NCSM Journal

of Mathematics Education Leadership

FALL/WINTER 2020 VOL. 21, NO. 2

www.mathedleadership.org

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership*** (JMEL) are interested in manuscripts!

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM. Categories for submissions include:

- **Case studies and lessons learned** from mathematics education leadership in schools, districts, states, regions, or provinces
- **Research reports** with implications for mathematics education leaders
- **Professional development** efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
- Other categories that support the NCSM vision will also be considered.

Submission Procedures

Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel. Manuscripts should be emailed to the Journal Editors, currently Drs. Brian Buckhalter and Erin Lehmann, at **ncsmJMEL@mathedleadership.org**.

Submissions should follow the most current edition of APA style and include:

1. A Word file(.docx) with author information (name, title, institution, address, phone, email) and an abstract (maximum of 120 words) followed by the body of the manuscript (maximum of 12,000 words)
2. A blinded Word file (.docx) as above but with author information and all references to authors removed.

***Note:** Information for manuscript reviewers can be found at the back of this publication.

NCSM

Officers:
Mona Toncheff, *President Elected*
Connie S. Schrock, *Immediate Past President*
Paul Gray, *President-Elect*
Shawn Towle, *First Vice President*
Sharon Rendon, *Second Vice President*

Regional Directors:
Katey Arrington, *Regional Director – S2*
Cathy Boutin, *Regional Director – E1*
Cheryl Cantin, *Regional Director – Canada*
Astrid Fossum, *Regional Director – C2*
Bernard Frost, *Regional Director – S1*
John SanGiovanni, *Regional Director – E2*
Rosa Serratore, *Regional Director – W2*
Steven Shadel, *Regional Director – C1*
Denise Trakas, *Regional Director – W1*

Appointees and Committee Chairs:
Becky Angus, *Social Media Chair*
Patricia Baltzley, *Marketing /Communications Chair*
Bill Barnes, *Historian*
Brian Buckhalter, *Journal Editor*
Natalie Crist, *Webeditor*
Kristine Cunningham, *Membership*
Ronni David, *Sponsor Liaison*
Mary Fitzgerald, *Conference Planner*
Kim Gill Interim, *Executive Director*
Linda Griffith, *Treasurer*
Ruth Harbin Miles, *Conference Coordinator*
Shelly Jones, *Positions Paper Editor*
Donna Leak, *Nominations Chair*
Erin Lehmann, Associate Journal Editor
Susan Nolte, *Civca Staff – Finance*
Jenny Novak, *Professional Learning Director*
Georgina Rivera, *Professional Learning Director*
Kim Romain, *Inspiration! Associate Editor*
Sarah Schuhl, *Secretary*
Pamela Seda, *Local Arrangements (2021)*
Jacob Sisson, *Affiliate Coordinator*
Tom Stricklin, *Sponsor Liaison*
David Van Vleet, *NCTM Rep*
Samantha Wuttig, *Awards Chair*
Gwen Zimmermann, *Inspiration! Editor*

Inquiries about the ***NCSM Journal of Mathematics Education Leadership*** may be sent to:
Brian Buckhalter at ncsmJMEL@mathedleadership.org

Other NCSM inquiries may be addressed to:
NCSM–Leadership in Mathematics Education
PO Box 3406
Englewood, CO 80155
Email: office@ncsmonline.org • ncsm@mathforum.org

Table of Contents

NCSM Vision

NCSM is the premiere mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

Purpose Statement

The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

- *Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education*
- *Fostering inquiry into key challenges of mathematics education leadership*
- *Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice.*

Comments from the Editors

Brian Buckhalter, *Buck Wild About Math, LLC*
Erin Lehmann, *University of South Dakota*

This is our first issue together as editors and we want to extend a thank you to Mona Toncheff for inviting us along this journey, Carolyn Briles, a former editor, and all of the past editors of JMEL for their dedication and guidance in supporting the membership of NCSM through our beloved journal. We are excited about the opportunity to advance the bold leadership direction of NCSM through the experiences and direction of our contributors.

What a time to be a teacher or a leader. The world is facing a global pandemic with COVID-19, and the look and feel of teaching is new to every stakeholder in education. Classrooms are forced to reinvent the experience of learning without compromising the integrity of teaching. With the goal of providing equitable access to high-quality math instruction, NCSM and NCTM are striving to provide direction for mathematics teachers and leaders so they can make informed decisions to meet the unique learning needs of all students (NCSM & NCTM, 2020). Teachers and leaders are forced to re-evaluate the norms that have been established in our educational systems for decades, illuminating discrepancies in resources and traditional barriers that may hinder this new form of learning.

In the midst of the everyday changes, one thing still holds true: ALL students deserve to thrive in high-quality mathematics learning environments, and teachers will still need support in helping them do so. During this time of reflecting on our past practices to form a foundation for the future of mathematical success for ALL students we invite you to reflect on your own practices and beliefs, both personal and professional, acknowledge the barriers that may be hindering effective teaching and learning in your system, and boldly take the steps- or at least one step- toward a vision of equity in learning for ALL.

Our first article, "Examining Potential Pitfalls That Hinder Productive Coaching Conversations" highlights the importance of conversations between instructional coaches and the teachers they support. Jakopovic acknowledges some of the barriers during this part of most coaching cycles and offers insights into how coaches and other mathematics education leaders can engage in productive coaching conversations that can foster reform-oriented shifts in teacher practice.

The second article of this issue "Making the 'Cut': One District's Strategy for Algebra Placement" is a reprint from our Spring 2007 issue. In this article, Grandgenett and Jackson examine the traditional timeline and entrance criteria in one school district for students' initial placement in Algebra courses. Under the development of a new selection procedure, nearly 67% of the students qualified for Pre-algebra placement and even made their selection process easier to administer, giving more students to earlier access to Pre-algebra and Algebra courses. ✪

Reference

National Councils of Supervisors of Mathematics (NCSM) & National Council of Teacher of Mathematics (NCTM). (2020). *Moving forward: Mathematics learning in the era of COVID-10.* https://www.mathedleadership.org/docs/resources/NCTM_NCSM_Moving_Forward.pdf

Examining Potential Pitfalls that Hinder Productive Coaching Conversations

Paula Jakopovic, *University of Nebraska-Omaha*

Abstract

Instructional coaching is a popular approach for providing professional development to classroom teachers that can help to enhance their knowledge and use of effective teaching practices. A growing body of research on mathematics instructional coaching, and mathematics coaching in particular, suggests that coaches can both increase teacher self-efficacy and use of research based instructional practices and potentially have a positive impact on student achievement (Campbell & Malkus, 2011; Ellington et al., 2017; Knapp, 2017). To help ensure the effective implementation of coaching initiatives, researchers continue to examine the types of coaching practices that are most effective at creating these shifts. This paper adds to the literature on some of the challenges that coaches face in their planning conversations with classroom teachers and provides insights into how coaches and other mathematics education leaders can engage in productive coaching conversations that can foster reform-oriented shifts in teacher practice.

Introduction

In recent years, the focus on teaching K-12 mathematics has increasingly shifted toward an aim to "build procedural fluency from conceptual understanding" (National Council of Teachers of Mathematics [NCTM], 2014, p. 10), in classrooms where "…students are effectively engaged in learning mathematics" (National Council of Supervisors of Mathematics [NCSM], 2014, p. 1). Rather than focusing only on memorized procedures and rules, or solely on constructing meaning through inquiry, recent publications on mathematics teaching, as well as standards documents, strive for the development of conceptual understanding to meet an end goal of fluency with procedures and processes. In response to this shift, changes have been made to curricular materials, resources, and more importantly, the expectations for what constitutes effective mathematics teaching (Council Board for the Mathematical Sciences [CBMS], 2012; NCSM, 2014; NCTM, 2018). These changes often require teachers to adjust their existing instructional knowledge and practices, and to learn and incorporate new ways of teaching mathematics into their daily practice.

For example, current research informed teaching practices such as attending to the cognitive demand of problems to make them accessible yet mathematically challenging for students, force teachers to think about their role in developing and enacting curricula in ways they previously have not (NCTM, 2014; Smith & Stein, 2018). Other practices, like engaging students in discourse and making student thinking a central part of instruction, push teachers to shift toward a student-centered practice where instruction was previously teacher-led (Chapin et al., 2013; NCTM, 2014, 2018). Although these sorts of ambitious teaching practices are promoted by mathematics researchers and educators at a national level (Ball et al., 2008; Carpenter et al., 2015; NCTM, 2014, 2018), helping teachers locally to envision and adopt them into daily practice can be challenging.

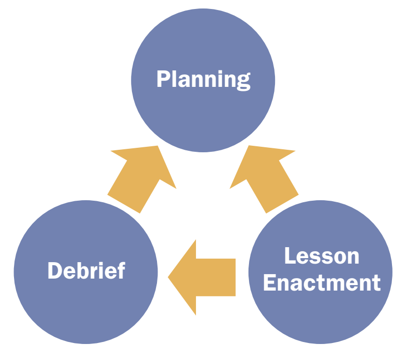
Studies suggest teacher professional development is a complex endeavor, one that should be situated within the context of the classroom, and embedded in real-time active learning, and utilize coaching/expert support as well as provide opportunities for teachers to receive feedback and reflect on practice (Darling-Hammond et al., 2017). Mathematics coaches present one potential avenue to provide contextualized support that is content specific. In this model of professional development, a knowledgeable teacher leader engages in planning with classroom teachers to facilitate engagement and reflection on effective mathematical teaching practices (Killion & Harrison, 2018; Knight, 2007; 2017; Sutton et al., 2011). Traditional professional development models, such as in-services and summer workshops, do not follow teachers back into their classroom practice, meaning the implementation of complex teaching innovations often meets with minimal transfer to long term practice (Darling-Hammond et al., 2017; Feiman-Nemser, 2001). Instructional coaching is often touted as a professional development model that has the potential to successfully mitigate some of these barriers by providing on site, in the moment supports that are tailored toward the needs of individual teachers (Campbell et al., 2013; Ellington et al., 2017; Knapp, 2017).

Examining the Literature on Mathematics Coaching

Definitions of Coaching

Instructional coaching is defined in a variety of ways in the literature. Killion and Harrison (2018) describe coaches as having as many as ten different roles that they may take up in their work with teachers. In addition to diversity in roles, there are also a range of instructional coaching models that coaches can subscribe to, including cognitive coaching (Costa & Garmston, 2015), instructional coaching (Knight, 2007; 2017), and content focused coaching (West, 2008). One common thread between these different coaching models is the articulation of an ongoing cycle of planning and discussion between coaches and teachers, which is embedded in daily teaching practice. More generally in the literature, these coaching conversations are part of the "three-part coaching cycle" (Bay-Williams et al., 2014). As the name suggests, this cycle is comprised of three components: planning the lesson, data gathering/lesson observation, and debriefing or reflecting after the lesson. These cycle components are dynamic in nature, and coaches work flexibly within their contexts to engage teachers in this framework. Figure 1 illustrates this cycle as it is typically enacted in the professional literature.

FIGURE 1.
The three-part coaching cycle.
Adapted from *Mathematics coaching: Resources and tools for coaches and leaders, K-12* (Bay-Williams et al., 2014).



Despite commonalities in different coaching models, the literature shows that the extent to which an instructional coach has a positive effect on teacher practice or student learning is not always clear. The mixed results of previous studies aided in the framing of my own study of coaches, in order to find a focused lens within which to study their work with teachers.

Understanding the Nature of Mathematics Coaching as "Effective" Professional Development

Although initial studies on the effectiveness of using mathematics coaches as a form of embedded, ongoing professional development were met with mixed results (Brosnan & Erchick, 2010; Campbell & Malkus, 2011; 2014; Chval, et al., 2010; Kretlow & Bartholomew; Murray et al., 2008), more recent studies highlight the potential of coaches to positively affect change. Mathematics coaching can lead to teachers' successful implementation of research based instructional practices, which positively impacts student learning (Campbell & Malkus, 2011; Knapp, 2017). Similarly, Frazier (2018) and Taylor (2017) found that coaching, and particularly mathematics coaching, can have a statistically significant impact on teacher competency/self-efficacy and student growth.

Simply installing a mathematics coach into a school is not enough to affect teachers' beliefs about mathematics instruction; studies have shown that teachers who are "highly engaged" with a coach make shifts in their perceptions about student learning in mathematics toward it being a sense making activity, and further coaches working with classroom teachers can translate to increases in student achievement (Ellington et al., 2017; Knapp, 2017). Understanding what makes for an effective mathematics coach is therefore a critical piece of the puzzle to ensuring the success of this type of professional development model. Schulte (2020) describes the need for coaches to have effective listening skills, questioning skills, and the ability to develop trusting relationships with teachers, and Russell et al. (2017) identify three key coaching practices: 1) engaging in deep conversations about the teaching and learning of mathematics that center on student thinking, 2) establishing clear content and pedagogical goals for coaching sessions, and 3) providing descriptive and evidence-based feedback to teachers. Russell et al. also present a framework for mathematics coaching that includes making pedagogical goals to help the teacher and coach achieve the mathematical goal of the lesson, then involves the teacher and coach engaging in "deep and specific discussion" of the lesson elements needed to support these goals (p. 154). Similarly, Desimone and Pak (2017) highlight five elements of effective coaching practice: a content focus, active learning opportunities for teachers, coherence in terms of content and goals, sustained duration, and collective participation within schools to develop a learning community. Such guidelines provide structure for instructional coach training programs and highlight important features of coaching implementation to maximize the potential success of these initiatives in a wide range of school contexts.

In examining how coaches can help to develop collective language and teaching practices, Gibbons et al. (2017) suggest that, during the deep conversations at the center of coaching practice, mathematics coaches should focus on using questions that are "carefully phrased" to help teachers attend to the mathematics and student thinking that are aligned to the goals of the coaching cycle. In terms of setting mathematical and pedagogical goals, the authors state, "Some coaching designs instruct coaches to ask teachers what they want to work on or improve and respond to those requests. However, as teachers start to develop ambitious instructional practices, they may not be positioned to identify their own learning needs" (p. 248). This suggests the need for coaches to differentiate their coaching practice based on the knowledge, experience, and readiness of individual classroom teachers. It also highlights another coaching practice that studies have found to be important, the skill of shifting between monologic (meaning is fixed and disseminated to create common meaning) and dialogic (meaning is dynamic and co-created through conversation) talk with teachers to meet mathematics content and pedagogical goals (Gibbons et al., 2017; Ippolito, 2010). Recent studies suggest that using a combination of both monologic, or directive coaching, and dialogic, or responsive coaching, is important to engage teachers in reflection on and shifts in teaching practice (Sailor & Price, 2015). A case study by Hammond and Moore (2018) found directive coaching to be successful in generating increased use of target instructional strategies as well as increased teacher self-efficacy; the authors suggest the importance of limiting the amount of directive suggestions a coach offers to a classroom teacher for this approach to be effective. This article seeks to examine what coaches say and do during these deep conversations with teachers in ways that both align with and deviate from the suggestions in the current literature to help coaches better understand how to maximize their coaching conversations with teachers and avoid common pitfalls in this practice.

Developing a Theoretical Framework for Examining Coaching Conversations

My study examined coaching conversations to look for instances where coaches helped classroom teachers attend to reform-oriented teaching strategies in their daily planning and teaching. I hoped to determine what coaches said and did that was more or less productive in helping teachers to reflect on and incorporate these strategies into their practice. To do so, it was necessary to clearly define what reform-oriented teaching strategies look like and find an appropriate framework to examine what mathematics coaches did to shift teachers' thinking and practice in productive ways. This section provides a definition of what is meant by effective mathematics teaching and details the development of an analytical framework for my study.

Mathematical Teaching Practices

In response to the shifts in content standards and curricular materials toward reform-oriented practice, *Principles to Actions: Ensuring Mathematical Success for All* was published by NCTM (2014) as a guiding document for mathematics teachers. The authors describe the need for successful mathematics programs to have "…effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p. 7). To do this, NCTM outlines eight mathematical teaching practices (MTPs) to guide the work of classroom teachers:

- Establish mathematical goals to focus learning,
- Implement tasks that promote reasoning and problem solving,
- Use and connect mathematical representations,
- Facilitate meaningful mathematical discourse,
- Pose purposeful questions,
- Build procedural fluency from conceptual understanding,
- Support productive struggle in learning mathematics, and
- Elicit and use evidence of student thinking.

This document provides teachers, teacher leaders, researchers, and other stakeholders with a common language and set of expectations about what constitutes "effective" mathematics teaching. At a broad level, coaches can begin to help teachers envision what their instruction can look like and provide supports to help teachers make shifts in their practice toward reform-oriented strategies (Bay-Williams et al., 2013; McGatha et al., 2018). This broader framework provided the foundation from which the analytical framework for examining coaching conversations in this study developed.

Mathematics Coaching Focal Areas

If a major goal of mathematics coaching is to help teachers incorporate the MTPs successfully into their practice, it is important for coaches to understand how they can maximize their time spent working with teachers. At the time of my study, research was just beginning to emerge on mathematics coaching frameworks that aligned well with my research questions, so I utilized research on effective teaching practices to develop an analytical framework for my study (Ball et al., 2008; NCTM, 2014). To establish this framework, I first categorized the MTPs within three broader focal areas: teaching practices focused on the overarching mathematical goals of lessons, practices focused around problem design and implementation, and practices that help to bring to the surface the mathematical thinking of students (Jakopovic, 2017). This categorization is illustrated by Figure 2 and provides an overview of what it looks like for teachers to engage in effective mathematics planning and teaching.

FIGURE 2.
Categorization of the mathematical practices for teaching into focal areas (Jakopovic, 2017).

Set Mathematical Goals	Use Intentional Mathematical Problem Design	Assess and Analyze Students' Mathematical Thinking
Establish math goals to focus learning	Implement tasks that promote reasoning and problem solving	Facilitate meaningful mathematical discourse
Build procedural fluency from conceptual understanding	Use and connect mathematical representations	
Support productive struggle in learning mathematics	Pose purposeful questions	Elicit and use evidence of student thinking

According to Campbell et. al (2013), for coaching conversations with classroom teachers to be "productive," a mathematics specialist must be able to "…make adjustments to her techniques as her familiarity with the teacher's level of mathematical understanding evolves" (p. 21). Gibbons et al.'s more recent work echoes the need for flexibility and differentiation on the part of the coach (2017). As the coach learns which of the MTPs will become the focal points of their work with a particular teacher, they can adapt their coaching practice to tailor it to the needs of each individual.

To better understand how a coach might break down this work, I then examined what types of specific planning and instructional tasks teachers engage in that help them to enact the teaching practices. To do so, I utilized the "mathematical tasks of teaching" developed by Ball et al. (2008) to classify the specific components of planning and instruction that are related to effective teaching practices. According to the researchers, teachers utilize specific types of knowledge to engage in a variety of tasks that pertain to the planning and implementation of mathematics instruction on an ongoing basis. These "mathematical tasks of teaching" (MTTs) require teachers to apply this knowledge in ways that are both specific and demanding in their daily practice. Similar to the MTPs, many of these tasks can be categorized more broadly as helping teachers to develop mathematical goals, design and implement lessons, and provide opportunities to examine student thinking. Figure 3 shows the nine MTTs that are well-aligned with the broader focal areas to help to illuminate what MTPs require to be enacted successfully. These focal areas and their underlying MTTs afforded me a framework for examining what teachers say and do during coaching conversations to determine what coach moves, if any, lead to reform-oriented and focused planning and teaching of mathematics.

FIGURE 3.
Categorizing Mathematical Tasks of Teaching within Teaching Practice Focal Areas
(Adapted from *Jakopovic, 2017).*

Set Clear Mathematical Goals

- Find examples to make a mathematical point
- Link representations to underlying ideas
- Appraise/adapt content of textbooks

Use Intentional Mathematical Problem Design

- Present mathematical ideas
- Pose productive questions
- Modify tasks

Assess and Analyze Students' Mathematical Thinking

- Recognize what is involved in using a representation
- Evaluate plausibility of student claims
- Give/Evaluate mathematical explanations

This framework is similar to the recent work of McGatha et al. (2018), which lays out a series of "professional learning focus areas" for mathematics coaches to attend to in their coaching conversations with teachers. As this paper will discuss, having a coaching focus (or lack thereof) during these conversations can dramatically change the trajectory of the conversation with teachers. It is worth understanding what and how coaches do to engage teachers in both focused and unfocused conversations, so that coaches and other professional developers can enhance the potential effectiveness of this work.

Methodology

The purpose of this qualitative study was to understand how mathematics coaches engage with teachers during coaching conversations to promote teacher reflection on the MTPs and MTTs in their daily practice. Like teaching, the work of an instructional coach is a complex practice (Bay-Williams et al., 2014; Ippolito, 2010).

This study was a single case study bounded by six elementary mathematics coaches from the same urban school district. Case studies allow the researcher to examine a contemporary and relevant issue, "in depth and within its real-world context," often begins with a theoretical framework to guide data collection and analysis and utilizes multiple data sources to allow for triangulation of findings (Yin, 2017, p. 14). As the goal of my study was to better understand how these coaching interactions influence teacher practice and why certain coaching conversations are more effective than others, qualitative case study design was well suited to examine this work (Yin, 2017). I utilized participant-observation, field notes, and interviews with the coaches and select teachers, to triangulate findings during my data analysis.

The major question guiding my research was: How do mathematics coaches craft the conversations they have with teachers during planned three-part coaching cycles in the way that they do in order to promote teacher conversation about and engagement with research-based mathematics teaching practices? The conversations that occur within the three-part coaching cycle were chosen as a particularly prominent platform that coaches use to initiate deeper conversations with teachers to examine in this study. A secondary question that guided this research was: What are the questions, statements, and moves that coaches make to support teacher thinking and instructional planning around these teaching practices? Campbell, et al. (2013) and Gibbons et al. (2017) suggest that asking good questions is at the heart of the strategies that coaches employ in their work with teachers. Mathematics coaches question teachers about student learning and about their practice. Coaches must know when to ask a question and when to wait for a better time and must be comfortable posing questions that have no immediate answer. Additionally, coaches can use other moves, such as offering suggestions and sharing examples, to help teachers think deeply about planning mathematics lessons that are centered on MTPs and MTTs. Developing a better understanding of the types of moves coaches make, and how they shape reflective conversations with teachers, can be fundamental to developing highly effective mathematics coaches. This study sought to analyze the sorts of moves mathematics coaches use during deep planning and reflection interactions with teachers to focus on MTPs and MTTs, and to what extent these moves appear to be successful in meeting these intended coaching goals.

Participant Selection

The participants of this study were six elementary mathematics coaches in a large, urban school district. Funding for the coaches was provided by two local philanthropic organizations, and this study was situated within broader research of mathematics initiatives in the participating school district. As well as being experienced former classroom teachers, each participant coach completed a mathematics education graduate program prior to becoming a coach, and they received an intensive ten-day coaching training from the Examining Mathematics Project at the onset of their new role. The six coaches served a total of eight elementary schools at the time of the study. Many of the schools where the coaches served were labeled as "low achieving" based on the results of statewide assessments, with three of the eight schools achieving at less than 50% proficiency on the 2014-2015 statewide mathematics assessment. Most of the schools served students of low socioeconomic status (SES), with six of the eight schools having rates at 85% and above for students receiving free or reduced lunch (a common indicator for determining SES) at the time of the study.

A total of 25 coaching cycles were observed and recorded in the participating elementary schools, which included work with 20 teachers during the spring of 2015. The teachers ranged in experience from one to twenty years of experience. Amy, Candy, and Mary worked with 2 to 3 teachers each in their observed coaching cycles, while Alex, Sharon and Emily worked with 5 to 6 teachers each. All coaches worked with a combination of beginning career teachers and veteran teachers with over 10 years of teaching experience. The backgrounds of the coaches also had some variability. Table 1 shows the background information on each of the coach participants at the time of the study.

Table 1: Participant Demographics at Time of Study

Coach Name*	Years Teaching Experience	Years Coaching Experience
Candy	10	3.5
Amy	17.5	1.5
Mary	16	1.5
Alex	10	1.5
Sharon	15	1.5
Emily	23	1.5

**Pseudonyms were used to protect the anonymity of participants*

Data Collection

I collected a large sample of coaching cycles with the intent of illuminating the complex work that coaches do with teachers. During the spring semester of 2015, I gathered evidence from 25 total coaching cycles, 22 of which included a face-to-face debrief (two cycles included debriefs via email and one cycle did not conclude with a debrief). I audio taped and transcribed the observations of coaching conversations for later analysis. I maintained field notes during my observation of the coaching cycles to better capture a complete view of the planning and debriefing conversations by including data, for example, that the audio-recordings could not capture. I kept notes of the classroom environment, mathematics and ideas that were shared in writing (both during the planning and debriefing and during lessons), and notes about non-verbal communications that occurred during the cycles. These observations of coaching cycles were supported by brief interviews with the coaches to clarify background information about the teacher, previous coaching work, and their goals for and reflections on the coaching cycle, as well as end of semester interviews with the coaches and select teacher participants. The complete scope of my data collection is shown in Table 2.

Table 2: Data Collection Spring 2015

Data Sources	Quantity of Data Collected
Observations of planning conversations	27
Observations of debrief conversations	22
Brief, informal conversations with coach	39
Researcher field notes on coaching cycles	25
Extended final coach interviews	6
Final teacher interviews	8

Data Analysis

I used qualitative methods to examine the data in two rounds of analysis. First, I coded transcripts of the planning and debriefing sessions for evidence of the Mathematical Tasks of Teaching (MTTs), to determine the types of teaching tasks that occurred in coaching conversations, as Table 3 illustrates (Ball et al., 2008). These teaching tasks illustrate the types of teaching moves that mathematics teachers engage in when trying to incorporate research-based instructional strategies. Specifically, I looked for instances of teachers discussing one of the MTTs during round one of coding.

Table 3: Definitions and Examples of Teacher Talk Coded as MTT (Mathematical Tasks of Teaching)

MTT	Definition	Example
Presenting Mathematical Ideas	Determining task design and set up (task, questions to pose, etc.). Determining, analyzing, or posing problems with the same/different structures (Selling et al., 2016).	"I like your idea of giving them the problem and just saying, 'Okay, I want you to try to work it out.' And so that way they are kind of working with the numbers. I'll give them time to try and figure it out, they'll talk about it with a shoulder partner…then I can go through it and we can talk about reasonableness. You know, how four-fifths is really close to another whole, so really technically it's almost two wholes there."
Finding examples to make a specific mathematical point	Matching task/problem to goal of the lesson. Matching word problems with a particular structure (Selling et al., 2016).	"I guess [I'd] like to help them see that, I mean, in some sense they could just reduce this to find the least common denominator… So would it make sense to say, 'Here's our two-sixths, we're adding one-third,' and then have them see if they can determine, before we get into this problem solving piece when we're just first introducing the idea of it, and having them sort of experiment to find a common denominator?"
Recognizing what is involved in using a particular representation	Anticipating benefits/drawbacks of using particular models for a given mathematical task. Anticipating how students might attempt to incorporate such models (both correctly and incorrectly). Selecting, creating, evaluating representations for a given operation or mathematical idea; analyzing representations for the same reason (Selling et al., 2016).	"I think a lot of my kids will figure that part out. Like, 'Oh you can't have that because the big number is on top,'…It's just going to be tricky for them because I taught them how to change a mixed number to an improper fraction, an improper fraction to a mixed number. They've seen that. I think what's going to be tricky for them is when they see the whole number out front right here and an improper fraction. That's going to start to confuse them and they're not going to know, 'Well what do I do with 11 when I'm dividing?'"

MTT	Definition	Example
Linking representations to underlying ideas and to other representations	Making explicit links between symbols, concrete pictures, diagrams, etc. Utilizing multiple models and helping students see connections between models. Connecting mathematical terms and ideas to analogies/metaphors/stories intended to help students understand mathematical concepts. Connecting or matching representations- matching to operations, to other representations, comparing the validity of two representations (Selling et al., 2016).	"Should we let them explore to find an answer? See if they can find multiple representations for it and then just kind of take a look at them and write them up there, two-thirds, four-sixths, and eight-twelfths and see if we can notice anything about those numbers…We say, 'Okay, these are what they are added together, what are we noticing about the denominators?' Or, 'What are we noticing about the numerators?'"
Evaluating the plausibility of students/ claims (often quickly)	Interpreting student ideas/claims (processing what they are saying and doing and offering appropriate feedback). Using and attending to student errors. Analyzing structure in student work (Selling et al., 2016). Making sense of student work in relation to instructional goals, mathematical structures, and multiple ways of solving (Kim, 2016).	"When we first start, you know, kids weren't talking about much but now they're eager to say, 'I disagree with that,' and, 'Okay, well why do you disagree with that?' 'Well, I saw that I didn't have enough so I had to regroup,' today one of my students said. 'I had to regroup the ones.' When actually he had to regroup the tens, and I was like, 'Really good job using the vocabulary. Let me make sure we're regrouping the tens because we have a tens stick and we're changing it…'"
Giving or evaluating mathematical explanations	Providing mathematical descriptions (with or without student help) that offer clear characterizations of the steps of a mathematical process. Teacher directing of explanations that includes attention to meaning (can include justification). Comparing, critiquing, and improving =mathematical explanations (Selling et al., 2016).	"They would say, 'Well, we're finding the difference,' which it didn't even say the difference in the problem. It said how many more. They're like, 'We know we're finding the difference and the difference means the answer to a subtraction problem.'"

I then refined the coded MTT teacher comments to the three broader focal areas of mathematical goals, problem design, and student thinking in order to look for trends across the data and connect these teaching tasks to the research based mathematical teaching practices (MTPs) advocated for in the current literature (NCTM, 2014). Recognizing that these two categorizations of the "what" of mathematics teaching overlap in terms of these larger themes, or focal areas, allowed me to then shift to examining what coaches did, if anything, to facilitate this teacher talk and reflection around the focal areas (as Figure 4 illustrates) in my next phase of coding.

In round two of my analysis, I re-examined the coded excerpts for evidence of whether something the coach said or did precipitated these instances of teacher talk around focal areas. For this process, I utilized open and axial coding (Strauss & Corbin, 1990) because, even though "coaching moves" have been broadly defined in the literature, little research currently examines the specifics of what coaches do and say to help teachers attend to MTPs and MTTs. Much of the current work is quantitative in nature, which can make it difficult for others to understand and replicate "productive" coaching conversations in their own contexts. Multiple current mathematics coaching resources tout that coaches help teachers to engage in the following: 1) setting goals, 2) posing ideas, 3) sharing ideas or suggestions, 4) actively listening, and 5) helping teachers to analyze student work (Bay-Williams et al., 2014; Campbell et al., 2013; Huguet et al., 2014).Through this second level of analysis, I used deductive and inductive methods (Saldaña, 2016) to develop a list of "coaching moves" from which I could examine the extent to which coaching conversations are more or less productive in helping teachers attend to these reform-oriented teaching practices (see Table 4).

FIGURE 4.
Aligning MTPs and MTTs to Three Broader Focal Areas

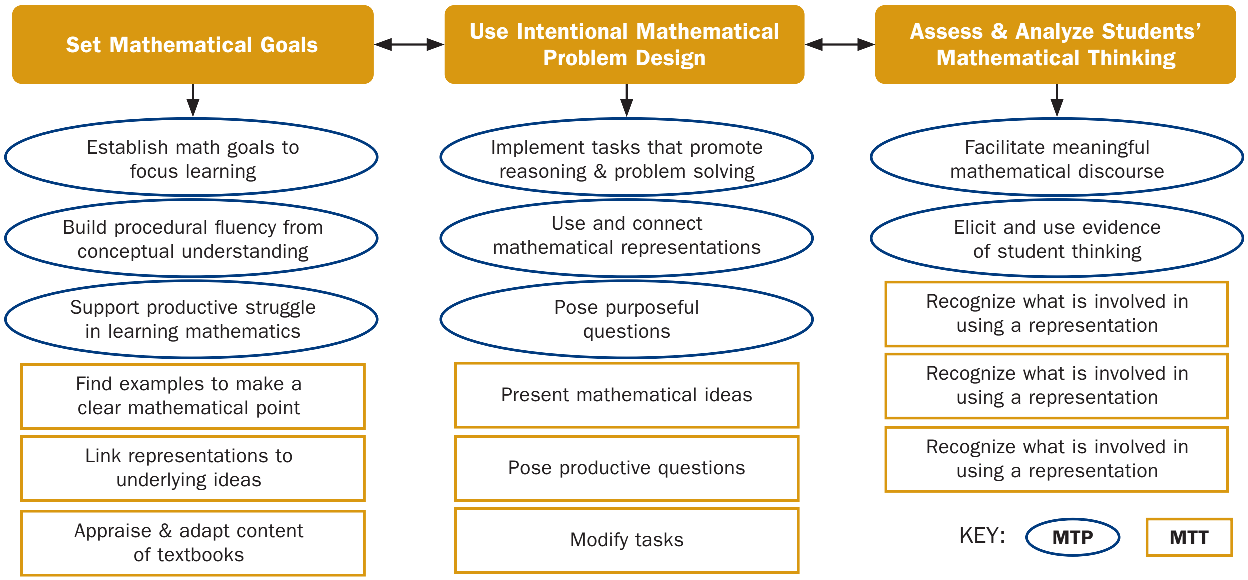


Table 4: Counts of Coaching Moves Coded for Mathematical Focal Areas

Focal Areas	Coaching Moves to Press Focal Areas	Coding Count
Mathematical Goals	QMG = Posing questions about mathematical goals	12
	QG = Posing questions about goals related to teaching	15
	SMG = Offering suggestions about mathematical goals	6
	RMG = Restating mathematical goals	14
Mathematical Problem Design	QPD = Posing questions about mathematical problem design	82
	QPS = Posing questions about the organization and set up of the problem/activity	14
	SPD = Offering suggestions about mathematical problem design	35
	SPS = Offering suggestions about the organization and set up of the problem/activity	8
Students' Mathematical Knowledge/ Thinking	QAS = Posing questions to anticipate student mathematical thinking/strategies	46
	QES = Posing questions about examples of student mathematical thinking/strategies	18
	QBS = Posing questions about student background knowledge of mathematics	16
	SES = Sharing examples of student mathematical thinking/strategies	25
Unrelated to MTT Themes	Q = Posing generic/other questions about the lesson	30
	SET = Sharing examples of teaching moves/collected data from lesson	4
	I = Interruption to conversation	6

In the final part of my analysis, I examined the extent to which categorizations of coaching moves coincided with the teacher making a remark that either focused on MTPs and MTTs or was more generically worded in response to the coach's move(s) (see Appendix A). I looked for trends in how the wording of individual coaching moves resulted in various patterns of teacher response, as well as how coaching moves across an entire conversation orchestrated more or less teacher comments attending to specific MTPs and MTTs. In addition to analyzing planning and debriefing transcripts, it was necessary to code and analyze my field notes and interview transcripts at this final phase of analysis. I cross-referenced the various data sources for each of these coaching conversations to better understand whether coaches had a particular focus walking into a coaching session, if they used a protocol to guide their coaching moves, and look for any other potential factors that might have influenced the direction of each conversation. Not only did examining a range of data sources help me to develop a better understanding of this complex work, it also allowed me to triangulate my findings in ways that increased the validity of my research (Yin, 2017). In particular, analyzing the brief interviews I conducted with coaches before and after each coaching session with a teacher allowed me to compare a coach's intended focus for the conversation with what actually occurred, and to hear the coach's perspective on how productive the coaching session was in helping the teacher engage in planning and conversation around the focal areas. For the purposes of this study, I use "more productive" to describe coaching moves that resulted in teachers reflecting on specific MTPs and MTTs during coaching conversations. I utilize "less productive" to refer to coaching moves that resulted in teacher responses that did not attend to MTPs and MTTs or that shifted the conversation in ways that became more coach-centered than teacher-centered (Appendix A).

Findings

Coaching Moves

In my data analysis, I found that coaches used a variety of coaching moves intended to help focus teacher thinking and planning around the three broader MTP focal areas, yet some of these moves were more productive than others in helping teachers attend to the focal areas. Understanding how coaching moves can guide the flow of conversation with classroom teachers and lead to teachers increased attention on the focal areas or not, can be instrumental in helping coaches learn how to maximize their time with teachers.

In the following section, I present several vignettes to illustrate some of this study's findings about potential barriers to productive coaching conversations that surfaced in my analysis, as well as provide examples of coaching moves that led to more productive work with teachers.

Unfocused, Surface Level Conversations

One potential barrier mathematics coaches can face is a lack of focus in their coaching conversations with teachers. I categorized 8 of the 47 planning or debriefing conversations (from the 25 coaching cycles) as "less productive" due to a repeated lack of the teacher attending to MTPs or MTTs (see Appendix B). In my analysis I found that the coaching moves utilized in these conversations were often not worded in ways that specifically focused on MTPs or MTTs (i.e. "What are you working on?" versus "What is your mathematical goal for this lesson?"), follow up moves were not used to press teachers for further discussion after brief responses (i.e. "talk moves"), and the coaching moves often lacked focus on the focal areas.

As McGatha et al. (2018) suggest, having a professional learning area as a focal point of the coaching conversation can help coaches guide teachers in making connections to mathematical and pedagogical topics in manageable chunks during the coaching cycle. As Appendix B illustrates, often in these scenarios the coaching moves were followed up by teacher responses that were generic (i.e. not tied to an MTT or MTP), focused on logistics and organizational issues rather than the mathematics, or, rather than answering the coach's question, the teacher's response went on an unrelated tangent. When coaches and teachers engaged in conversations that did not have a clear direction related to one or more of the focal areas, and coaches did not keep the flow of the conversation focused on these themes, it tended to result in a sparse amount of teacher talk about MTTs or MTPs. In other words, the conversation did not go "deep" (Russell et al., 2017), rather tended to scratch the surface of topics related to the lesson. In these more surface level conversations, even when coaches did use coaching moves that were specific to the focal areas and teachers did not respond in kind, there were seldom follow up probes used by the coach to make further attempts to engage the teachers in conversation around mathematical goals, problem design, and/or student thinking. Additionally, when half or less of the coaching moves were specifically worded around focal areas, teacher responses were likewise focused on effective teaching practices less than 50% of the time.

To help illustrate this, Vignette 1 provides an example of one of these "less focused" conversations. Vignette 1 occurs after the coach, Amy, observes a 2nd grade math lesson reviewing multidigit addition and subtraction in preparation for a chapter test. This coaching conversation is represented in Appendix B as "Debrief 15." Amy meets to debrief with the classroom teacher to revisit the lesson. The two plan and debrief lessons fairly regularly.

> **AMY:** So, what did you think? How did it go?
>
> **Teacher:** I don't know.
>
> **AMY:** Do you think they got it?
>
> **Teacher:** I think, I get, like we've talked about this with the math talk where I could keep going on and on. So, then I feel like we were on that one problem for like a bajillion years and yeah, so, I don't know. I feel like, sometimes I need to reign it in.
>
> **AMY:** I get caught up in that too, like when I saw that Student One had that other way of solving the problem, he added to, I was noticing that he added instead of subtracting to solve.
>
> **Teacher:** Yeah. And I would've loved to spend more time on that so we could clarify, but…
>
> **AMY:** But then it takes so long, so it's just about balancing the time.
>
> **Teacher:** I also noticed Student Two was like, "I disagree" and I was like excited to hear the math talk moves, and I wanted him to show us, but then we'd already spent like what, fifteen, twenty minutes on that one problem. Instead I told him, "Well you can go talk to Student Three in the back and help him understand this problem."

In this initial part of the conversation, Amy has an opportunity to capitalize on the focal area of "analyzing student thinking," or to follow up about the flow of the student discourse during the lesson. Instead, she allows the conversation to shift from brief comments about student to student interaction, rather than using follow up questions and comments to help the teacher dig in and examine what students are saying and doing mathematically and how to adjust the pacing of student led conversation in the moment.

This is perhaps a missed opportunity on the part of the coach to use focused coaching moves to help the teacher think more deeply about the MTPs and about what it is in her teaching practice (the use of specific MTTs) that can help students express their mathematical reasoning. For example, Amy could have posed a follow up question, such as, "What is it that you would want to clarify, for him or his peers?" as a way to press the teacher think about the possibility of helping students see the inverse nature of adding and subtracting and how students could benefit from exploring the relationship between addition and subtraction strategies further. Alternatively, she could have asked the teacher a follow up question regarding how teachers decide how and when to "reign it [the math talk] in." Instead, Amy lets the teacher shift away from focusing on a specific instance mathematical thinking or use of math talk moves and the thread of conversation shifts as a result of the lack of follow up probes on the part of the coach.

In the next part of the conversation, Amy moves on to further discussion about mathematical discourse. Again, this is one of the professional learning focal areas and an MTP that a focused series of coaching moves could help the teacher to examine deeply. Rather than discussing what features of the "math talk" (Chapin et al., 2013) led to more or less productive conversation around subtraction algorithms during the lesson, Amy's questions once again lack focus and depth to help the teacher examine her own practice. Amy begins with a suggestion about planning for a future lesson, then asks the teacher to reflect on what she "noticed" about students' mathematical work.

> **AMY:** I think they're getting a lot out of the math talk. It's just trying to figure out how much time to spend on it, and we said math talk is going to be messy. The whole thing is that it's about quality over quantity. So, maybe what we can plan for next time is to focus on one problem more intensely like that, and have different students share their strategies, but then move on. Because today we went straight to the other problem and I thought they were kind of losing focus. What did you notice about the math that they were doing?
>
> **Teacher:** I think they're doing okay. I like that when we added that addition problem in, how most of them caught on that it was not a subtraction situation…. although some did subtraction to solve. Before I give the test today, I'll just have to say, "Make sure you're looking at what type of problem you're doing" so they pay attention.

AMY: Did you get to see what Student Four did? What did he do on that last problem? I thought that was interesting,

Teacher: Oh, where he just took each part one at a time? The ones, the tens and the hundreds and was adding them?

AMY: Did he use that strategy on the other problems?

Teacher: I didn't notice.

AMY: I thought it was good that he was at least making a connection, that he had to look at those place values with the hundreds, tens, and ones and split them apart. So, how do you think the test will go today?

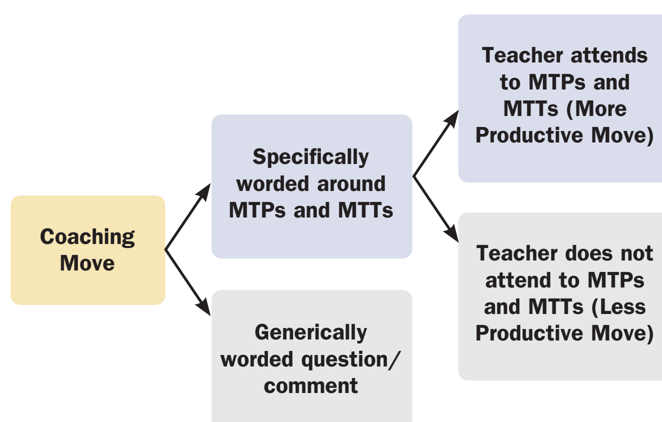
Here the teacher focuses more on the problem content than the strategies students were using and discussing during the lesson. Rather than pressing the teacher further, Amy shifts from trying to engage the teacher in conversation about student strategies and math talk to the upcoming assessment. Interestingly, when I met with Amy after the coaching session and asked about her goals going into the debrief, she said, "My goals, well for myself with her, was just to continue to work on that, not so much the math talk, but the management of making those kinds of decisions, when and where and how much. Let kids explain" (Amy Post Interview Session 15). Although Amy brings up the idea of math talk and student strategies in the debrief, she herself moves the conversation away from this self-reported "goal" to talking about the assessment. Appendix B illustrates how the final part of the conversation appears disconnected from the topics of student thinking and discourse and shifts towards anticipating how students will perform on the test instead.

Throughout this coaching conversation, Amy misses multiple openings to press the teacher further to discuss student strategies and mathematical discourse, which results in shifts from topic to topic, with little focus on specific details related to student thinking or math talk/goals. When examining the conversation in terms of productivity, even when Amy uses coaching moves that are worded in ways that focus on one of the three focal areas, the teacher's remarks are often brief and unspecific in nature. Additionally, Amy does not utilize follow up coaching moves (i.e. questions, comments, suggestions) to engage the teacher in deeper reflection or conversation of these topics during the conversation. As a result, the teacher's attention to specific MTPs and MTTs remains at a shallow, disconnected level of reflection.

This vignette highlights one example from my data that suggests a potential need for mathematics coaches to be conscientious in the ways they help to structure coaching conversations with teachers. When comparing Vignette 1 with other coaching conversations in my study, it became evident that it is helpful for coaches and teachers to know which focal areas they plan to focus on during the session, and that coaches may need to use particular types of coaching moves to help facilitate meaningful conversation (I share an example of this later in the findings). Without focus, conversations with classroom teachers may involve only surface level discussion about MTPs and MTTs, or wander off-topic, which can fail to help teachers learn to attend the focal areas more intentionally in their planning and instruction.

Along with focus, coaches must also be prepared to use a range of follow up coaching moves (such as prompting, probing, and offering examples/suggestions to stimulate additional talk around MTTs) to help teachers learn to attend to the focal areas during these conversations when an initial question or suggestion does not do so (see Figure 5). When coaching moves result in a teacher response not attending to MTPs or MTTs, if the coach does not try again to engage them on a topic, the coach often loses out on an opportunity for productive talk to occur. The next section builds on this idea of knowing how and when to use follow up moves to respond to the direction the teacher and coach take the coaching conversation.

FIGURE 5.
The Potential Flow of Coaching Conversations (Adapted from Jakopovic, 2017).



Unbalanced and Unresponsive Conversations

Even when coaches come in with a focus for the coaching conversation, it is possible for the balance of the conversation to go awry. I categorized 6 of the 47 coaching conversations as "less productive" due to a shift in balance from the teacher to the coach doing the majority of the talking (see Appendix C). In these instances, if focal areas were discussed, the talk was led by the coach, and when teachers picked up on these cues to talk about the mathematics, it was typically for only one turn of the conversation. With the exception of one of these six conversations, teacher responses attended to an MTT that related to the mathematical focal areas approximately 50% of the time, meaning that half of the time teachers responded with generic comments or comments about organization and logistics of the lesson. Another feature of these coaching conversations was the use of a checklist or questioning "script" on the part of the coach to help facilitate the conversations. In my field notes for several of these sessions, I noted the focus of the coach tending to be on the script, rather than using follow up probes and questions to elicit additional ideas and information from the teacher in a conversational way. The literature on instructional coaching suggests that the dialogue between a coach and teacher should be a partnership, and that the coach needs to employ active listening in order to be responsive to the teacher throughout the conversation (Gibbons et al., 2017; Ippolito, 2010). Thus, a second finding of this study is the confirmation that when coaches are unresponsive to teachers' thinking, or begin to overtake the thread of the conversation, this can also lead to less productive talk by teachers around mathematical focal areas. Part of the goal of this study is to examine what this looks like in practice, which the third vignette will illustrate.

Vignette 2 takes place when Sharon meets to debrief with a 5th grade teacher after observing a lesson where fraction tiles are used to compare mixed numbers. Sharon and the classroom teacher have only met a few times prior to this coaching conversation, and the relationship is still fairly new. Sharon leads off trying to engage the teacher in reflecting on the goal of the lesson.

> **SHARON:** So, let's talk about yesterday's lesson. You were having students compare mixed numbers using fraction tiles.
>
> **Teacher:** Yes, there was a lot going on in the textbook, so I decided to just focus on having students represent the numbers using the tiles.

In this beginning excerpt, Sharon perhaps misses an opportunity to press the teacher about the intentional use of manipulatives and tie back to MTPs and MTTs related to lesson design and student understanding. Where she could have asked a question such as, "Why did you feel the tiles were the best representation to focus on?" to probe the teacher further, Sharon moves right along in the next series of turns to a new topic.

> **SHARON:** It's so important to make those adjustments to help students be successful in understanding the content. So, something I noticed during the lesson is that you often tried to get students to explain their thinking, and sometimes, in the moment, it can be tricky to come up with just the right questions. You would ask students things like, "How do you know?" or "Why do you think that?" What did you notice about how students responded to these questions?
>
> **Teacher:** Yeah, I think sometimes they have a hard time putting the math into words.
>
> **SHARON:** I could tell it was something you were being really intentional about trying to do during the lesson though, which is great. So, the types of questions you were posing fall into a category of "Encouraging Reflection and Justification Questions," and can be really nice follow ups to ask after posing an anchor question to get students thinking. And you feel pretty comfortable using those types of justification questions?
>
> **Teacher:** I think they come pretty naturally to me, yeah.

Sharon misses a chance here to help the teacher focus on how these question types can elicit evidence of student thinking, which could have helped the teacher connect these "teacher moves" back to impact on student learning. Without anchoring the practice of questioning back to the lesson itself and the evidence of student thinking the teacher observed during the lesson, it may be unclear to the teacher why Sharon is so focused on questioning.

Prior to the coaching session, Sharon had expressed to me that this was her personal goal for the coaching session, explaining, "I was going to obviously see if there was something she wanted first. I didn't really hear anything come out [during a scheduling conversation], so I jumped in with the [math] talk" (Sharon, Pre-Interview, Session 2).

Rather than helping the teacher to reflect on what she is already doing to help students who struggle to put "the math into words," Sharon continues sharing information about question types that she brought with her to the debrief meeting, sticking to her agenda rather than focusing on the aspects of the lesson the teacher appeared interested in discussing. A shift in balance happens at this point, and the coach begins to dominate the remainder of the conversation.

> **SHARON:** So, there are other question types as well, ones that might be a little more challenging to think of in the moment, but that are equally important to helping students learn how to explain their reasoning and make connections in math. So, I was thinking that, if you were to use the probing questions you are already good at in conjunction with some of the other question types, it could have a powerful impact on students in your classroom. For example, if I lead off with a question like, "What do you notice about the size of the pieces and the denominator in each of those examples?" and then follow up with probing questions as students explain, it could help them make those mathematical connections that we are aiming for when we use the manipulatives.
>
> **SHARON:** Okay.
>
> Coach: So that question is an example of the type, "Making the Mathematics Visible," where our goal is to help students see mathematical structures and make connections between the physical representation and the abstract. It might also be that students are not sure what the proper language and terminology is to talk about their thinking, so you could also include questions that help to draw that information out from the group. "What is the top part of a fraction called? The bottom part? What do these parts of the fraction mean?" those sort of questions can help remind students of the vocabulary and concepts that are embedded in this problem.

As the vignette continues beyond these turns, Sharon continues to provide information to the teacher, without providing time for the teacher to process or to connect to examples from her current practice. The teacher's responses become more brief and non-committal ("Got it," "Makes sense," "Okay.") and it is evident that the focus of the conversation has shifted completely away from the lesson and the teacher's initial conversation about manipulative use. As Sharon continues, it is unclear to what extent the information being shared is helpful or being internalized by the teacher because she is no longer engaged in the dialogue.

Vignette 2 provides an example of a small subset of data points in my study where coaching can become unbalanced and unresponsive in nature (Sailors & Price, 2015; Ippolito, 2010). It illustrates the importance of not only utilizing coaching moves that focus around MTPs and MTTs, but also the importance of being responsive to both the goals of the teacher during the conversation, as well as the direction of the conversation itself. Whether it was a lack of noticing on the part of the coach or relying on a list of pre-prepared questions to guide the conversation, when coaches failed to do this, often the conversation became one sided in my data, with the coach doing the majority of the talking. This is not dissimilar to Jackson's (2018) principles for an effective teaching mindset, which includes starting where students are in their current understanding, as well as the adage that teachers should "never work harder than your students." Similarly, coaching conversations can be less productive when the coach does all of the work during the conversation and fails to meet teachers where they are in their current understanding of effective teaching practices. If Sharon had abandoned her agenda, and instead probed the teacher to reflect on the intentional decision to use fraction tiles, she could have engaged in a rich discussion about mathematical goals and problem design. This discussion could have helped the teacher begin to develop an intentional noticing of why these decisions are so important to fostering students' mathematical understanding. Instead, the vignette presents another missed opportunity for a coach to engage in productive conversation about mathematics focal areas with a teacher.

Moving Toward Productive Coaching Conversations

In the first two vignettes, I illustrate examples of coaching conversations where coaches either engage in surface level discussion about MTTs and MTPs or fall into the trap of being more directive than responsive in their coaching. The final finding of my study suggests that coaches who actively listen and are engaged in what classroom teachers are saying (or not saying) can be more responsive in the moment. If coaches utilize coaching moves and follow up moves flexibly and strategically to press teachers to consider their practice in ways that lead to successful incorporation of the MTPs, this can result in more productive coaching conversations. I categorized 33 of the 47 coaching conversations as being more productive in engaging teachers in dialogue around the three mathematical focal areas. In all of these instances, the coach was able to guide the teacher's focus toward the MTTs and MTPs more successfully, and in nearly all of these instances, coaches used follow up questions and probes to help teachers clarify and extend this thinking. The final vignette illustrates a third finding from my study, which is that when coaches facilitate the use of coaching moves in more productive ways and maintain balance in the conversation, they can help co-construct meaning from these reflections on and in practice with the teacher (Schon, 1983).

In Vignette 3, coach Amy is meeting to debrief a lesson on graphing with a 2nd grade teacher. The two often engage in coaching cycles together, and despite having a brief amount of time to meet, Amy uses a series of coaching moves aimed at helping the teacher reflect.

> **AMY:** Alright, so we don't have a lot of time, but what's something that you feel like went really well?
>
> **Teacher:** I'm glad that when the groups finally started working together, they were realizing and kind of picking things out of each other. Like, you would hear one person explain how, then the others would also attempt to explain to me how instead of just sitting back and having one person explain.
>
> **AMY:** Right.
>
> **Teacher:** Because that's happened in the past, so I can tell that they're starting to feel more comfortable and not just with the graphs, but also in using math talk and explaining things to me.
>
> **AMY:** Good. Yeah, I know at one point they were struggling with making the graph, and at one point during the planning session, you had even talked about giving them a graph template. So, how do you feel about that now since you didn't?
>
> **Teacher:** Right. I'm glad I didn't. Especially when we did the second table, because they did get it, and they just needed that little extra time, so I'm really glad that I didn't. They're getting there. So, I'm glad that I didn't. I had them in my hand, remember?
>
> **AMY:** I do.

Amy intentionally poses questions that tie the planning session to the lesson enactment during the initial turns in the conversation, and these coaching moves help the teacher to consider her students' mathematical understanding, one of the three focal areas. The teacher acknowledges the value of allowing students time to engage in "productive struggle," and how she maintained a student-centered focus during the lesson (NCTM, 2014, 2018). Rather than shifting focus after this initial remark from the teacher, Amy continues to press on with follow up probes and questions to help the teacher reflect more deeply about the specific MTTs she used to help the lesson go successfully. She purposefully restates event from the lesson for the teacher to reflect on, and then poses questions to help the teacher connect the outcomes back to specific aspects of planning and teaching tied to the focal areas.

> **AMY:** So, what do you think you did, for the groups that were really successful in the end, what moves did you make that led to that success for them? Because it really is tough to make a graph on your own. But what did you do that helped them with that?
>
> **Teacher:** I think when it came to deciding like what to put where or what the interval should be, I just asked them, "Well, what is the reason that you did this?" and had them explain that to me. I think when they were explaining it to me why they were having hard time, for example, "Well, we just have enough room to go to seventeen," and then I explained to them, "Well, how high do you really need to make it?" After that, I didn't have to tell them the answer. They kind of figured that out on their own just by answering those questions.
>
> **AMY:** I actually noticed that throughout the lesson, as you worked with the different tables you were kind of probing and questioning and trying to get them there without taking over their thinking.
>
> **Teacher:** Right.
>
> **AMY:** Is that purposeful on your part that you?
>
> **Teacher:** Yes, I've been trying really hard to do that as a teacher, when they are at a loss for words, without just like flat out telling them, "Okay, do this next."

The remainder of the coaching conversation continues in a similar manner, with Amy probing the teacher about specific elements of the lesson that draw the sometimes unconscious decisions of the teacher out to the forefront, and by actively listening and responding to the reflections of the classroom teacher to guide the direction of the conversation (Gibbons et al., 2017; Ippolito, 2010; Schulte, 2020).

Vignette 3 provides a counterpoint that illustrates when discussion about mathematics is focused, connected directly to the mathematical focal areas, it can begin to help teachers identify the specific teacher actions that lead to successful mathematics lessons. Amy's purposeful and flexible use of initial and follow up coaching moves in this vignette allow the teacher to begin to notice the aspects of her planning and teaching that led in this instance to productive struggle, high student engagement, student discourse, and the successful implementation of a lesson that meet her mathematical learning goals. By highlighting these MTPs in a reflective coaching conversation, mathematics coaches can help teachers to develop this sort of noticing of the aspects of planning and teaching that will facilitate them in becoming more intentional about the continued incorporation of the MTPs in their future practice.

Discussion and Potential Implications

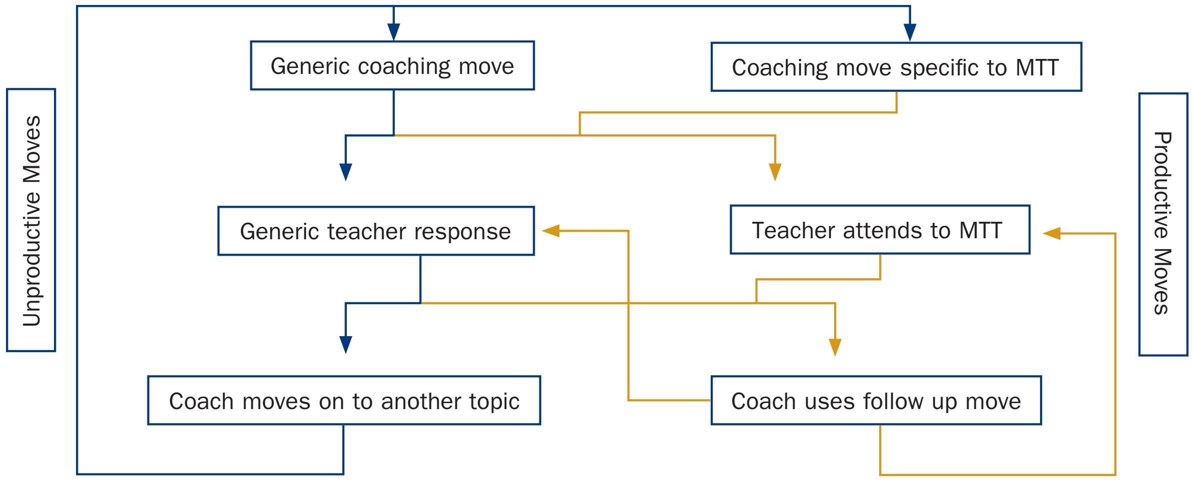
In this study, I found that the types of coaching moves used by coaches, whether those moves are worded in ways that specifically connect to the three MTP focal areas, and the extent to which coaches are responsive to teachers during coaching conversations can influence the overall "productivity" of the coaching conversation. Conversations where coaches use a range of moves and are explicit in connecting those coaching moves to MTPs and MTTs, often help teachers attend to these features of reform-oriented teaching practices better than those moves that are not. Similarly, when coaches are responsive to the comments of teachers, they are sometimes better able to guide the conversation toward MTPs and MTTs, and help teachers see how to regularly plan with the focal areas in mind in their practice. As we begin to better understand how what coaches say and do during these conversations influences what teachers attend (or fail) to in their practice, we can continue to improve the potential impact of this professional development model.

Much in the way that teaching is a complex process (Cochran-Smith & Lytle, 1990), instructional coaching is complex (Ellingson, et al., 2017; Killion & Harrison, 2018; Knapp, 2017). One idea that came out of this study is that, alone, coaching moves may be neither inherently productive or unproductive, it is how they play out within the conversation, and how each of the actors (the coach and the teacher) reacts to these moves, that determines their impact on the conversation direction. Figure 6 illustrates the patterns of coaching moves over the course of a coaching conversation that led to more or less productive talk around the mathematical focal areas in this study. In my larger study (Jakopovic, 2017), I categorized coaching conversations along a range of less to more productive and found that where the conversation goes after the initial coaching move sometimes matters more than the initial coaching move itself. When coaches fail to listen to where the teacher is in their thinking and in the reflection process, they may miss opportunities to engage them productively.

The idea that the moves themselves are not the only determining factor in the success of coaching conversations suggests that coaches may need to develop certain skills that can help them to enact these conversations productively. Coaches must learn to craft coaching moves that highlight the MTPs and MTTs they are trying to help teachers attend to and reflect on. They must also recognize when follow up moves are needed to go beyond surface level conversations and help teachers engage in reflective talk about their teaching practice, as well as to keep the conversation teacher-centered and retain a balanced dialogue.

To do this effectively, coaches must come into the conversation with classroom teachers with specific mathematics and pedagogical goals in mind (Desimone & Pak, 2017; Russell et al., 2017). These goals may be co-constructed with the classroom teacher prior to the coaching conversation or designed by the coach depending on the experience level of the teacher in working with MTTs and MTPs as effective teaching strategies in mathematics (Gibbons et al., 2017). From there, the coach must be prepared to engage teachers in "deep conversations" about mathematics teaching and students' mathematical ideas (Russell et al., 2017) that are clearly focused around these goals (Desimone & Pak, 2017). As the literature suggests, this requires the planning of carefully phrased questions to ensure the coherence of the dialogue between coach and teacher (Gibbons et al., 2017). It also requires the coach to employ active listening skills to determine whether teachers

FIGURE 6.
Potential patterns of coaching interactions between coaches and teachers.



are attending to the mathematics focal areas these questions are tied to or not, so that they may utilize follow up probes and coaching moves to press for deeper talk about ambitious teaching practices and student thinking. For example, in the coaching conversation illustrated by Vignette 3, Amy asks the teacher specifically about her decision to have students create their own graphs without a template, then uses a series of follow up probing questions that allow the teacher reflect on her goal of having students create their own graphs. This facilitates the teacher thinking about the benefits of engaging students in productive struggle, one of the MTPs tied to the focal area "Mathematical Goals" in my framework, and one of her mathematics and pedagogical goals for the lesson.

Researchers and instructional coaching experts alike describe this "responsive coaching" as critically important to promoting active participation on the part of teachers in this work (Campbell et al., 2013; Gibbons et al., 2017; Ippolito, 2010; Knight, 2017). When coaches are responsive, they begin to cycle on the right side of the diagram in Figure 5, as the coach and teacher begin to dig more deeply into an idea through continued focus on one of the mathematical focal areas. This requires preparation and practice on the part of the coach, both in terms of planning for and engaging in focused, coherent, and deep mathematical conversations with teachers. In Vignette 2, when Sharon began directing the conversation, she did not notice that the teacher was no longer actively engaged in co-constructing the conversation. Contrastingly, in Vignette 3 Amy posed follow up questions, such as, "So what do you think you did…that led to that success for them?" and provided specific examples of her own noticing of teaching moves that fostered productive struggle (e.g. "I actually noticed…you were kind of probing and questioning and trying to get them there without taking over their thinking."). Engaging in responsive coaching that keeps the teacher actively engaged can help them learn what features of their practice to attend to in order to engage in ambitious mathematics teaching. Coaches can pose follow up questions, offer suggestions, or share their own wonderings about students' mathematical thinking as opportunities to take the conversation deeper with teachers.

When coaches use productive moves like this to maintain a focus on the MTPs and work toward a balanced, responsive conversation with classroom teachers, they can begin to shift the mindset and focus of classroom teachers about planning and enacting mathematics lessons over time. In their research, Sherin and van Es (2009) found that teachers often focus their attention on a range of lesson elements during the complex act of teaching, such as student behaviors and engagement. In my study, when coaches consistently use focused, productive coaching moves, and maintain a responsive stance in their conversations with teachers, I found they can also help teachers to shift their professional noticing toward the MTPs and reform-oriented teaching practices. Van Es et al. (2017) describe these shifts as helping teachers learn to attend to student thinking and learning in ways that can transcend planning for or reflecting on a single lesson. If a potential goal of coaching is to help teachers develop a mindset for planning and reflecting in and on practice, rather than treating daily instruction as a series of isolated actions, it is important to better understand the potential impact coaches can have on teacher noticing beyond the scope of the current study.

The findings of this study suggest that the ways in which mathematics coaches engage with teachers during coaching conversations can lead to more or less development of teacher noticing of the aspects of mathematical goal setting, lesson planning, and examining student thinking in ways that can engage them in using reform oriented teaching practices (Ball et al., 2008; NCTM, 2014). In particular, there are specific skills coaches can develop in their practice that may positively affect this work with teachers, including:

- Developing effective skills in fostering two-way communication;
- Developing teachers' professional vision and noticing of reform-oriented teaching practices;
- Developing the coach's own professional vision and noticing of the teacher as a learner;
- Understanding and developing productive patterns of coaching moves and a responsive practice; and
- Developing a range of follow up coaching moves to increase teacher noticing of and use of MTPs.

To help coaches develop these skills and practices, the following suggestions stem from the findings of this study:

- Co-construct a mathematical goal for the lesson and a pedagogical goal for the coaching cycle with the classroom teacher prior to meeting;
- Pre-plan carefully constructed questions that attend to the coaching cycle goals and are coherent;
- Be flexible in the use of these questions and prompts during the conversation to ensure the teacher is an active and engaged participant in the coaching and planning process;
- Employ active listening skills while engaging in conversations with teachers and be prepared to utilize follow up coaching moves to help "deepen" the focus on the mathematics and student thinking;
- Brainstorm possible probes ahead of time to help pivot in the moment during dialogue with teachers;
- Be aware of the balance between being responsive to the needs of the teacher and being directive about the need to employ ambitious and equitable teaching practices for all mathematics learners;
- And, be consistent in the use of focused coaching moves as a way to help teachers learn what is worth attending to in order to develop ambitious mathematics teaching practices.

Coaches can engage with a number of resources, such as the toolkits provided by McGatha et al. (2018) to assist with goal setting and planning of intentional coaching moves and questions prior to meeting with classroom teachers to help engage in productive conversations that center on the mathematics focal areas defined in this study. By planning and enacting coaching cycles that keep elements of effective coaching practice in mind (Desimone & Pak, 2017), and through deep conversations with classroom teachers (Gibbons et al., 2017), mathematics coaches can help teachers develop noticing of the features of their teaching practice that can engage learners in learning conceptually rich mathematics.

In this study, examining coaching moves used during planning and reflective conversations with teachers helped generate a framework for examining how mathematics coaching moves can be "productive" in engaging teachers around MTPs. In some instances, coaches guided conversations that focused around many layers of reform based teaching practice, including discussion of mathematical goals, designing meaningful problems, and anticipating and analyzing students' mathematical thinking. Productive coaching moves that were found to rely not only on the phrasing and patterns of interactions on the part of the coach, but also those of the classroom teacher.

Limitations and Future Directions

This study was designed as a case study, examining individual instances of coaches having conversations with teachers to plan and reflect around MTPs and MTTs. As such, additional research is needed to further test and develop the definitions and theories presented here. The small sample size of coaches from a single school district, as well as the limited ability to observe coaches working with the same classroom teachers for multiple cycles, are limitations of the current study that should be tested with additional and more robust studies to inform and better shape the arguments presented herein. In particular, the goal of this study was to develop a framework for examining coaching conversations, however additional research needs to further test the reliability of the framework with larger sample sizes and using diversified participant pools. The boundary of this case study was the planning and debriefing conversations themselves, which presents another limitation of this work. Future research needs to be done that examines the connection between these conversations and the corresponding enacted lessons to see if teacher noticing and reflection translates to instructional practice. Additionally, understanding how coaches make both planned and in the moment decisions when engaging in deep conversations is an aspect of mathematics coaching that needs to be better understood.

Although existing studies have attempted to quantify coaching effectiveness, less work has illustrated what coaches actually say and do to make this happen or not). This study suggests that the research of Sherin and van Es (2009) of developing the professional noticing of teachers in ways that shift their professional vision toward enacting and reflecting on MTPs in their practice could be worthwhile to further pursue in future studies on mathematics coaching. This enactment of coaching conversations is not unlike the orchestration of productive mathematics conversations with students (Smith & Stein, 2011). Having these sorts of discussions requires planning on the part of the teacher. Similarly, mathematics coaches may need to anticipate and plan purposeful questions to move their conversations with teachers forward as well. By analyzing examples of lived coaching conversations and providing evidence to the mathematics educational community, it is the hope that coaches will have tools and resources to better visualize and enact the elements of effective mathematics coaching practice presented by the professional literature. ✪

References

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389-407.

Bay-Williams, J., McGatha, M., McCord Kobett, B., & Wray, J. (2014). *Mathematics coaching: Resources and tools for coaches and leaders.* Pearson.

Brosnan, P., & Erchick, D. (2010). Mathematics coaching and its impact on student achievement. In P. Brosnan, D. Erchick, & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American chapter of the interational group for the psychology of mathematics education.* Ohio State University.

Campbell, P., & Malkus, N. (2014). The mathematical knowledge and beliefs of elementary matheamtics specialist-coaches. *ZDM Mathematics Education, 46,* 213-225.

Campbell, P., Ellington, A., Haver, W., & Inge, V. (Eds.). (2013). *The elementary mathematics specialists' handbook.* National Council of Teachers of Mathematics.

Campbell, P., & Malkus, N. (2011). The impact of elementary mathematics coaches on student achievement. *The Elementary School Journal, 111*(3), 430-454.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (2015). *Children's mathematics: Cognitively guided instruction.* Heinemann.

Chapin, S., O'Connor, C., & Anderson, N. (2013). *Classroom discussions in math: A teacher's guide for using talk moves to support the common core and more, grades K-6* (3rd Ed.). Math Solutions.

Chval, K., Arbaugh, F., Lannin, J., Van Garderen, D., Cummings, L., Estapa, A., & Huey, M. (2010). The transition from experienced teacher to mathematics coach: Establishing a new identity. *The Elementary School Journal, 111*(1), 191-216.

Cochran-Smith, M., & Lytle, S. (1990). Research on teaching and teacher research: The issues that divide. *Educational Researcher, 19*(2), 2-11.

Conference Board of the Mathematical Sciences (2012). *The mathematical education of teachers II.* American Mathematical Society and Mathematical Association of America.

Costa, A., & Garmston, R. (2015). *Cognitive coaching: Developing self-directed leaders and learners.* Rowman & Littlefield Publishers.

Darling-Hammond, L., Hyler, M., & Gardener, M. (2017). *Effective teacher professional development.* Learning Policy Institute. https://static1.squarespace.com/static/56b90cb101dbae64ff707585/t/5ad7aa45758d464041c2130e/1524083271897/Effective_Teacher_Professional_Development_REPORT.pdf

Desimone, L., & Pak, K. (2017). Instructional coaching as high quality professional development. *Theory into Practice, 56*(1), 3-12.

Ellington, A., Whitenack, J., & Edwards, D. (2017). Effectively coaching middle school teachers: A case for teacher and student learning. *The Journal of Mathematical Behavior, 46,* 177-195.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustatin teaching. *Teachers College Record, 103*(6), 1013-1055.

Frazier, R. (2018). The impact of instructional coaching on teacher competency, job satisfaction, and student growth. [Doctoral dissertation, University of Colorado Colorado Springs]. Mountainscholar.org.

Gibbons, L., Kazemi, E., & Lewis, R. (2017). Developing collective capacity to improve mathematics instruction: Coaching as a lever for school-wide improvement. *The Journal of Mathematical Behavior, 46,* 231-250.

Hammond, L., & Moore, W. (2018). Teachers taking up explicit instruction: The impact of a professional development and direct instruction coaching model. *Australian Journal of Teacher Education, 43*(7), 110-133.

Huguet, Marsh, & Farrell (2014). Building teachers' data-capacity: Insights from strong and developing coaches. *Education Policy Analysis Archives,* (2252), 1-31.

Ippolito, J. (2010). Three ways that literacy coaches balance responsive and directive relationships with teachers. *The Elementary School Journal, 111*(1), 164-190.

Jackson, R. (2018). *Never work harder than your students and other principles of great teaching* [2nd Ed.]. Association for Supervision and Curriculum Development.

Jakopovic, P. (2017). Understanding what makes for productive coaching moves to help teachers attend to mathematical tasks of teaching (Doctoral dissertation). Retrieved from ProQuest. (Accession Number ED578444).

Killion, J., & Harrison, C. (2018). Coaches' multiple roles support teaching and learning. *Tools for Learning Schools, 21*(1), 1-18.

Kim, Y. (2016). Interview prompts to uncover mathematical knowledge for teaching: Focus on providing written feedback. *The Mathematics Enthusiast, 13*(1&2), 71-92.

Knapp, M. (2017). An autoethnography of a (reluctant) teacher leader. *Journal of Mathematical Behavior, 46,* 251-266.

Knight, J. (2007). *Instructional coaching: A partnership approach to improving instruction.* Corwin Press.

Knight, J. (2017). *The impact cycle: What instructional coaches should do to foster powerful improvements in teaching.* Corwin Press.

Kretlow, A., & Bartholomew, C. (2010). Using coaching to imrpove the fidelity of evidence-based practices: A review of studies. *Teacher Education and Special Education, 33*(4), 279-299.

McGatha, M., Bay-Williams, J., McKord Kobett, B., & Wray, J. (2018). *Everything you need for mathematics coaching: Tools, plans, and a process that works for any instructional leader.* Corwin Press.

Murray, S., Ma, X., & Mazur, J. (2008). Effects of peer coaching on teachers' collaborative interactions and students' mathematics achievement. *The Journal of Educational Research, 102*(3), 203-212.

National Council of Supervisors of Mathematics (2014). *It's TIME – themes and imperatives for mathematics education: A leadership framework for Common Core Mathematics.* Solution Tree Press.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2018). *Catalyinzing change in early childhood and elementary mathematics: Initiating critical conversations.* National Council of Teachers of Mathematics.

Russell, J., Correnti, R., Stein, M.K., Bill, V., Hannan, M., Schwartz, N., Booker, L., Pratt, N., & Matthis, C. (2020). Learning from adaptation to support instructional improvement at scale: Understanding coach adaptation in the TN mathematics coaching project. *American Educational Research Journal, 57*(1), 148-187.

Sailors, M., & Price, L. (2015). Support for the improvement of practices through intensive coaching (SIPIC): A model of coaching for improving reading instruction and reading achievement. *Teaching and Teacher Education, 45,* 115-127.

Saldaña, J. (2016). *The coding manual for qualitative researchers* (3rd Ed.). Sage.

Schon, D. (1983). *The reflective practitioner: How professionals think in action.* Basic Books.

Schulte, J. (2020). The impact of instructional coaching programs on teacher effectiveness: Perceptions from the field. [Unpublished doctoral dissertation, Baker University].

Selling, S., Garcia, N., & Ball, D. (2016). What does it take to develop assessments of mathematical knowledge for teaching? Unpacking the mathematical work of teaching. *The Mathematics Enthusiast, 13*(1&2), 35-51.

Sherin, M., & van Es, E. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education,* 60(1), 20-37.

Smith, M., & Stein, M. (2018). *5 practices for orchestrating productive mathematics discussions* [2nd Ed.]. National Council of Teachers of Mathematics.

Sutton, J., Burroughs, E., & Yopp, D. (2011). Mathematics coaching knowledge: Domains and definitions. *NCSM Journal,* 13(2), 12-20.

Taylor, J. (2017). The effectiveness of instructional mathematics coaching: A study on how the implementation of instructional mathematics coaches in elementary schools impacts student achievement and promotes teacher self-efficacy. [Doctoral dissertation, University of St. Francis]. Proquest.

Van Es, E., Cashen, M., Barnhart, T., & Auger, A. (2017). Learning to notice mathematics instruction: Using video to develop preservice teachers' vision of ambitious pedagogy. *Cognition & Instruction,* 35(3), 165-187.

West, L. (2008). Content coaching: Transforming the teaching profession. In *Mentoring, coaching, and collaboration* (pp. 163-211). Corwin Press.

Yin, R. (2017). *Case study research and applications: Design and methods* (6th Ed.). Sage Publications Inc.

APPENDIX A
Examples of More/Less Productive Coaching Moves Around the Focal Areas

Mathematical Focal Areas	Examples of Productive Coaching Moves (Based on the Professional Literature)	Examples of Coded Excerpts from the Data of Coach Move & Teacher Response
Mathematical Goals	• Posing questions around mathematical goals • Offering ideas and suggestions around mathematical goals • Prompting use of data/ student work analysis to inform mathematical goals	**More Productive:** **Coach:** Specifically, what is your objective for this lesson? When you get done, what do you really want to know? **Teacher:** I want to know if they can look at a ten frame, recognize the numbers and write that number, and then if they know the teen numbers in order. Because I know they know one through ten in order. **More Productive:** **Teacher:** I had him do it on the board and he wrote 39 and this box equals 62. And he said, "Well, I was looking and knew 9 plus something couldn't be 2, so I knew it had to be 12. So I put the 10 up here, and then I knew 9 plus 3 was 12. **Coach:** So what you said, I was just thinking, this seems like a great moment where, how do we get this to transfer to some other kids? … Do we say something like, 'Can someone explain to me again how he did this?" **Teacher:** Like have someone else explain it? I like that because I keep telling the class to pay attention, We're learning from our friends right now. Watch all the different ways to solve these problems." …but like everyone is just sitting there waiting. **Coach:** I think this is such a big moment, you need to spread it. **Teacher:** Yeah, I think that if someone else explains it, it will help me check for comprehension too. **Less Productive:** **Coach:** I know we didn't get as far as we wanted…with show me ten more, ten less…I mean there were some that obviously were stumped but then it was good that they all seemed [eventually] to catch on. **Teacher:** So yeah, I thought that was good. Overall I thought it was alright. **Less Productive:** **Coach:** So, my first question is, how do you think it went with them coming up to [describe] the circle and with the [descriptive] writing? **Teacher:** I think that the circle thing was fine. I think maybe in retrospect maybe just choosing just a couple and then having the rest… they all wanted to go reach and touch it [the circular objects]… **Coach follow up:** I was pleasantly surprised with the writing though. **Teacher:** Mm hmm. They did pretty good with that. The ones that I figured would be able to handle it, handled it really well. And the other ones were trying to copy, which isn't out of the ordinary for any of our writing. **Coach:** For kindergarten. And that's why I purposefully wrote two of them up there because I knew some of them weren't going to be able to do it. **Teacher:** Right.

Mathematical Focal Areas	Examples of Productive Coaching Moves (Based on the Professional Literature)	Examples of Coded Excerpts from the Data of Coach Move & Teacher Response
Mathematical Problem Design	• Posing questions around mathematical problem design • Offering ideas and suggestions around mathematical problem design • Prompting use of data/ student work analysis to inform problem design	**More Productive:** **Coach:** So what are you thinking of doing…have you already looked at it and decided which activities you feel are most beneficial? **Teacher:** What I would really like to do, I know it gives the whole template here, but what I'd really like to do is have the kids kind of create their own. **More Productive:** **Coach:** Alright. So talk me through, what are your thoughts for tomorrow on line graphs? **Teacher:** With line graphs, talking, just kind of starting off showing them this, um, this table of data. And then talking about what patterns they notice and what this could possibly be a table for and so building that knowledge of why we have, why we would take this table. I mean is this something that we will be talking, what would that even be for? And then extending down to line graphs, and then talking about scales and intervals… we're going to make a quick diagram of what scales are, of what intervals are, and then just going into this section talking about how we can relate them to ordered pairs. And a big piece that I want to talk about is this, the break in the scale. Because that's something that's, I think a lot of students might not key into if not explicitly mentioned. **Coach follow up:** Okay so that was another question I was going to ask. Any other misconceptions that students might have here other than that disruption of the scale? Anything else that you're…? **Teacher:** Well something, and this is, it's not the same interval both ways. I mean this, we're going up by two every time and this I'm going by one in time and it's, I think that's something that they're really going to have just some struggles with is knowing which one I plot on my x-axis and which one I plot on my y-axis. **Coach follow up:** But, would this be an opportunity then to talk to them about, as long as you keep this on the x or y-axis, it would be okay… **Less Productive:** **Coach:** You talked about baskets…so at each basket would there be the same objects? …So everyone in that group is measuring the same object? **Teacher:** I was thinking of different objects in each basket, but what do you think? …I mean, what would be the best organized, easy management piece for the kids? **Less Productive:** **Coach:**So you had a lot of the questions that really um, built the background, reviewed the vocabulary, those kind of things. What, what questions did you feel really got at the mathematics, I guess? **Teacher:** Um, when they were figuring out how to label it, I guess. One person knew you had to start at zero fourths, and then I think that's when everyone was like, "Oh yeah!" [Coach moves on to another topic]

Mathematical Focal Areas	Examples of Productive Coaching Moves (Based on the Professional Literature)	Examples of Coded Excerpts from the Data of Coach Move & Teacher Response
Examining Student Mathematical Knowledge and Ideas	• Posing questions around students' mathematical knowledge or ideas • Offering ideas and suggestions around students' mathematical knowledge or ideas • Prompting use of data/ student work to analyze students' mathematical knowledge or ideas	**More Productive:** **Coach:** "How might did it grow?" That was the question they came up with and they…counted every inch and said it grew 17 inches because they weren't thinking of it as linear growth. Did you notice? **Teacher:** I thought about that during the lesson too, because you're exactly right. What we've done so far with graphs has been…things you can just add up. **More Productive:** **Coach:** So what if we start out by just saying a question of, how much paint do you think you're going to have? **Teacher:** Well almost two ounces, because four-fifths is close to another one whole. **Coach follow up:** Mm hmm. So two and two, about four. So do you think your kids, some of your students will come to that idea? If you just ask the question? **Teacher:** Yes. I think a couple of my really high students will, but I don't think a lot of them. I think if I say it, well four-fifths, is that going to be closer to zero or to another whole? I think a lot of them will be able to say oh well if I shade it, it's almost another whole. I don't know. Maybe they would get that. **Less Productive:** **Coach:** So do you think your average kid knows if…they have an answer that's reasonable, that makes sense or not? **Teacher:** No. I'd say no. **Less Productive:** **Coach:** As you reflect [on the lesson], what surprised you? **Teacher:** One of the things that surprised me was when I saw some of the kids take their fingers and mimic the tile size to estimate without actually measuring first – to mimic the size of the tile as a way of measuring. Also, I was pleasantly surprised that the estimates for the most part were very reasonable. **Coach follow up:** What do you think came easily for the students and what was mathematically difficult and why? [disconnected probe] **Teacher:** Easily, I think was measuring it with the tiles. They did a great job putting them bumper to bumper. What was hard is they wanted to jump ahead without estimating. I had to step in and strongly encourage the students to estimate before measuring. I need to do a reteach lesson on estimating. There were some who were stressed about being wrong, I had to give them a pep talk to estimate.

Mathematical Focal Areas	Examples of Productive Coaching Moves (Based on the Professional Literature)	Examples of Coded Excerpts from the Data of Coach Move & Teacher Response
Setting Pedagogical Coaching Goals	• Posing questions around students' mathematical knowledge or ideas • Offering ideas and suggestions around students' mathematical knowledge or ideas • Prompting use of data/student work to analyze students' mathematical knowledge or ideas	**Coach:** What would you think about, how would you think about if we added an objective kind of just for you? **Teacher:** Sure. **Coach:** Not really for the kids. So find, the objective for the kids is you know, vocabulary and introducing all that stuff, but probably an objective for you would be looking at this as, "Hey this is where my kids are, this is a formative assessment, and these are some things…" And I can sit with you after they're done. **Teacher:** Yeah, I think that's what I need help with, because I can see the answers, I can see they missed so many problems but knowing what to do with them as a class. Like individually it's a little easier, but it's okay, my whole class did…what are my? What am I going to do because of that?

APPENDIX B
Unfocused/Surface Level Coaching Conversations

Coaching Session	Coach Moves & Teacher Responses	M 1	M 2	M 3	M4	M 5	M 6	M 7	Moves/ Responses Phrased around Focal Areas
Planning 14	Coach Move [a]	Q	RMG	Q	SPD	SPS	QPS	QAS	5/7
	Teacher Response [b]	PI	PI	SR	SR/G	G	OL	CD	4/7
Debrief 14	Coach Move	QPD+	QES	QPD	QES	QAS			5/5
	Teacher Response	PI, EE	EE	G	CD	__			3/5
Planning 15	Coach Move	Q	QAS	RG	QAS	SPS, I			2/5
	Teacher Response	B b bG	EC, T	G	G	SC/G			1/5
Debrief 15	Coach Move	Q	I	QAS	SES	SPS	SPS		2/6
	Teacher Response	G	__	G	G	EC	G		1/6
Planning 9	Coach Move	QBS	Q	QMG	SPD+	QPD			4/5
	Teacher Response	G	PI	FE-	G	G			2/5
Debrief 9	Coach Move	RMG	SET	SPD	SET				2/4
	Teacher Response	SR	SC/G	PI /G	T				1/4
Planning 8	Coach Move	QG	SPD	QPS	I				2/4
	Teacher Response	AQ	AQ	PI/G	T				2/4
Planning 4	Coach Move	QPD	QG	SPS	QPS				3/4
	Teacher Response	OL	UN	OL	OL				0/4

[a] Coaching Moves: I = Interruptions; Q = Generic questions; QAS = Questions to anticipate student thinking; QBS = Questions about student background knowledge; QES = Questions about examples of student thinking; QG = Questions about teaching goals; RG= Restating teaching goals; QMG = Questions about mathematical goals; QPD = Questions about problem design; QPS = Questions about organization and set up; RMG = Restating mathematical goals; SES = Sharing examples of student thinking; SET = Sharing examples of teaching moves/collected data; SPD = Suggestions about problem design; SPS = Suggestions about organization and set up

[b] Teacher Responses: AQ = Productive mathematical questions; CD = Choosing and developing definitions; EC = Evaluating the plausibility of students' claims; SC= Anticipating student claims; EE = Giving/evaluating mathematical explanations; FE = Finding examples to make a mathematical point; G = Generic comments; LR = Linking representations to underlying ideas/representations; OL = Considering organization and logistics; PI = Presenting mathematical ideas; SR = Selecting representations for particular purposes; T = Tangents; UN = Using/critiquing mathematical notation and language

Note : + Includes follow up question or example from coach

APPENDIX C
Unbalanced/Non-Responsive Coaching Conversations

Coaching Session	Coach Moves & Teacher Responses	M 1	M 2	M 3	M4	M 5	M 6	M 7	M 8	M 9	Moves/ Responses Phrased around Focal Areas
Planning 7	Coach Move [a]	QBS	QBS+	Q	SMG SPD+	QPS	RMG	SPD	SPD	SPD	7/9
	Teacher Response [b]	SC, PI	SC	OL	G	PI	LR	AQ	PI, OL	OL	5/9
Debrief 7	Coach Move	SES	QPS	QPD	SET	I	SET	SES	QES	QMG	8/8
	Teacher Response	G	OL	SC	LR		AQ	SC	SC	LR	6/8
Debrief 10	Coach Move	QES	QMG	QAS+	QES+	QPD+	Q	QBS	QES		7/8
	Teacher Response	EE	G	SC	LR	G	OL	G, PI	SC		4/8
Planning 18	Coach Move	QMG	Q	QAS	QBS	QAS	QPS				5/6
	Teacher Response	SR, PI	OL	G	LR	SC	OL				3/6
Debrief 18	Coach Move	Q	RMG	QMG+	QPD	SES	Q	SPD+	SES		6/8
	Teacher Response	SC	SC	G	PI	PI	CD	G	G		4/8
Planning 19	Coach Move	QBS	QAS	QPS	QPS	SPD	QPS	QAS	I	QPD	8/8
	Teacher Response	CT, RR	SC	EE	AA, OL	G	OL	G	__	UN	3/8

Note: See Appendix B for key

Making the "Cut": One District's Strategy for Algebra Placement

Neal Grandgenett, Ph.D., *University of Nebraska at Omaha*
Roberta Jackson, Ed.D., *Westside Community Schools, Omaha, Nebraska*

Abstract

Some of the most discussed issues in mathematics education today involve Algebra and its instruction. These issues include the optimal timeline for when students first take a formal algebra course, the related selection process for getting into that first course and what algebra instruction should generally look like throughout the curriculum. Algebra is being recognized as a key "gate-keeper" course for high school and college success and has even been called an emerging "civil rights issue" by some researchers and authors. When to place students into an algebra class and how to ensure that a student is ready for Algebra are both critical curriculum decisions for a district. In many districts, algebra placement is a process that may be undergoing considerable revision along with how algebra is integrated across the curriculum. This article describes one district's approach for evaluating and revising their placement strategy for admitting students into their first middle school algebra course.

"Not every child has an equal talent or an equal ability or equal motivation, but all children have the equal right to develop their talent, their ability and their motivation."

~ John Fitzgerald Kennedy, 1963

John Kennedy's famous civil rights quote that "all children have the equal right to develop their talent, their ability, and their motivation" was made in a speech to the American people in a radio address on the morning of June 11, 1963. That was the morning that President Kennedy sent in the Alabama National Guard to open up the University of Alabama to two well-qualified black students. Access to a college education, for all qualified students was of course one of the most important civil rights issues of that day. In many ways, that civil rights issue is still with us in mathematics education and is often represented within the discussions of when students take Algebra and how they study it throughout their K12 coursework.

In mathematics education, the timeline for when students take Algebra, the related selection process, and what algebra instruction should look like throughout the K12 curriculum are some of the most discussed issues in the profession today. For example, algebra instruction and placement have been strongly represented in the last several National Council of Teachers of Mathematics and National Council of Supervisors of Mathematics annual conferences, with numerous sessions and presentations dedicated to algebra instruction. Another example of this professional dialogue is the new 2006 document by the National Council of Teachers of Mathematics, called "Curriculum Focal Points" which details topics of particularly important focus for pre-kindergarten to grade 8 mathematics instruction. This document has algebra well identified as a focus area, with consistent references to "number operations and algebra" as focal points from first grade through fifth grade, and an emphasis on "algebra" itself as one of the key focal points in grades 6-8. Algebra is obviously continuing to become an ever more important topic in K12 mathematics instruction.

The importance of algebra is also increasing as computer technology impacts the ways in which we have to teach mathematics (Heid, 2005; Hegedus & Kaput, 2004). Instructional tools such as graphing calculators, computerized algebra programs and homework helping websites are allowing schools and teachers to more effectively provide the instructional depth to algebra that it deserves in its growing importance in the K12 mathematics curriculum (Heid & Edwards, 2001). In fact, professional associations such as the Association of Mathematics Teacher Educators are commonly mentioning algebra as an instructional area particularly compatible with new technologies of instruction (Association of Mathematics Teacher Educators, 2006).

In a direct reference to the civil right passions of the 1960's, algebra has even been called an emerging "civil rights issue" for the next decade (Checkley, 2006; Moses, 2000; Moses, 1994). From a research perspective, an early understanding of algebra has been shown to be a key (and perhaps THE key) predictor for success in high school mathematics coursework and even entry into college (Burris, Heubert, Levin, 2004). A study by Horn and Nunez (2000) illustrates the importance for students in taking the advanced mathematics coursework that follows an early algebra placement. In their study, students of parents who never attended college more than doubled their chances for enrolling in a four-year college when taking coursework past Algebra 2. A well-prepared student that gets into an "early algebra sequence"may well have a distinct academic advantage over a student who does not get into that sequence. In addition, a poorly prepared student who fails at an early algebra course, may well be doomed to struggling in mathematics or even discarding mathematics as something that they are only minimally interested in learning (Schoenfeld, 2002).

Thus, how a school district selects students to enter a formal algebra course and when that selection process occurs is becoming critically significant within a district's mathematics program. With an awareness of just how important such an algebra selection process can be for students, the Westside Community Schools and the University of Nebraska at Omaha carefully examined Westside's algebra selection process by reviewing past placement data, holding a series of collaborative discussions, and then modifying the selection process to try to be as fair as possible to students within the context of limited district resources. This article describes an evidence-based investigation of Westside's algebra placement process and the related changes that the district made in its placement procedures as a result of this inquiry.

The Historical Context at Westside

First, it is important to get a sense of the Westside Community Schools. The district is an urban school district of approximately 6,000 students, 1,400 of whom are not residents of the district, but rather attend through Nebraska's school choice program. Eighty-six percent (86%) are white. Approximately 20% of the students qualify for free or reduced price lunch. The district has a K-12 curriculum with ten elementary schools (grades K-6), one middle school (grades 7-8), and one high school (grades 9-12). The district has always prided itself on having a strong and vibrant mathematics program, which has been recognized within the context of several awards, including students qualifying for the National Math Counts competition for five consecutive years, several students achieving perfect scores on the American Mathematics Competition and a high number of student qualifiers in the state's annual mathematics competitions.

During 2001, the Westside Community Schools adopted a new mathematics curriculum at the elementary level in order to better challenge their elementary students in mathematical problem solving as well as other higher level mathematics skills. The curriculum blends basic skills development with conceptual understanding activities in a mix that has been shown to be a positive component of effective mathematics instruction in several districts across the country (Cavanagh, 2006). The Westside program was carefully planned and adopted with considerable input from teachers, parents and even students (Grandgenett, Jackson,Willits, 2004). The elementary program revisions also included the adoption of Everyday Mathematics instructional materials, which appeared to align well with district desires to better challenge students. Elementary teachers also went through an extensive professional development program to help prepare them for a more challenging elementary curriculum. This professional development process also systematically included the early integration of algebra's big ideas, such as variables, patterns and functions, and proportions and proportional reasoning as recommended by authors such as Greenes (2004).

Teachers and students have embraced this revised elementary curriculum. Along with better preparing students for mathematical problem solving, reasoning, and mathematical connections, the curriculum also carefully covers introductory algebra topics which are well integrated into all grade levels at the elementary level. For example, in the Everyday Mathematics curriculum, algebra-related topics appear in each elementary grade and are indexed within the instructional materials (Everyday Learning Corporation, 2002).

Like most school districts today that have worked hard to develop an effective elementary mathematics program, placement into a formal algebra or pre-algebra course (leading to Algebra) at the middle school level has now surfaced at Westside as an important focus area for further revisions within the K-12 mathematics program. The district's strong elementary preparation in algebra readiness has only increased a need to offer strong middle school coursework options for students. Thus, the early integration of algebra concepts at the elementary level has essentially encouraged a more systematic approach to algebra at the middle school. This need for a careful transition for algebra instruction is consistent with research that suggests that successful instructional efforts for algebra should be well paced and systematic across the curriculum (Noddings, 2000; Steen, 1992).

In the National Research Council's 2005 report "How Students Learn," a total o f 179 out of the 600 pages are dedicated to the learning of mathematics. Within this extensive discussion, Fuson, Kalchman, and Bransford (pgs. 217-256) reinforce that there are three important principles for teachers to follow in helping provide a foundation for the learning of 1) teachers must engage student prior understandings; 2) teachers must help students build a deep foundation of factual knowledge, give students a conceptual framework, and help them to organize knowledge; and 3) teachers need to help students take a metacognitive approach in taking control of their own learning within challenging coursework.

Challenging coursework has always been a strong component of Westside's mathematics program and student selection for such coursework has always been an important district concern. Historically, in the Westside district, two assessments were used to identify students who were perceived as "ready" for a challenging Pre-algebra course in the middle school after an aggressive elementary school curriculum. Students who received a score above the established cut scores were placed in Pre-algebra and others were placed in the "regular" 7th grade mathematics curriculum. This practice had a long history but no real documentation of the validity of the assessments or the predictive capability of the established cut scores. One of the primary assessments was even a "district-made" test that was initially constructed nearly 20 years ago by a group of middle school teachers and revised periodically over the years based upon the further input of later teachers.

The tests and the cut scores used for algebra placement had essentially not changed for more than a decade, but in recent years the proportion of students qualifying for Prealgebra had steadily increased. The following table shows the percentage of students that took the placement tests each year and the percent qualifying within the district during the four years before changes were made in the selection process.

Pre-algebra Testing

Year	6th Grade Enrollment	Number Taking Test	Percent Taking Test	Number Qualifying	Percent Qualifying
2001-2002	405	250	61.7%	137	33.8%
2002-2003	422	384	91.0%	248	58.8%
2003-2004	468	420	89.7%	283	60.5%
2004-2005	452	390	86.3%	258	57.1%

Although the tests and qualifying scores hadn't changed generally between 2001 and 2005 other things had. Historically, letters were sent to parents of students identified by sixth grade teachers as potential candidates for Prealgebra. These parents were invited to have their child take the screening tests at the middle school on a Saturday morning or designated weekday evening, a practice that was eventually found to penalize students whose parents were not aware of, or initially interested in, providing this opportunity for their children. Procedures were then changed in the spring of 2002. Middle school teachers and counselors continued to administer the tests, but the tests were given during the school day at each elementary school and all students were encouraged to take the tests. As mentioned previously, the elementary curriculum had also changed during this period. The new curriculum placed greater emphasis on problem solving, reasoning, mathematical connections and had students apply their mathematical understanding to a greater extent than the previous curriculum. The curriculum also systematically introduced the "big ideas" of algebra at the lower grade levels. Standardized test scores in mathematics went up after the adoption of the new curriculum and teachers believed that the new curriculum also may have positively impacted students' performance on the Pre-algebra screening test.

As the numbers of students placed in Pre-algebra increased, middle school teachers recognized that the students arriving in these classes were representing a wider range of backgrounds and also observed that some students within this increased pool of students appeared to be struggling more than in the past. Two additional concerns led administrators to the conclusion that the placement tests and cut scores needed to be carefully examined. First, the validity of the tests themselves was in question. One test was a basic teacher-developed computational mathematics test, which had been refined over time, but without any formal reliability and validity testing. The other test was the Orleans Hanna, a commercially published assessment of algebra readiness (Harcourt Brace and Company, 1998). However this more established test was not being used in connection with student grades as the test publisher prescribed. Secondly, there was no documentation of the formal procedures used to set passing scores on either of the assessments. There essentially was no evidence that the tests, or the established cut scores, were effective predictors of student success in Pre-algebra. Thus, the district felt it was time to carefully examine and better formalize the algebra placement process.

Looking at the Situation Statistically

To look at the algebra placement situation statistically and to better examine the algebra placement process, Westside partnered with the University of Nebraska at Omaha, to review the existing data related to the district's seventh grade mathematics placement process and compare the statistical power of the historical cutoff procedure with an alternate procedure thought to be more consistent with the new mathematics program. These two contrasting selection procedures included 1) the current use of the district constructed mathematics survey test (called the Westside Survey Test) and the commercially prepared Orleans Hanna Test, and 2) a potential alternate procedure using student grades and the Orleans Hanna Test. The alternate procedure using grades in combination with Orleans-Hanna scores, was also an assessment strategy recommended by the publisher of the Orleans-Hanna Test. In this context, grades were changed to a numerical score (again following Orleans-Hanna), using a scale of 0-12 for each grade assigned from F (assigned 0 points) to A+ (assigned 12 points). A total of 373 past student records were available to help investigate the relative statistical power of these two procedures.

As a first step in the statistical investigation, correlations were conducted to examine the overall relationships of various fifth grade and sixth grade mathematics variables (i.e., scores on mathematics assessments administered in fifth or sixth grade) with seventh grade mathematics achievement as represented by grades (see table). The

SAMPLE CORRELATIONS (6th GRADE)	r
Total 6th Grade Score	0.62
Mathematics Grade	0.60
Reading Grade	0.56
*Grades and Orleans Hanna Test Combined	0.55
Social Studies Grade	0.53
*Survey Test and Orleans Hanna Test Combined	0.43
Survey Mathematics Test	0.42
Orleans Hanna Raw Score	0.40
Science Grade	0.37
SAMPLE CORRELATIONS (5th GRADE)	**r**
Gr 5 SAT9 Total (Complete) Battery	0.45
Gr 5 SAT9 Total Math	0.42
Gr 5 SAT9 Math Proc	0.40

district also had a practical desire to have the qualifying procedure include a written test to aid in parent discussions. Another desire by the district was to somewhat emphasize the 6th grade scores since these scores would be more closely associated in time to the seventh grade year. In examining the correlations, it appeared that the potential alternate selection procedure of combining semester “report card grades” with the Orleans Hanna Test was a viable alternative to the earlier procedure.

Multiple regression procedures were then used to compare the relative strengths of the two data models: the new model (Grades + *Orleans Hanna*) with the old model (Survey Test + *Orleans Hanna*) in their predictive relationships to student grades in seventh grade mathematics. The new model of combining grades and the *Orleans Hanna* scores was found to be statistically stronger when considering its effectiveness for achievement predictions within the available sample of 373 past student records. The new model accounted for 38% of the variance in scores, approximately double that of the old model, which accounted for only 19% of the variance. Actually, these findings are quite consistent with research that suggests that combinations of coursework grades and testing can be useful in predicting future mathematics performance (Burris, Heubert, Levin, 2004; Fenton, 2002).

Again using the historical data, the relative effectiveness of the two cutoff score strategies were then examined by considering how many "true predictions" and "false positives" the different cutoff score procedures represented while looking at the historical distribution of the 373 scores. For purposes of this comparison process, the following operational definitions were used:

> **True Prediction:** This term referred to the situation where a student made the cutoff score and then was successful in seventh grade mathematics.
>
> **False Positive:** This term referred to the situation where a student made the cutoff score, but was then unsuccessful in seventh grade math.
>
> **Successful in seventh grade Math:** A student was considered to be successful in seventh grade math if they received a grade of at least a "B" in their seventh grade math course.

As mentioned earlier, the current cutoff score procedure used a combination of tests that included the Orleans Hanna Test and a district created mathematics survey test. This traditional cutoff score process included the following criteria identified in district communications to parents:

> "Students who are recommended for enrollment in the Pre-algebra course demonstrate the knowledge to be successful in Pre-algebra by meeting one of two criteria: 1) a score of 60% or higher on the Orleans-Hanna Algebra Prognosis Test and a score of 70% or higher on the Westside Mathematics Survey Test or 2) a combined average score on the two tests of 67% or higher.”

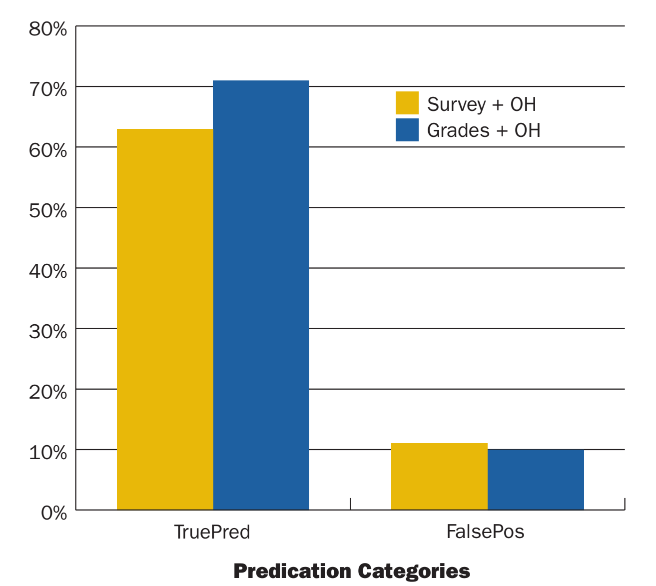
This traditional cutoff score procedure predicted 63% of the sample's mathematics achievement (true prediction). About 11% of the sample was false positives (student made cutoff score but then struggled). It was also found by examining the 373 records that the two options within the criteria for qualifying (meeting the cut score on both tests or the mean of the two) statistically overlapped and were not both needed. All students either met both criteria or neither.

The recommended new student selection model used the Orleans Hanna Test and student grades. This selection process included a procedure recommended by the test publisher for combining student grades in four subjects (Math, Science, Social Studies, Reading/Writing). This approach uses the scale of 0-12 for each grade assigned from F (0 points) to A+ (12 points), and when combining all four grades, this point summation then accounts for a total grade value ranging from 0 to 48. This grade value is then combined with the Orleans Hanna Test scale of 0-50, to give an overall combined score ranging from 0 to 98. When examining the historical data, the new cutoff score procedure was found to be potentially superior based on this past data and a cutoff score of 64 was considered to be statistically optimum. Using this cutoff score, the prediction of student success (true prediction) was generally maximized and the false positives were relatively minimized (student makes cutoff score but is unsuccessful). This cutoff score predicted 71% of the population successfully, with 10% false positives.

Based on this analysis, the new cutoff score process was expected to statistically increase the true prediction of student success by roughly 8% while also potentially decreasing the false positives (student makes cutoff score but then struggles) by roughly 1%. These two approaches are compared side by side on the graph (next page).

Using the historical sample of 373 students to “predict” how many students would be expected to make the new cutoff score, it was determined that the new cutoff score

SURVEY TEST + ORLEANS HANNAH PREDICTIONS VS. GRADES + ORLEANS HANNAH PREDICTIONS



process would most likely have about 67% percent of the district's students expected to quality for the initial middle school algebra course.

In essence, by using the new assessment procedure (combining student grades and the Orleans Test) it was concluded that there would be a more effective assessment process than the current procedure (using the Westside Survey Test and Orleans Hanna). The analysis of the historical data suggested that the new procedure would be more accurate, have slightly less of a chance of admitting students who would then struggle and would admit a few more students into the program. This new procedure would also make use of a test with greater demonstrated reliability and validity than a district constructed test.

The New System in Action

As expected, the new selection procedure resulted in nearly 67% of the students qualifying for Pre-algebra and has made the selection process easier to administer. Adding students' grades to the selection process using the numerical assignments as recommended by the Orleans Hanna Test is continuing to be monitored. Including grades and assigning the overall grade score to have an equal weight to the test itself, resulted in 35 students qualifying for Pre-algebra that would not have on the basis of the test score alone and disqualified 9 students that would have qualified on the basis of the test alone. The performances of these students are now being carefully observed.

As one might expect, we are finding that more advanced middle school mathematics coursework has significant implications for the mathematics curriculum throughout the secondary years. Increasing the number of students taking Algebra as eighth graders has the direct effect of increasing the number of students in advanced level mathematics in high school. The student who takes Pre-algebra as a seventh grader typically goes through a secondary course sequence that concludes with Calculus as a senior. Currently approximately 25% of the district's seniors take Calculus, roughly the same percentage that took Pre-algebra as seventh graders. Beginning with the new selection process for Pre-algebra in the 7th grade (and then Algebra in the 8th grade) the number of Calculus students at the high school level will potentially double.

As the district continues to review and adjust its mathematics placement process, some particularly talented students may well eventually become potential candidates for Calculus III as seniors. Historically the district has paid tuition for such students to enroll in Calculus III at a local University, but this will not be of interest for large numbers of students since Calculus III is required for only a few university majors. AP Statistics is being added to the high school course offerings to provide another option, but almost certainly, as more students are placed into early advanced coursework, the demand for higher-level mathematics courses in high school will grow.

Teacher perceptions continue to be mixed with the initial implementation of the selection process. Some teachers are skeptical that a larger percentage of students are able to handle Algebra and would still prefer a cut score resulting in fewer students being placed into the Pre-algebra sequence. Fewer identified students would indeed mean fewer students placed in Pre-algebra who do not perform well. However, it would also increase the number of students in seventh grade "General Mathematics" who might have been more appropriately placed in Pre-algebra.

The larger number of Pre-algebra students has also resulted in a scheduling challenge at the Middle School. Rather than six sections of seventh grade Pre-algebra, as was the case prior to the new selection process there are currently 11 sections. This change brings staffing and staff development implications. Teachers who have previously taught only seventh grade mathematics must be prepared to teach more challenging courses.

Although the greater numbers of accelerated students has required significant changes in middle school scheduling and staffing, the change has been particularly positive for scheduling in one important respect. Having a traditionally small number of accelerated students resulted in that group of students also taking other core curriculum courses such as English, Science and Social Studies together. This traditional procedure had the unfortunate effect of tracking throughout the system. With a larger number of students, it has been possible to schedule those students in a way that they can be better integrated throughout the system, minimizing the tracking across the middle school curriculum.

Next Steps: Where Do We Go from Here?

The changes related to algebra placement have been significant, but they are only just beginning. We will continue following the effectiveness and practicality of this new selection process. As greater numbers of students are placed and complete the courses, the statistical analyses we will conduct should be able to provide a more complete picture of how the new placement process is working. Curriculum review and staff planning is ongoing. High school staff and administrators have been involved throughout the change process and are fully aware of the implications. As more accelerated students advance through the system, significant changes will need to occur at the high school level. The high school will likely need to add Calculus III and certainly more sections of advanced mathematics classes will be needed. Who will teach these advanced level classes? That discussion is currently underway. Teachers who have taught Algebra and Geometry in the past will undoubtedly be asked to also teach these higher-level mathematic courses.

Finally, it is important that we continue the philosophical debate. There are those district educators who believe that only a very select group of students should be accelerated or take more advanced mathematics coursework. While at the other extreme, some educators believe that all seventh grade students should take Pre-algebra and that there should be no placement tests at all. We see such debate within the district as healthy and an important key to providing the best and most appropriate mathematics program for all students. Although we are still evolving toward a truly equitable and effective algebra placement strategy, we believe that we have made an important step forward with this revised and more inclusive placement process. As suggested by the John Kennedy, we also believe that "all children have an equal right to develop their talent, their ability and their motivation." Hopefully, the students in the Westside Public Schools are a step closer to realizing this important right with our mathematics curriculum. ✪

References

Association of Mathematics Teacher Educators. (2006). *Preparing teachers to use technology to enhance the learning of mathematics.* An AMTE position statement, published January, 2006. Accessible at http://www.amte.net/.

Burris, C., Heubert, J.P., Levin, H.M. (2004). Math acceleration for all. I*mproving Achievement in Math and Science, 62*(5), 68-71.

Cavanagh, S. (2006). Big cities credit conceptual math for higher scores. *Education week.* January 11, 2006, pgs 1-3.

Checkley, K. (2006). "Radical" math becomes the standard: Emphasis on algebraic thinking, problem solving, communication. *Education Update*, newsletter within the Association for Supervision and Curriculum Development, 48 (4), 1-2, 8.

Everyday Learning Corporation (2002). *Everyday Mathematics.* A University of Chicago School Mathematics curriculum available through McGraw Hill Companies: Desoto, Texas.

Fenton, R. (2002). Selecting students for pre-algebra: Examination of the relative utility of Anchorage pre-algebra screening tests and the state of Alaska standards Benchmark 2 Mathematics Study. *Anchorage School District Report 2002*, Eric Document Service, ED 466644.

Fuson, K.C., Kalchman, M., Bransford, J.D. (2005). Mathematical understanding: An Introduction. *How students learn: History, mathematics, and science in the classroom.* Committee on How People Learn, A targeted report for teachers from the National Research Council, Donovan, M.S. and Bransford, J.D. (Eds). Washington, DC: The National Academies Press.

Grandgenett, N. F., Jackson, R, Willits, C. (2004). Evaluating a New Mathematics Curriculum: A District's Multi-Stakeholder Approach. *Journal of Mathematics Education Leadership*, Volume 7, Number 1, pgs 13-21, Spring, 2004.

Greenes, C. (2004). Algebra: Its elementary! *ENC Focus, 12*(23), pgs 1-8.

Harcourt Brace and Company. (1998). The Orleans-Hanna Algebra Prognosis Test. San Antonio, Texas: Harcourt Brace.

Heid, M.K, Edwards, M.T. (2001). Computer algebra systems: revolution or retrofit for today's classrooms? *Theory into Practice, 40* (Spring 2001), 128-36.

Heid, M. K. (2005). Technology in mathematics education: Tapping into visions of the future. In Masalski, W.J. and Elliott, P.C. (Eds.) *Technology-supported mathematics learning environments*, the Sixty-Seventh Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Hegedus, S.J., Kaput, J.J. (2004). An introduction to the profound potential of connected algebra activities: Issues of representation, engagement and pedagogy. *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education, 3*(1), 129-136.

Horn, L., Nunez, A.M. (2000). Mapping the road to college: *First-generation students' math track, planning strategies and context of support* (NCES 2000-153). Washington, DC: U.S. Department of Education. Available: http://nces.ed.gov/pubs2000/2000153.pdf.

Moses, B. (2000). Exploring our world through algebraic thinking. *Mathematics Education Dialogues*, April 2000. Reston, Virginia: National Council of Teachers of Mathematics.

Moses, R.P. (1994). Remarks on the struggle for citizenship and math/science literacy. *Journal of Mathematical Behavior, 13*, 107-11.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics.* Reston, Virginia: National Council of Teachers of Mathematics.

Noddings, N. (2000). Algebra for all? Why? *Mathematics Education Dialogues*, April 2000. Reston, Virginia: National Council of Teachers of Mathematics.

Schoenfeld, Alan H. (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. *Educational Researcher, Vol. 31, No. 1* p13-25.

Steen, L.A. (1992). Does everybody need to study algebra? *Mathematics Teacher, 85* (4), 258-260.

JOURNAL OF MATHEMATICS EDUCATION LEADERSHIP

1. Manuscripts should be consistent with NCSM mission.

   NCSM is a mathematics education leadership organization that equips and empowers a diverse education community to engage in leadership that supports, sustains, and inspires high quality mathematics teaching and learning every day for each and every learner.

2. Manuscripts should be consistent with the purpose of the journal.

   The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

   - Strengthening mathematics education leadership through the dissemination of knowledge related to research issues, trends, programs, policy, and practice in mathematics education;
   - Fostering inquiry into key challenges of mathematics education leadership;
   - Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice; and
   - Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as to strengthen mathematics education leadership

3. Manuscripts should fit the categories defining the design of the journal.

   - Case studies of mathematics education leadership work in schools and districts or at the state level and the lessons learned from this work
   - Research reports with implications for mathematics education leaders

4. Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice.

5. Manuscripts should be consistent with the NCTM Principles and Standards and should be relevant to NCSM members. In particular, manuscripts should make the implications of its content on leadership practice clear to mathematics leaders.

6. Manuscripts are reviewed by at least two volunteer reviewers and a member of the editorial panel. Reviewers are chosen on the basis of the expertise related to the content of the manuscript and are asked to evaluate the merits of the manuscripts according to the guidelines listed above in order to make one of the following recommendations:

   - Ready to publish with either no changes or minor editing changes.
   - Consider publishing with recommended revisions.
   - Do not consider publishing.

7. Reviewers are expected to prepare a written analysis and commentary regarding the specific strengths and limitations of the manuscript and its content. The review should be aligned with the recommendation made to the editor with regard to publication and should be written with the understanding that it will be used to provide the author(s) of the manuscript feedback. The more explicit, detailed, and constructive a reviewer's comments, the more helpful the review will be to both the editor and the author(s).

Information for Reviewers

Please contact the journal editor if you are interested in becoming a reviewer for the Journal at ncsmjmel@mathedleadership.org.

LEADERSHIP IN MATHEMATICS EDUCATION
NCSM NETWORK COMMUNICATE SUPPORT MOTIVATE

Membership Application/Order Form

Use this form to renew a membership, join NCSM, update information, or order items. Complete this form and return with payment. The information you provide will be used by the NCSM office for member communication, mailing lists, and the NCSM Membership Directory. Membership Dues are currently $85.

NCSM sometimes provides its mailing list to outside companies. These companies have been approved by NCSM to send catalogs, publications, announcements, ads, gifts, etc. Check here to remove your name from mailing lists. In addition, by checking this box, only your name without contact information will be included in the NCSM Directory. ☐

PLEASE PRINT LEGIBLY OR TYPE

First Name________________________________Middle _________

Last Name ____

Employer____

This is my complete address: ☐ Home ☐ Work

Title ____

Address ____

Telephone ____

Please check all that apply. I currently work as: (Optional)

☐ State/Provincial Department of Education Employee
☐ Government Agency (NSF, DOE, etc.)
☐ Member of Local Board of Education
☐ Superintendent
☐ District Mathematics Supervisor/Leader
☐ Principal
☐ Department Chair
☐ Grade-Level Leader
☐ Teacher Leader
☐ Author
☐ Coach/Mentor
☐ Consultant
☐ Curriculum Leader/Specialist
☐ Education Technology Provider
☐ Pre-Service Educator
☐ Professional Developer
☐ Publisher
☐ Teacher
☐ Other______________________________

Please check all that apply. I am a leader in mathematics education at the following levels:

☐ National
☐ Regional (more than one state/province)
☐ State/Province
☐ District/County/City
☐ Building
☐ University/College
☐ Senior High School
☐ Junior High/Middle School
☐ Elementary School
☐ Pre-Kindergarten
☐ Other______________________________

Since designations vary over time, check the one you feel best describes you:

☐ African American/Black ☐ Asian American ☐ European American/White ☐ Mexican American/Hispanic/Latino
☐ Native American ☐ Pacific Islander ☐ Bi-Racial/Multi-Racial ☐ Other__________________

Check the area you serve:
☐ Rural ☐ Suburban ☐ Urban

Do you influence purchasing decisions?
☐ Yes ☐ No

Age: ☐ under 25 ☐ 25 - 34 ☐ 35 - 44 ☐ 45 - 54 ☐ 55-64 ☐ over 64

Work Experience:
☐ First year in position
☐ 2-5 years in position
☐ 8-10 years in position
☐ 11-20 years in position
☐ 21-30 years in position
☐ over 30 years in position
☐ retired

Please check all that apply. Which of the following characterize the community you serve?
☐ High percent poverty
☐ High percent of English language learners
☐ Racial and ethnic diversity
☐ None of the above

Qty.	Item*	Member	Non-Member	P&H**	Sub-Total
____	**Monograph: Future Basics: Developing Numerical Power**	$15	N/A	N/A	________
	PRIME Leadership Framework				
____	1-4 copies (each)	$16	$18	$4.95	________
____	5-9 copies (each)	$15	$17	$10.70	________
____	10-15 copies (each)	$14	$16	$14.50	________
____	16-99 copies (each)	$13	$15	**	________
____	100 or more (each)	$12	$14	**	________
____	NCSM Member Pin	$2			$________
			Merchandise Total:		$________
	Membership Dues	**$85**			$________
			Total order:		$________

*Availability of products and prices are subject to change without notice.**
****Postage/Handling**: Books are sent by USPS. For orders of 16 or more copies, contact NCSM Member & Conference Services for a postage and handling price. Outside the U.S. or for expedited orders, call for shipping prices.

Please return this form to:
NCSM Member and Conference Services
6000 E. Evans Avenue 3-205, Denver, CO 80222
Phone: 303.758.9611; Fax: 303.758.9616
Email: office@ncsmonline.org Web: mathedleadership.org

Payment Method: ☐ Visa ☐ MasterCard ☐ Discover Card ☐ Check/M.O. *(U.S Funds only)* ☐ P.O.**

Purchase Order # ______________________________________

Credit Card # ____

Cardholder Name_____________________________ Exp ____/____

Cardholder Signature ___________________________________

Purchase orders will be accepted for PRIME orders only. A purchase order number must be included. ***Please note:* an invoice will NOT be sent. Should you need an invoice, please use this order form.

Emeritus Membership: Please check the NCSM website (mathedleadership.org) for eligibility requirements.

NCSM
PO Box 3406
Englewood, CO 80155

Presorted Standard
U.S. Postage
PAID
Brockton, MA
Permit No. 301

LEADERSHIP IN MATHEMATICS EDUCATION

NCSM

NETWORK
COMMUNICATE
SUPPORT
MOTIVATE