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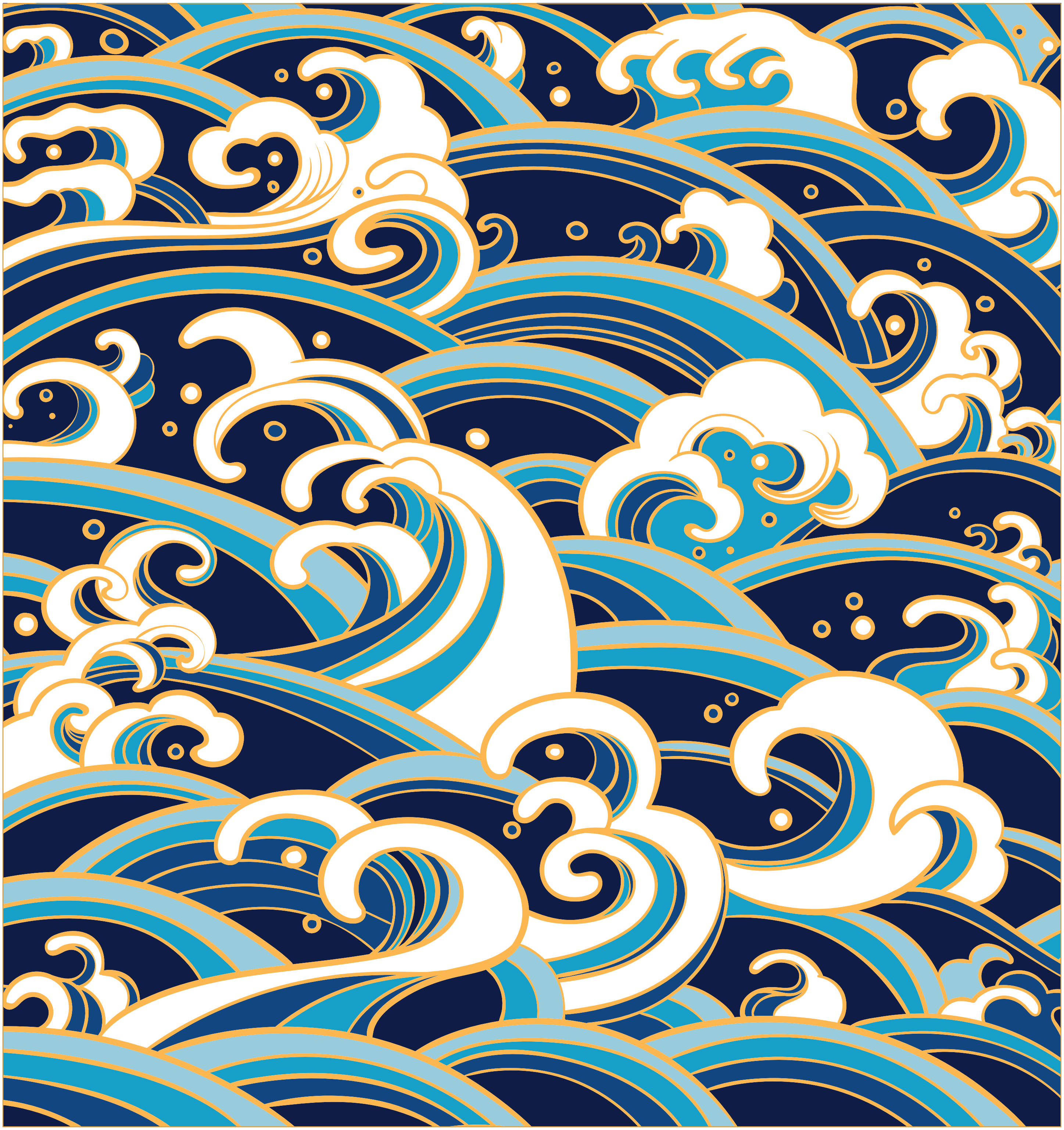

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Discontinuity in Enacted Scope and Sequence of Middle Grades Mathematics Content

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Abstract

The 2010 release of the Common Core State Standards for Mathematics (CCSSM) initiated a tremendous effort to align the mathematics curriculum across the United States. However, the work of enacting these standards, including determining the order to teach grade-level standards, was often left to local schools and district experts to determine. These decisions were influenced by several factors and often formalized in scope and sequence documents, which outlined the order in which grade-level standards would be taught and the amount of time devoted to specific mathematical content and skills. In this paper, we report the analysis of eight Grade 8 mathematics teachers' scope and sequence documents and the underlying factors that influenced their development. Given the discrepancies apparent across these eight documents, we discuss the implications stemming from these curricular decisions and recommend district leadership consider the connections across mathematical content when making decisions regarding the sequencing of topics in any grade level.

Introduction

There is widespread agreement among scholars that curricular coherence is important. The National Mathematics Advisory Panel (2008) defined a coherent curriculum as "marked by effective, logical progression from earlier, less sophisticated topics into later more sophisticated ones" (p. xvii). The Third International Mathematics and Science Study (TIMMS) acknowledged that curricular coherence was the foremost predictor of student performance (Schmidt, Wang, & McKnight, 2005). Furthermore, there are "strong theoretical reasons to expect that a coherent approach to learning, in which learners are supported in deepening their developing ideas by connecting them to multiple contexts of use, should be effective" (p. 525). NCTM further elaborates the significance of curricular coherence as they define mathematical connections in *Principles and Standards for School Mathematics* (2000) as the ability to "recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (p. 64).

To create this curricular coherence, schools and districts often develop scope and sequence documents that specify the order of the mathematics content that teachers should teach throughout the school year as well as the amount of instructional time teachers should devote to these topics. While the Common Core State Standards for Mathematics (CCSSM) outlines the knowledge and skills that should be the focus of instruction at each grade level, it also allows for flexibility for the implementation of these standards: it does not dictate the curriculum, teaching methods, or the scope and sequence of topics at any grade level (CCSSI, 2011). Consequently, pedagogical decisions for how to teach the content in the CCSSM remain open to interpretation (Munter, Stein & Smith, 2015).

When teachers use a mathematics textbook, the book has the potential to become the default scope and sequence for the mathematics topics based on the placement of content in the chapters. However, teachers' lack of understanding in terms of the cognitive intentions of the curricular materials often contributes to an incoherent curricular use of the materials (Confrey, Gianopulous, McGowan, Shah, & Belcher, 2017). In addition, researchers have found that many teachers are using online materials to a greater extent and are often modifying their existing textbook sequences (Larson, 2016; Webel, Krupa, & McManus, 2015). The past president of NCTM, Matt Larson stated, ". . . [the] undercutting of curricular coherence by the introduction of disjointed tasks that are of questionable quality, do not fit within the mathematical learning progression and are not coherent" (Larson, 2016, p. x). While Larson specifically refers to online curricular selection, this stance can also be considered with regards to all tasks selected for instruction.

Building a coherent scope and sequence of mathematical topics is critical for developing students' understanding. As an example, students in Grade 8 should learn about rational and irrational numbers (CCSSM 8.NS.A1-2). These Grade 8 students also learn about the Pythagorean theorem; specifically, how to find a missing side length of a triangle and how to find distances between two points on a coordinate grid (CCSSM 8.G.B7-8). While these two topics are in different domains (number systems and geometry, respectively) they are naturally connected to each other: when students find the length of a missing side on a triangle they find rational or irrational numbers. Therefore, if students are to have opportunities to make connections between these two topics, they should be taught either together or in sequence with each other. Conversely, if teachers teach rational and irrational numbers in isolation from the Pythagorean Theorem, or from other content that may provide connections across content areas, students may miss a valuable opportunity to learn and to make important connections across the curriculum that has the potential to deepen their understanding of the content.

With the discretion to sequence topics given to local schools and districts, the question arises concerning the variance in arrangements of topics in a given grade level across schools. In this paper, we present data from eight Grade 8 teachers' scope and sequence documents across four states (AR, MI, NV, and UT) to answer the following research questions:

1) Where is the topic of geometric transformations sequenced within the Grade 8 curriculum across four states, and in what ways does the sequence allow Grade 8 students to make connections across mathematics content?
2) What influences Grade 8 teachers' decisions regarding the sequencing of the mathematics content?

Methods

Eight middle school teachers, from seven different school districts representing four states (AR, MI, NV, UT), were selected to participate in the study. The teachers submitted their scope and sequence documents and participated in an interview about the development of this document and rationale for their decisions related to the sequencing of Grade 8 mathematical content. These teachers participated in a larger National Science Foundation study focused on describing teachers' curricular reasoning for their mathematical decisions. All four states adopted CCSSM, and thus the content in Grade 8 was identical. We compared the scope and sequence documents submitted by the teachers to identify when content was taught during the academic year. We analyzed the interview data to identify the influences on teachers' decisions to sequence the content in the way they did. Finally, state assessment information from the four states was analyzed to triangulate the interview data and the scope and sequence documents.

Results

In this section, we present results as they pertain to the two research questions under investigation. In particular, we share our findings related to the sequencing of mathematics topics in Grade 8 that we identified through scope and sequence documents, as well as the teachers' thoughts gleaned from the interview data. Additionally, we discuss our investigation of the connections between scope and sequence documents and state assessment documents as an influencing factor for these decisions.

Scope and Sequence

In our NSF project, we primarily focused on examining teachers' curricular reasoning as it pertains to mathematical decisions related to the teaching of geometric transformations in Grade 8. Table 1 provides an overview of the reported scope and sequence of key topics per quarter (approximately 9 weeks) throughout the school year across the eight teachers. As reported in Table 1, two teachers taught geometric transformations during Quarter 1, one teacher each taught geometric transformations during Quarter 2 and Quarter 3, and four teachers taught geometric transformations during Quarter 4.

Table 1: Scope and Sequence of 8th Grade Content per Quarter

Teacher	Quarter 1	Quarter 2	Quarter 3	Quarter 4
AR$_1$ (HA)	**Geometric Transformations**	Linear Functions; Bivariate Data	Rational/Irrational Numbers; Pythagorean theorem; Angle/Triangle Relationships; Exponents	Solving Equations; Volume;System of Linear Equations; Congruence/ Similarity
AR$_2$ (MH)	**Geometric Transformations**	Angle/Triangle Relationships; Solving Equations; Congruence/ Similarity; Rational/ Irrational Numbers	Linear Functions; Solving Equations	System of Linear Equations; Pythagorean theorem; Volume; Bivariate Data
MI$_1$ (SJ)	Exponents; Rational/ Irrational Numbers; Pythagorean theorem; Solving Equations; Linear Functions	**Geometric Transformations**; Linear Functions; Systems of Linear Equations	Exponents; Volume; Angle/Triangle Relationships; Pythagorean Theorem	Bivariate Data
MI$_2$ (MT)	Rational/Irrational Numbers; Linear Equations; Linear Functions	Linear Equations; Bivariate Data; System of Linear Equations	**Geometric Transformations;** Exponents	Pythagorean theorem; Volume; Angle/Triangle Relationships
NV$_1$* (TC)	Rational/Irrational Numbers; Exponents	Solving Equations; Linear Equations; Systems of Linear Equations; Linear Functions	Angle/Triangle Relationships; Linear Functions; Bivariate Data	**Geometric Transformations;** Dilations; Pythagorean theorem; Volume
NV$_2$* (SS)	Rational/Irrational Numbers; Exponents	Solving Equations; Linear Equations; Systems of Linear Equations; Linear Functions	Angle/Triangle Relationships; Functions; Bivariate Data	**Geometric Transformations;** Dilations; Pythagorean theorem; Volume
UT$_1$ (BS)	Rational Numbers; Solving Equations; Exponents	Irrational Numbers; Exponents; Linear Functions; System of Linear Equations	Bivariate Data; Pythagorean theorem; Angles/Triangle Relationships	**Geometric Transformations;** Congruence/Similarity; Volume
UT$_2$ (FJ)	Solving Equations; Linear Equations	Linear Equations; Systems of Linear Equations; Bivariate Data, Linear Functions	Exponents; Volume	**Geometric Transformations;** Rational/Irrational Number; Pythagorean theorem; Congruence/ Similarity; Angle/Triangle Relationships

** Note: NV1 and NV2 are teachers at the same school.*

A similar variation of topic placement is evident across other content as well. In particular, the quarter in which teachers taught Bivariate Data, Congruence and Similarity, Rational and Irrational Numbers, Pythagorean Theorem, and Angle and Triangle Relationships (e.g., parallel lines cut by a transversal) is not consistent among the eight teachers. Furthermore, the placement of content relative to other concepts indicates that among these eight teachers in four states, there does not appear to be a consistent placement of topics throughout the school year.

Teachers' rationale for the placement of geometric transformations in the school year varied. We found that teachers who taught transformations in either quarter 3 or 4 were surprised when their students made connections to mathematics concepts they had taught earlier in the year, often without prompting from the teacher. One teacher (TC) was teaching Pythagorean triples when a student connected this idea to dilations. The student said, "well of course there are going to be other triples because you are just dilating the triangle so you just multiply the three sides of the triangle by a scale factor and you will have a similar triangle." As a result of students making these impromptu connections and once realized, the teacher decided to capitalize on this in the future. Another teacher (FJ) stated, "I would say that I would keep it [geometric transformations] at the end because I like tying everything together in a nice bow."

The two teachers who taught geometric transformations at the beginning of the school year (HA and MH) intended to make connections throughout the year. One teacher (HA) focused on the various connections she anticipated making instructionally throughout the school year. She added that Grade 7 students at her school can take an Accelerated Math course which would prepare them to take Algebra I in Grade 8. The Accelerated Math course included a unit on geometric transformations, so those students who discovered they were not ready for Algebra I in Grade 8 could transfer back into the regular Grade 8 Math course after the first quarter. Therefore, HA's reasoning for beginning the school year with geometric transformations was to ease the transition of these students back into the Grade 8 Math course without missing any new mathematical content during quarter 1 of the school year. The other teacher (MH) remarked that she intended to use geometric transformations (which was the focus for the entire first quarter of the year) as a springboard for the mathematics content taught throughout the year. While both teachers decided to teach geometric transformations as their first unit of the school year, their rationale for doing so was different. On the other hand, teacher (MT) who taught transformations in quarter 2 indicated that she had chosen to teach geometric transformations prior to the Thanksgiving break because it was a short unit, and the unit could be completed before the break. MT seemed not to consider curricular connections when placing the transformation unit and based her decision on the school calendar.

When directly asked how their scope and sequence documents were developed, the typical response from all eight teachers was that a group of teachers in a school or the district worked together to unpack the mathematics standards and then determine which mathematics content went together best. The teachers indicated that their knowledge of which content went together was largely based on their past experience as well as curriculum materials being used.

Influence of State Assessments on Scope and Sequence

Many of the teachers indicated that the content and timing of state assessments also impacted the placement and timing of the teaching of topics. In Table 2, we list the assessments used by each state. For example, the M-STEP (Michigan Student Test of Educational Progress) only assesses algebra topics in Grade 8, which the two teachers in our study indicated was a reason why they pushed the majority of geometry content to quarters 2 and 3. However, the Arkansas teachers taught transformations during quarter 1 and indicated that the state assessment impacted the placement of transformations in their scope and sequence. One teacher from Arkansas indicated that she believed the ACT Aspire was very geometry-intensive. This knowledge of the assessment also impacted her decision to teach geometric transformations during quarter 1.

In addition to conducting interviews centered on the scope and sequence document, we independently analyzed the assessment frameworks across the four states. We then compared teachers' scope and sequence documents to ascertain ways the state assessment might have influenced the placement of mathematics content. From our analysis of the official websites for each of the assessments, we identified several instances of misalignment between teacher's interpretation of the mathematics standards and the assessed content on the state assessment. For example,

Table 2: Assessments Used by State

State	Assessment
AR	ACT Aspire
MI	M-STEP (Michigan Student Test of Educational Progress) and MME (Michigan Merit Exam)
UT	SAGE (Student Assessment of Growth and Excellence) https://www.schools.utah.gov/assessment/assessments
NV	Smarter Balanced Assessment Consortium (SBAC)

Standard 8.G.A.3 asks students to "describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates" (CCSSI, 2010). Of the three states that assess geometry content, only the SAGE in Utah focused explicitly on assessing students' ability to provide coordinate rules for transformations, and the SBAC assessment used by Nevada explicitly notes that such rules are a "non-targeted construct" of the assessment. Yet all eight teachers in our study were explicit about the need to teach students the coordinate rules for transformations; they believed these rules were implied by the standard 8.G.A.3 and assessed on the state assessments.

Implications

From this study, we found that the same mathematical topics and concepts are taught among the eight teachers interviewed over the course of the year, which is not surprising given that each state included in the study adopted CCSSM. However, it is also apparent that among the teachers there is very little agreement regarding the sequence in which the content is taught across the school year. These discrepancies are understandable given that CCSSM does not dictate the ordering of content throughout the year. However, knowing that these discrepancies exist, we posit two areas of implications: implications for curriculum developers, district leaders, and state leaders, and implications for teachers.

Implications for Curriculum Developers and Leaders

We maintain that the sequencing of mathematics content matters for building connections of topics within the school year for students. From our small sample, we found that the scope and sequence documents were usually developed based on a small group of teachers' past experiences with different curricula and the sequences that were most familiar. This has implications for curriculum developers and district leaders. While we agree that there is no "right order" for mathematics content to be sequenced within a given year, there are sequences that naturally lead to connections more easily than other sequences that can unnecessarily make connections more challenging. We challenge curriculum developers to use learning trajectories research to determine sequences of mathematics topics in their curricula. While curriculum is typically sequenced to match the order of the chapters, we know that many teachers do not use curricula in this way. We suggest that curriculum developers could provide teachers with multiple sequences that could connect content in different meaningful ways.

While most districts have a scope and sequence document or framework, we suggest that the process for sharing and using the scope and sequence documents needs to be modified. In most districts this document is developed by a small committee of instructional leaders and teachers and then is disseminated to all teachers within the district. We believe that district leaders need to be more proactive and transparent in having discussions with teachers across the district about why topics are sequenced in the way they are and what connections are expected if the sequence is followed. These discussions could help teachers understand the connections that are expected across content rather than focusing on teaching a list of topics for assessments. We also believe that state and district leaders need to offer professional development for teachers to learn about different connections across mathematics content. Professional development needs to support teachers and their instruction with this knowledge. If state and district leaders were to discuss the reasons why content was sequenced as it is, then this may help teachers make decisions that would better support students making connections among the mathematics content taught.

Additionally, we found important differences among state-level assessments across the four states in our study resulting in disconnects between the curricular materials teachers use and their state assessments. That is, teachers are teaching topics (e.g., using coordinate rules for teaching transformations) while the state assessments explicitly labeled such topics as a "non-targeted construct." These discrepancies indicate that teachers likely need more exposure to the mathematical content of state assessment frameworks, and state-level coordinators need to build coherence between the content in state assessments and scope and sequence documents within schools or districts.

Implications for Teachers

The sequencing of mathematics within a grade and across grade levels is a vitally critical decision if teachers are to assist students in connecting mathematical content. We found in our study that teachers' decision-making for scope and sequence was primarily based on their current curriculum materials and personal experiences teaching the content. None of the eight teachers indicated that a learning trajectory was used to influence the sequencing of mathematics content in their classroom. Such lack of learning trajectory usage is an example of research in the field of mathematics education that is not reaching, nor directly impacting those who have the most influence on students' opportunity to learn – the mathematics teachers. We recognize teachers may need professional development focused on learning trajectories including why certain mathematical topics should be taught prior to other mathematical topics. In addition, in an era where traditional textbooks are not as prevalently used as in the past, and where many teachers are not using textbooks at all, all stakeholders (e.g., curriculum specialists, teachers, curriculum developers) need to consider the connections between different content when making sequencing and instructional decisions.

Furthermore, teachers need instructional support in sequencing curriculum that provides multiple learning opportunities for students to make mathematical connections. One consequence of the inconsistent curriculum sequence is that some students have many opportunities to connect mathematical concepts while others do not. For example, one of the teachers who started the school year teaching geometric transformations chose to do so because she saw many of the Grade 8 standards connecting to this idea (regardless of strands). She discussed that throughout the school year, she continued to build upon the "transformational thinking" from the first unit of the year within the subsequent topics, including integrating algebraic thinking and concepts into the geometry unit. Additionally, some of the teachers who taught geometric transformations later in the school year used this unit to connect multiple concepts together that had been addressed earlier in the school year.

Conversely, one teacher who began the school year with geometric transformations did not teach the concept of congruence and similarity until near the end of the school year. By placing these topics as bookends for the school year rather than in short proximity to one another, building connections between the two ideas would be harder to do. Hence students lost an opportunity to deepen their understanding of the connections between congruence and transformations. While we did not specifically look at the relationship between student achievement and specific curricular sequences, opportunities for rich mathematical connections are lost without a supporting sequence.

Conclusion

Although this is a limited study of eight teachers across four states, we have demonstrated that there is little agreement regarding the sequencing of content at one specific grade level. We assume that this phenomenon is not specific to Grade 8 or to the four states under investigation. Our study points to the incoherence among mathematics curriculum and confirms previous research indicating that among middle-grades curricular materials, there are important points of incoherence with the sequencing and development of topics (Olson, 2014).

Though the standards outlined in CCSSM are sequenced in multiple ways, sequencing must support student learning of the content and naturally build connections across topics. These curricular decisions regarding the placement of topics within the school year, as well as the duration of time spent on each topic, must be made with student learning in mind rather than for non-academic reasons that may pressure curricular decisions. Void of connections, students might believe that mathematics is simply an accumulation of topics and ideas. We maintain that the sequencing of topics within the school year is a vitally critical decision if teachers are to assist students in connecting the mathematical content. While we do not argue that there is one "right order" for the content to be sequenced, we do suggest there are sequences that naturally lead to connections more easily made than other sequences.

In order to work toward meaningful mathematical education, we must ask how content is connected and not simply what content is taught. We believe that our field is at an important juncture in understanding that it is not enough to teach topics across disparate disconnected lessons. From our work, in particular from our current research couched within geometric transformations, it is necessary for teachers, district leaders, parents, and policy makers to understand that the more mathematical topics students are taught in disconnected lessons, the less opportunities they have to learn and be exposed to rich mathematics.

In our research, as well as our collective professional development work, we have observed teachers who are eager to deepen their understandings of mathematical connections and learning trajectories. We believe district leaders who design and deliver professional development to support teachers in deepening mathematical meanings need support to help school administrators recognize the importance of understanding these connections. In the end, we encourage all stakeholders who are involved in making curricular decisions for their school, district, or state to consider the importance and implications of sequencing mathematical content at the forefront of all decision-making and to do so with the goal of promoting connections and understanding across topics to deepen student learning. ✪

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