NCSM Journal

of Mathematics Education Leadership

FALL 2018 VOL. 19, NO. 2

National Council of Supervisors of Mathematics www.mathedleadership.org

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership (JMEL)*** are interested in manuscripts addressing issues of leadership in mathematics education which are aligned with the NCSM Vision.

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM.

Categories for submissions include:

- Case studies and lessons learned from mathematics education leadership in schools, districts, states, regions, or provinces
- Research reports with implications for mathematics education leaders
- Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
- Other categories that support the NCSM vision will also be considered.

Submission Procedures

Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.

Manuscripts should be emailed to the Journal Editor, currently Carolyn Briles, at ncsmJMEL@mathedleadership.org.

Submissions should follow the most current edition of APA style and include:

1. A Word file (.docx) with author information (name, title, institution, address, phone, email) and an abstract (maximum of 120 words) followed by the body of the manuscript (maximum of 12,000 words)
2. A blinded Word file (.docx) as above but with author information and all references to authors removed.

***Note:** Information for manuscript reviewers can be found at the back of this publication.

Inquiries about the ***NCSM Journal of Mathematics Education Leadership*** may be sent to:
Carolyn Briles
19019 Upper Belmont Place
Leesburg, VA 20176
Email: ncsmJMEL@mathedleadership.org

Other NCSM inquiries may be addressed to:
National Council of Supervisors of Mathematics
PO Box 3406
Englewood, CO 80155
Email: office@ncsmonline.org • ncsm@mathforum.org

Table of Contents

NCSM Vision

NCSM is the premiere mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

Comments from the Editors

M. Carolyn Briles, *Loudoun County Public Schools*
Nancy Drickey, *Linfield College*

Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. (NCTM, 2000, p. 18)

Creating an environment conducive to learning mathematics is critical in helping students grow mathematically. As math education leaders, we know it when we see it. We celebrate classrooms that are rich in mathematical thinking and student discourse. We cheer high-level tasks that challenge students while still encouraging them. But how do we help others establish and nurture these environments? How do we help teachers, administrators and the education community build an environment conducive to learning mathematics? The articles in this issue of JMEL address these questions.

One aspect of this environment is the classroom teacher's focus on mathematics. At the elementary level, teachers who are mathematics content specialists add strength to the curriculum and instruction of learners beginning their journey into mathematics. NCSM specifically recommends "the use of Elementary Mathematics Specialists (EMS) in PK–6 environments to enhance the teaching, learning, and assessing of mathematics in order to improve student achievement" (NCSM, 2010). As leaders, how do we encourage schools to incorporate this model? How can we help a school or district transition from single generalist teachers to a model of team teaching with mathematics content specialists?

Markworth, Brobst, Parker and Ohana prepare us for that discussion with their article, "Exploring Elementary Content Specialization: Benefits and Cautions, Pitfalls and Fixes." They followed both mathematics content specialists and generalist teachers at the elementary level and compared key factors of instruction. They address the challenges they identified in the implementation of elementary content specialization and offer suggestions as to how to support schools, teachers, and parents through the transition.

Focusing on the older student, Glassmeyer and Roth address the mathematics learning environment in high school with respect to productive struggle. Supporting productive struggle in learning mathematics is one of the eight effective teaching practices recommended by the National Council of Teachers of Mathematics (NCTM, 2014). But how do teachers do that? What does productive struggle look like in the secondary mathematics classroom, and how can we, as leaders, help teachers create that environment?

Glassmeyer and Roth help us understand this in their article "Characterizing How Expert Algebra Teachers Promote Productive Struggle." The authors combined a productive struggle framework and a cognitive demand framework to analyze how nine National Board Certified teachers promoted productive struggle in their classrooms. Their research offers insights into teacher responses that are helpful in supporting struggle as well as an analysis method for giving teachers feedback about their own classroom.

As you read each article in this issue of JMEL, we hope that you recognize the critical role of environment in mathematics instruction. We encourage you to consider the learning environment for mathematics that you help create whether that environment is your own classroom, a department or grade level within a school, or a division within an entire district. We hope that this issue gives you ideas for making that environment an even better place for learning mathematics and that, as leaders, you will share those ideas with others. ✪

References

National Council of Supervisors of Mathematics. (2010). The role of elementary mathematics specialists in the teaching and learning of mathematics. Retrieved from https://www.mathedleadership.org/docs/ccss/JointStatementOnMathSpecialists.pdf

National Council of Teachers of Mathematics. (2010). Principles and standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

Exploring Elementary Content Specialization: Benefits and Cautions, Pitfalls and Fixes

Kimberly A. Markworth, Ph.D., *Western Washington University*
Joe Brobst, Ed.D., *Old Dominion University*
Ruth Parker, Ph.D., *Mathematics Education Collaborative*
Chris Ohana, Ph.D., *Western Washington University*

Abstract

Teaching elementary mathematics well is a significant challenge for self-contained classroom teachers who are responsible for teaching all content areas. This article reports on research findings regarding elementary content specialization (ECS), in which elementary teachers share classes of students in order to specialize in certain content areas, oftentimes manifested through a team teaching model. The research findings from this study relate to four takeaways: focus, professional development, instructional time, and student support. In addition, potential pitfalls and corresponding fixes with implementing ECS are identified and discussed. Teachers, specialists, and administrators considering ECS through team teaching may use these results, takeaways, and recommendations to weigh the benefits and challenges of ECS, as well as plan for best practice and potential pitfalls.

It is easy to argue that the tasks and challenges of teaching elementary mathematics have changed over the past three decades. Building on the ground-breaking standards movement begun in the 1980's, *Principles to Actions* identifies eight effective mathematics teaching practices:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematics discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking. (National Council of Teachers of Mathematics, 2014, p. 10)

These practices present arduous demands for teachers and their instruction. When coupled with rigorous standards for students and the expectation that educators engage *all* students as learners of mathematics, the challenge of being an effective mathematics teacher becomes one that requires strong content and pedagogical content knowledge, ongoing professional development, and reflective inquiry.

Traditionally, elementary teachers are expected to be generalists, required to teach all content areas to their students in a self-contained classroom setting. An elementary teacher is expected not only to rise to the expectations of teaching mathematics well, but also to similar expectations in the other content areas. These changing expectations for elementary teachers in all content areas may be making self-contained models of instruction difficult to sustain.

Team teaching in the elementary grades is not a new concept, and elementary teachers are likely already familiar with some of the varied arrangements of team teaching that are possible. In some locations, these arrangements are ubiquitous – a school-wide arrangement for some grade levels with its own tradition and the kinks already worked out. For example, in the fifth grade where I started my classroom teaching, teams of three classroom teachers would share students. I taught science to three classes and mathematics to two classes while the other two team teachers shared responsibility for language arts and social studies. (For descriptions of various team teaching models, see Markworth, Brobst, Ohana, & Parker, 2016, or Markworth, 2017.)

In other schools, the determination to maintain the traditional self-contained classroom structure is strong. Resistance to alternatives may be grounded in concern for the developmental appropriateness of elementary students receiving their instruction in a structure that resembles those used with older students. Or, teachers may be so used to self-contained instruction that consideration of a different structure has never been given much thought.

Several educators have argued for the increased implementation of content specialization at the elementary level by citing potential advantages (Chan & Jarman, 2004; Gerretson, Bosnick, & Schofield, 2008; Reys & Fennell, 2003; Wu, 2009). With teachers focusing on specific content areas, they can have additional time to develop cohesive and meaningful lesson plans around a subject. Professional development can target specific content areas and the instructional practices that will support students' learning. Teachers can teach subjects that they are both enthusiastic about and feel competent to teach. Potential advantages for students include increased access to expert instruction and the ability to benefit from multiple teachers' teaching styles.

In our NSF-funded research project, *Every Day Every Child*, we examined local cases of Elementary Mathematics Specialists (EMS) as full-time teachers of two or more classes of students for mathematics in order to:

- characterize different specialist instructional models (see Markworth, Brobst, Ohana, & Parker, 2016, or Markworth, 2017), and
- provide evidence regarding the impacts of these alternatives to self-contained classrooms on teachers and students (discussed here and in Markworth, Brobst, Ohana, & Parker, 2016).

The practice-based goal of this paper is to communicate practical generalizations from our research – benefits and cautions, pitfalls and fixes – to inform teacher and administrators' decisions regarding the use of EMS as full-time classroom teachers in a non-self-contained setting.

Methods

Participants

We use the term *EMS teachers* henceforth to distinguish these classroom teachers from EMS who serve as coaches and/or interventionists, as identified by McGatha and Rigelman (2017). All of the EMS teacher participants in this research (n=24) taught either 2 or 3 classes of students in mathematics. Nine (9) of the EMS teachers specialized only in mathematics; the rest of the EMS teachers (15) specialized in additional subject areas such as Reading, Writing, Science, and Social Studies. Of the 24 EMS teacher participants, 1 teacher taught 1st grade, 3 teachers taught 3rd grade, 8 teachers taught 4th grade, 10 teachers taught 5th grade, and 2 teachers taught multiple grades.

A comparison group of teachers consisting of self-contained elementary teachers was recruited first by identifying schools with similar demographic and socio-economic student populations and then by identifying self-contained teachers within those schools with a similar number of years of experience to EMS teacher participants. The resulting comparison group (n=17) was comparable in age and years of experience. Differences in the number of teacher participants in each group related to attrition over the course of the two-year data collection process and changes to teacher assignments. Additional information regarding the selection and comparison of participants can be found in Markworth, Brobst, Ohana, & Parker, 2016.

Data Collection and Analysis

Our project team of two mathematics educators and two science educators conducted semi-structured interviews, online teacher surveys, and six video-recorded classroom observations per participating teacher. For the EMS teacher participants, these were conducted over an 18-month time span; the comparison group of teachers, which was recruited in the second year of the project, completed these tasks during a single academic year.

Interviews. Each participant took part in an interview using a semi-structured interview protocol. When there were multiple teacher participants at the same school, they were invited to complete this activity in a focus group. All interviews were audio-recorded and transcribed for analysis.

Qualitative coding of all interview data was accomplished using NVivo qualitative analysis software. Initially, all interviews were coded using two lenses: a temporal lens and a stakeholder lens. Then, transcripts were coded according to a combination of *a priori* themes (e.g., collaboration, curriculum, resources, and standards) and emergent themes (e.g., flexibility, continuity, and content integration). Data from this analysis was used to identify the four takeaways and potential pitfalls discussed below, as well as to triangulate the findings from the quantitative analysis of the surveys.

Survey. Participants also completed a two-part online survey. Surveys consisted of original questions as well as questions drawn from existing instruments, primarily the 2012 National Survey of Science and Mathematics Education (Horizon Research, 2012). Questions covered a range of topics: demographics; educational and teacher preparation; teaching responsibilities; factors related to the teacher's current teaching position; factors influencing the initial impetus and continuation of the specialist model; teacher beliefs about mathematics instruction; enthusiasm and preparedness for all subject areas; and professional development experiences and needs.

Survey data was analyzed using the IBM SPSS Statistics 23 software package. First, descriptive statistics were generated. Next, comparisons between the means of EMS teachers and self-contained teachers' responses were conducted using independent sample t tests (two-tailed) along with integrated Levene's tests for equality of variances.

Classroom Observations. Each participant completed a total of six video-recorded classroom observations. These were divided into three pairs of two consecutive days. Lessons were scored by the two mathematics educators on the project using the Reformed Teacher Observation Protocol (RTOP). The two scorers established inter-rater reliability by independently scoring 11 videos and negotiating a score. This helped to clarify their scoring procedures and establish common understanding of the RTOP criteria. A linear regression between the original scores demonstrated that the R^2 value of 0.9529 met the project's expectations for inter-rater reliability and was consistent with the reported inter-rater reliability of the RTOP instrument (0.954).

The first five videos for each classroom teacher were scored by one of the mathematics educators, sharing each participant's videos between the two scorers. Next, we analyzed the spread of teachers' scores for their first five videos; the individual teachers' scores demonstrated little variance. Three teachers had greater variance than others, greater than 15% of the total possible points on the RTOP. Thus, for these three teachers, we scored their sixth videos. An independent samples t-test was performed to compare the math specialist and self-contained teachers on their total RTOP scores as well as the five sub-categories.

Additional information regarding the project's data collection and analysis of interview and survey data can be found in Markworth, Brobst, Ohana, & Parker, 2016, and Markworth, 2017. Interview protocols and teacher surveys are available at the project website https://cse.wwu.edu/smate/edec-instruments. Information about the RTOP, including its psychometric properties and a manual for training and implementation, can be found at http://www.public.asu.edu/~anton1/AssessArticles/Assessments/Biology%20Assessments/RTOP%20Reference%20Manual.pdf.

Findings

We have distilled the findings from these data collection activities and data analyses into the following takeaways for teachers and leaders who may be considering alternatives to the self-contained classroom: focus, professional development, instructional time, and student support. In the following sections, we discuss our findings and how they relate to each of these generalizations.

Takeaway 1 - Focus

The most obvious and appealing benefit to engaging in a team teaching model is likely its impact on a teacher's ability to focus on fewer content areas. Team teaching results in fewer "preps" – content areas for which the teacher needs to prepare. In a self-contained classroom, the teacher needs to be knowledgeable about and well-prepared to teach the primary content areas of mathematics, science, social studies, reading, writing, along with others such as spelling, technology, art, etc. EMS teachers, by taking on two or more classes of students for mathematics, give up one or more of these content areas, allowing them to focus their energy and time on fewer areas.

Planning Time. Elementary teachers have a set amount of planning time during their day, and all elementary teachers know that substantial planning occurs outside of these hours. EMS teachers in our study reported an average of 270 minutes planning for their mathematics instruction per week, a statistically significant difference from the average of 159 minutes per week spent by self-contained teachers. EMS teachers also demonstrated greater satisfaction with their planning time, being significantly more likely to agree with the statements, "I have enough time to plan for all of the subjects I teach," and "I have enough time to plan for my math instruction."

More time spent planning has the potential to translate to richer mathematics lessons. Amy (pseudonym), a 5th grade teacher who teaches mathematics to three classes of students, discussed a substantial impact on her instruction:

> I think it's really helpful just to be able to have more time to plan and really dig deeper into the standards…. [You] can really dig deeper and find cooler activities, more interactive activities than just doing a worksheet or something on paper…. I spend a lot more time planning…. This way all my time is focused on math and increasing student understanding. Where before it's like, "Okay. We'll do the best we can and move on."

Generally, EMS teachers did not find that fewer subject areas reduced overall planning time. Rather, they indicated that their time was better, or more deeply spent. Melia, another fifth-grade teacher, specializing in both mathematics and science, had similar thoughts:

> I think you can plan deeper lessons so it's not like I don't…if say a 45-minute planning day, I don't feel like, "Well now I only need 30 because I'm teaching the same thing twice." I'm able to take that lesson deeper. I still need the same amount of time. Does that make sense?

EMS teachers can focus their planning time on particular content areas, delving deeper into standards, rich learning experiences, and differentiation for their diverse learners.

A Mathematics Classroom. A focus on mathematics also allows teachers to create a more content-focused learning environment for students. A self-contained elementary teacher's classroom is a colorful jumble of sights and stations related to all content areas. Although this is often pleasant, it restricts teachers' ability to create a laboratory of learning in which posters of strategies and vocabulary resources linger indefinitely, mathematics manipulatives and activities remain accessible, and students' opportunities to make mathematical connections are cultivated. As Shirley stated about her experience, "I liked being able to establish my classroom as focused on math, so I had math stations up that I didn't have to take down because I needed room for something else. The whole room was a math lab." When teachers can focus on mathematics, the opportunities arise to create more cohesive and supportive learning environments for students.

Take-Away 2 – Professional Development

Content focus can also positively narrow a teacher's engagement in professional development. Instead of spreading oneself thin at professional development for all of the content areas, an EMS teacher has flexibility to pass over professional development in some content areas and extend it in mathematics. Eliza (4th grade), for example, found that professional development in mathematics and collaboration with colleagues was more warranted in this role:

> And I do think since I have started teaching a math block it has legitimized my commitment of time to in-service in the summer, to in-service PD during the school year, to my collaborative work with the rest of the grade levels in the building.

Additionally, professional development can have more substantial effects on teachers' practice. Not only can EMS teachers attend more mathematics-focused professional development, they have additional opportunities to apply and refine new learning through lesson repetitions. Consider a 5th grade teacher's reflection on trying to enhance effective teaching practices in all content areas:

> And I think the specialist model – especially for intermediate – because there's so many things that teachers have to know really deeply that making change is hard when you're trying to make change in so many different subject areas.

Professional development is more worthwhile both in what an EMS teacher can attend and the impact it can have on the teacher's instruction.

The importance of engaging in ongoing professional development for EMS teachers cannot be understated. *Every Day Every Child* conducted its research with EMS teachers who had no specific preparation in mathematics and where additional, sustained preparation was limited. Our comparison of the quality of instruction revealed that, in fact, the EMS teachers' quality of instruction was slightly lower than that of their self-contained counterparts – though this difference was not statistically significant. Although the EMS teachers generally reported enthusiasm for teaching mathematics and greater satisfaction with planning time, their instruction was not markedly different. It may be that teachers either were not purposefully selected for this role – based on demonstrated, high-quality instruction – or not supported through professional development to develop more effective teaching practices.

In our research related to the quality of instruction, we found that the potential benefits of specialization were not realized. Clearly, access to and engagement in high-quality, sustained, mathematics-focused professional development will be a critical tool for achieving the potential for expert instruction with EMS teachers. It is not enough to enjoy teaching mathematics, and the EMS teacher on a team should not be chosen solely on this criterion. Instead, selection should be based on teachers' demonstrated competence with teaching mathematics and commitment to improving their practice through professional development.

Take-Away 3 – Instructional Time

One common concern for team teaching structures is the instructional time that is lost with transitions between classrooms. Our study relied on teachers' self-reported data of instructional and transition time; however, teachers' accounting of their students' time in instruction and time lost to transition provides valuable information about the planning of students' schedules in team teaching models.

Comparisons of the data in Table 1 indicate that there were no statistically significant differences for time spent in mathematics instruction or time lost to transitions. Interestingly, each of the measures is in favor of the team teaching structure with more instructional time and less time lost to between- or within-classroom transitions. In several cases in our study, we found that teams had intentionally arranged schedules to capitalize on transitions. For example, a team of teachers might switch classrooms after/before a special or recess in order to minimize lost instructional time. As Saralynn, a 4th grade teacher, describes:

Table 1: Average time spent in mathematics instruction and transitions

	Elementary Mathematics Teachers	Self-Contained Teachers
Minutes spent per class of students in mathematics instruction per week	361	331
Total minutes of between-classroom transition time per day	27	29
Total minutes of within-classroom transition time per day	11	15

> In between the two classrooms they only switch once. All the other switches happen because of recesses and the interventions…. But my partner and I have designed that schedule specifically with the block of Literacy to limit the number of switches in a day. It takes up a lot of time.

The EMS teachers in our study indicated that, once scheduled, this instructional time was well-protected. Because these teachers were unable to run over by a few minutes or come back to something later in the day, they were more aware of how they used their instructional time. However, being unable to run over by a few minutes or come back to something later in the day were significant challenges for EMS teachers, as they lacked the flexibility to stretch or add time to meet their instructional needs.

Take-Away 4 – Student Support

As the number of teachers on a team increases, the number of students served by the teachers on this team likewise increases. This is a great concern to those who question the developmental appropriateness of team teaching structures in elementary grades. At the root of this concern is the preservation of the singular relationship between students and their self-contained teachers and all of the social, emotional, and academic support this provides.

EMS teachers in our study described benefits to the students from team teaching that cannot be provided in a self-contained classroom. They indicate that having multiple team teachers allows for multiple sets of eyes to pick up on issues that students may be having. As one 5th grade teacher explained:

> There's been students we've been concerned about – not just academically but really concerned about their behavior, not they're active and disruptive but more like socially concerned and then we've been able… are you seeing this in your classroom as well?… I can think of three students that we have all had a pulse on much more this year in the first three months of school that I think we've been connected with.

EMS teachers may also find that something works for a student in one classroom that the other teachers may replicate to better support a student's needs.

The physical act of moving to different classrooms during the day also allows for "fresh starts" for the students and teachers. Salome, a 5th grade science specialist in our study, captured what she called a "clean slate" with each transition:

> This way the teacher and the kids, every 75 minutes, you've got a clean slate, somebody who is not done with you yet. And I think that speaks to a lot of the kids who have historically been troubled kids, had problems sitting and focusing and working … it's a completely different world every 75 minutes. And I think that helps a lot of the kids stay engaged.

When students have a situation develop in one classroom, a new setting allows them to put aside the situation and begin again.

The comparison of self-contained and EMS teachers' survey results suggests that EMS teachers believe they are as capable of meeting the needs of their students. With the exception of knowing the strengths and weaknesses of the students in English language arts, there were no significant differences between self-contained and EMS teachers' perceptions of their abilities to know and meet the needs of the whole child (Table 2). It may be that students in team teaching situations benefit when the team focuses on meeting these needs. One multi-grade EMS teacher indicated that her knowledge of and relationships with students were both strengthened:

Table 2: Knowing and meeting the needs of the whole child

Please provide your opinion about each of the following statements: (1 – strongly disagree to 6 – strongly agree)	**EMS Teachers (Mean)**	**Self-Contained Teachers (Mean)**
I know the strengths and weaknesses of each of my students in math.	5.29	5.41
I know when each of my students is struggling or succeeding in math.	5.38	5.47
I have enough time with my students to meet their needs in math.	3.58	3.35
I know the strengths and weaknesses of each of my students in English language arts.	4.13*	5.41*
I know when one of my students is struggling with organization.	5.42	5.71
I know the social and emotional needs of each of my students.	5.42	5.41
I know when one of my students is having a bad day.	5.71	5.71
I have enough time to meet the social and emotional needs of all of my students.	3.33	3.35
I have enough time to meet with other teachers and support staff about the needs of my students.	3.67	3.06

> It's really nice to have that connection and that connection with all the kids. I love that. And then having that…just that knowledge about the students who are…. if one of my homeroom students is having a hard time in my room, the three of us talk about that and we can talk about what that same student is doing in science and what that same student is doing in writing…. There's three of us who spend time with that child who have things to bring to the table. And so it's just all of us getting to know all of our students on a deeper level. I had worried that I wouldn't have as strong of a connection with my homeroom but what's happened is that I have a strong connection with all the 4th and 5th graders now.

EMS teachers consistently reported more attention on individual students and their needs during their collaborative planning time, since their ability to focus on content planning was limited.

The four take-aways of this study – focus, professional development, instructional time, and student support – indicate that team teaching offers great potential as an alternative to the traditional, self-contained classroom. EMS teachers find that specialization in mathematics provides opportunities to focus in their planning time, classroom environment, and professional development. Despite these advantages, there is also evidence that EMS teachers' quality of instruction may require a sustained commitment to high-quality, mathematics-focused professional development. In contrast to concerns raised relating to team teaching models, EMS teachers report that instructional time is not negatively impacted by transitions between classrooms, and they are as capable of meeting the social, emotional, and academic needs of their students.

Pitfalls and Fixes

Despite the potential that team teaching offers teachers and students, team teaching is a significant instructional shift. Several months may be needed to examine and weigh the possible team teaching structures, consider the strengths of the classroom teachers, negotiate the scheduling with stakeholders, and plan for other logistical challenges. In Table 3 (next page), we highlight pitfalls that may be encountered and potential fixes for each.

With thoughtful preliminary planning, a new team of teachers can avoid many challenges that may otherwise doom team teaching from the start. Team teaching has many stakeholders, including parents, specialists, and other teachers. Examining the possible impact for all stakeholders and establishing a team working relationship between the teacher members of the team can support the new team in getting off to a positive start.

Conclusion

Our investigation of EMS and self-contained teachers suggests that team teaching has the potential to create opportunities for more students to be impacted by passionate, knowledgeable, elementary mathematics teachers. Rigorous standards (e.g., the Common Core State Standards for Mathematics), as well as recent calls for effective teaching practices, demand that elementary schools use effective mathematics teachers to their maximum benefit. If implemented thoughtfully and with a continuing commitment to improving instruction, team teaching may make this possible.

This continuing commitment to improving instruction is critical to utilizing a content specialization model to its maximum benefit. Twenty states (including the District of Columbia) offer a Mathematics Specialist Certification, and many institutions in these states have initiated programs that support the development of effective mathematics teachers, interventionists, and coaches (Rigelman & Wray, 2017). Elementary schools that commit to content specialization in mathematics should simultaneously commit to supporting their EMS teachers in becoming certified as specialists (where available) or engaging in professional development that supports their ability to engage all students in the eight effective teaching practices (National Council of Teachers of Mathematics, 2014). Otherwise, the benefits of content specialization may be limited to factors related to teacher satisfaction (e.g., adequate planning time) and not extend to better instruction or improved student learning.

With the lingering challenges that students in the United States still encounter with learning mathematics, it may be time to challenge the traditional, self-contained structure that is prevalent in elementary classrooms. And it might be an ideal time to thoughtfully and carefully experiment with the professional development and effective use of EMS teachers. ✪

Table 3: Pitfalls and fixes associated with initiating team teaching

Pitfalls	Fixes
Consistency – Students may experience different behavioral and academic expectations between classrooms. Students' preferences for teachers and teaching styles can vary significantly between teachers.	Team teachers often describe their relationships with other members of the team as a marriage. Take this into consideration when choosing members of a team. Teachers' behavioral and academic expectations should be fairly consistent. Yet, teachers' strengths and weaknesses should be balanced and contribute to an overall team composition. Team members should be able to capitalize on other team members' strengths and compensate for other team members' weaknesses.
School Buy-In – Teachers and administrators consistently remarked on the challenge of creating a schedule that would work with specials, lunch, other grade levels, etc.	Before planning a team teaching schedule, discuss the prospect with all other school faculty and administration. Explain why you would like to try an alternative to self-contained classrooms, along with its benefits and challenges for different stakeholders. Get everyone on board, because they may all have to make small sacrifices to make it work.
Parent Buy-In – Parents will likely be concerned about their children's ability to adjust to a situation involving multiple teachers, classrooms and transitions. Understandably, they do not want their children to be lost, literally or figuratively.	Present parents with a clear plan and rationale for the change in structure. This rationale may include the take-aways discussed above. Teachers should also explain how they have thought through potential pitfalls, and how they plan to make sure that the transition to the new structure is smooth and the team is effective in meeting students' needs. In addition, it may be worthwhile to explain how and why particular teachers were chosen to teach the content areas.
Schedule – Students may lose focus and time in instruction with multiple between-classroom transitions during the day.	Schedule the between-classroom transitions to coincide with other transitions during the day, such as transitions to and from lunch, recess, or specials. Plan ahead for how students will transition their materials (e.g., books, binders, pencils) between classrooms.
Flexibility – Rigid team teaching schedules limit flexibility during the day. Transitions may be rushed, thereby interrupting content or assignment completion.	Schedule blocks of instruction with individual teachers. For example, if one teacher teaches both mathematics and science, schedule these back to back to allow for some flexibility between times for these content areas. Support team members' efforts to follow up with students in your homeroom who may need extra time to complete an assignment. Consider flexing time schedules every week or two to make up time that some classes may be missing.
Social, Emotional, and Academic Needs of Students – With more students, teachers may find it challenging to get to know all of their students.	Plan team events that develop students' and teachers' sense of a team community. Use common planning time to collaborate around students' social, emotional, and academic needs. Value other teachers' perspectives and relationships with students.
Home-School Communication – Parents need to understand the communication from school, and know how to communicate with their children's teachers.	Have a team parent meeting and/or directions sent home about communication. Address the following questions: • How will homework be communicated to students and parents? What steps will the team teachers take to ensure that homework is reasonable? • How will other school announcements be communicated to students and parents? What steps will the team teachers take to ensure that communication is not overwhelming or contradictory?
Conferences – The demands on teachers for additional conferences are significant.	Develop a conference plan that allows for all parents to receive information about their children in each content area without expecting whole-team conferences for each child (unless this additional time is planned for). Encourage requests for whole-team conferences for any concerned parent.
Content Collaboration – Elementary mathematics teachers have fewer opportunities to collaborate with other teachers around content at their grade level.	Identify other grade-level content teachers throughout the district with whom to collaborate. Common planning time is difficult with teachers outside of your school, but the establishment of an online or after-school collaborative group can alleviate the feelings of content isolation. Alternatively, collaboration with elementary mathematics teachers at other grade levels can be a good source of professional development of understanding how content develops and aligns vertically.

References

Chan, T. C., & Jarman, D. (2004). Departmentalize elementary schools. *Principal, 84*(1), 70.

Gerretson, H., Bosnick, J., & Schofield, K. (2008). A case for content specialists as the elementary classroom teacher. *The Teacher Educator, 43*(4), 302-314.

Horizon Research, Inc. (2012). 2012 National survey of science and mathematics education: Science teacher questionnaire. Retrieved February 16, 2014, from http://www.horizon-research.com/2012nssme/sample-page/instruments/

Markworth, K. A. (2017). Elementary mathematics specialists as elementary mathematics teachers. In M. B. McGatha & N. R. Rigelman (Eds.), *Elementary Mathematics Specialists: Developing, refining, and examining programs that support mathematics teaching and learning* (p. 203-210). Volume 2 of the AMTE Professional Book Series. Charlotte, NC: Information Age Publishing, Inc.

Markworth, K. A., Brobst, J., Ohana, C., & Parker, R. (2016). Elementary content specialization: Models, affordances, and constraints. *The International Journal of STEM Education.* Retrieved from http://stemeducationjournal.springeropen.com/articles/10.1186/s40594-016-0049-9.

McGatha, M. B., & Rigelman, N. R., Editors. (2017). *Elementary mathematics specialists: Developing, refining, and examining programs that support mathematics teaching and learning.* Charlotte, NC: Information Age Publishing, Inc.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* NCTM: Reston, VA.

Reys, B. J., & Fennell, F. (2003). Who should lead mathematics instruction at the elementary school level? *Teaching Children Mathematics, 9*(5), 277–282.

Rigelman, N. R., & Wray, J. A. (2017). Current state of mathematics specialist state certification and standards. In *Elementary mathematics specialists: Developing, refining, and examining programs that support mathematics teaching and learning*, edited by Maggie B. McGatha and Nicole R. Rigelman, 33-38. Charlotte, NC: Information Age Publishing, Inc.

Wu, H.-H. (2009). What's sophisticated about elementary mathematics? *American Educator, 33*(3), 4-14.

Characterizing How Expert Algebra Teachers Promote Productive Struggle

David Glassmeyer, *Kennesaw State University*
Joel Roth, *River Ridge High School*

Abstract

While frameworks for analyzing teacher actions have been developed, little research describes how expert teachers promote productive struggle in their classrooms. In this paper, we report findings from using a productive struggle framework and a cognitive demand framework to characterize how nine National Board Certified algebra teachers promoted productive struggle in a lesson. After analyzing videos of their lessons, we found these teachers, whom we label as expert teachers, used a high-cognitive demand task and gave responses that promoted students' productive struggle, often maintaining the high-cognitive demand of the task. This analysis provides mathematics teacher educators and leaders concrete evidence of how expert algebra teachers promote productive struggle in the classroom. We also discuss implications useful for educators and administrators who decide on algebra curricula and professional development, including tailored information for how teachers can respond to student struggle in ways that promote the high cognitive demand of algebra tasks.

Introduction

Mathematics education research has described students actively struggling to learn mathematical concepts as essential in fostering conceptual understanding of the material. This process is often called productive struggle, and research has identified the important role mathematics teachers play in promoting students' productive struggle (Warshauer, 2014, 2015). For example, the way a teacher responds to a student when they are struggling can impact the cognitive demand of the task. Specifically, the cognitive demand is lowered if a teacher responds by telling the student the answer as opposed to raising the cognitive demand if a teacher responds by providing the student a reason-provoking statement.

Two key frameworks have been developed and used to describe teacher actions influencing productive struggle in the classroom. First, Stein and Smith (1998) developed a cognitive demand framework to characterize the tasks teachers choose to incorporate in the classroom and the resulting cognitive demand the tasks invoke from students. Second, Warshauer (2014) developed a productive struggle framework measuring teacher interactions with students during episodes of struggle. While these two frameworks have been used in a variety of mathematics education settings, they have not been used jointly to quantitatively measure teachers' selection of a task and the interactions with students during the task, and determine correlations in the change in cognitive demand. Using both frameworks within a single study could reveal important findings about how a teacher might select a low-level cognitive demand task but implement it in a way that increases the cognitive demand by promoting student struggle from his/her responses to episodes of students struggle. Alternatively, a high-level cognitive demand task might be implemented in a way that reduces the cognitive demand through teacher responses to student struggle that include telling or directed guidance.

Furthermore, little is known about the ways specific types of teachers incorporate tasks and respond to student struggle. For example, how do expert teachers enact tasks and promote student struggle? Are the tasks and ways expert teachers respond to student struggle different than novice teachers? Another example is examining how teachers of a specific mathematical subject differ in task implementation and response to student struggle. What kinds of tasks do algebra teachers enact, and how do they tend to respond to student struggle? Are there differences between algebra teachers and teachers of other mathematical subjects (geometry, trigonometry, etc.)? Answers to these questions are currently unknown, and knowing more about the patterns of specific groups of teachers could help (a) mathematics teacher educators provide tailored professional development to improve the level of cognitive demand within these teachers' classrooms, (b) designers of algebra curricula incorporate tailored advice on how teachers can respond to student struggle in ways promoting the high cognitive demand of the task, and (c) education researchers routinely incorporate these two frameworks to understand teacher actions.

This study contributes to the literature by reporting findings from using the productive struggle framework and cognitive demand framework to characterize how nine national board certified algebra teachers, whom we call expert mathematics teachers, promoted productive struggle in a lesson. We investigate the following three research questions: (1) What types of student struggle are typical during an algebra task implemented by expert mathematics teachers? (2) How do expert mathematics teachers respond when students struggle within an algebra task? (3) What relation, if any, exists between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task? We incorporated qualitative methods to answer these questions by using Warshauer's (2014) productive struggle framework and Stein and Smith's (1998) cognitive demand framework to code videos of student-teacher interactions within algebra classrooms. From these codes, we characterized the episodes of struggle and types of teacher responses. Then we used quantitative analysis to determine correlations between characterizations of student struggle and the associated cognitive demand of the task.

Productive Struggle Framework

Similar to Vygotsky's (1978) Zone of Proximal Development, Towsend (2018) offered the idea of students' *zone of productive struggle*. The idea was to encourage students to dig deeper into algebraic relationships and experience productive struggle. While students were offered the opportunity, the teacher needs to ensure students are not working outside their zone which may make them feel overwhelmed. Teachers sometimes have difficulty encouraging productive struggle, as this aim might seem at first counterintuitive towards the goals of the lesson. But research suggests that when properly structured and implemented, productive struggle can lead to success for both the teacher and the students (Edwards, 2018; Freeburn & Arbaugh, 2017; Lobato, Clarke, & Ellis 2005). For example, when teachers went through and experienced productive struggle as students, they saw the importance of the process, group discussions, enjoyed the process, and developed confidence that productive struggle supports mathematical goals such as conceptual understanding and problem solving (Murawska, 2018).

A productive struggle framework was developed by Warshauer (2014) using three principal areas of mathematics education research to build her productive struggle framework. First, the framework draws upon literature surrounding the important role struggle plays in students learning and understanding mathematics (Hiebert & Grouws, 2007). Second, the framework incorporates literature documenting how characteristics of mathematical tasks impact students' struggle (Smith & Stein, 1998). Third, the framework relies upon "the ways teachers respond to students' struggles in classroom interactions to capture episodes of struggle, episodes within the stages initiation, interaction and resolution" (Warshauer, 2014, p. 377) and the impact of teacher responses on the cognitive demand (Henningsen & Stein, 1997; Herbel-Eisenmann & Breyfogle, 2005). Using these areas of literature, Warshauer (2014) used an embedded case study to identify and describe the student struggle, teacher actions in response to the struggle, and the resulting impact on cognitive demand.

Table 1: Types of Struggle Experienced by the Student

Kind of Struggle	Descriptors
Get started	• Confusion regarding what task is asking • Forgetting how to solve a type of problem • Gesturing uncertainty and resignation • No work written down
Carry out a process	• Unable to progress on a problem due to inability to use or process a formulated representation, carry out an algorithm, or recall needed facts or formula
Uncertainty in explaining and sense-making	• Difficulty in explaining or making sense of their work • Express uncertainty • Unclear reasons given for their choice of strategy
Express misconceptions and errors	• Misconception related to mathematical content in problem • Performing an arithmetic or technological error

Adapted from Warshaur, 2014.

Table 2: Types of Teacher Responses

Teacher response	Descriptors	Dimensions
Telling	• Supplying information • Directing students towards a strategy • Correcting an error • Referring or referencing simpler problem	• Cognitive demand lowered • Removed struggle efficiently • Suggested an explicit idea
Directed Guidance	• Redirect student thinking • Narrow down possibilities for action • Direct an action • Break down problem into smaller parts • Alter problem to an analogy	• Cognitive demand lowered or maintained • Teacher builds on student thinking
Probing Guidance	• Ask for reasons and justification • Offer ideas based on students' thinking • Seek explanation that could get at an error or misconception	• Cognitive demand maintained • Encouraged student's self-reflection • Questioned and built on student thinking • Used as basis for guiding student
Affordance	• Ask for detailed explanation • Build on student thinking • Press for justification and sense-making with group or individually • Afford time for students to work	• Cognitive demand maintained or raised • Acknowledged, questioned, and allowed student time • Built on student thinking, perhaps by clarifying and highlighting student ideas

Adapted from Warshaur, 2014.

The result of her work was the productive struggle framework which provides the means to classify students' struggles, teacher responses, outcomes of the struggle, and changes in cognitive demand. While literature about the cognitive demand of the activity was incorporated into the literature, specific focus on the task is not included in the Productive Struggle Framework. Table 1 summarizes Warshaur's (2014) four characterizations of student struggle: *(1) getting started, (2) carrying out a process, (3) uncertainty in sense making and explaining,* or *(4) expressing misconceptions and errors.* Table 2 summarizes Warshaur's four characterizations of teacher response: *(1) telling, (2) directed guidance, (3) probing guidance,* or *(4) affordance*. Table 3 summarizes Warshaur's outcomes: *(1) productive, (2) productive at a lower level,* or *(3) unproductive.* The methodology section details how we interpreted and applied these categories to analyze video data.

Table 3: Outcome of Struggle

Outcome Type	Descriptors
Productive	• Maintained the intended goals and cognitive demand of the task. • Supported students' thinking by acknowledging effort and mathematical understanding. • Enabled students to move forward in the task execution through student actions.
Productive at a lower level	• Lowered somewhat in the cognitive demand of the intended task. • The teacher rather than the students actively guided the students through the struggle. • The students passively following a directed guidance.
Unproductive	• Students continued to struggle without showing signs of making progress toward the goals of the task. • Reached a solution but to a task that had been transformed to a procedural one that significantly reduced the task's intended cognitive demand. • Students simply stopped trying.

Adapted from Warshaur, 2014.

Table 4: Changes in Struggle

Changes	Descriptors
Factors Associated with the Maintenance of High-Level Cognitive Demands	• Teacher uses scaffolding, questioning, comments, and feedback to press for student reasoning, explanation, justification, and conceptual connections. • Teachers supports students in monitoring their own progress and the modeling of high-level performance. • Teacher allows sufficient time for task.
Factors Associated with the Decline of High-Level Cognitive Demands	• Teacher emphasizes complete and correct answers rather than the meanings and understanding of the concepts. • Teacher provides their own thinking and reasoning at the expense of student reasoning. • Teacher reduces the complexity of the task by providing explicit procedures or proscribed routines. • Teacher accepts unclear or incorrect student explanations. • Teacher expectations are not clear or appropriate for high-level cognitive activities or does not maintain classroom environment suitable for high-level cognitive activities. • Teacher does not allow sufficient time for task or too much time is allowed, resulting in off-task behavior. • Teacher selects a task that is inappropriate for the group of students (e.g., students do not have prior knowledge needed or task expectations are not clear enough to put students in the right cognitive space).

Adapted from Stein and Smith, 1998.

Cognitive Demand Framework

The learning processes in mathematics are quite sensitive to the selection of the task by the teacher. Lappan and Briars (1995) state "there is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics." (p. 138). Tasks that represent higher levels of thinking are especially important because they provide students the opportunity to think and reason in complex and meaningful ways (Stein and Smith, 1998).

Stein and Smith (1998) identified four ways a task can be approached on the same topic, each with a different kind of cognitive demand on students. The first category of tasks is called *memorization tasks*, which are lower-level cognitive demand tasks requiring students to use previously learned factors, rules, formulas, or definitions. These tasks have explicit and clear direction too short to use procedures and have no connection to underlying facts, rules, or formulas. The second category is *procedures without connections*, another lower-level cognitive demand task that include algorithmic use of procedures with little ambiguity or connections to underlying concepts. These tasks focus on producing correct answers rather than developing mathematical understanding through explanations. The third category is *procedures with connections*, which are higher-level cognitive demand tasks that have students use procedures to develop understanding of mathematical concepts through multiple representations and engagement with conceptual ideas. The fourth category is *doing mathematics*, which is a higher-level cognitive demand task that has students use complex and non-algorithmic thinking to explore and understand the nature of mathematical concepts, processes, and relationships. These tasks necessitate students to self-monitor and self-regulate, to access prior knowledge and apply it to the task, and to examine constraints of the task (Smith & Stein, 1998).

In addition to the selection of a task, teachers' implementation of the task also impacts the cognitive demand. The seminal Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUSAR) project found that "having the opportunity to work on challenging tasks in a supportive classroom environment translated into substantial learning gains on an instrument specially designed to measure exactly the kind of student learning outcomes advocated by NCTM's professional teaching standards" (Stein and Smith, 1998, p. 16). Based on this work, the researchers created three ways teacher-student interactions can impact the cognitive demand of a task: *(1) factors associated with the decline of the task, (2) factors associated with maintenance of the task,* or *(3) factors associated with the increase of the increase of the task*. Table 4 details the characteristics of each categorization and the associated teacher-student interaction descriptors. These descriptors align with Warshauer's (2015) productive struggle framework. For example, when teachers address students' struggles by supplying information they are essentially removing the demand. Teachers can direct student actions or use probing guidance to address students' struggles in ways that maintain the intended cognitive demand. Teachers can also have an opportunity to increase the intended level of cognitive demand when they can take a procedure with connections task and utilize opportunities to incorporate doing mathematics (Warshauer, 2015).

Methodology

Our participants included nine National Board Certified teachers who taught algebra during the time of the study. The designation of National Board Certification (NBC) in the United States was created to reward the most accomplished teachers and is proclaimed to be the most respected professional certification available in K-12 Education. The NBC standards contain five propositions based on research of what accomplished teachers should emulate in the classroom, such as being committed to students and their learning (proposition 1) and managing and monitoring student learning (proposition 3) (NBC, 2017). To gain certification, teachers must also show evidence of participating in professional learning communities and of ongoing reflection on teaching.

Before applying for NBC, an individual must have a bachelor's degree, a valid teaching license, and three years of teaching experience. The certification process for Mathematics – Adolescence and Young Adulthood involves (a) taking a computer-based mathematical content knowledge assessment, (b) submitting instructional materials and work samples with commentary, (c) submitting two videos showing evidence of how the teacher's classroom practices and learning environment contributed to student engagement and to meet the mathematical goals of the lesson, and (d) submitting a portfolio demonstrating evidence of how the teacher develops and applies knowledge of students to plan and impact student learning.

In this study, we examined nine algebra videos from nine teachers that were submitted as part of part (c) of the NBC process. NBC instructions were that these videos should focus on student engagement and the teaching practices and format used to help students meet the mathematical learning goals for the lesson (National Board for Professional Teaching Standards, 2016). Thus, these videos provided valuable data on task selection and the teacher-student interactions of implementing the lesson. Since the teachers used the videos to become National Board certified, we thus call our participants expert mathematics teachers. Therefore, the videos were appropriate for answering our research questions about how students struggle on an algebra task selected by expert mathematics teachers, how mathematics teachers respond to student struggle, and how teachers' responses impact the associated changes in the cognitive demand of the task.

The researchers of this study used publicly available CBS videos of nine expert algebra teachers implementing a lesson with their students; four were male and five were female. We selected these nine teachers because they were the only teachers in the available data set who submitted a video using an algebra lesson. The videos were between ten and twenty minutes, and the teacher decided which lesson and segment were recorded and shared. We also collected the associated tasks that were implemented in the video. The video and associated task documents of the nine teachers comprised the data collection for this study.

Data Analysis

We used Hiebert and Grouws' (2007) definition of productive struggle as students' "effort to make sense of mathematics, to figure something out that is not immediately apparent," (p. 287). To analyze the written documents for student struggle, we used Stein and Smith's (1998) four levels of cognitive demand to code the type of enacted algebra task (1). The lesson was coded as one of the cognitive demands: *memorization* (1a), *procedures without connections* (1b), *procedures with connections* (1c), and *doing mathematics* (1d). The first two levels (*memorization, procedures without connections*) are considered lower levels of demand, and the second two levels (*procedures with connections, doing mathematics*) are considered higher levels of demand (Stein and Smith, 1998).

We analyzed the video data using Warshauer's (2014) productive struggle framework for identifying and coding the four elements of each struggle episode: (2) the struggle experienced by the student, (3) the teacher response, (4) the outcome resulting from the response, and (5) the subsequent change in cognitive demand demonstrated by the student. To code elements (2) through (4), we identified the unit of analysis as an episode of struggle using Warshauer's (2014) definition: (a) the time, beginning with an indication of student uncertainty, confusion, or teacher-directed question; (b) the corresponding teacher response and teacher-student interactions; and (c) the outcome, either productive or unproductive struggle.

In each episode, we coded the type of struggle experienced by the student (2) using one of Warshauer's characterizations: confusion about an approach or what the task was asking, which was coded as *get started* (2a), an inability to carry out an algorithm, implement a process that is generally algebraic in nature (2b), which was coded as *carry out a process*, difficulty explaining their work or making sense of their work, which was coded as *uncertainty in explaining and sense-making* (2c), and *an expression of a misconception or error* (2d).

In each episode, we coded the teacher's response (3) using Warshauer's categorizations: when the teacher supplies information, directly corrects an error, or suggests a strategy, coded as *telling* (3a), when the teacher redirects student thinking, directs an action, or narrows down the possibilities for action, which was coded as *directed guidance* (3b), when the teacher asks for reasons and justification or seeks an explanation that could get at an error or misconception, which was coded as *probing guidance* (3c), and when the teacher asks for a detailed explanation, presses for justification and sense making, or builds on student thinking, which was coded as *affordance* (3d).

In each episode, we coded the outcome of the struggle (4), using Warshauer's three categorizations: when the student or group of students work through the struggle while maintaining the intended level of cognitive demand or are at least able to continue engagement, which was coded as *productive* (4a), when the struggle is addressed by reducing or removing the struggle or making the task easier, which was coded as *productive at a lower level* (4b), and when the students are unable to proceed past the struggle or the teacher completely removes the struggle and fundamentally changes the original intentions of the task, which was coded as *unproductive* (4c). Finally, in each episode, we coded the level of cognitive demand following the outcome (5) using Warshauer's three categorizations: *lowered* (5a), *maintained* (5b) or *increased* (5c) adapted from Warshauer (2014).

In an effort to answer the third research question regarding what relation, if any, exists between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task, we performed statistical tests in Statistical Package for the Social Sciences (SPSS). The first analysis we performed was a Spearman Rank Correlation. The Spearman Rank Correlation was done because the categories to be analyzed consisted of ordinal variables. We then ran a Kruskal-Wallis H Test with the teacher as the factor to see if the individual teacher was a significant factor in relation to the types of responses given to students.

Results

After coding each episode of struggle based on the type of struggle experienced by the student, we found 29 of the 58 (50%) episodes of student struggle were *uncertainty in explaining* and sense making, 20 of the 58 (34%) episodes were *carrying out a process*, 5 (9%) were *getting started*, and 4 (7%)

FIGURE 1.
Results from the nine videos of expert teachers enacting an algebra lesson.

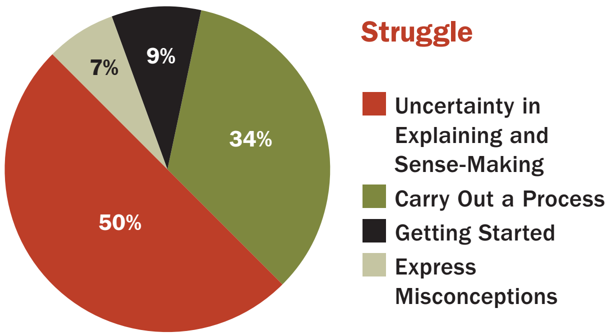


FIGURE 2.
Results from the nine videos of expert teachers enacting an algebra lesson.

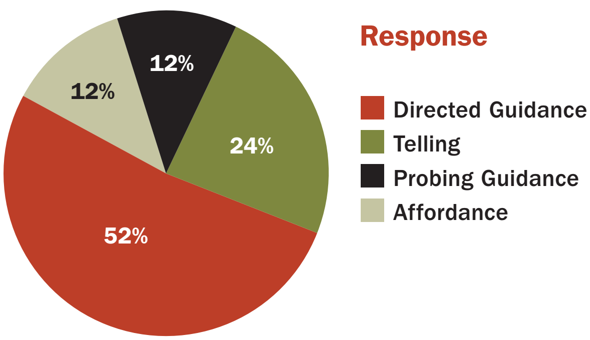
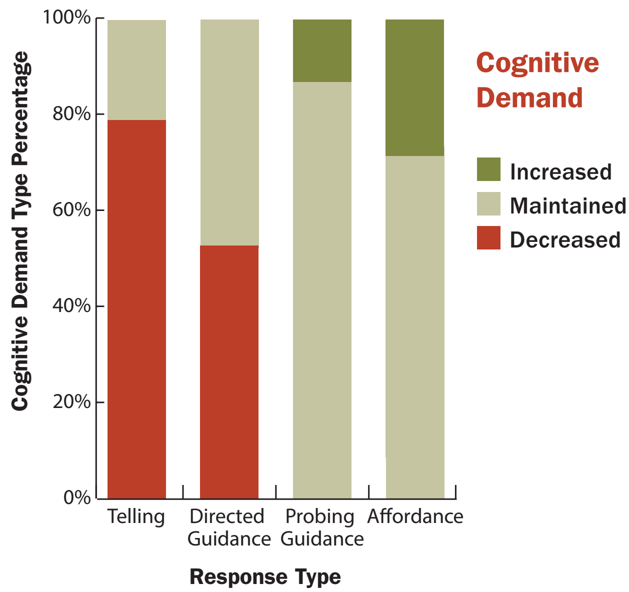


FIGURE 3.
A breakdown of how each type of teachers' response impacted the subsequent cognitive demand of the task.



were *misconceptions*. Figure 1 gives a visual representation of the findings, showing the high prevalence of *uncertainty in explaining and sense making* experienced by students.

After coding each episode of struggle based on teachers' responses, we found 30 out of 58 (52%) episodes were responded to with *directed guidance*, 14 of the 58 (24%) episodes were *telling*, 7 (12%) were *probing guidance*, and 7 (12%) were *affordance*. Figure 2 shows a visual representation of how the majority of teachers' responses were directed guidance.

We found a pattern between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task (Figure 3). Of the 14 *telling* responses, 11 (79%) decreased the level of cognitive demand, 3 (21%) maintained the level of cognitive demand, and 0 increased the level of cognitive demand. Of the 30 *directed guidance* responses, 16 (53%) decreased the level of cognitive demand, 14 (47%) maintained the level of cognitive demand, and 0 increased the level cognitive demand. Of the 7 *probing guidance* responses, 0 decreased the level of cognitive demand, 6 (86%) maintained the level of cognitive demand, and 1 (14%) increased the level of cognitive demand. Of the 7 *affordance* responses, 0 decreased the level of cognitive demand, 5 (71%) maintained the level of cognitive demand, and 2 (29%) increased the level of cognitive demand.

When the *telling* and d*irected guidance* responses are combined and the *probing guidance* and *affordance* responses are combined, a clearer pattern emerges. Of the 44 *telling* and *directed guidance* responses, 27 (61%) decreased the level of cognitive demand, 17 (39%) maintained the level of cognitive demand, and 0 increased the level cognitive demand. Of the 14 *probing guidance* and *affordance* responses, 0 decreased the level of cognitive demand, 11 (76%) maintained the level of cognitive demand, and 3 (24%) increased the level cognitive demand.

A Spearman Rank Correlation analysis verified this pattern, showing the relationship between student struggle and teacher responses to be statistically significant (Table 5). This provided evidence that lower level struggles (*getting started, carrying out a process*) were addressed by teachers *telling* or *giving directed guidance.*

Table 5: Correlations Using Spearman Rank Across the Variables Task, Struggle, Response, Outcome, and Cognitive Demand.

		CORRELATIONS				
Spearman's rho		**Task**	**Struggle**	**Response**	**Outcome**	**Cognitive Demand**
Task	Correlation Coefficient	1.000	.017	-.026	-.034	.086
	Sig. (2-tailed)	.	.899	.847	.799	.519
	N	58	58	58	58	58
Struggle	Correlation Coefficient		1.000	.296*	.300*	.192
	Sig. (2-tailed)		.	.024	.022	.149
	N		58	58	58	58
Response	Correlation Coefficient			1.000	.652**	.584**
	Sig. (2-tailed)			.	.000	.000
	N			58	58	58
Outcome	Correlation Coefficient				1.000	.774**
	Sig. (2-tailed)				.	.000
	N				58	58
Cognitive Demand	Correlation Coefficient					1.000
	Sig. (2-tailed)					.
	N					58

** Correlation is significant at the 0.05 level (2-tailed).*

*** Correlation is significant at the 0.01 level (2-tailed).*

The Spearman Rank Correlation test also revealed a significant correlation between the way teachers responded to students' struggles and the impact on cognitive demand (Table 5). The correlation of .584 and significance $p<.01$ indicates teachers' response to student struggle can greatly affect the potential changes in cognitive demand. Thus, the *telling* and *directed guidance* responses were less likely to maintain or raise the cognitive demand of the task in comparison to *probing guidance* and *affordance* responses.

A strong and statistically significant correlation was found between the teachers' response to a struggle and the associated outcome of the episode ($r=.65$, $n=58$, $p<.001$). This was unsurprising because the outcome of an episode (productive, productive at a lower level, or unproductive) was a direct result of the interactions from the response. We found a statistically significant correlation between the outcome of an episode and the resulting cognitive demand of the task ($r=.77$, $n=58$, $p<.001$). This is again unsurprising

FIGURE 4.

Kruskal-Wallis H test across the variables response, task, struggle, outcome, and cognitive demand.

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Task is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.000	**Reject the null hypothesis.**
2	The distribution of Struggle is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.003	**Reject the null hypothesis.**
3	The distribution of Response is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.209	Retain the null hypothesis.
4	The distribution of Outcome is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.234	Retain the null hypothesis.
5	The distribution of Cognitive Demand is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.004	**Reject the null hypothesis.**

because if an episode resulted in a productive outcome, the productive aspect influenced the cognitive demand of the task.

A Kruskal-Wallis H test was conducted using the nine expert teachers as a grouping variable to determine if the kinds of student struggle differed across the teachers, the way teachers responded to the struggle differed, differences in the outcome of the episode across the teachers, or differences in the cognitive demand across the teachers. Since each teacher used a different task that had an original cognitive demand rating associated with it, we did not compute how the original task differed across teachers.

The Kruskal-Wallis H test showed that there was a statistically significant difference in struggle score between the different teachers (test statistic = 8.520, $p = 0.003$). When specific teachers were compared, the test revealed teachers eight and nine were significantly different in the types of struggles students encountered ($p=.001$). The test also showed that there was a statistically significant difference in cognitive demand score between the different teachers (test statistic = 22.378, $p = 0.004$). When specific teachers were compared, the test revealed teachers two and seven were significantly different in how they responded to student struggle ($p <.001$). Figure 4 details these findings as well as the lack of significance between the types of responses across the same category of teachers and the outcome across categories of teachers.

Discussion

By examining an algebra lesson from nine expert mathematics teachers, we found student struggle was evident in all lessons. The first research question asked what types of student struggle are typical during an algebra task implemented by expert mathematics teachers. We found students struggled in ways that aligned with Warshauer's (2014) framework, most often by *struggling to carry out a process*, or *getting started*. These types of struggle are perhaps unsurprising because algebra tasks tend to incorporate many processes that students must perform, or about which students must reason. We note students struggled to carry out algebraic processes within expert mathematics teachers' classrooms and that a teachers' response to this struggle is the determining factor for the cognitive demand of the task.

The second research question asked how expert mathematics teachers responded when students struggled within an algebra task. We found teachers responded in ways that aligned with Warshauer's (2014) framework, most often by offering *directed guidance* or *telling*. These responses can perhaps be explained by the nature of the tasks and the types of student struggle to which the teachers were responding. The algebra tasks primarily caused students to struggle to carry out a process, specifically an algebraic process. The teachers' response to give them the solution method is perhaps a natural resolution to the struggle, as this is a quick way to have students overcome the problem and move forward within the task.

We found an interesting relationship when answering our third research question regarding the relation between teachers' responses and changes in the cognitive demand of the task. We founded *directed guidance* or *telling* responses lowered the cognitive demand of the task 61% of the time and maintained the cognitive demand of the task 39% of the time. *Directed guidance* or *telling* responses were never observed to raise the cognitive demand of the task. While the *probing guidance* and *affordance* responses were less commonly provided by the expert teachers, when these responses were given, they always maintained the cognitive demand of the task (76%) or raised the cognitive demand of the task (24%). This suggests that even though providing probing guidance and affordance responses to students' struggle during algebra tasks might be more time consuming, it is an important part of the learning process, as suggested by other researchers (Lewis, & Özgün-Koca, 2016; Stephan, Pugalee, Cline, & Cline, 2016; Townsend, Slavit, & McDuffie, 2018; Zeybek, 2016).

This study has a limitation of the small sample size (n=9) having been selected as the only algebra teachers available to us. Having collected data from more expert algebra teachers or multiple lessons from the nine expert algebra teachers would have increased the number or episodes of student struggle, ensuring saturation had been achieved, and likely have had a positive impact on the validity of the findings (Fusch & Ness, 2015).

Implications

One implication of this research for educational leaders is that Warshauer's (2014) framework can be useful in describing student struggle and teacher responses. We propose that educational leaders categorize exemplars of how teachers provide *probing guidance* and *affordance* responses to students *struggling to carry out a process or* *getting started*. This would provide constructive and practical content for professional development sessions to help algebra teachers to foster productive struggle in their classroom while maintaining the cognitive demand of tasks.

We also made note of a few teaching characteristics that were not captured in Warshauer's (2014) framework that may be lurking variables useful for educational leaders and educators to consider. First, we saw some teachers used wait time more effectively than others to have students resolve the struggle. Examining the wait time duration and context in comparison to the resolution of the uncertainty might help researchers understand the role this plays in promoting students' productive struggle. Second, the use of groups during the task allows students to provide each other responses to their struggle, thus avoiding an episode or response from the teacher. This type of environment seemed to provide students opportunities to resolve their struggle in comparison to settings where the teacher was the sole person attending to students' struggle. Teachers should be encouraged to consider going through a productive struggle experience to see first-hand the dynamics of the process, group work, and how the entire experience can impact problem solving (Murawska, 2018). Like other researchers (Warshauer, 2014), we suggest teachers incorporate group settings to better promote productive struggle while maintaining the cognitive demand of the task.

A second implication of this work is that mathematics teacher educators can use this data to provide evidence and examples for pre-service and novice teachers on what productive struggle looks like in the classroom and ways to best respond to student struggle to maintain or increase the cognitive demand of the task. Encouraging struggle can be a difficult thing to do for teachers often trained to remove impasses for students. Finding the appropriate strategies to implement opportunities for struggle are an essential aspect of teaching (Freeburn & Arbaugh, 2017; Lobato et al., 2005). New teachers could be influenced by seeing expert teachers incorporating student struggle as a regular, positive, and necessary occurrence in the classroom. Practicing *probing guidance* and *affordance* responses to students rather than *directed guidance* or *telling* responses could be beneficial for teachers in all stages of their career.

A third implication of this work is for educational leaders who select or have input on deciding algebra curriculum materials. The need for high cognitive demand tasks is well established in the literature. Creating appropriate opportunities for productive struggle can be an issue of equity. Task selection for productive struggle opportunities can be important when considering differentiation for diverse learners (Lynch, Hunt, & Lewis, 2018). We contend that in addition to providing high cognitive demand tasks within curricula, educational leaders should also provide suggested *probing guidance* and *affordance* responses that algebra teachers can use with students. Having these examples in the teacher editions of algebra textbooks might give teachers ideas on how to avoid *directed guidance* or *telling* responses, thus maintaining or raising the cognitive demand of the task.

In conclusion, we recommend three kinds of future studies. First, we suggest similar research examining expert teachers within other mathematical topics, such as geometry, trigonometry, and calculus, to see if comparable tasks and responses to student struggle occur. Second, this study had only high cognitive demand tasks used by the expert teachers; we recommend investigating other expert algebra teachers who incorporate low cognitive demand tasks and determining the types of responses given to episodes of student struggle. Third, comparing novice and expert teachers' responses to student struggle within mathematics classrooms would provide useful information about how to tailor teacher education, professional development, and curricular materials to teachers at various stages in their careers. ✪

References

Arbaugh, F., & Freeburn, B. (2017). Supporting productive struggle with communication moves. *Mathematics Teacher, 111(3)*, 176-181.

Barlow, A. T., Duncan, M., Lischka, A. E., Hartland, K. S., & Willingham, J. C. (2017). Are your students problem performers or problem solvers? *Teaching children mathematics, 23*(9), 550-558.

Barlow, A. T., Gerstenschlager, N. E., Strayer, J. F., Lischka, A. E., Stephens, D. C., Hartland, K. S., & Willingham, J. C. (2018). Scaffolding for access to productive struggle. *Mathematics Teaching in the Middle School, 23*(4), 202-207.

Edwards, C. (2018). Productive struggle. *Mathematics Teaching in the Middle School, 23*(4), 183-183.

Fusch, P. I., & Ness, L. R. (2015). Are we there yet? Data saturation in qualitative research. *The qualitative report, 20*(9), 1408.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 524-549.

Herbal-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning, 1*, 371-404.

Lappan, G., & Briars, D. (1995). How should mathematics be taught. *Prospects for school mathematics*, 131-156.

Lewis, J. M., & Özgün-Koca, S. A. (2016). Fostering perseverance. *Mathematics Teaching in the Middle School, 22*(2), 108-113.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, 101-136.

Lynch, S. D., Hunt, J. H., & Lewis, K. E. (2018). Productive struggle for all: Differentiated instruction. *Mathematics Teaching in the Middle School, 23*(4), 194-201.

Murawska, J. M. (2018). Seven billion people: Fostering productive struggle. *Mathematics Teaching in the Middle School, 23*(4), 208-214

National Board for Professional Teaching Standards (2017). Retrieved from http://www.nbpts.org/

National Board for Professional Teaching Standards (2016). Component 3: Teaching practice and learning environment component at-a-glance. Retrieved from http://www.nbpts.org/wp-content/uploads/Component_3_AAG.pdf

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all.* National Council of Teachers of Mathematics.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics teaching in the middle school, 3*(4), 268-275.

Stephan, M., Pugalee, D., Cline, J., & Cline, C. (2016). *Lesson Imaging in Math and Science: Anticipating Student Ideas and Questions for Deeper STEM Learning.* ASCD.

Townsend, C., Slavit, D., & McDuffie, A. R. (2018). Supporting all learners in productive struggle. *Mathematics Teaching in the Middle School, 23*(4), 216-224.

Vygotsky, L. S. 1978. *Mind in Society: The Development of Higher Psychological Processes.* Boston: Harvard University Press.

Warshauer, H. K. (2014). Productive struggle in middle school mathematics classrooms. J*ournal of Mathematics Teacher Education, 18*(4), 375-400.

Warshauer, H. K. (2015). Strategies to support productive struggle. *Mathematics Teaching in the Middle School, 20*(7), 390-393.

Zeybek, Z. (2016). Productive struggle in a geometry class. *International Journal of Research in Education and Science, 2*(2), 396-415.

JOURNAL OF MATHEMATICS EDUCATION LEADERSHIP

Information for Reviewers*

1. Manuscripts should be consistent with NCSM mission.

   NCSM is a mathematics education leadership organization that equips and empowers a diverse education community to engage in leadership that supports, sustains, and inspires high quality mathematics teaching and learning every day for each and every learner.

2. Manuscripts should be consistent with the purpose of the journal.

   The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

   - Strengthening mathematics education leadership through the dissemination of knowledge related to research issues, trends, programs, policy, and practice in mathematics education;
   - Fostering inquire into key challenges of mathematics education leadership;
   - Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice; and
   - Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as to strengthen mathematics education leadership.

3. Manuscripts should fit the categories defining the design of the journal.

   - Case studies of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
   - Research reports with implications for mathematics education leaders
   - Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice.

4. Manuscripts should be consistent with the NCTM Principles and Standards and should be relevant to NCSM members. In particular, manuscripts should make the implications of its content on leadership practice clear to mathematics leaders.

5. Manuscripts are reviewed by at least two volunteer reviewers and a member of the editorial panel. Reviewers are chosen on the basis of the expertise related to the content of the manuscript and are asked to evaluate the merits of the manuscripts according to the guidelines listed above in order to make one of the following recommendations:

   a. Ready to publish with either no changes or minor editing changes.

   b. Consider publishing with recommended revisions.

   c. Do not consider publishing.

6. Reviewers are expected to prepare a written analysis and commentary regarding the specific strengths and limitations of the manuscript and its content. The review should be aligned with the recommendation made to the editor with regard to publication and should be written with the understanding that it will be used to provide the author(s) of the manuscript feedback. The more explicit, detailed, and constructive a reviewer's comments, the more helpful the review will be to both the editor and the author(s).

---

* Please contact the journal editor if you are interested in becoming a reviewer for the *Journal*.

LEADERSHIP IN MATHEMATICS EDUCATION
NCSM NETWORK COMMUNICATE SUPPORT MOTIVATE

National Council of Supervisors of Mathematics
www.mathedleadership.org

Membership Application/Order Form

NCSM has five categories of membership including individual, electronic, student, emeritus and institutional. You may also join online at mathedleadership.org for the individual and electronic memberships. Complete this form and return with payment. The information you provide will be used by the NCSM office for member communication, mailing lists and the NCSM online Membership Directory.

PLEASE PRINT LEGIBLY OR TYPE

First Name__________________________Middle ________

Last Name ____________________________________

This is my mailing address: ☐ Home ☐ Work

Address ______________________________________

____

City______________________State______________

Province/Postal Code_____________Country ____________

NCSM sometimes provides its mailing list to outside companies. These companies have been approved by NCSM to send catalogs, publications, announcements, ads, gifts, etc. Please check the box below to remove your name from mailing lists. In addition, by checking this box, only your name without contact information will be included in the NCSM Directory. ☐

Employer______________________________________

Title ____

Telephone (__________)___________________________

Fax (__________) _______________________________

Email ____

Please check all that apply. I currently work as:

☐ State/Provincial Department of Education Employee
☐ Superintendent
☐ Graduate Student
☐ Department Chair
☐ District Mathematics Supervisor/ Curriculum Director
☐ Building Administrator
☐ Teacher Leader
☐ Coach/Mentor
☐ Consultant
☐ Higher Education
☐ Other___________________________

Since designations vary over time, check the one you feel best describes you:

☐ African American/Black ☐ Asian American ☐ European Descent ☐ Hispanic Descent
☐ Native American ☐ Pacific Islander ☐ Bi-Racial/Multi-Racial ☐ Other_______________

Check the area you serve:
☐ Rural ☐ Suburban ☐ Urban

Do you influence purchasing decisions?
☐ Yes ☐ No

Age: ☐ under 25 ☐ 25 - 34 ☐ 35 - 44 ☐ 45 - 54 ☐ 55-64 and over

_____NCSM Member Pin $2	$ __________
Merchandise Total:	**$** __________
Individual Dues $85	**$** __________
Electronic Membership Dues $70	**$** __________
Student Membership* $55	**$** __________
Institutional Membership $100	**$** __________
Institution Name: ______________________	
TOTAL ORDER:	$ __________

Payment Method: ☐ Visa ☐ MasterCard ☐ Discover Card
☐ Check/Money Order *(U.S Funds only)*
☐ Purchase order***

**Purchase Order # ________________

Credit Card #________________________Exp ____/ ____

Cardholder Name_________________Billing Zip________

Cardholder Signature _____________________________

Please Note: An invoice will NOT be sent. Please use this form as your invoice.

*__Student Membership:__ Requirements for student membership are 1) This is your first membership in NCSM (student members are eligible for one renewal at the student member rate for a maximum of two years and; 2) You are currently enrolled in a graduate program to become a mathematics leader (proof of enrollment must be provided). This membership must be approved by the Membership & Marketing Committee.

Emeritus Membership: Requirements for emeritus membership are being age 65 and a member in good standing of NCSM for a minimum of 15 years or more. Visit mathedleadership.org for additional information.

***__Purchase Orders:__ POs must be paid within 60 days or the membership will be suspended until payment is received.

Institutional Membership: With an Institutional Subscription membership, your organization will receive all of NCSM's print material during the time of your membership. Institutional Subscriptions can only be renewed by mail or fax.

NCSM Tax ID: 39-1556438

Please return this form to:
NCSM Member and Conference Services
6000 E. Evans Avenue 3-205, Denver, CO 80222
Phone: 303-758-9611; Fax: 303-758-9616
Email: office@mathleadership.org **Web: mathedleadership.org**

National Council of Supervisors of Mathematics
PO Box 3406
Englewood, CO 80155

Presorted Standard
U.S. Postage
PAID
Brockton, MA
Permit No. 301

LEADERSHIP IN MATHEMATICS EDUCATION

NCSM

NETWORK
COMMUNICATE
SUPPORT
MOTIVATE