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of Mathematics Education Leadership

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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership (JMEL)*** are interested in manuscripts addressing issues of leadership in mathematics education which are aligned with the NCSM Vision.

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM.

Categories for submissions include:

- Case studies and lessons learned from mathematics education leadership in schools, districts, states, regions, or provinces
- Research reports with implications for mathematics education leaders
- Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
- Other categories that support the NCSM vision will also be considered.

Submission Procedures

Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel.

Manuscripts should be emailed to the Journal Editor, currently Carolyn Briles, at ncsmJMEL@mathedleadership.org.

Submissions should follow the most current edition of APA style and include:

1. A Word file (.docx) with author information (name, title, institution, address, phone, email) and an abstract (maximum of 120 words) followed by the body of the manuscript (maximum of 12,000 words)
2. A blinded Word file (.docx) as above but with author information and all references to authors removed.

***Note:** Information for manuscript reviewers can be found at the back of this publication.

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NCSM Vision

NCSM is the premiere mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

Purpose Statement

The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

- *Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education*
- *Fostering inquiry into key challenges of mathematics education leadership*
- *Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice.*

Comments from the Editors

M. Carolyn Briles, *Loudoun County Public Schools*
Brian Buckhalter, *Buck Wild About Math, LLC*

Sophisticated forms of teaching are needed to develop student competencies such as deep mastery of challenging content, critical thinking, complex problem-solving, effective communication and collaboration, and self-direction. In turn, effective professional development (PD) is needed to help teachers learn and refine the pedagogies required to teach these skills. (Darling-Hammond, Hyler, & Gardner, 2017, p. v)

Preparing students to be contributing citizens in today's dynamic world is challenging work. Access to information has changed what we need to learn and the best ways to learn it. Preparing teachers to prepare learners has become more and more complex, and for in-service teachers already in the classroom, professional development (PD) is the main conduit for this process.

As leaders in math education, we owe our teachers a positive and effective learning experience. Their time is valuable, and we must use it well. So what makes effective PD? In their research, Darling-Hammond, Hyler, & Gardner (2017) found seven characteristics of programs that benefit both teachers and their students. They found PD that makes lasting positive change:

- is content focused
- incorporates active learning
- supports collaboration
- uses models and modeling of effective teaching practices
- provides coaching and expert support
- offers opportunities for feedback and reflection
- is of sustained duration.

This issue of JMEL focuses on effective professional development. While the programs described in this issue embody effective PD, our authors go one step further and ask what we as leaders can learn about teachers through their PD experiences.

In our first article, "Equity-Focused Professional Development for Algebra I Teachers in Urban Districts," Bonner asks if sustained PD that focuses not only on math content but also on equity-based practices can develop more culturally responsive math educators. In addition to answering this question, her observations include an interesting phenomenon that she calls the professional development gap (PDG), a phenomenon in which teachers who have been consistently using instructional practices learned in PD revert back to traditional teaching methods. Her research identifies precipitating events for these changes and seeks to understand why PD, even the most effective, sometimes doesn't stick.

Our second article, "Transitioning Face-to-Face Mathematics Professional Development to Synchronous Online Implementation: Design Considerations and Challenges," deals with another equity issue – the issue of access to effective PD, especially for teachers in rural areas. Amador, Callard, Choppin, Gillespie, and Carson share their experiences and lessons learned in redesigning their successful, in-person Teaching Labs PD for an on-line experience. Each iteration of their model gives a unique insight into the needs and challenges of classroom teachers.

Finally, in our third article, "Teacher Interpretations of the Goals of Mathematics Professional Development and the Influence on Classroom Enactment," Walker explores how teachers' perceptions of the goals of PD affect what is implemented and, just as importantly, what is not implemented in their classroom. He follows four secondary math teachers as they choose and teach lessons designed during a long-term PD program.

As a leader in mathematics education, what is your role in the professional development of teachers? How can your leadership improve that experience for them, and in turn, for their students? We hope that these articles will help you answer those questions. Professional development is a key part of our mission at NCSM as we seek to inspire high-quality mathematics instruction for each and every learner. As leaders, we cannot forget that those learners include teachers. ✪

References

Darling-Hammond, L., Hyler, M. E., Gardner, M. (2017). *Effective teacher professional development.* Palo Alto, CA: Learning Policy Institute. Thousand Oaks, CA: Corwin.

Equity-Focused Professional Development for Algebra I Teachers in Urban Districts

Emily P. Bonner, *University of Texas at San Antonio*

Abstract

Student data show that there is a need to develop a more culturally responsive mathematics teaching force. As such, we developed a framework for equity-focused professional development (EFPD) for mathematics teachers through which we hope to improve student access to mathematical knowledge. In this paper we present our EFPD framework, program, and initial results related to culturally responsive mathematics teaching. Further, we describe our process for tracking teacher progress. In this context, we present struggles that we have faced in implementing this framework in an effort to contribute to ongoing discussions about the ways in which the educational system in general and the current political climate in education impact EFPD.

American students' average scale mathematics scores on the National Assessment of Educational Progress (NAEP) have consistently increased since 1990, yet the gaps in performance across ethnic groups persist (NAEP, National Center for Education Statistics 2017). This disparity in performance outcomes, in addition to the need to think about mathematics education more comprehensively (Gutiérrez & Dixon-Román, 2011), has highlighted the need for designing learning environments that address the educational needs of an increasingly diverse student population. Professional learning for mathematics teachers in the form of equity-focused professional development (EFPD) has the potential to address this problem.

In this work, EFPD for mathematics teachers is characterized as professional development that fosters culturally responsive teaching practices that "draw meaningfully on the cultures, languages, and experiences that students bring to classrooms to increase engagement and academic achievement for students" (Dutro, Kazemi, Balf & Lin, 2008, p. 271) in an effort to diminish the existing achievement gaps and counter the dominant deficit discourse surrounding underserved students in mathematics classrooms. As such, EFPD provides in-depth content support for teachers while explicitly addressing and centering race, class, and identity in the program. The shift towards culturally responsive mathematics teaching is foundational, and in-depth content knowledge supports teachers enacting more equitable teaching.

Culturally responsive mathematics practice (CRMT) (Bonner, 2014; Gay, 2000; Gonzalez 2009; Ladson-Billings, 1994), has roots in, " . . . a pedagogy of opposition [that is] committed to collective, not merely individual empowerment" (Ladson-Billings, 1995, p. 160). The literature base in culturally responsive teaching (CRT) provides a theoretical framework within which innovative practice can develop; however, systemic structures complicate the ability for teachers and teacher educators to enact culturally relevant practice in meaningful, holistic ways. In teacher education there are hallmarks of CRT that are important to teacher practice in general, not just in mathematics. As such, culturally responsive teachers operate from a foundationally critically conscious framework that underlies practice. Culturally responsive teachers are committed to learning about and from students (Bonner, 2014; Villegas & Lucas, 2002) to capitalize on students' funds of knowledge (Moll et al., 1992) in the classroom. This requires teachers to focus on developing culturally connected ways of communicating with students so that transmission of knowledge, and therefore power, can be transferred in the classroom more seamlessly (Bonner, 2012). Through these practices teachers develop an asset-based view of students (Villegas & Lucas, 2002), implicitly and explicitly value the knowledge that students bring to the classroom, and help them to see how their knowledge base is valuable in operating in various settings (Gay, 2010).

Culturally responsive teachers build relationships with students by attending to the development of students' complex identities in and out of the classroom (Aguirre, Mayfield-Ingram, & Martin, 2013). This means disrupting deeply held beliefs about students that may have been ategorized as "low" or "at risk" and rejecting deficit language. Students from all backgrounds have shown resilience in a variety of settings (Martin, 2000) and are capable of brilliance in mathematics if given the opportunity (Turner & Celedon-Pattichis, 2011). As such, culturally responsive teachers utilize communication, knowledge, and relationships to disrupt the dominant narrative and create pathways and access for traditionally underserved students to thrive in mathematics and beyond. While much work on CRT has been done, there is little that speaks to professional development for in-service secondary mathematics teachers as a tool for developing culturally responsive practice.

Purpose of the Study

The purpose of this study was to determine if an ongoing professional development program that specifically focused on building mathematics content and equity-based practice was effective in developing more culturally responsive mathematics educators. The goal of that program was to improve the educational experiences of traditionally underserved students in mathematics classrooms. The study presented here explores the successes and struggles of this EFPD program and is meant to contribute to discussions in the literature around equity-focused professional development of mathematics educators. As such, this paper aims to present a comprehensive overview of our framework for EFPD and present findings related to ongoing struggles experienced within this framework that relate not only to this topic but also to larger conversations about the impact of professional development on teacher practice, particularly as it relates to underserved populations.

Description of the Program

The City Mathematics Collaborative[1] (CMC) is a program that provides long term (at least two years) professional development to mathematics teachers teaching in schools with high populations of traditionally underserved students. The program emerged due to state-wide needs in mathematics education and is federally funded. The program has served over 100 in-service Algebra I teachers who teach in one of several high-need urban districts, each of which serves traditionally underserved students from low socioeconomic neighborhoods. Teachers in these districts were recruited in teams (by district), and have been targeted for professional development based on district need (districts with high percentages of failing students are given priority), teacher content knowledge (number of advanced mathematics courses taken), years of service, or certification issues (alternatively certified or not certified in instructional area). Below is general information to give the reader a snapshot of the teachers involved in the project. These are averages over six years (three two-year iterations) of the project:

- Teachers have completed an average of nine hours in college level mathematics content courses.
- 20% of teachers have an undergraduate degree in mathematics content.
- 80% of teachers were alternatively certified.
- 10% of teachers were not certified in mathematics.
- Teachers have an average of seven years of experience (years of experience range from 1-25).
- Districts are among the lowest performing in the city in mathematics.

The CMC has two major components: a 45-hour summer course (three hours per day for three weeks) and 65 hours of professional development during the academic year (sessions are held roughly one Saturday per month). The summer course focuses largely on developing teachers'

[1] A pseudonym has been used

mathematics content knowledge but includes several other unique components. For example, master teachers from urban districts infuse the content-focused instruction with research-based, culturally responsive practices. Further, participants engage in an online component, eCommunity of practice, in which they are prompted to discuss issues of equity, reflect on topics from class, and work as a team to develop culturally responsive habits and action research plans that will help to investigate inequity and promote equity in their schools.

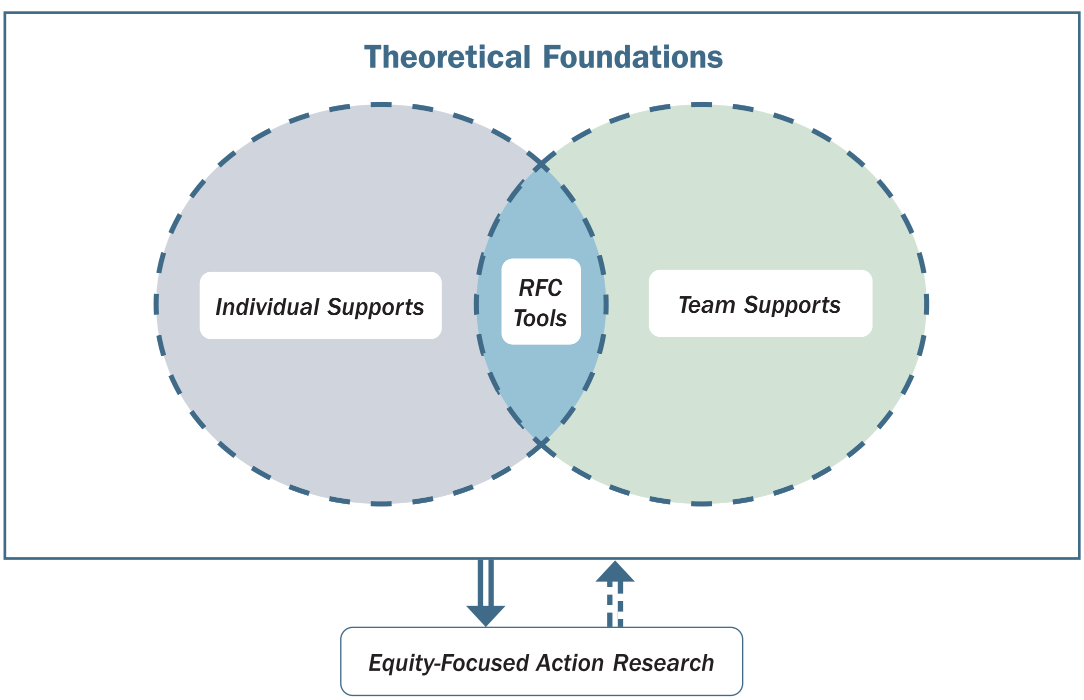
During the academic year, content of Saturday sessions is determined by specific district and teacher needs. For the cohort in this study, the most notable sessions centered on using technology in the teaching of Algebra I (calculators, GeoGebra, Wii gaming systems), and teacher planning and alignment. Further, teachers continued to engage in the eCommunity of practice and worked to build an ePortfolio throughout the academic year.

Throughout the academic year, data about the project were gathered from multiple sources, including interviews with participants (a minimum of every six months), classroom observations, the eCommunity of practice discussions and reflections (individual and group), and field notes from professional development sessions. Data were transcribed and deidentified before coding. A three-tiered coding scheme (open, axial, selective) and constant comparison (Glaser & Strauss, 1967) were utilized to unearth themes from the data. These themes gave us insight into the broad spectrum of data that we collected and helped us to identify patterns that emerged. We will report on this program and the ways in which various aspects and experiences impacted teacher practice and student learning as well as the components of the program that were not successful in impacting teacher practice.

Equity-Focused Professional Development Framework

Given the unique population of students served by our teachers, we explicitly focused our professional development sessions on issues of equity in mathematics including components that would contribute to a greater attention to these issues among teachers. Our initial framework is presented in Figure 1 and includes several foundational pieces: ready for classroom (RFC) tools (Gage, 1974), theoretical foundations, individual support (Fullan, 1991) and team building (Lieberman & Miller, 1991; Calderón, 1999) in the context of ongoing professional development and research. It is in the intersection of these foundational experiences that we hope to see meaningful outcomes such as equity-focused action research which may be useful in helping teachers to identify and challenge educational

FIGURE 1. *EFPD framework*



inequities that they see in practice (Cornell, 2012). Here we will provide details about the major aspects of our framework as a context for our research results so far and ongoing "struggles."

The EFPD framework is rooted in literature- and practice-based foundations on which we focus when developing professional development sessions and other experiences. In the short term, the goal in structuring the program around these areas is that together these foundations will serve as catalysts for more meaningful, deep, equitable practice among teachers. In the long term, the goal is for teachers to take these foundations forward together and facilitate change on their campuses and in their districts. It should be noted that although mathematics content knowledge is not its own category, it underlies all activities.

Theoretical foundations. All of the work that we did in facilitating professional development sessions and other supporting activities was rooted in theoretical foundations. Most readily, we centered discussions around culturally responsive teaching (Gay, 2000; Ladson-Billings, 1995), highlighting the following practices as central to this work:

- Learn about and honor cultural heritages, which affect students' ways of communicating, ways of learning, dispositions, and attitudes,
- Honor cultural heritages, which affect teachers' ways of teaching,
- Communicate consistently high expectations through challenging tasks, respect, and high level discourse that is culturally connected,
- Design instruction to promote student engagement and build bridges between lived and abstract mathematical concepts,
- Challenge the status quo, and provide opportunities for students to do the same.

As the facilitators and mentors operated from an equity perspective, these theoretical foundations were not only discussed, but also modeled and centered throughout the program. This was done in explicit ways, such as discussions around readings and classroom events, and in implicit ways, such as through targeted questioning that guided teachers to think about moves from an equity perspective. For example, during the first professional development session that we held, teachers read Wheatley's *Willing to be Disturbed* (2002, sessions 1 and 2) to set the stage for difficult discussions, teamwork, and individual growth. We also utilized Aguirre, Mayfield-Ingram, and Martin's (2013) guiding questions: "What mathematics, for whom? For what purposes?" (p. 5) to guide our discussion (sessions 1 and 2). These questions reinforced the central idea of constructing knowledge together about problems that have yet to be solved. Teachers also discussed McIntosh's (1989) White Privilege Inventory and the strengths and weaknesses of this type of tool (session 3). This facilitated discussions about race, privilege, and status, and the ways that these constructs affect students and schooling. Teachers also completed seminal readings such as chapters from Geneva Gay's (2010) *Culturally Responsive Teaching* (session 5), Sensoy & DiAngelo's (2012) *Is Everyone Really Equal?* (session 5), and Gloria Ladson Billings' (1994) *The Dreamkeepers* (sessions 5 and 6) during the project.

As we moved through the program, we kept these conversations and aforementioned bullet points as foundations for our work and continually referenced them as guides for best practice. Notably, this affected the ways in which teachers (and teacher educators) were more careful when using deficit language to describe learners. Ultimately, we saw these theoretical foundations facilitate paradigm shifts towards culturally responsive practice. We also held online discussions related to these ideas. For example, if a teacher taught a lesson and encountered an issue that called into question an issue of equity or access, he or she might post a thought question on our discussion board, and others could contribute or discuss as they were able. This allowed for more continuous dialogue in a safe space throughout the program.

Individual and team supports. At the campus level, we provided teachers with implementation support at both the individual and team level. For example, a teacher mentor made regular visits to each participating campus. Each mentor was assigned to particular campuses for which they were primarily responsible. Some crossover was intentionally built in to encourage collaboration. Mentors traveled to assigned campuses and classrooms to provide specific feedback to participants in the course of teaching. This included observing, providing feedback on particular areas of interest to the teacher and/or project, co-teaching with participants, and providing emotional support. Project directors also visited each classroom at least one time per semester. In addition, peer observations and feedback were also encouraged, and we noted that these interactions occurred voluntarily, even when the mentor was not present.

To complement the individual support that teachers received, we provided team support at the campus and district levels. At the campus level, teacher mentors found common times where teachers could discuss specific lessons and action plans as well as ways in which they could support each other. These meetings operated in a fluid manner depending on schedules and new issues that may have come up. They often functioned as a support group to build community. When possible, campus administrators such as department chairs were invited to attend and contribute to these conversations. At the district level, teacher visits were facilitated between campuses. This allowed for discussions about vertical and horizontal alignment and helped teachers to see what was happening across the district.

Ready for classroom tools. In the course of recruiting teachers, we learned that participants desired tangible "tools of the trade" (Gage, 1974) that were physical manifestations of the theoretical ideas we were advancing. As such, we sought to provide professional development sessions that would provide these ready for classroom tools. To cue thinking (McTighe & Lyman, 1988) and facilitate discussion among the teachers, we had participants read short, key articles in preparation for a session and then engaged them in online discussions. These online discussions provided the bridge necessary to facilitate the development of praxis, that point where theoretical discussion meets practical application.

Many of our initial workshops focused on this area and addressed topics such as using an NSpire calculator to teach functions and using tools such as GeoGebra to facilitate problem-based learning. Further, participants spent much of the summer working with two master teachers who shared many ideas for projects and other instructional tools.

Equity-focused action research. The foundations of our framework are meant to serve as springboards to more meaningful experiences and actions in the classroom, particularly in terms of equitable practice. Our focus, therefore, is in accomplishing these outcomes as a result of providing the foundations. In looking at our foundations, for example, we have stated that teachers came to our program hungry for RFC tools. In our view, this was a great opportunity to provide teachers not only with these tools, but also with knowledge in theoretical foundations that would allow them to take RFC tools and adapt them for their particular population. Through this process, teachers were involved in innovating[2] to develop new tools and ideas about curriculum. Given the districts' focus on packaged curricula, we saw this as an area that needed particular attention. Further, we hoped that providing individual support and tools, such as calculators, computers, and literature, would help teachers to innovate in other ways such as using technology as a tool to promote equity.

In order to support this type of innovation beyond the project, we engaged teachers in action research projects to inform the most effective types of instruction for their particular population. These types of projects had both an individual and group component, and they allowed teachers to focus on areas that were of particular interest to them. For example, a teacher could choose to investigate whether a particular computer program (an RFC tool) supported a student's understanding of equivalent fractions. Alternately, a team of teachers could develop a community-based lesson and implement it across classes to determine if that type of lesson had an effect on student engagement and achievement.

The results of teachers' research studies were shared across the CMC project and beyond. As such, teachers learned to use a sustaining tool that allowed them to design classroom research projects to inform instruction and promote equity. Further, teachers began to advocate for themselves and each other using data collected in classrooms. For example, one group of five teachers from a particular campus found that providing access to a particular computer program for three minutes per day helped students to master basic skills, thus increasing achievement across the board. As a team, they disaggregated their data to show the administration that this was most beneficial to traditionally underserved students and advocated that this should be available to students across campus to promote a more equitable environment.

Outcomes and Discussion

The purpose of this study was to determine if an ongoing professional development program that specifically focused on building mathematics content and equity-based practice

[2] We define teacher innovation as the implementation, on any scale, of a new idea, activity, or teaching method in or out of the classroom.

was effective in developing more culturally responsive mathematics educators. Across all of our data several themes emerged that provide some insight into this type of work with this particular population.

Sustained Support

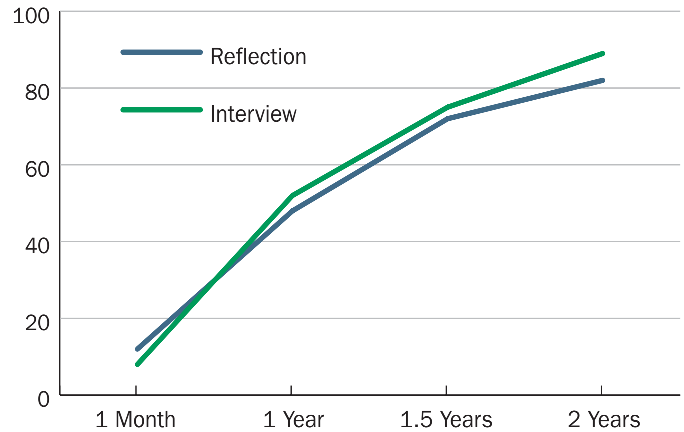
As detailed in Figure 2 and Table 1, we saw shifts in pedagogy and approaches to teaching over the two years that teachers were engaged in the program. It is important to note, however, that many of these shifts occurred very gradually, with the most notable movement towards CRT happening towards the end of the project. This pattern was most evident in interview and online reflection data collected from participants. On average, the number of participants who discussed some aspect of culturally responsive teaching in subsequent instances of interviews and reflections grew substantially. Figure 2 shows quantitatively (by count) the drastic increase in discussion of CRT tenets across the project. We believe these sharp increases were due to continued and in-depth discussion of these ideas in individual, team, and online settings. Our observation data showed similar trends, but the interview and reflection data were particularly relevant as these data came directly from the teachers themselves.

Sustained support also decreased deficit language used by teachers in the program. After one year in the program, teachers' use of deficit language in online reflections decreased by 78%. This was a major shift away from using the words "low", "at risk" or "below grade level" to describe students.

Innovation

Generally, teachers who were in the project for one year showed more willingness to innovate in culturally responsive ways in the classroom. This finding is supported by classroom observations. In interviews and surveys, teachers indicated that they were more concerned with "whether a student gets the concept, not just the answer" than they were at the beginning of the project. Further, teachers reported that although it was "scary to try new things, it is great to embrace what the kids know and roll with it." It is important to note that it took many months for us to see any changes in practice. This supports the notion that long term, sustained professional development, as opposed to day long sessions, are more likely to have an impact on classroom practice.

FIGURE 2. *Participant (on Average) Articulation of CRT across Years in Project*



In relation to the culturally responsive practices of participants, we found through classroom observations and interviews that 82% of participants (1) exhibited a greater awareness of the role of culture in the mathematics classroom, and (2) exhibited a greater ability to verbalize about culturally responsive mathematics teaching. These transitions are shown in Table 1. Interviews cited here were roughly 1.5 years apart. While this does not always imply action on the part of the teacher, it was apparent that willingness to discuss issues surrounding culture greatly increased over time. This made it possible for us to more readily discuss issues of equity in recent group sessions.

Sample responses for four participants are shown in Table 1. These findings were triangulated with observational and online discussion data. Participant C, for example, began the school year relying heavily on direct instruction. After looking at data on student achievement and purposeful attention to student engagement, this teacher has incorporated structured discussions around mathematical tasks into his class more readily. Though this teacher still heavily relies on teacher-centered approaches, our data show that he is now more engaged in online discussions with others about ways to innovate in his classroom and is more attentive to student engagement. Participant D emerged as a leader in the group and, eventually, in various communities in the city. For example, her mural projects which combine art and mathematics have been widely publicized, and she speaks of these as emancipatory practices for students. She has spoken at university and conference events about decolonization and racism in the education system.

Table 1: Sample Interview Responses

Participant	Interview #1 (beginning of project)	Interview #3 (1.5 years into project)	Observations (o) discussions (d)
Participant A	"It is not really an issue in my class. In math I treat [all students] the same."	"I've learned to adapt, that every student has a different learning style and you've got to try to adapt and cater to that… as difficult as it might be, but it helps the student."	Small groups pulled to engage in problem solving (o) Teacher says "It was smart when you"…(o) "Our students bring so much knowledge to teh [sic] table, I never thought to use their culture (in instruction) before." (d)
Participant B	"I think it plays a part but I just try to be fair…make sure everyone gets the same chance. I don't focus on it."	"I see that every district is different and every classroom is different. And it's more of a multicultural and how to deal with everybody and how to deal with differences, than necessarily how to deal with one particular culture."	"When I use problem tasks the students can come at it from different angles and strategies. Sometimes they get frustrated but that's good" (d) Whole lesson is gradual release of responsibility; no student discourse (o, beginning of project) 5 minutes, introduce task [task listed], 20 minutes students working in groups with teacher questioning strategies…last 10 minutes students share strategies (o, end of project)
Participant C	"I make sure everybody gets the same opportunity to work hard."	"I realized we have the cultures of pretty much, you know, the whole inner city…which is rich now I see. And with that I get to talk to parents, I see how it is at home and can use that in my teaching."	
Participant D	"I hold them all to that [high] expectation. No excuses even if they are from that side [of the city]".	"[Now] I'm able to relate a little bit more. [For example] when we're talking about graphs and we're talking about intersections I ask them "what part of the city do you live on?" and they tell me and I use a name from that part of the city. So I'm using street names from that part of the city, and they're like "oh, that's what it means?" and they're like "Yea, that's an intersection". So,I kind of go to where they live, kind of their mentality, even when I talk to the parents.	Teacher discussing mural project with students (teacher and students will be painting a mural in the neighborhood called "Always Learning" that combines mathematics and art, o) Switches between Spanish and English when explaining concepts(o) "My struggles in the past are what makes me who I am and why I can relate. These kids don't see their value until you help them see their value."

Team Building

The EFPD of this program increased CRT on a team level through intentional focus. Overall, our data show that the main factors in galvanizing teams were providing ideas for innovation and creating a space where teachers can discuss, debate, and plan to implement such ideas in a structured, team-focused setting. Teams indicated that not only had they "never [before] really had the opportunity to sit down collectively as a group and talk about what we got out of [a session or lesson] and debrief about our classroom," but also that they "were able to discuss the ways that we would implement these tools in our classrooms immediately so that students can benefit." Participant reflections also provided data to this effect. Teachers from one team described disaggregating student data and noticing different trends that correlated with race, class, and/or gender. In explicitly focusing on these categories in their teams, teachers began to deconstruct their practice to determine what they could do to provide access to the students who, as was shown by data, had not been successful in classroom mathematics.

Professional Development "Gap"

While issues continually emerge in most PD projects, one area, which we have termed the "professional development gap," was determined to be particularly notable because of frequency and severity. For a variety of reasons, teachers in our project struggled to implement innovative practices in their classrooms for sustained periods of time. As such, we found what we are calling a "professional development gap" (PDG). DuFour (2005) coined the phrase "knowing-doing gap" (KDG) to refer to the disconnect that exists between teacher knowledge of best practice (in terms of student engagement and achievement) and actual classroom practice which may not align with what teachers know. Our PDG builds on this idea but does not assume that teachers inherently "know" best practices, since pedagogy and instructional techniques are often defined within a district politically.

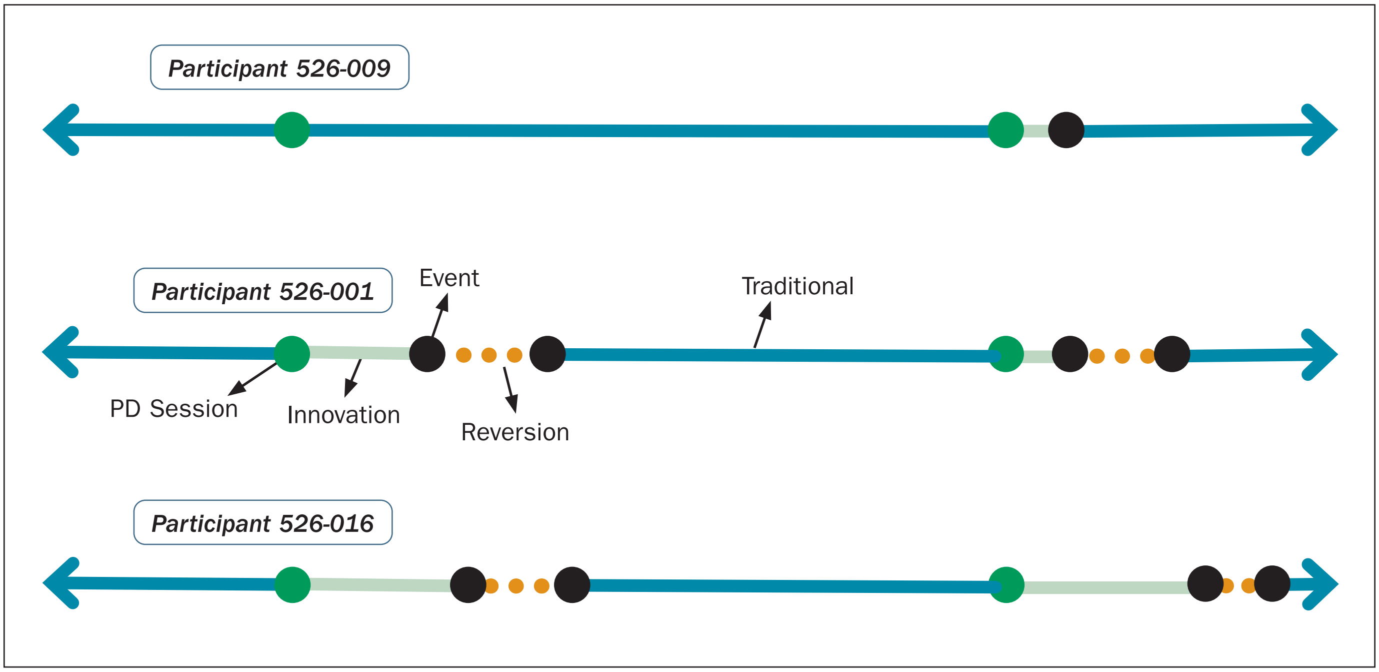
As our project indicates, we believe that teachers need ongoing, sustained professional development, especially in mathematics (Birman et al., 2007) that provides a space not only for teachers to gain new knowledge, but also to discuss, plan, and collaborate with other educators in ways that are constructive in terms of navigating the test-driven climate in many schools. Further, teachers need support at the classroom level to put these ideas into practice in the midst of scripted curricula and district mandates. The PDG, then, refers to the lapse that some teachers experience, largely due to school-related factors, back to traditional teaching methods that are not conducive to student achievement or engagement between professional development and support sessions.

When we noticed the trend that led us to the PDG, we began tracking individual teacher practice in terms of innovation, implementation of non-traditional practice (learned through the project or otherwise), and student engagement (as observed). Examples of these data points are shown below in Figure 2. As discussed earlier, teachers in this project were recruited because of gaps in background knowledge and low student achievement. Initial observations showed that 17 out of 18 (94%) of teachers employed largely traditional, "banking" style (Friere, 1970) teaching methods. As such, when tracking practices, we deemed "innovation" as any observed deviation from this traditional model for a sustained period of time (at least one lesson). "Reversion" (orange, dotted line) refers to largely traditional practice with hints of innovation. For example, a teacher in "reversion" may engage students in a problem-based task, but then walk them through the content step-by-step.

Results of this tracking from three participants are presented in the figures that follow. Though the three figures do not align in terms of time, we believe that there are many implications of these findings. In order to bring clarity to our model, we will briefly walk the reader through the first pieces of our findings related to participant 526-001. The timeline begins on the left-hand side of the model. Here, we first observed this participant employing largely teacher-led, lecture-based lessons. Specifically, the teacher would stand at the front of the classroom and use a projector to take notes, which students were expected to copy as she went. Students were rarely engaged, and engagement was usually related to behavior. For example, the teacher would notice a student sleeping and would call the student's name in the middle of the lesson in an effort to "correct the behavior" (as stated by teacher 526-001).

Where the first green dot occurs, the teacher attended a professional development session focusing on problem-based learning and computer applications for the project. During the debrief portion of that session, the teacher showed a strong interest in the topic and developed an idea for a problem that was relevant to her students. In the weeks that followed, we observed her implementing this

FIGURE 2. *Participant tracking sample data.*



idea through a two-day lesson, and she discussed the experience with others online. She indicated that she would be designing a similar lesson designed around the following week's content. During the second of these innovative lessons, the first precipitating event occurred. A student in her class began engaging in off-task behavior during the group work portions of the lesson. The teacher responded to the student but expressed some concern in terms of losing control of the class. At the end of that day, the teacher walked the students through the solution to the problem rather than letting them struggle with it for another day.

To the observers, this indicated reversion. At the beginning of day two of the lesson, the second precipitating event occurred. The teacher received the unit test written by the district coordinator that was to be given the next day. The teacher saw that the test consisted largely of non-contextual problems and became uncomfortable with her lesson plan for the day. She decided to build a review worksheet based off of the test and work through it with students in traditional fashion during class. This indicated a full reversion back to traditional instruction. In this, as in most cases, a systemic or "top-down" issue was the precipitating event that triggered the reversion. This speaks to the pull that teachers feel between trying new, potentially relevant, teaching methods with underserved students and preparing students for district and state mandated tests.

We discovered that in delving more deeply into the issue of these precipitating events, we were able to more readily deconstruct teacher experiences in the program. Teacher innovation, for example, was mediated by professional development. Though not all teachers innovated after each session, when they did innovate, it was always precipitated by a professional development session. Likewise, when teachers began to revert back to traditional methods (i.e. the "reversion" stage), this reversion was mediated by some event, as was the final reversion back to traditional methods. Though these were varied, events were almost exclusively systematic and often political. One example is given above with the issues surrounding tests being written at the district level without teacher input. Other examples of events that led to reversion include:

- Two teachers had to shift focus due to an upcoming benchmark.
- One teacher was in a school that adopted a new curriculum and was "not given much room to stray" from the scripted lessons.
- One teacher stated, "I was going to try more [problem solving] but my principal really wants us to focus on [the state test] right now [in January].
- Four teachers indicated that "whatever strategies, or materials, or resources, [we get], we need to utilize them according to what the school wants. And sometimes

> it's very difficult to incorporate them in the classroom, the strategies that I'm getting [from the PD]".

While these findings are especially relevant to the PDG, it seems that these precipitating events speak to the current political climate in which teachers are working. These struggles, then, are systemic and must be approached in systemic ways.

Timing of Innovations

Another interesting point for discussion stems from our findings related to the timing and content of the professional development sessions and how these relate to teacher innovations in the classroom. Particular sessions seemed to lead to more and longer periods of innovation, and this could have been due to a variety of factors. For example, one particularly successful session in terms of observed innovations occurred in December. According to interview data, part of this success was due to teacher interest in and relevance of the topic. However, we must consider the effects of the more flexible time of year and the gaps in mandated curricula that seem to occur around the holidays and after end-of-course testing is completed.

This particular topic is important to include in discussions among mathematics teacher educators as these types of data may give us clues as to how to best pique teacher interests, align with mandated curricula, and time professional development sessions for greatest impact. The fact that so many decisions at the school level are based on testing and other political structures, coupled with the fact that mathematics teachers in high-need schools need support in navigating these structures, makes these discussions imperative.

Implications and Further Discussion

Our data support that sustained professional development with an explicit focus on culturally responsive practice and equity can have an impact on teacher practice. With ongoing support and tools for practice such as action research, teachers in this study were more likely to sustain culturally responsive practice. These findings are based on a specific professional development program for secondary mathematics teachers, but they also have implications for all professional development programs. In developing and implementing these programs, it is imperative that mathematics teacher educators are aware of and report out about the complications that affect outcomes of professional development. Mathematics teacher educators should intentionally design these programs to engage teachers in challenging discussions about CRT, equity, and connections to practice.

This work adds to the literature base in that it provides data gleaned from professional development, rather than pre-service teacher education, specifically with secondary mathematics teachers. At this level, it is often not clear how to translate research and theory to practice, but our data show that there is real potential in doing this type of work in a long-term PD format. CRT does not always have to be embedded in mathematics tasks; rather, teacher practice in relation to deficit thinking, ways of communicating with students, and developing critical consciousness can be affected in ways that transform classrooms and student learning.

Phenomena such as the professional development gap should be discussed in mathematics education outlets so that we can collaboratively build successful programs while having ongoing discussions about issues that may arise. Through engagement in pragmatic conversations about programmatic nuances, we can more readily understand the scope of success and struggles in teacher education programs. These are important, timely, and systemic issues that, through investigation and discussion, can provide important information about how to develop a more culturally responsive teaching force.

It is through these professional, pragmatic discussions that we can (1) identify roadblocks and possible complicating factors that add to the complexity of our work, including political and curricular factors, (2) begin to identify and test possible solutions to these "roadblocks", and (3) implement sustained, professionally-based strategies that will allow for the greatest amount of teacher and student success. This will allow for an ongoing, productive discussion about the vast and expanding literature base on professional development and how it might apply in various settings and within various political contexts.

This project ultimately empowered teachers to question traditional mathematics practice and consider the questions posed by Aguirre, Mayfield-Ingram, and Martin (2013): "What mathematics? For whom? For what purpose?" (p. 5). Teachers were challenged to enact culturally responsive practice in a politically challenging context and to remember that they too are learners with assets that are vital to the success of their students. ✪

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Transitioning Face-to-Face Mathematics Professional Development to Synchronous Online Implementation: Design Considerations and Challenges

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Abstract

To make professional learning experiences more accessible to teachers, professional development providers redesigned a face-to-face professional learning experience – a Teaching Lab – for an online platform utilizing synchronous modalities. To design the online version of the Teaching Lab, our team employed design principles derived from research on high-quality professional development and from theories of technology use in education. We describe these design principles, the multiple iterations of the Teaching Lab, and the challenges we faced in the design process. We consider the roles of technology as replacement, amplification, transformation, or hindrance with respect to the online model. We conclude with a discussion of the technology framework to offer suggestions and considerations for mathematics education leaders who design professional learning opportunities.

Providing access for rural teachers to high-quality professional development has been a consistent problem. Challenges such as distance, cost, and availability of substitute teachers have plagued the efforts of professional development providers and rural district leaders alike. Capitalizing on the affordances of technology, our professional development project team designed a fully online professional learning model, providing an in-person experience from a distance. The purpose of this article is to share our experience as professional development designers so that others in similar roles can learn from the challenges we faced moving an existing program to an online format. We describe the design and implementation of online demonstration lessons, which we termed Teaching Labs. We previously used Teaching Labs as a face-to-face professional learning experience in which we worked with small groups of mathematics teachers to plan, implement, and reflect on lessons taught by a facilitator, often in conjunction with a broader set of professional learning experiences. The Teaching Labs encompass features similar to the studio model (e.g. Higgins, 2013; TDG, 2010), lesson study (e.g. Fernandez & Yoshida, 2004) or demonstration lessons (e.g. Barlow & Holbert, 2013; Strayer et al., 2017). These models give teachers an image of high-quality instruction, provide immediate and practical takeaways, deepen their understanding of pedagogical principles, and orient them to an inquiry stance. Our goals for the Teaching Labs were for teachers to observe how to elicit and build from student thinking about important mathematical ideas.

In prior implementations of face-to-face Teaching Labs, a facilitator from our professional development team worked with a small group of teachers to plan a lesson together, hypothesizing how the lesson would engage students with key mathematical ideas. The facilitator taught the lesson in one of the participating teachers' classrooms while the small group of teachers observed with a particular focus on student reasoning. Afterward, the facilitator and teachers debriefed the lesson experience together focusing on student thinking and learning outcomes. Throughout this article we use the term facilitator to refer to our project personnel who both taught the Teaching Lab lesson and also facilitated the professional learning experience for the participating teachers. We purposefully use the term facilitator instead of coach or instructor to denote the collaborative and mindful way we hoped to guide teachers to develop their own noticing and discourse practices.

Through funding from the National Science Foundation, we created an online version of the Teaching Labs to make them available to middle school mathematics teachers in rural contexts. Although our focus was to support rural teachers, we believe lessons learned about our transition from face-to-face professional development to an online model would be beneficial for educational leaders and professional development designers. This article describes our design rationale and iterative efforts to transform the Teaching Labs into fully online experiences. We articulate the challenges and opportunities entailed by this transformation, with the goal of advancing the conversation of online professional development of mathematics teachers, especially those who are not geographically proximate to sites that offer high-quality professional development. We believe administrators, coaches, and professional development providers have similar struggles. Sharing our story and the considerations and challenges we faced as we transitioned to an online model can help the field continue to explore new ways that technology can support teachers.

Professional Learning Context

In our project, the Teaching Labs were situated within a larger three-part online professional learning model that used both synchronous and asynchronous modalities to provide learning opportunities designed to meet or exceed face-to-face learning opportunities. We designed the model to support teachers to improve their discourse practices and to use their knowledge of student thinking to make instructional decisions (e.g., Jacobs, Lamb, & Philipp, 2010; Smith & Stein, 2011). The three components included: a) online course modules, b) Teaching Labs, and c) online video coaching. The online course modules were designed to support teachers to improve discourse practices in their classrooms based on the work of Smith and Stein (2011). The Teaching Labs, the focus of this paper, were the second component of the three-part model. Coaching, the third component, followed a Content-focused Coaching approach (West & Staub, 2003) transformed into a fully online experience. The three parts of the model overlapped temporally, took place across two academic years, and included multiple Teaching Labs (for a full description of the entire model, see Choppin et al., in press). The teachers in our project were 16 middle grades mathematics teachers from rural contexts.

Research Base for Design of Teaching Labs

The Teaching Labs were based on *lesson analysis* (Yeh & Santagata, 2015), in which we treated lessons like experiments; teachers conjectured how students would engage with mathematical tasks and how teacher moves would elicit and focus attention on student thinking. To accomplish these goals, we designed and implemented the online Teaching Labs around two principles: increase teacher focus on student thinking and use video effectively.

The first principle, *increase teacher focus on student thinking*, relates to our specific goals for the Teaching Lab. One of the primary purposes was to move teachers away from primarily evaluative reflections on classroom practice to more objective and knowledge-based reflections (e.g., Sherin & van Es, 2009). The goal of the Teaching Labs was similar to the focus on professional noticing described by Sherin and colleagues (Sherin & van Es, 2009; van Es & Sherin, 2008) in which video was used to develop teachers' ability to notice and interpret student thinking and the nature of classroom interactions. Our goal was to have teachers notice how the qualities of the tasks, in conjunction with facilitator's instructional decisions during the lesson, combined to expose student thinking, so they could focus on productively leveraging student thinking to make important connections. We aimed to support teachers to engage in detailed and complex analyses of student thinking in order to make connections between tasks, facilitator discourse moves, and the productiveness of student thinking. In short, we hoped to initially have participating teachers focus on objective aspects of student thinking (e.g., strategies, use of representations, interactions with others) and features of practice emphasized in our project (e.g., how the facilitator elicited student strategies and organized classroom discussion) as the basis for principled observations of classroom practice (Mason, 2002).

The second principle, *use video effectively*, relates to the structure of the professional conversations and the use of video recordings which researchers describe as having a number of affordances. Video allows educators to reflect on classroom practice without having to observe lessons in real time (Sherin, 2004) as well as allowing for a focus on specific aspects of practice, afforded in part by the ability to pause or replay the video (Borko, Jacobs, Eiteljorg, & Pittman, 2008). Video consequently supports collaborative learning "focused on reflection, analysis, and consideration of alternative pedagogical strategies in the context of a shared common experience" (Borko et al., 2008, p. 419).

Technology Framework Used to Describe Design Processes

To describe the transformation of our Teaching Lab design from face-to-face to online, we turn to the Replacement, Amplification, or Transformation (RAT) framework (Hughes, Thomas, & Scharber, 2006), which builds on longstanding theories in technology education (e.g. Pea, 1985; Reinking, 1997). Replacement refers to technology use that replaces but does not change instructional practices, learning processes, or content goals. Amplification refers to technology use that increases efficiency or productivity in an educational setting but largely maintains the existing form. Transformation builds heavily on the work of Pea (1985) and refers to technology use that leads to or supports instruction, the learning process, or goals in a way that is fundamentally different from what could be accomplished without the technology. The RAT framework is a tool for critical decision-making concerning technology integration in an educational context. Researchers have used the framework in empirical research to explore how prospective or practicing teachers integrate technology in their classrooms (e.g Hsieh & Tsai, 2017; Van Zoest, Stockero, & Kratky, 2010). Within the field of mathematics teacher education, various researchers have used the RAT model to characterize learning opportunities technology provides (e.g. Amador, Weston, Estapa, Kosko, & De Araujo, 2016; Coleman, 2017; Thomas & Edson, 2017, 2018; Van Zoest, Stockero, & Kratky, 2010; van Bommel, & Palmer, 2018). Additionally, Kimmons, Miller, Amador, Dejardines, and Hall (2015) applied the RAT model in a prospective teacher context and added Hindrance (H) to the model, recognizing that the use of technology may hinder learning opportunities. Thus, the RATH (Replacement, Amplification, Transformation, or Hindrance) model was formalized to more holistically capture all potential outcomes of technology integration. We provide this lens to illustrate how we considered technology integration as we moved our face-to-face Teaching Lab to an online version and believe others could apply a similar process in their own context as they consider transitions to online professional development.

Teaching Lab Implementation

We describe four iterations in the design of our Teaching Labs. We highlight the challenges we faced in moving the Teaching Labs to an online environment and the design considerations that resulted from the affordances and constraints related to the platforms and tools we used. To illustrate how we made the transition from face-to-face to online Teaching Labs, we share our design decisions and rationales, as well as reflections on each iteration. We had four design iterations: Iteration 1: Face-to-Face Design; Iteration 2: Original Online Design; Iteration 3: Intermediate Online Design; and Iteration 4: Current Online Design.

Iteration 1: Face-to-Face Design

Our face-to-face Teaching Lab engaged teachers in a facilitated day-long professional learning experience that included the following three components: a) pre-lesson discussion, b) lesson observation, and c) debrief discussion. Prior to meeting with the full group of teachers, the facilitator consulted with the teacher in whose class the lesson would be taught to determine a lesson goal, to select or design a high-cognitive demand task, and to craft a lesson plan. On the day of the lesson, the facilitator shared the mathematical learning goals of the lesson and the lesson plan draft with the full group of teachers. The full group then discussed the lesson plan and the mathematical tasks, anticipated student thinking, and proposed possible modifications to the lesson design to better support student learning. Prior to the lesson implementation, each teacher established a personal focus for their observation to support more productive noticing (e.g. Jacobs et al., 2010; van Es & Sherin, 2008). For example, one teacher may have decided to focus on teacher questioning and student responses, while a second teacher may have decided to focus on student interactions within small groups.

In the second component of the face-to-face Teaching Lab, the facilitator taught the lesson while the participating teachers observed. During the lesson, teachers were encouraged to move about the classroom to collect detailed observation notes about student thinking and instructional moves but not to engage with students. The final component of the Teaching Lab was the facilitated debrief discussion during which teachers shared their observations based on their area of focus and were supported to reflect on implications for their own practice.

Iteration 1 Reflections. Although we found face-to-face Teaching Labs were effective to support teachers' learning, they posed two logistical issues. First, all teachers had to travel to the site of the lesson and spend the full day there which proved burdensome for teachers, particularly those in rural contexts. Second, substitute teacher shortages in the region made it difficult for teachers to be out of their classrooms. In addition to the logistical issues, there was a pedagogical issue in the face-to-face version. It was difficult to control what teachers attended to during the live lesson implementation; some teachers paid attention to aspects that were significant to student learning and some did not. The varied nature of the teachers' areas of focus affected the productivity of teacher noticing with the goal of focusing attention on student thinking.

Iteration 2: Original Online Design

In the first online iteration, we attempted to replace the three components of our face-to-face Teaching Labs in an online space with the primary goal of alleviating the logistical concerns related to travel and the need for substitute teachers. Although we identified the video conferencing technology Zoom as a reasonable replacement to host synchronous pre-lesson and debriefing discussions, it was not possible for us to have a synchronous lesson observation due to scheduling conflicts amongst the teachers. As a result, we separated the three components of the face-to-face Teaching Lab so they occurred on different days. We scheduled a 60-minute synchronous pre-lesson discussion with teachers using Zoom which occurred after the school day. The structure and goals of this pre-lesson discussion directly mirrored those of the face-to-face Teaching Labs. In order to disseminate lesson materials, we set up a shared Google folder in which we uploaded the lesson plan, the task description, and other supporting documents. In addition, because participants' viewing of the enacted lesson was limited to the video recording rather than an in-person observation, we did not require that the participants decide up-front what they were going to focus on for their observation.

The facilitator then implemented the lesson in a participating teacher's classroom and project personnel video-recorded the lesson. Our professional development team then viewed the video and created a note-catcher that included prompts to focus teachers' viewing on particular instructional moves or student responses. We then made available the unedited video and note-catcher to the teachers within two to three days of when the lesson was taught. The teachers viewed the recording asynchronously to fit their schedules. Approximately one week later, the teachers and the lesson facilitator met synchronously via Zoom for a 60-minute debrief discussion during which teachers shared their observations, reflected on what they had noticed, and described implications for their own practice—a conversation very similar to the face-to-face debrief discussion.

Iteration 2 Reflections. For this first fully online Teaching Lab, many teachers indicated that they appreciated not having to travel to participate. Many also noted that they appreciated not having to miss school time, as all activities took place outside of the teachers' school day. However, this initial online design presented new challenges. First, feedback from teachers indicated they felt overwhelmed by the process. Instead of attending a one-day professional learning experience, they now had three separate components that they needed to schedule: the synchronous pre-lesson discussion, the asynchronous viewing of the lesson video, and the synchronous debrief discussion. This feedback was of particular concern for the project team because participation in all three components of the Teaching Lab was important. A second challenge was that the process of recording the lesson, sharing it with teachers, and providing ample time to view the lesson created a time lapse between the phases of the Teaching Lab. Teachers commented that it was difficult to remember the conversations from the pre-lesson discussion when watching the video or engaging in the debrief discussion. A third challenge teachers communicated was that watching a full lesson on video from one vantage point was far less engaging than observing a full lesson in a face-to-face setting.

Iteration 3: Intermediate Online Design

Based on the challenges noted in Iteration 2, we worked to design Iteration 3, in which we moved our thinking to consider how we could use affordances of the technology to re-conceptualize how we implemented Teaching Labs. We wanted technology not to serve simply as a replacement but as an enhancement to the experience (Hughes et al., 2006). The first major design adjustment was to move all three parts of the Teaching Lab into a single, two-hour synchronous session to alleviate the challenges with teacher scheduling and the extended time between components of the Teaching Lab. The resulting Teaching Lab design consisted of a 40-minute pre-lesson conversation, 35 minutes for teachers to watch clips of the lesson video and create notes of their observations, and a 45-minute debrief discussion to share thinking around each clip and reflect on implications for individual practice. This all took place in one synchronous online session using Zoom.

This alteration required the project team to plan, teach, and video-record the lesson prior to the Teaching Lab synchronous session. This decision also required an adjustment to the original intention of the pre-lesson discussion because the lesson was already planned and implemented prior to engaging teachers in the pre-lesson discussion. Like the face-to-face pre-lesson discussion, this online pre-lesson discussion focused on anticipating student thinking in order to prepare teachers to productively notice thinking as they viewed video clips (e.g. Sherin & van Es, 2009; van Es & Sherin, 2008).

In addition to these design changes, we also thought about how to use video for the lesson observation in ways that would create more thoughtful observations and productive conversations. To ameliorate the limitations related to lesson observations inherent when using one camera, we used two cameras, with one focused on the teacher and one on students. We also started to take advantage of the fact that we could determine the aspects of the lesson viewed by teachers. Instead of providing teachers with the complete lesson video, the professional development team carefully selected and organized smaller video clips that strategically highlighted different phases of the complete lesson. For example, the launch phase of a task during a particular Teaching Lab lesson took approximately 15 minutes during the live lesson implementation. We edited out less useful moments during this launch (e.g., passing out papers, private work time), and created an eight-minute clip that provided teachers with an image of this lesson launch that included facilitator moves and student interactions. Through this process, we condensed a full lesson video into four or five clips that totaled approximately 30 minutes, though we still provided a complete image of the lesson—one of our intentions of a Teaching Lab.

To further deepen teacher reflection and foster rich discussion around the lesson images, the project team created a focused set of questions for each clip. These questions were consolidated into a *capture sheet* that was provided to the teachers for the viewing of the video. Figure 1 shows an excerpt from a capture sheet used for a five-minute clip of the opening lesson discussion. This particular example was designed to focus teachers' noticing on the connection between the activity and student engagement in the upcoming task. This type of purposeful focus enhanced the lesson debrief discussion.

Iteration 3 Reflections. Based on teachers' feedback, the design changes implemented in Iteration 3 were well received. Teachers appreciated a more limited commitment in terms of the number of sessions and the compactness of the three components in terms of keeping track of the discussions. In addition, facilitators reported this design had positive effects on participation and engagement. The modifications made in this online Teaching Lab design reflected efforts by the project team to use technology to amplify (i.e. Hughes et al, 2006) the learning opportunities for teachers. We shifted our question from, "How can we use online technology tools to best replicate a face-to-face Teaching Lab?" to "How can we leverage online technology

FIGURE 1. *Excerpt from a Capture Sheet.*

Video Clip #1: Initial Lesson Launch and Opening Discussion (0:00 - 5:23)

Focus for Clip #1: This clip contains the teacher's initial launch to the task which includes some small group conversation followed by a whole group discussion. As you watch both components, how might this initial activity support student engagement in the task?

Time Stamp	Thoughts/Ideas/Evidence

tools to transform our Teaching Lab design to something that is not possible in a face-to-face Teaching Lab?" This new question drove our thinking for Iteration 4, described below.

Iteration 4: Current Online Teaching Lab Design

Building on the successes of Iteration 3, the project team worked to address the challenge of how to maximize the affordances of technology for teacher learning in an online Teaching Lab to make the process transformational (i.e. Hughes et al., 2006). Though perhaps obvious in hindsight, we began to think about how the use of video for the lesson observation allowed us to pause the lesson at any time and engage teachers in discussion. Consequently, Iteration 4 intertwined the observation and debrief components, rather than having teachers watch the entire series of video clips without interruption and then engage in a single debrief discussion. As in the previous iteration, we engaged teachers in a pre-lesson discussion, but now we asked them to watch a single clip followed by a shorter debriefing conversation, and repeated this with subsequent lesson clips and focus questions. This allowed us to focus teachers' noticing and to highlight instances of practice.

The ability to pause the video at any time during the lesson also allowed for unique discussions not possible during face-to-face debriefing examples. For example, we edited a video clip in which the facilitator approached a group of three students who appeared to be stuck while working on a cognitively demanding task. The facilitator asked clarifying questions to understand the students' strategies. We paused the video at that moment, provided teachers with the students' strategies, and posed questions that positioned teachers to consider possible instructional moves on the capture sheet, as seen on the capture sheet in Figure 2.

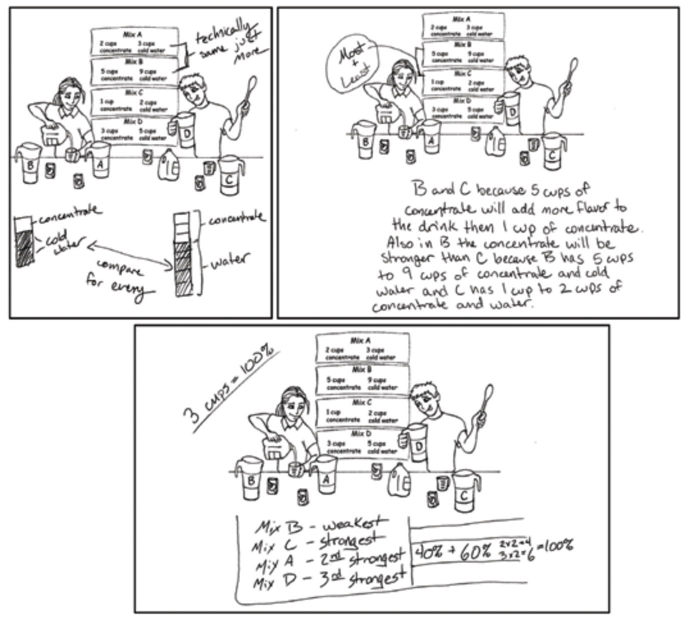
We placed the teachers in breakout rooms within Zoom (which allow for small group conversations) and asked them to examine the student work and determine questions they would use to assess and advance the thinking of the students. After ten minutes, each group shared their questions and strategies for interacting with the group of students. Teachers then watched the next clip, which showed how the facilitator responded to the students. Showing the follow-up clip allowed teachers to reflect on the affordances and drawbacks of the facilitators' actions as well as compare it to the possibilities that they generated (see Figure 3).

FIGURE 2. *Excerpt from the Capture Sheet showing student work.*

Video Clip #2 (Part 1): Small Group Discussion (8:31-11:08)

Focus for Clip #2 (Part 1):
Keeping in mind the lesson goals, what might you ask these students to assess and advance their thinking if you were the teacher?

Assessing Questions	Advancing Questions



Iteration 4 Reflections. As in Iteration 3, teachers expressed appreciation that this next iteration allowed them to engage in a Teaching Lab online and in one sitting. In addition, the opportunity to reflect on the lesson video at key moments allowed for new opportunities to deepen teachers' engagement. Of particular importance to this design was the ability to connect key moments in the video to discussion moves (Smith & Stein, 2011); we were able to create and pause video clips in ways that problematized specific practices and allowed teachers to consider their own actions. Teachers were given an opportunity to pause and reflect on how they might respond to these particular students in a way that would both assess and advance student learning in relation to the lesson goals. In addition to these connections, Iteration 4 provided richer discussions about the productiveness of facilitator moves related to student learning than had previously been the case. By pausing and problematizing these key moments,

FIGURE 3. *Excerpt from the Capture Sheet for critical reflection of the teachers' questions.*

Video Clip #2 (Part 2): **Small Group Discussions (11:08 - 13:09)**

Focus for Clip #2 (Part 2):
What are affordances and drawbacks to the teacher's questions and actions?

Affordances	Drawbacks

the facilitator not only made their practice public, but also made it more vulnerable and open to discussion.

Current Design Challenges and Future Directions

The professional development team identified three areas of challenge for scaling up our design to engage more teachers: a) the preparation time for facilitators, b) expense of the implementation, and c) ability to record lessons in a teacher's classroom. In Iteration 4, we estimated the facilitators spent an average of 70 hours per Teaching Lab in: the planning, implementing, and recording of the lesson; selecting and editing the video clips; developing the capture sheet; and planning for and facilitating the synchronous Teaching Lab sessions. These tasks were costly in terms of compensating the professional development personnel and making the design feasible for future professionals to implement. Furthermore, the logistics of coordinating and teaching a lesson in a teacher's classroom was challenging due to travel logistics, camera operators, and student assent/parent consent requirements because of video recording.

These challenges led us to consider a different possibility for video use in the Teaching Lab and to consider the use of previously recorded videos. However, these changes require consideration of teacher learning and engagement. For example, if the lesson video was no longer from one of the participating teacher's classrooms, would this cause a loss of ownership or authenticity for the teachers? How much impact does the authenticity of the video have on teachers' noticing and reflection on the lesson? Can we use available lesson videos from the Internet, which would reduce the cost but further remove the authenticity of the video? As we move forward with this work involving teachers, we continue to consider these challenges and opportunities.

Technology Characterization and Design Principles

As we consider the challenges and affordances of each of the Teaching Lab iterations, we remain focused on the technological aspects of the process and the affordances of technology use as well as the design principles germane to the project. Table 1 *(next page)* shows our iterations in relation to the RATH framework (Hughes et al., 2006; Kimmons et al., 2015) and the design principles as a means to further describe how we consider the various approaches as related to technology integration.

Through this process, we applied the RATH framework to a professional learning design context, building on the traditional use of this framework (Hughes et al., 2006; Kimmons et al., 2015). We consider this a contribution of this work and suggest other professional development providers coordinate their efforts with the RATH framework to consider how the decisions they make with technology replace, amplify, transform, or hinder the experiences they design for teachers.

Conclusion and Recommendations

The iterative design process of our Teaching Labs provides professional development designers insight about how to transition from face-to-face professional learning to an online space. We were able to recognize the affordances and constraints related to technology and capitalize on the advantages to arrive at a transformational experience that would otherwise not have occurred (i.e. Hughes et al., 2006). We were able to leverage video to hone the focus on specific aspects of teaching practice (Sherin, 2004) and transition teachers from primarily evaluative reflections on classroom practice to knowledge-based interpretations and responses (i.e. Sherin & van Es, 2009). We accomplished this through selecting edited video clips, designing a capture sheet specific to each lesson video, and structuring the learning environment. At the same time, the facilitator purposefully guided the teachers to develop their own noticing and discourse practices (Coles, 2013). An initial review of data collected during this process indicate that learning outcomes from our online version were comparable

Table 1: Overview of iterations of the transformation of Teaching Labs

Iteration	Coordination with RATH Framework (Amador, 2015)	Rationale for RATH Characterization (Replace, Amplify, Transform, Hinder)	Relation to Design Principles (increase teacher focus on students' thinking; use video effectively)
Iteration 1: Face-to-Face Design	No digital technology	This process was void of digital technology	Knowledge-based reflections and noticing were scattered dependent on the teachers' focus during the live lesson; video was not used
Iteration 2: Original Online Design	Replacement with Hindrance	Technology was used to replace aspects of Iteration 1; teachers felt constrained by the limited perspective of the video when having to watch the lesson from one vantage point	Knowledge-based reflections and noticing were from the perspective of one camera angle, which constrained opportunities; video was used to try to replace in-person observation, but actually hindered noticing because of one vantage point
Iteration 3: Intermediate Online Design	Amplification	Edited video with two camera angles provided opportunity for the actual lesson length in the video to be reduced, which focused attention; capture sheets supported video to highlight aspects of video.	Knowledge-based reflections and noticing were more focused with edited videos from two camera angles; edited video was helpful for focus on students' thinking and the capture sheets augmented the use of video for reflection
Iteration 4: Current Online Teaching Lab Design	Transformation	The use of edited clips including two camera angles coupled with stopping and starting video and capture sheets created opportunities to anticipate student thinking and provided an experience that would not otherwise be possible without the technology	Knowledge-based reflections and noticing were focused because of ability to pause video and discuss and the ability to incorporate capture sheets that supported reflection; edited video was helpful for focus

to outcomes from the original face-to-face design for the teachers who participated. This is encouraging for supporting teachers in rural areas who may not otherwise have access to high-quality professional learning opportunities.

We recommend that mathematics education leaders thinking about moving a face-to-face professional learning experience to an online space consider the features of the face-to-face experience that are essential for the intended learning. At the same time, we encourage others to consider design principles that resonate with their intended learning outcomes. As evidenced in our experience, this transition was not straightforward and we faced many challenges along the way. As we move forward with this work, we will continue to explore additional avenues for improvement to our design. We will also consider how new and advancing technologies may also influence our work and perhaps lead to transformative experiences that we have yet to imagine. We encourage others to think about how they can leverage technology to provide learning experiences for teachers that otherwise may not be possible. ✪

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Teacher Interpretations of the Goals of Mathematics Professional Development and the Influence on Classroom Enactment

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Abstract

This multiple-case study is an investigation of how four high school teachers interpreted the goals of a professional development (PD) program and how these interpretations influenced their instructional practices during observed lessons. The teachers participated in PD that focused on using standards-based pedagogy and mathematical tasks with higher-level demands. Each teacher stated an interpretation of the goals that was consistent with the PD, but concentrated on one of the objectives for each of the goals. The teachers' interpretations of the goals influenced the lessons they taught and their use of ideas from the PD.

National standards such as the National Council of Teachers of Mathematics (NCTM) (1989, 2000), the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGACBP & CCSSO) (2010), and the National Research Council (NRC) (2001) have provided visions for mathematics teaching and learning in K-12 schools. These visions contain goals for students that include rigorous content, reasoning, modeling, communicating, connecting, constructing arguments, and supporting conclusions (NCTM, 2000; NGACBP & CCSSO, 2010). These standards-based visions include new ideas that can be challenging for teachers and schools to enact (Coburn, Hill, & Spillane, 2016; Munter, Stein, & Smith, 2015; NCTM, 2014). For example, teachers must learn new content, gain experience with different instructional techniques, and implement new assessment methods (Reys, Reys, Lapan, Holliday, & Wasman, 2003).

Research-based professional development (PD) can help mathematics teachers overcome the challenges of standards-based pedagogy (Lappan, 1997; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010), but studies indicate that the outcomes of PD programs are inconsistent. Some teachers are able to teach in a manner consistent with the goals of the PD that are focused on standards-based instruction, some teachers' instruction reflects portions of the goals, and other teachers struggle with using the reform concepts in their classroom (Coburn et al., 2016; Cook, Walker, Sorge, & Weaver, 2015; Munter et al., 2015). One explanation for the inconsistencies is that teachers attempting to use standards-based instructional strategies adapt to the new visions by interpreting and constructing understandings based on the way instruction is currently done (Coburn et al., 2016; Munter et al., 2015; Roth McDuffie, Choppin, Drake, Davis, & Brown, 2018). Research has reported examples of teachers who perceive that they are providing standards-based instructional practice, but observations by researchers reveal that what the teachers perceive is not consistent with this type of instruction (e.g. Cohen, 1990; Roth McDuffie et al., 2018; Spillane & Zeuli, 1999).

Due to the influence that teachers' interpretations have on the enactment of standards-based pedagogy in the classroom, it is important to learn more about the interpretations teachers develop as a result of PD experiences. In particular, understanding how teachers' interpretations of PD goals on standards-based instruction influence classroom practice could help PD providers and mathematics teacher supervisors understand inconsistencies concerning attempts to improve instructional practice. The question investigated in this research is: How do teachers interpret the goals of a standards-based mathematics PD program and how did their interpretations influence the enacted mathematics lessons?

Standards-Based Mathematical Practices

An important aspect of this research was PD aimed at helping teachers learn about and enact *standards-based visions for mathematics instruction*. Two sets of practice standards were used to define standards-based mathematics instruction. The first set of practice standards was the set of eight standards for mathematical practice (SMPs) identified by the NGACBP & CCSSO. The SMPs "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGACBP & CCSSO, 2010, p. 6). The SMPs are (a) making sense of problems and persevering in solving them, (b) reasoning abstractly and quantitatively, (c) constructing viable arguments and critiquing the reasoning of others, (d) modelling with mathematics, (e) using appropriate tools strategically, (f) attending to precision, (g) looking for and making use of structure, and (h) looking for and expressing regularity in repeated reasoning. The second set of practice standards used in the PD was the set of eight mathematics teaching practices (MTPs) that "provide a framework for strengthening the teaching and learning of mathematics" (NCTM, 2014, p. 9). The MTPs include (a) establishing mathematics goals to focus learning, (b) implementing tasks that promote reasoning and problem solving, (c) using and connecting mathematical representations, (d) facilitating meaningful mathematical discourse, (e) posing purposeful questions, (f) building procedural fluency from conceptual knowledge, (g) supporting productive struggle in learning mathematics, and (h) eliciting and using evidence of student thinking.

Teachers who use the SMPs and MTPs to provide standards-based instruction in K-12 mathematics classrooms have specialized roles. One of the main responsibilities of a teacher in a standards-based mathematics classroom is to plan, establish, and sustain the mathematical learning environment. They are responsible for creating an environment where students can actively build mathematical understandings and share concepts (NCTM, 2000, 2014). Students have important roles in standards-based mathematics classrooms that are negotiated and developed through their participation over time (McClain & Cobb, 2001). For example, students are expected to make conjectures and share mathematical thinking, use reasoning to explain solutions to all members of the class, persevere in solving mathematical problems, and use mathematics to model experiences (Boaler, 2002; NGACBP & CCSSO, 2010).

Principles of effective PD for K-12 teachers of mathematics (e.g. Loucks-Horsley et al., 2010; Sztajn, 2011) were used to describe the PD program. These principles recommend establishing clear goals that incorporate school needs along with national, state, and local standards as a framework to support change (Sztajn, 2011). Goals and objectives provide benchmarks to monitor progress toward the vision of teaching and learning promoted by PD (Loucks-Horsley et al., 2010). Goals that reflect features that teachers find valuable are an important consideration because changes in instructional practice can be linked to perceptions about PD (Chapman, 2011; Martin & Gonzalez, 2017; Walker, 2018).

Methods

To address the research question, information was needed about teachers' interpretations of the goals of a PD program and how they enacted instruction in relationship to their interpretations of the goals. A multiple-case study design (Merriam, 2009) was used to analyze the influences on instructional practice in the complex social units of classrooms and schools. The role of the researcher was observer as participant (Merriam, 2009). The researcher was not involved in the design or execution of any parts of the PD or any mathematics lesson taught. Findings are reported as a case for each teacher. A summary of standards-based mathematical practices observed is provided with each lesson. These summaries provide additional information about the enacted lessons, but are not a focus of the research question. Findings and implications are presented following the cases.

Participant Selection

The PD program for this research was Teaching Algebra with Practice Standards (TAPS). TAPS was identified because it focused on helping teachers implement standards-based mathematical practices. It was a three-year program funded by a mathematics partnership grant. TAPS included partnerships between four Midwestern universities and four school districts, all from the same state. Faculty and graduate students specializing in mathematics education from all four university partners worked together to plan the PD activities. Each university was paired with a neighboring school district for the delivery of the activities.

This research focused on the Springfield School Corporation (SSC), which was one of the four partner school districts. There were fifteen SSC teachers who volunteered to participate in the first year of TAPS. Each of the fifteen teachers taught mathematics in grades six through twelve. All fifteen of the teachers received continuing education units and stipends for work done outside of school time.

The teachers participating in this multiple-case study were a subset of the fifteen teachers. At the beginning of the TAPS summer institute, the researcher presented the opportunity to participate in this research to all fifteen of the teachers. They were informed that participating in the research would require them to complete surveys, interviews, and allow the researcher to conduct classroom observations. As an incentive, teachers who participated in the research were credited with up to ten hours of independent work required by TAPS. Four teachers volunteered to participate. All of the research participants were high school teachers from Springfield High School (SHS) in SSC.

Data Collection

Data for this research were collected in 2015-2016 during the first year of TAPS. Data sources used to construct a description of the PD included the written PD proposal, field notes taken by the researcher during the PD sessions, and email interview responses from the PD facilitators. Data for instructional practices consisted of two enacted lessons for each teacher that were observed and videotaped. The researcher asked the teachers to self-select the lessons that were observed. The criteria for selection was that the observed lessons were developed during the PD workshops and consistent with the goals of the PD. The observations provided evidence of mathematics instruction that was intended to be consistent with the PD goals. The Wisconsin Longitudinal Study observation tool (Shafer, Wagner, & Davis, 1997) was used to organize data collection during the observations. The tool was adapted to identify evidence of the SMPs and MTPs during classroom instruction.

The four participating teachers were interviewed five times using protocols adapted from Shafer, Davis, and Wagner (1997) and Shafer, Davis, and Wagner (1998). The first interview took place in the summer after the PD was completed to learn about each teacher's interpretations of the PD goals and how they anticipated using the PD during the upcoming school year. The first interview included a question that asked each teacher to state the goals for TAPS in her or his own words. The next two interviews took place during the first half of the school year. These interviews were before and after the first observed lesson. Interview questions provided information about the planned lesson, the enacted lesson, and how ideas about standards-based instruction from the PD were included. The final two interviews took place during the second half of the school year. These interviews were before and after the second observed lesson.

Data Analysis

Classroom observation data were used to describe the alignment between each teacher's enacted instruction and the SMPs and MTPs. Lessons were classified as no evidence, sometimes, or yes for each of the SMPs and MTPs. No evidence was used when there were no classroom events or only one classroom event that aligned with a practice standard descriptor. Sometimes was used when there were two or three classroom events that aligned with a descriptor. Yes was used when there were more than three classroom events that aligned with a descriptor.

Each of the teacher interviews were transcribed. An inductive approach of comparative pattern analysis was used to create a category coding system for the transcripts (Merriam, 2009). The categories were further examined for sub-categories (Lincoln & Guba, 1985; Patton, 2002). For example, one of the coding categories for the transcribed interviews was "Teacher Role." Sub-categories for "Teacher Role" included: monitor or listener, source of mathematical knowledge, ensurer of correctness, and facilitator. Selected quotes in each of the teacher cases were representative of a coding category.

An independent education researcher checked the reliabilities of the observation classifications and the coding system. The researcher was trained on the classification and coding systems and completed independent coding. The reliability of the observation analysis was checked by calculating a Krippendorff (2004) alpha value of 0.8223. The reliability of the coding system was checked by calculating the percent of agreement, which was 90%.

The PD Program

TAPS was the PD program in this research (see Methods, Participant Selection). Goals for TAPS were developed jointly by the universities and school district partners from the analysis of a needs assessment. One area of need was teachers' knowledge and skills for teaching algebra. The assessment revealed that teachers needed to learn about research-based learning tasks, learn about research-based instructional strategies (including differentiated instruction), and have time to improve the mathematics programs based on these topics and student data (TAPS Proposal, pp. 6-7). A second area of need was students' algebraic knowledge and skills. The student passing rates for the four district partners was 25% below the state passing rate on state standardized mathematics tests. Each of the school districts also noted limited opportunities for students to engage in authentic learning tasks to enhance their algebraic understandings. Despite the limited opportunities, each district expressed a desire to learn more about authentic learning tasks and how to include them into the curriculum (TAPS Proposal, pp. 4-6).

Based on the needs assessment, TAPS identified two goals for the program (TAPS Proposal, p. 3). The first goal was to enrich teachers' knowledge and skills for teaching algebra. Objectives for the first goal included: (a) engaging in solving rich algebra tasks to enhance algebraic understanding and habits of mind (e.g., abstracting from computation, doing and undoing, and building rules to represent functions); (b) collaborating to locate and develop algebra activities, including modifying textbook tasks to increase cognitive demand, relate algebra to STEM and other real-world contexts, and address SMPs; (c) enacting research-based pedagogical strategies (e.g., productive discourse, multiple representations) within a system of structured reflection and feedback from critical friends; and (d) participating in a collaborative action-research project in which teachers identify their own focus for enhancing their classroom practice.

The second goal was to improve students' algebraic knowledge, algebraic skills, and disposition toward algebra. Objectives for the second goal included: (a) assessing and building upon students' prior knowledge of algebraic concepts; (b) engaging students in solving rich algebra tasks to enhance algebraic understanding and habits of mind; (c) providing opportunities for students to make meaning of algebra, including its conceptualization beyond symbolic manipulation and value as a tool for inquiry in STEM and other real-world contexts; and (d) improving students' performance on standardized and class-level assessments and motivation to engage with algebraic concepts.

Features of the PD

The PD was a year-round program that started with a ten-day summer institute in June 2015. Three follow-up sessions took place during the school year. In addition to the organized PD meeting times, each teacher was expected to teach lessons based on the standards-based mathematical practices, complete two observations of another teacher teaching a lesson from the PD, and provide data for research being conducted by the PD facilitators. Each teacher had an opportunity to participate in 86 hours of PD.

The standards-based mathematical practices were shared with the teachers at the beginning of TAPS as the vision for mathematics instruction for the PD. The SMPs were described to the teachers as descriptors of what students have an opportunity to do when learning mathematics (Field Notes, 2015-06-09). The MTPs were described to the teachers as descriptors for what teachers have an opportunity to do when teaching mathematics (Field Notes, 2015-06-09). In addition, the PD focused on the use of mathematical tasks with higher-level demands (Stein, Smith, Henningsen, & Silver, 2000). Teachers participated in mathematical tasks, discussed the characteristics of mathematical tasks, worked in small groups to create three tasks that would be used during the upcoming school year, and presented tasks to each other.

The PD facilitators used sample lessons and activities about patterns, relationships, and generalizations. Additionally, the PD facilitators provided active learning opportunities for teachers (Desimone, 2009; Loucks-Horsley et al., 2010) including journal responses, small and large group discussions, teacher peer observations, video study, student-like participation in mathematical tasks, and presentation of tasks with feedback from the group.

Summer Institute

The summer institute ran in conjunction with the SSC summer school program. This allowed the participating teachers to gain experience using the standards-based mathematical practices and mathematical tasks with the summer school students. It also provided an opportunity for the teachers to observe each other and to discuss the observations. The morning summer school sessions lasted three hours. The afternoon summer institute work-sessions also lasted three hours.

On the first day of TAPS, the facilitators discussed the goals of the PD with the teachers. The facilitators reviewed the standards-based mathematical practices with the teachers and shared that they would focus on developing and implementing activities aligned to these practices. The PD facilitators summarized the goals for the teachers as knowing more about algebra, teaching algebra, and ways to improve teaching algebra (Field Notes, 2015-06-08). These discussions were consistent with the program goals, but did not present the goals with the same detail as the TAPS proposal.

Most of the institute days included a reflection question that the teachers wrote about in reflection journals. The prompts included questions such as: "What do you see as the major challenges in teaching algebra?" (Field Notes, 2015-06-08) and "What connections are there between algebra topics, between algebra and other math, and between algebra and other non-math topics?" (Field Notes, 2015-06-12). After the personal writing, the teachers would discuss the questions in small groups and as a whole group.

In addition to the reflection questions, time was dedicated to understanding mathematical tasks. Teachers reviewed examples of mathematical tasks, sorted them as higher-level or lower-level (Stein et al., 2000), and developed characteristics of tasks that could be used as identifiers. For example, the teachers described higher-level mathematical tasks as having multiple steps, requiring justification, and allowing the opportunity for more than one correct answer. They described lower-level mathematical tasks as requiring only basic computation, having few steps, and being limited to the use of a formula or memorization (Field Notes, 2015-06-08).

An important feature of the PD was the time devoted to discussing and understanding the standards-based mathematical practices. For example, on the seventh workshop day the reflection question was: "Which MTPs do you feel most competent implementing in your classroom? Which do you wish you were better at?" Teachers responded during the whole group discussion:

> Teacher 1: I would like to be better with productive struggle and questioning.
>
> Teacher 2: I would like to get better with struggle without losing them, allow kids to struggle without stepping in.
>
> Teacher 3: I need to improve not jumping in to help.
>
> Teacher 4: It takes mistakes to learn. (Field Notes, 2015-06-16)

When teachers had time to work on the mathematical tasks for their classroom, the facilitators regularly asked the teachers to reflect on which of the standards-based mathematical practices were aligned with the task and to find ways to include more of the SMPs and MTPs.

Follow-Up Sessions

The three follow-up sessions took place after school in October, February, and April. Teachers met with facilitators for two hours. They shared the use of mathematical tasks in their classrooms and learned more about the standards-based mathematical practices. The follow-up sessions included reflection questions, readings from *Making Sense of Algebra* (Goldenberg et al., 2015), sample mathematical tasks led by the facilitators, and time for teachers to work on mathematical tasks for use in their classrooms.

The reflection questions, readings, and sample mathematical tasks provided opportunities for the teachers to learn more about the standards-based mathematical practices. For example, during the February meeting one of the PD facilitators shared how he selected and modified a presented mathematical task to align with the mathematical practices:

> Facilitator: Here is how I thought about the [standards-based mathematical practices] when I designed the task; the task included persevere because the scaling

> was not given to you; we had to reason abstractly because you had to go between context and numbers and solve the inequality; and you had to look for structure using shapes within shapes. (Field Notes, 2016-02-02)

Teacher Case Studies

Teacher 1: Doug Collins (DC)

Doug Collins was a male with thirteen years of teaching experience. This was his second year at SHS and he taught Algebra 1 and Geometry during the 2015-2016 school year. Mr. Collins had a bachelor's degree in mathematics and he was working on a master's degree in mathematics education. When asked to describe the goals of the PD in his own words, Mr. Collins stated that they were "to try to help improve the algebra one end-of-course exam scores at [SHS]" (DC Interview, 2015-09-15). His interpretation of the goals of the PD was to help students pass the state accountability and graduation test they took at the end of their algebra one course.

Doug Collins: Enacted lesson #1. Mr. Collins' first observed lesson was a task he developed during the PD Summer Institute on writing and solving multi-step equations. He described the academic standards that would be included in the lesson as following order of operations, solving equations, and checking solutions as reasonable (DC Interview, 2015-09-15). The task was done as review before an upcoming test. When asked about the purpose of the lesson, Mr. Collins replied, "I am hoping that the students get more experience with solving equations, showing all of their work, because I have students who don't like to do that, and hopefully to help build their confidence" (DC Interview, 2015-09-15).

Each student was given an algebraic expression on either a gold or a green piece of paper. Mr. Collins explained that a student with a gold sheet should find a student with a green sheet. They would set their algebraic expressions equal to each other and then find a value for the unknown that would make the equation true. Mr. Collins told the students that they should work together, show all of their work, and check to see if the solution made the equation true. He also stated that the students should complete at least five equations with five different partners.

The students worked in pairs on this task for thirty-five minutes. They checked answers with each other, explained methods used to find an answer, and used calculators to check answers. Students asked questions such as, "Can you do that?" and "Do you understand why I added seven?" (DC Observation, 2015-09-18). Mr. Collins moved around the room checking work done by students and helping students find new partners. He made comments to encourage the students to work together such as, "If you don't agree you will need to check with your partner" (DC Observation, 2015-09-18). At the end of the class, Mr. Collins asked the students to return to their seats and collected their work.

Mr. Collins's first lesson included some elements of standards-based mathematical practices emphasized by the PD. In comparison to the SMPs, evidence was seen of students *making sense of problems and persevering to solve them*. During the partner work, the students worked together to find and check solutions to algebraic equations. There was also evidence of students *constructing viable arguments and critiquing the reasoning of others*. This occurred as the students worked with different partners and explained how they found the solutions. When considering the MTPs, there was evidence of Mr. Collins *promoting reasoning and problem solving* and *facilitating meaningful mathematical discourse*. The teacher *promoted reasoning and problem solving* by providing challenging problems and having the students explain their work to each other and check the answers to see if they made the algebra equation true. *Facilitating meaningful mathematical discourse* was observed when he encouraged the students to talk with their partners and explain the steps for finding solutions.

Doug Collins: Enacted lesson #2. The second observed lesson took place at the end of a unit on quadratic equations. The topic for the lesson was using data to determine if relationships were linear or quadratic (DC Interview, 2016-04-25). Mr. Collins explained that the academic standards that would be addressed in this lesson were recognizing different types of equations, graphing ordered pairs, writing equations, and interpreting data and graphs (DC Interview, 2016-04-25). When asked where this lesson fit within the unit he was teaching, Mr. Collins stated:

> DC: It's at the tail end. We actually just got done. They are actually testing tomorrow on exponential equations, graphing them, solving word problems on

them. So we've done all the math and now … here's an example of how [quadratic equations] can apply. (DC Interview, 2016-04-25)

For the lesson, Mr. Collins used a mathematical task presented by the PD facilitators, a modified version of "Bridge Strength" from *Thinking with Mathematical Models* (Lappan, 2005). At the beginning of the lesson, Mr. Collins asked the students to find a partner and to gather pennies, a cup, three strips of four different-length strips of paper, and books for suspending the strips to create a bridge (DC Observation, 2016-04-27). Mr. Collins passed out a work packet to each student and told them that they would need to read the packet so they would know how to do the activity for the day. Students were instructed to run through all of the experiments first and collect all of the resulting data (DC Observation, 2016-04-27). After all data were collected, the packet had fourteen questions for the students to answer about the experiment.

The students worked in pairs. They suspended the paper bridges, placed a cup on the bridge, placed pennies in the cup until the bridge collapsed, and recorded the number of pennies required to collapse the bridge in a data table. After a bridge collapsed, the students increased the thickness or the length of the bridge and repeated the process. Most of the teacher-to-student interactions involved clarifying how to set up the bridges or how to collect the data (DC Observation 2016-04-27). The student-to-student interactions included clarifying methods to collect and represent the data (DC Observation, 2016-04-27). When data collection was complete, the students made graphs and answered questions in the packet about the data to determine if relationships were linear or quadratic and to make predictions.

Some of the standards-based mathematical practices were observed during Mr. Collins's second lesson. For the SMPs, evidence was seen of students *making sense of problems and persevering to solve them*. This occurred during the small group work. Students were given a higher-level mathematical task and worked in small groups to make sense of the problem and answer related questions. The students also *modeled with mathematics* when they organized their data into tables and used the data to make graphs of the relationship between length or thickness and the weight of collapse. There was evidence of some of the MTPs.

Mr. Collins was observed p*romoting reasoning and problem solving*. This occurred as the students made sense of the problem and made predictions using the collected data. There was also evidence of *facilitating meaningful mathematical discourse* as the students worked in small groups, clarified terminology with each other, and explained reasoning about the graphed relationships.

Doug Collins: Interpretation of goals and enacted lessons. Mr. Collins interpreted the goals of the PD to be a means to help students pass a state accountability and graduation test. His interpretation focused on TAPS' second goal: To improve students' algebraic knowledge, algebraic skills, and disposition toward algebra. His description of the goals in his own words was very close to the objective to improve students' performance on standardized and class-level assessments. When he identified the lessons to be observed that were consistent with the goals of the PD, both enacted lessons were a review or an extension to prepare students for an upcoming test.

Teacher 2: Kathy Gibson (KG)

Kathy Gibson was a female teacher with eleven years of teaching experience; ten of the years were at SHS. She taught Pre-Calculus and was the mathematics department chair. Her bachelor's degree was in mathematics education and she was working on a master's degree in mathematics education. After the PD summer institute, Ms. Gibson was asked to describe the goals of the PD in her own words.

> KG: Well, I don't know. I guess I would say the goals for me would have been to get more activities and more things that I could use in class that had a higher depth of knowledge questions and how I could improve in that area. I guess that was my main goal.
>
> Interviewer: What do you think the goals were for the presenters? What do you think [PD facilitator] was trying to accomplish or [other PD facilitator]? Do you think it was the same thing?
>
> KG: I don't know. (KG Interview, 2015-09-17)

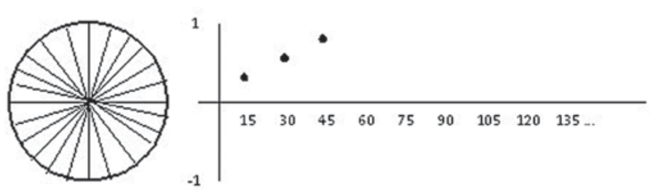
Initially, Ms. Gibson answered with her personal goals for the PD. She wanted to get more activities to use in her class with higher depth of knowledge questions. When she was asked what the goals were for the program, she replied that she did not know.

Kathy Gibson: Enacted lesson #1. Ms. Gibson's first observed lesson was an introduction to graphing sine and cosine functions. The lesson included a mathematical task that she developed during the PD sessions. When asked about the purpose of the lesson, Ms. Gibson replied, "It is discovering the graph of a sine function from the unit circle" (KG Interview, 2015-09-17).

The lesson started when students received a packet with instructions and questions and were told by Ms. Gibson that they would need to read the packet in order to know what to do (KG Observation, 2015-09-21). The students collected the needed materials for the lesson which included a large sheet of paper, a protractor, a compass, a meter stick, a piece of yarn about two meters long, and several pieces of uncooked spaghetti. The students worked in groups of three or four on the task.

Students used the compass to draw a unit circle with a radius equal to the length of one of the spaghetti noodles on one end of the large piece of paper. The students used the protractor to mark fifteen-degree increments around the circle and created a Cartesian plane next to the unit circle. The x-axis was labeled with the degrees of the circle and the y-axis was labeled with the vertical distances from the horizontal diameter of the circle to each of the given degrees (Figure 1). Students used additional spaghetti pieces to measure the perpendicular heights at the fifteen-degree increments and transferred the ordered pairs to their graph. The resulting graph was a sine curve. Once groups completed the sine curve, they followed similar steps to create a cosine curve.

FIGURE 1. *Task comparing unit circle to sine curve.*



The students worked together to make sense of the instructions, agree on terminology, use tools to construct a sine or cosine curve, and respond to questions in the packet. Student comments included, "If the spaghetti is the radius, then the circle is two-spaghetti wide," and "The curve follows the same pattern" (KG Observation, 2015-09-21).

Many groups noticed patterns with the different lengths. For example, students noticed that the perpendicular distance to the point on the circle at 45 degrees was the same as the distance at 135 degrees. Ms. Gibson walked around the room checking on the progress of the groups and asking questions to monitor student thinking. She asked the students questions about the activity like, "Do you see any patterns in the graph of the sine curve?" or "How are the graphs of sine and cosine the same and how are they different?" (KG Observation, 2015-09-21).

After completing the graphs, the students discussed their work. One group displayed a graph with a sine and cosine curve in the front of the class that was used as a reference during the discussion. The class discussed questions such as, "What is the period or the wavelength of the sine curve?" and "What are the zeros of the graph?" Students shared their thinking about the graphs such as, "It repeats after 360 because 0 and 360 are coterminal" (KG Observation, 2015-09-21). Ms. Gibson finished the whole class discussion by explaining that these were the parent graphs for the sine and cosine functions and the class would learn more about the properties of these functions.

Ms. Gibson's first observed lesson included many elements of standards-based mathematical practices; a few are highlighted here. One SMP observed during this lesson was *making sense of problems and persevering to solve them.* During the partner work, the students worked together to understand the instructions and work on the mathematical task, consider the relationship between the unit circle and the two trigonometric functions, and answer questions about the characteristics of the functions. A second observed SMP was *looking for and expressing regularity in repeated reasoning* during the lesson. This occurred when the students noticed patterns in the vertical distances at different degree measures around the circle (e.g., the sine values at 45 degrees and 135 degrees are equal). For the MTPs, evidence was seen of *using and connecting mathematical representations* and *posing purposeful questions*. Students had opportunities to *connect mathematical representations* by making the sine and cosine curves in proximity to a unit circle and using non-standard methods for measurement to find values of sine and cosine at different angles. Ms. Gibson *posed purposeful questions* during small group work and during the whole class discussion when she asked the students about patterns and asked them to compare the graphs.

Kathy Gibson: Enacted lesson #2. The topic for the second observed lesson was solving non-linear systems of equations. Similar to the first observed lesson, she used a task that she developed during the PD. Ms. Gibson planned for the students to work in small groups and present solutions to the whole class so that they would "communicate and talk to each other about their ideas" (KG Interview, 2016-02-10).

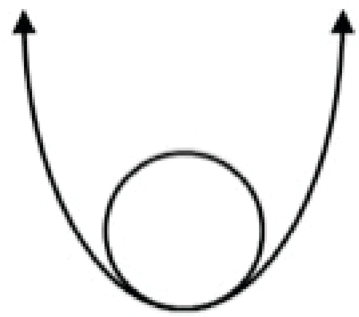
At the beginning of the second lesson, Ms. Gibson told the students that they could work in small groups to answer two questions: (a) How many different possible intersection points are there if a line and a circle are graphed in the same coordinate plane? and (b) Write a set of equations for each of the possibilities you have and find the intersection points for each (KG Observation, 2016-02-16). The small groups had as many as four students and some students chose to work individually.

The students discussed mathematical ideas in relationship to the questions. For example, they discussed what it meant for a line to consist of an infinite number of points and the possibility of a circle and line intersecting at one point (KG Observation, 2016-02-16). Students made drawings to demonstrate the different intersection possibilities. Ms. Gibson moved around the room to monitor the different student groups. She discussed ideas in the lesson with the students such as how to find the points of intersection. She also challenged their current understandings by asking questions such as, "Can you do this in a different way?" (KG Observation 2016-02-16).

When groups finished the first questions, Ms. Gibson asked them to find the number of possible intersections between a parabola and a circle. Groups discussed the possibility of zero, one, two, three, and four intersections between the parabola and circle. Some debated the possibility of a circle and a parabola intersecting at an infinite number of points if the circle aligned "just right" with the vertex of a parabola (Figure 2).

With about fifteen minutes remaining in the class, Ms. Gibson announced that the groups were going to share solutions (KG Observation, 2016-02-16). Different groups shared equations that were examples of a line and a circle intersecting or a parabola and a circle intersecting. The groups justified the points of intersection and explained how they selected the equations that they used. For example, one student explained:

FIGURE 2. *Image of parabola and circle debated as having infinite points of intersection.*



> Student: We centered our circle around zero so it would be easier to work with. For one intersection we put our circle right underneath the parabola so it just hit at one point. From there we slowly started moving our circle up until it hit [the parabola] two, three, or four times. (KG Observation, 2016-02-16)

There was evidence of many of the standards-based mathematical practices in Ms. Gibson's second observed lesson. The lesson included opportunities for students to *construct viable arguments* as they argued the possibilities of different intersections and justified their reasoning during the small group work and during the whole class discussions. The students *used repeated reasoning* when they developed patterns for moving or changing properties (e.g. slope, radius, intercepts) of the lines, parabolas, or circles. MTPs evident during this lesson included *using mathematical tasks that promote reasoning and problem solving* and *posing purposeful questions.* Ms. Gibson started the task with two challenging questions that required the students to use reasoning and problem solving to conduct a mathematical investigation and apply many conceptual mathematical ideas. In addition, Ms. Gibson posed questions focused on exploring other solution methods and justifying solutions to other members of the class.

Kathy Gibson: Summary. When asked about the goals of the PD program, Ms. Gibson replied with her personal goal, to get more activities to use in her class with higher depth of knowledge questions. Her personal goal was similar to the first goal of the PD, to enrich teachers' knowledge and skills for teaching algebra. It centered on the objective to develop activities that would address the standards-based mathematical practices. Each of the lessons identified by Ms. Gibson for observation embodied her personal goal. In both lessons, Ms. Gibson was observed *posing purposeful questions*, which is consistent with her goal to include higher depth of knowledge questions.

Teacher 3: Laura Henderson (LH)

Laura Henderson was a female high school mathematics teacher with four years of teaching experience. This was her second year teaching at SHS and she taught Algebra 1. Her bachelor's degree was in mathematics education. Ms. Henderson described the goals of the PD in the following way, "I think the goals are to align the [SMPs and MTPs] and the content standards to make an algebra class more enriching to take it to that next level for the kids" (LH Interview, 2015-09-02).

Laura Henderson: Enacted lesson #1. The topic for Ms. Henderson's first observed lesson was a review of order of operations. She planned to have the students work in small groups "to thoroughly explain themselves and understand the content" (LH Interview, 2015-09-02).

At the beginning of the first lesson, Ms. Henderson gave a worksheet to each student that included a diagram with the numbers one through ten organized the way bowling pins are organized on a bowling lane. She explained to the students that they were going to play order of operations bowling (LH Observation, 2015-09-04). The students would roll four six-sided dice and use the four numbers with the order of operations to equal different values one through ten. For example, if 1, 2, 4, and 5 were rolled the students could do $5 \times 4 \div 2 - 1$ to get 9 and could do $5 + 4 - 2 - 1$ to get 6. If students could not find a way to get all of the values one through ten, then they could roll the dice a second time and use the new outcomes to get the remaining values.

Students were told to work with a partner and show all of their work on the worksheet. Students asked clarifying questions such as, "Can we use exponents?" or "Do we have to use all of the numbers?" (LH Observation, 2015-09-04). Most of the student discussions centered on checking answers and explaining how they used the four numbers to find values between one and ten. Ms. Henderson walked around the room checking the work done by the students. She reminded them to use grouping symbols and exponents (LH Observation, 2015-09-04). Some groups were not able to find a combination to get one of the values between one and ten. Ms. Henderson provided hints to groups when they only had a few numbers remaining. For example she visited one group and said, "You need an eight? What about six times five, divided by three, minus two?" (LH Observation, 2015-09-04).

The class discussed the task as a whole class during the last ten minutes. During the whole class review, Ms. Henderson asked the students to share the craziest equations they found. One student shared, "I had five to the power of one, minus two, minus two" (LH Observation, 2015-09-04). She asked if anyone used a square root, but none of the students shared an example. The students handed in their worksheets at the end of the class.

One standards-based mathematical practice was observed during Ms. Henderson's first lesson included. There was evidence of the SMP *attend to precision*. The main focus of the activity was applying the rules for order of operations to find different integer answers. Students manipulated numbers, performed calculations, and checked their work to calculate the different integer answers.

Laura Henderson: Enacted lesson #2. For the second observed lesson, Ms. Henderson planned for the students to work on data analysis and creating approximate best-fit lines. She felt that the lesson was aligned to the PD because she was using a mathematical task that was shared by the facilitators (LH Interview, 2016-04-29). The lesson involved students comparing data about the length and the width of different bird eggs (Mathematics Assessment Resource Service, 2011).

The lesson started when Ms. Henderson introduced the task.

> LH: You are going to work on a task involving bird eggs. It involves bivariate data, which we have talked about. You will need to read the questions and answer them the best that you can. If you get confused, I will clarify the question for you. After a while, you can work with a partner and compare what you have with what they have. There isn't just one right answer for these. Just because someone has a different answer doesn't mean that you are completely wrong. (LH Observation, 2016-05-04)

Ms. Henderson handed out a packet with a scatter plot graph of data comparing the length and the width of different bird eggs. Students were instructed to use the data to answer a series of questions about the relationship.

The students worked independently on the task at the beginning. After about twelve minutes, Ms. Henderson asked the students to work with a partner. The students compared answers and explained their solution methods to each other (LH Observation, 2016-05-04). Ms. Henderson walked around the class checking work done by the students and listening to the small group discussions. She asked some groups clarifying questions such as, "What can you do to check the equation?" (LH Observation, 2016-05-04).

The students returned to their seats after fifteen minutes. At this point, Ms. Henderson led a whole class review of answers. She asked questions such as, "How did you start this?" and "What do we do next?" (LH Observation, 2016-05-04). The students responded and explained steps in their solutions. For example, Ms. Henderson asked one student how he added a point to the graph given an egg with a length of 57 millimeters and width of 41 millimeters. The student explained how he graphed the point with length on the x-axis and width on the y-axis (LH Observation, 2016-05-04). In other cases Ms. Henderson re-read the questions from the packet and explained how to do the problems. For example, one question asked which egg had the greatest ratio of length to width. She explained that the students needed to create ratios for five different eggs and see which ratio was the largest (LH Observation, 2016-05-04). After the whole class review of the answers, the students passed in their work from this task.

Ms. Henderson's second lesson included some of the s tandards-based mathematical practices. In comparison to the SMPs, there was evidence of students *making sense of problems and persevering to solve them.* Ms. Henderson facilitated instruction by requiring the students to read and understand the mathematical task, limiting the amount of direction and answer-giving to students, and having the students work independently and in small groups. There was evidence of *attending to precision* when the students compared and checked solutions and estimated the length or width of eggs not on the provided scatter plot using an estimated best fit line. For the MTPs, Ms. Henderson implemented a task that included opportunities for the students to *use and connect mathematical representations*. This was evident when the students represented the data graphically and algebraically.

Laura Henderson: Summary. Ms. Henderson described the goals of the PD in her own words as aligning the SMPs and MTPs to mathematics content standards to improve her algebra class. Her goal was aligned with objectives from the first and the second PD goals: developing algebra activities that would address the SMPs and MTPs and engaging students in solving rich algebra tasks to enhance understanding. Each of the observed lessons reflected her interpretation of the PD goals because each of the lessons was an attempt to include practices like *attending to precision* and *making sense of problems and persevering to solve them* as students worked on tasks and explained the mathematical content.

Teacher 4: Ruth Lawrence (RL)

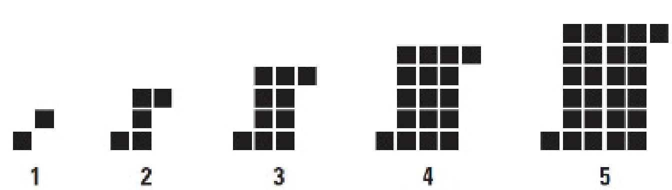
Ruth Lawrence was a female teacher with nine years of teaching experience, eight of them at SHS. At the time of this research, she was teaching Algebra 1 and Geometry. Ms. Lawrence had a bachelor's degree in mathematics. After the summer PD, she was asked to describe the goals for the PD program.

> RL: I think the goals were to expose us to mathematically rich tasks. And I think also we were supposed to learn about the SMPs and the MTPs and maybe how to keep those in our focus while we are teaching throughout the school year. (RL Interview, 2015-09-14)

Ms. Lawrence identified two key aspects or the PD, to help teachers learn about mathematical tasks and to help teachers learn how to use the standards-based mathematical practices in the classroom.

Ruth Lawrence: Enacted lesson #1. The first lesson involved creating algebraic representations of patterns using the S-Pattern task (Institute for Learning: Learning Research and Development Center, 2015) that was shared during the summer PD (Figure 3). Ms. Lawrence stated that the purpose of the lesson was "to explore this activity and represent a pattern with a quadratic equation" (RL Interview, 2015-01-07). The lesson took place over two days.

FIGURE 3. *Sequence of figures following the S-Pattern.*



At the beginning of the lesson, Ms. Lawrence asked the students to get into pairs, explained that the worksheet included a sequence of figures with patterns, and asked the students to answer questions about the patterns (RL Observation, 2016-01-08). The students described different patterns in the picture with their partners. Students asked their partners questions such as, "How do you explain that?" and justified statements such as, "It is F – 1 because it is the fifth figure, but there are only four squares in the row" (RL Observation, 2016-01-08). When the students shared their patterns and solutions, they frequently asked each other if they understood. When students did not understand, they would ask for an explanation (RL Observation, 2016-01-08).

Ms. Lawrence walked around the room checking on the student work. She encouraged the students to consider other patterns by asking questions such as, "You are doing different patterns. Are you noticing anything else?" (RL Observation, 2016-01-08). Ms. Lawrence also probed for student understanding with questions such as, "Does the height change too? Can you relate it to that?" (RL Observation, 2016-01-08). The groups answered questions about representing the total number of squares algebraically until the end of the class on the first day.

When the students returned the next day, Ms. Lawrence asked different students to come up to the front of the class and share the equations they created. They explained how they developed the parts of the equations based on the figures. For example, one student shared:

> Student: [Writes T = (F – 1) (F + 1) + 2 on the board]. F is figure number and T is total squares. If we talk about figure five, five would go where the F's are. [Draws a circle around the 4 × 6 rectangle in the middle of the picture, excluding the two individual squares on the top right and bottom left]. This is F + 1 [points to the side with length 6] and this is four [points to the top with length 4] and five minus one is four. And you add these two [points to the two excluded individual squares] at the end. So it is 26 squares. Six times four is twenty-four and add two. (RL Observation, 2016-01-09)

Many of the standards-based mathematical practices were observed throughout Ms. Lawrence's first lesson. The alignment with the practices was largely due to her use of a high-level mathematical task, providing opportunities for students to make sense of the problem, working in small groups, encouraging students to explain their thinking, and encouraging students to compare their solutions to other solutions. One of the most common SMPs was *reasoning abstractly and quantitatively* when the students developed equations to represent the relationship between the figure number and the number of squares in each figure and described patterns to answer the questions for the mathematical tasks. Students *constructed viable arguments* during the small group and whole class discussions as they explained their thinking and clarified statements if ideas were unclear. Two of the MTPs observed during this lesson were *connecting mathematical representations* and *facilitating meaningful mathematical discourse*. The students *connected mathematical representations* by creating algebraic equations to represent the relationship between figure numbers and number of squares. In some cases, students created data tables for the figures. Ms. Lawrence *facilitated meaningful mathematical discourse* by having the students work in pairs and asking them to explain patterns or solutions to their partners.

Ruth Lawrence: Enacted lesson #2. The topic for the second observed lesson was writing linear, quadratic, and cubic algebraic expressions. M.s Lawrence felt that the lesson was aligned to the PD because the activity included algebraic habits of mind and productive struggle (RL Interview, 2016-01-12). The mathematical task for the second lesson was the painted cube problem (Lappan, Fitzgerald, & Fey, 2006), which was shared by the PD facilitators during the summer. This lesson also took place over two days.

Ms. Lawrence started the lesson by passing out a worksheet with a data collection table and asking the students to get into small groups. Using a model of a 3x3x3 cube, she explained that the outside of a cube with edge length three could be painted and the unit cubes would have paint on zero, one, two, or three sides (RL Observation, 2016-01-15). Ms. Lawrence showed the class how to fill in their data table for the cube with edge length three such that eight unit cubes had paint on three faces, twelve unit cubes had paint on two faces, six unit cubes had paint on one face, and one unit cube had paint on no faces (RL Observation, 2016-01-15). The students were told to complete the worksheet about painted cubes with edge length two, three, four, five, six, and any length "n".

The students used snap-cubes to build models and worked in groups of two or three to complete the data table and the worksheet. Students asked questions such as, "Are there twenty-four cubes with one face painted?" and "Is it always times six?" (RL Observation, 2016-01-15). They explained answers to their partners while pointing to models and counting cubes located in the correct positions for three, two, one, or zero faces painted.

Ms. Lawrence walked around asking questions about different ways to work on the problem such as, "Is there a faster way to find the number of cubes in the middle instead of adding nine and nine and nine?" or "Can you think about the cubes in groups?" (RL Observation, 2016-01-15). She also shared ideas to help the students find patterns for a cube with any unit length. For example, she told a student, "I want you to go back and see how you found these [the number of cubes with paint on one side]. That will help you see the pattern" (RL Observation, 2016-01-15).

During the second class period, Ms. Lawrence told the class that they needed to explain patterns and write algebraic expressions for a cube with an edge length of "n" and be prepared to discuss their expressions with the whole class (RL Observation, 2016-01-16). During the whole class discussion, one student wrote the number of unit cubes with paint on one face for the different lengths of the large cube as 0, 6, 24, 54, and 96. She explained that each of the numbers is a multiple of six. Zero is 6×0, six is 6×1, twenty-four is 6×4, fifty-four is 6×9, and ninety-six is 6×16 (RL Observation, 2016-01-16).

Again, many standards-based mathematical practices were observed during Ms. Lawrence's second lesson. The students *modeled with mathematics* by creating data tables, algebraic equations, and graphs from the cube models. They also *looked for and made use of structure* by using patterns and algebraic expressions to represent mathematical relationships they found with the cubes. With respect to the MTPs, Ms. Lawrence *promoted reasoning and problem solving* by using a higher-level mathematical task and allowing the students to work in small groups to make sense of the problem. She also provided opportunities for the students to *connect mathematical representations* as they created models, completed data tables, wrote algebraic equations, and graphed the relationships. The students negotiated an understanding of these relationships during the small group and whole class discussions.

Ruth Lawrence: Summary. Ms. Lawrence interpreted the goals of the PD as exposing the teachers to mathematical tasks and using the standards-based mathematical practices as a focus throughout the school year. Her interpretation of the goals aligned with the first goal: To enrich teachers' knowledge and skills for teaching algebra. The description she provided was similar to the objective to develop activities that would address the SMPs. Both of the lessons Ms. Lawrence identified for observation that were consistent with the goals of the PD included mathematical tasks shared during the PD and contained many standards-based mathematical practices due to instructional strategies used like questioning and small group expectations.

Findings

There were two goals for TAPS. The first goal was to enrich teachers' knowledge and skills for teaching algebra. To help accomplish this goal, the PD providers engaged the participating teachers in solving algebra tasks to enhance algebraic understanding, asked each teacher to develop tasks or lessons that would address the standards-based mathematical practices, and provided a system for structured reflection and feedback. The second goal was to improve students' algebraic knowledge, algebraic skills, and disposition toward algebra. The PD providers addressed this goal by helping teachers engage students in solving rich algebra tasks and provide opportunities for students to make meaning of algebra.

When the teachers were asked to describe the goals of the PD in their own words, each teacher stated a goal that was consistent with the PD goals, but concentrated on one of the objectives. The teachers' interpretations of the goals influenced which lessons were selected for observation and the enactment of the lessons (see Table 1). Doug Collins described the goal of the PD as helping him improve student scores on the state accountability exams. Both observed lessons were review or extension activities where students applied learned content before an upcoming unit test. Kathy Gibson's goal, to include activities with higher depth of knowledge questions, was an influence on the lessons that were observed. During the observations there was evidence of Ms. Gibson *posing purposeful questions* and the students *constructing viable arguments.* The observed standards-based mathematical practices were consistent with using higher depth of knowledge questions. Laura Henderson's interpretation of the goal of the

PD was to align the standards-based mathematical practices with content standards and improve her algebra class. Her lessons included a few of the practices, but she included strategies like small group work and questioning that encouraged students to explain their work about order of operations and bivariate data. Ruth Lawrence's interpretation of the goal was to include mathematical tasks and the standards-based mathematical practices from the PD. The influence of her interpretation of the goals started with her choice to use mathematical tasks from the PD for each of her lessons. In addition, her enacted instructional strategies such as providing opportunities for students to make sense of the problem, working in small groups, and encouraging students to explain their thinking provided opportunities for the practices to be a part of the lesson.

Table 1: Comparison of teachers' interpretations of PD goals and observed lessons.

Teacher	Interpreted goal	Observed lessons
Doug Collins	Help improve the algebra one end-of-course exam scores	Review lessons for unit tests
Kathy Gibson	Get more activities with higher depth of knowledge questions	Mathematical tasks with instances of *posing purposeful questions*
Laura Henderson	Align the standards-based mathematical practices and the content standards to make an algebra class more enriching	Mathematical tasks with partner work, students were observed *attending to precision*
Ruth Lawrence	Expose the teachers to mathematical tasks and use the standards-based mathematical practices	Lessons involved mathematical tasks shared during the PD and many practices were observed due to enacted instructional strategies

Similar to the research demonstrating that teachers adapt to new visions for mathematics instruction by interpreting and constructing understandings (Coburn et al., 2016; Munter et al., 2015; Roth McDuffie et al., 2018), each teacher in this research interpreted the goals for the PD. The teachers interpreted the goals in a way that was consistent with the goals for TAPS, but tended to concentrate on an objective related to one of the two goals. Their interpretations of the goals are in line with findings described by Ball (1996) and Loucks-Horsley et al. (2010) where teachers focused on elements of PD that addressed curricular, pedagogical, and/or outcome needs for her or his class.

Discussion and Implications

This research describes how four teachers interpreted the goals of a mathematics PD program and how those interpretations influenced enacted lessons. The enacted lessons were lessons that the teachers selected to have observed because they felt the lessons were consistent with the goals of a PD program. The observed lessons were similar to the teachers' interpretations of the goals. A multiple-case study cannot suggest that the interpretations and aligned instructional patterns found are comprehensive for all teachers, but these patterns illuminate how teachers' interpretations of the original goals of the PD program can influence enacted lessons.

An implication of this research is that PD providers need to spend time learning about teachers' interpretations of the goals of PD. Similarly, when mathematics supervisors are working with teachers to enact a new vision for mathematics instruction, they should spend time learning about teachers' interpretations of the vision. Learning about teachers' interpretations of goals or a vision can help PD providers and mathematics supervisors in two ways. First, learning about the interpretations can help providers and supervisors identify misunderstandings or misalignment with the desired outcomes. This offers an opportunity to develop learning experiences that will enrich understandings about the goals or vision. If PD providers and mathematics supervisors feel that the teachers' interpretations are too narrow or too broad, they can work with teachers adjust their interpretations. This would help the providers/supervisors and teachers build a consistent vision and move in a similar direction towards achieving desired instructional practices. Second, because the teachers' interpretations influence enacted instruction, providers and supervisors should learn about the interpretations of goals or visions before performing classroom observations. Knowing a teacher's interpretation could provide insight about enacted instructional strategies for lessons, especially when the lessons are selected for observation by the teachers. PD providers and mathematics supervisors can use the interpretations of the goals and evidence from observed lessons to discuss the goals for mathematics instruction and work with teachers to promote strategies that are consistent with the goals.

This research focused on teachers' interpretations of goals while participating in PD and how those interpretations influenced enacted lessons. Many additional questions about the relationship between the interpretations and standards-based mathematical practices are not understood with this data, but would be helpful if investigated. For example, what influence could the interpretation of goals have on the enactment of specific standards-based mathematical practices? In addition, it would also help to know if the teachers' interpretations of the goals were associated with the development of the goals based on the needs of the school and the teachers, each teachers' prior experience with classroom situations (especially standards-based mathematical practices), or other influencing factors. Additional research on how teachers interpret the goals of a mathematics PD program would add to the understanding about impact on classroom practices. ✪

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