs].

The focus on Instructional Activities seemed to support teacher learning in a way that helps them to feel more prepared for implementing what they are learning about mathematics instruction in their own classrooms and to develop a sense of alignment with their colleagues.

Multiple enactments of instructional activities. In their reflections and interviews, teachers also highlighted how having multiple opportunities to enact the same Instructional Activity together supported their learning. Math Labs gave them "space to try it and see what it looks like in practice." Teachers often highlighted this chance for multiple enactments as the key difference from other professional development experiences. One teacher told us, "It's not like another [professional development] where you go and you listen and you hear about this stuff and then you happen to remember weeks later you can apply it in your situation." Teachers described how the experience supports them to be more comfortable with improving how they make sense of students' ideas. One teacher reflected, "Getting the chance to see kids in the moment. And be really responsive with those lessons when you try something out and then reflecting on it. . . Getting to do that with kids live, is priceless." Teachers also get a chance to try out new activities with support rather than trying them alone in their own classrooms. One teacher explained how different it would be to only receive instructions on how to do an Instructional Activity without trying it out. "None of us like to get up in front of the class and struggle through something and not know what we're doing . . . So the Labs, I think, make us better prepared."

Shifts in perspectives. What seemed to be powerful for teachers was that the interactions they have with students shift because of what they experience during Math Labs. Teachers described seeing that students are more capable of engaging in rigorous mathematical activity and discourse than they thought. For example, one teacher described the impact of seeing a colleague supporting students to engage in a rich conversation. "And when you see someone who is being successful with the same group of kids I have, [I ask myself]--what are they doing that I'm not doing yet? . . . Just really seeing that I need to push myself more to draw out more from my students, more discussion, more thinking." Teachers described the experiences as sparking

new awareness of their students' mathematical capabilities and challenging the preconceived notions they have about their students' capabilities. For example, one teacher described observing a colleague ask a student questions that she would not have thought of, which revealed aspects of the student's thinking she would not have expected. In this way, Math Labs offered teachers an opportunity to be in classrooms with students as learners, which enabled them to see their learners in new ways.

Collaborative enactments. A key element of the enactments in classrooms for teachers was that they feel truly collaborative in nature. Teachers described a willingness to take risks in the classroom alongside their colleagues. As one teacher described, "Being flexible is a huge part of a Math Lab. Being open to your colleagues' thoughts and opinions about what they think you should try next or what you should do to make it better." Teachers often highlighted this as a key difference for them from other professional development experiences. "I really felt okay to make mistakes. I've been in some other [professional development] where I felt like the response was, 'Oh, she's just not getting it, she's not doing it right.'" For teachers, this experimentation with colleagues can change how they feel about their own practice in their classrooms. Teachers described how Math Labs help them feel more willing to talk with colleagues and administrators outside of the professional development session about challenges they are facing in the classroom. As one teacher reflected, "Practicing together has taught me, we're okay making mistakes. We all make mistakes. And we're not perfect, just because we're teachers."

Student Learning

We have had the opportunity to document some aspects of student learning through our collaborations with schools and districts who use Math Labs. We do not claim, however, that Math Labs by themselves produce gains in student learning. We do believe, though, that they can be part of intentional and long-term plans for transforming instruction and creating meaningful workplace learning opportunities for teachers. With that caveat, we have seen significant changes in student learning.

In one diverse urban school serving students from communities who have been historically marginalized, where all teachers (K-5) participated in an average of six Math Labs each school year across three years and had considerable supports from school leaders, students' standardized test

scores improved from the 5th percentile in the state to the 78th percentile in three years. In the same school, achievement gaps between black and white students and between English speakers and bilingual speakers diminished significantly or were closed in that same period of time. We also interviewed students about their mathematical thinking at five different points in time over three years (Kazemi, Gibbons, Lomax, & Franke, 2016). Students' responses showed dramatic improvement in both accuracy and sophistication of strategies at each grade level. Third-through fifth-grade students, whose teachers focused on fractions much of the time in Math Labs, showed marked growth in their understanding of fractions. An analysis of students' responses to an equal sharing fraction task (Lewis, Gibbons, Kazemi, & Lind, 2015) revealed that students in three different cohorts developed more sophisticated strategies for partitioning and sharing, created more accurate representations of their partitions, and used more accurate fraction language and notation (Lewis, 2016). That analysis also found that at the start of each subsequent school year, the cohort of students entering a particular grade level brought with them more sophisticated strategies for partitioning and sharing as well as more accurate representations. This range of ways of measuring achievement cannot convey the full story of the experience teachers and students had at this school as they transformed the school culture into a place where students and teachers felt heard and seen. Still, these achievement gains are consequential for students.

Facilitation Demands

Our studies of school improvement have clearly shown that Math Labs are not a silver bullet to be mechanically implemented in order to change school cultures and students' learning experiences. Math Labs can serve, though, as an important part of transforming school cultures towards more equitable learning environments for students who have been historically marginalized. We end this article with observations about key facilitation demands of Math Labs. Getting Math Labs off the ground takes intentional work setting norms of risk-taking and deprioritizing practice. The facilitator plays an important role in shaping the tone of Math Labs and the vulnerability that teachers experience. The teacher educator also sets the tone for students during classroom visits by positioning students as important contributors to teachers' learning. To help do this important work, we have found that facilitators need to develop familiarity and adeptness with

the Instructional Activities used in Math Labs. Visiting teachers' classrooms independently of Math Labs to try out Instructional Activities and to model the kind of back and forth exchange that characterizes teacher time outs is important to establishing productive relationships with teachers and students. To make good choices in identifying teacher learning goals in Math Labs, this experience in teachers' classrooms is vital for facilitators, because discussions in Math Labs are informed with particular teachers' and students' needs in mind.

Conclusion

For Math Labs to be consequential over the long term, our studies of teacher learning have indicated that building leaders coordinate the work in Math Labs with other collaborative spaces in the school such as faculty meetings, grade-level meetings, and individual coaching support (Gibbons, Kazemi, & Fox, 2017; Gibbons, Kazemi, & Lewis, 2017). Math Labs, by themselves, are not a comprehensive solution to teacher learning needs. Our studies have also shown that Math Labs require skilled facilitation (Fox, 2018). Not surprisingly, shifts occur in how teachers and facilitators talk about practice as they learn together over time. At first, teachers and facilitators may focus more on technical aspects of what the Instructional Activities are or grapple with how to elicit students' ideas. Over time, though, as participants develop norms of trust

and experimentation, they can pursue more complex and persistent problems of practice, develop more coherent instructional practices within and across grade levels, and tackle emerging questions about student learning (Rigby, Kazemi, Lenges, Forman, & Fox, 2018). Like all efforts to achieve equitable learning experiences of students, Math Labs can be a useful resource if teachers and students are empowered to experience school as spaces where their ideas are heard.

AUTHOR NOTE:

Math Labs have inspired ways of structuring professional learning in other areas. At the University of Washington, we have adapted Learning Labs for (1) science, literacy, and social studies professional learning; (2) mentor professional development in teacher preparation; (3) incorporating technologies into teaching; and (4) learning culturally responsive and anti-oppressive pedagogies. We have begun to develop tools for teacher educators and school leaders who wish to implement similar professional learning opportunities for teachers. For resources to support facilitating Math Labs, go to TEDD.org and explore the following sections:

- Setting norms
- Facilitating Collaborative Planning
- Rehearsals
- Teacher time out
- Planning labs

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First Name _____ Middle _____

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This is my complete address: ☐ Home ☐ Work

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Telephone _____

Please check all that apply. I currently work as: (Optional)

- | | | |
|--|---|--|
| <input type="checkbox"/> State/Provincial Department of Education Employee | <input type="checkbox"/> Department Chair | <input type="checkbox"/> Education Technology Provider |
| <input type="checkbox"/> Government Agency (NSF, DOE, etc.) | <input type="checkbox"/> Grade-Level Leader | <input type="checkbox"/> Pre-Service Educator |
| <input type="checkbox"/> Member of Local Board of Education | <input type="checkbox"/> Teacher Leader | <input type="checkbox"/> Professional Developer |
| <input type="checkbox"/> Superintendent | <input type="checkbox"/> Author | <input type="checkbox"/> Publisher |
| <input type="checkbox"/> District Mathematics Supervisor/Leader | <input type="checkbox"/> Coach/Mentor | <input type="checkbox"/> Teacher |
| <input type="checkbox"/> Principal | <input type="checkbox"/> Consultant | <input type="checkbox"/> Other _____ |
| | <input type="checkbox"/> Curriculum Leader/Specialist | |

Please check all that apply. I am a leader in mathematics education at the following levels:

- | | | |
|--|--|--|
| <input type="checkbox"/> National | <input type="checkbox"/> Building | <input type="checkbox"/> Elementary School |
| <input type="checkbox"/> Regional (more than one state/province) | <input type="checkbox"/> University/College | <input type="checkbox"/> Pre-Kindergarten |
| <input type="checkbox"/> State/Province | <input type="checkbox"/> Senior High School | <input type="checkbox"/> Other _____ |
| <input type="checkbox"/> District/County/City | <input type="checkbox"/> Junior High/Middle School | |

Since designations vary over time, check the one you feel best describes you:

- | | | | |
|---|---|--|---|
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| <input type="checkbox"/> Native American | <input type="checkbox"/> Pacific Islander | <input type="checkbox"/> Bi-Racial/Multi-Racial | <input type="checkbox"/> Other _____ |

Check the area you serve:

☐ Rural ☐ Suburban ☐ Urban

Do you influence purchasing decisions?

☐ Yes ☐ No

Age:

☐ under 25 ☐ 25 - 34 ☐ 35 - 44
☐ 45 - 54 ☐ 55-64 ☐ over 64

Work Experience:

- | | |
|--|--|
| <input type="checkbox"/> 11-20 years in position | <input type="checkbox"/> 21-30 years in position |
| <input type="checkbox"/> First year in position | <input type="checkbox"/> over 30 years in position |
| <input type="checkbox"/> 2-5 years in position | <input type="checkbox"/> retired |
| <input type="checkbox"/> 8-10 years in position | |

Please check all that apply. Which of the following characterize the community you serve?

- ☐ High percent poverty
☐ High percent of English language learners
☐ Racial and ethnic diversity
☐ None of the above

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