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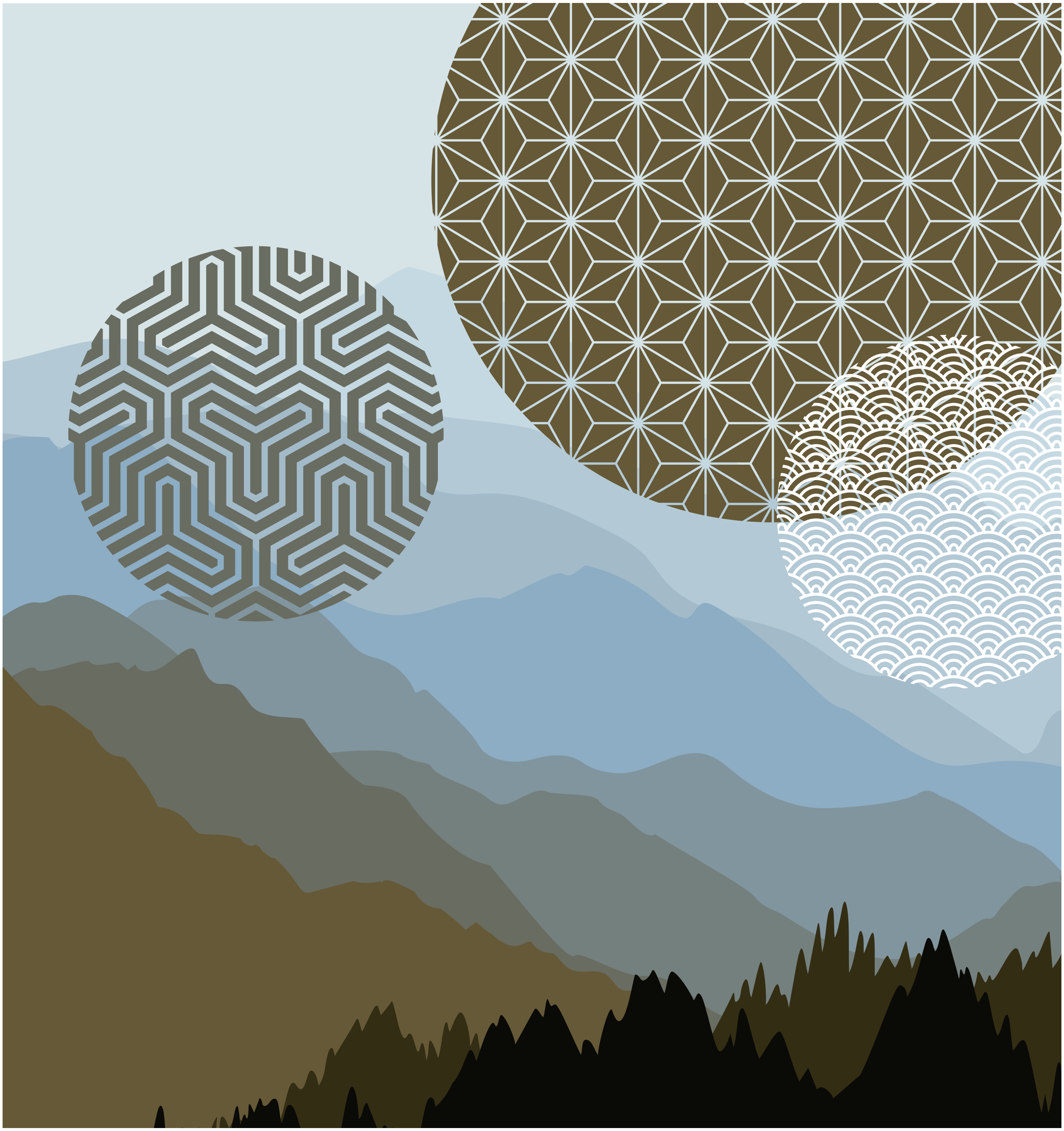

Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership (JMEL)*** are interested in manuscripts!

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM. Categories for submissions include:

- **Case studies and lessons learned** from mathematics education leadership in schools, districts, states, regions, or provinces
- **Research reports** with implications for mathematics education leaders
- **Professional development** efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
- Other categories that support the NCSM vision will also be considered.

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Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel. Each manuscript will be reviewed by two volunteer Each manuscript will be reviewed by two volunteer Each manuscript will be reviewed by two volunteer Each manuscript will be reviewed by two volunteer Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel. Manuscripts should be emailed to the Journal Editors, currently Drs. Erin Lehmann and Paula Jakopovic, at **ncsmJMEL@mathedleadership.org**.

Submissions should follow the most current edition of APA style and include:

1. A Word file (.docx) with author information (name, title, institution, address, phone, email) and an abstract (maximum of 120 words) followed by the body of the manuscript (maximum of 12,000 words)
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NCSM Vision

NCSM is the premiere mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

Purpose Statement

The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

- *Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education*
- *Fostering inquiry into key challenges of mathematics education leadership*
- *Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice.*

Comments from the Editors

Erin Lehmann, *University of South Dakota*
Paula Jakopovic, *University of Nebraska at Omaha*

"As an instructional leader, the decisions and actions you make matter to the teaching and learning of mathematics" (NCSM, 2019, p. 15).

We were hoping this letter from the editors would have come at a time when the words, "new normal, social distancing, pandemic, unprecedented, uncertain, etc." had been words from a very harsh and forgettable past. Unfortunately, this is not the case for most educators as distance learning and hybrid instruction continue to prevail. We encourage our mathematical leadership community to support, encourage, and advocate for the resilient teachers as they look to you for guidance and direction.

In this issue of *JMEL*, authors provide experiences (both successful and unsuccessful) of leadership within the classroom. Increasingly, mathematics teacher leaders are a prime example of how distributive leadership can be used to develop high-quality teaching and a mathematics classroom where all students can thrive. This form of continuous improvement is job-embedded and has been found to increase teacher retention (Sulit, 2020). While school leaders cannot do much about the staff shortages hitting most schools, they can find alternative ways to meet the professional learning needs of mathematics teachers. These are the times in which innovation and creativity are needed and where mathematics leaders can support and empower our teacher leaders (Cobanoglu, 2021).

The first article is titled, "Implementing Lesson Study: Challenges Identified by Emerging Teacher Leaders." Within the context of a professional development project, Barlow, Willingham, Lischka, Stephens, and Hartland supported emerging teacher leaders as they facilitated teachers' engagement in the lesson study process. In this paper, they share the self-identified challenges met as leaders of lesson study in their school settings. In addition, they share lessons learned in response to these challenges. Implications for mathematics education leaders are included.

In the second article, "Leadership from Within the Classroom: Opportunities and Challenges for Elementary Mathematics Specialists," Conner, Nguyen, Sheffel, and Webel describe the leadership opportunities and challenges experienced by eight Elementary Mathematics Specialists (EMS) who all remained in their primary role as classroom teachers after obtaining their specialist certificates. Drawing on Gigante and Firestone (2008), the authors categorize the EMS' leadership tasks in terms of whether they supported colleagues in increasing their knowledge of teaching mathematics. After describing the leadership tasks and how they came about, the authors describe four challenges at least some participants faced in enacting leadership from their classroom teaching roles. Finally, they share recommendations for ways different stakeholders can support EMS in taking on leadership tasks while remaining full time classroom teachers.

In the final article, "Inclusion and Intervention: Understanding "Disability" in the Mathematics Classroom," Jasien and Hays share how all students' learning—including students with learning and intellectual disabilities—is deepened when students with multiple ability levels engage in teamwork on high cognitive demand tasks, yet we know little about supporting teachers in inclusive mathematics classrooms. This knowledge void presents challenges for mathematics education leaders who wish to foster inclusion. Synthesizing a small but growing body of mathematics education research, this manuscript is a resource for leaders supporting teachers in inclusive, standards-based classrooms. In particular, this manuscript articulates (1) why productive struggle is essential for students with disabilities, (2) progressive definitions of disability and inclusion, and (3) conceptual descriptions of pedagogy in inclusive mathematics classrooms. It is followed by an appendix filled with tangible strategies that mathematics education leaders can adopt and adapt in their own contexts.

We wish you a wonderful New Year of happiness, health, and peace. ✪

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Implementing Lesson Study: Challenges Identified by Emerging Teacher Leaders

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Abstract

Within the context of a professional development project, we supported emerging teacher leaders as they facilitated teachers' engagement in the lesson study process. In this paper, we share the self-identified challenges met by this group as as they led lesson study in their school settings. In addition, we share lessons learned in response to these challenges. Implications for mathematics education leaders are included.

Introduction

As mathematics education leaders, we know that professional development is key to supporting effective mathematics instruction (Sztajn et al., 2017) and must be ongoing, embedded, and sustainable (Loucks-Horsley et al., 2010). With the support of external funding, our professional development project, Project IMPACT (Implementing Mathematical Practices And Content into Teaching), provided countless hours of professional development across a total of seven years. The project included in-school experiences, such as demonstration lessons (Loucks-Horsley et al., 2010), to enhance teachers' pedagogical content knowledge and summer institutes focusing on teachers' specialized content knowledge (Ball et al., 2008). We feel confident that our project met the expectations of being ongoing and embedded, as its design was informed by research (Desimone, 2009; Loucks-Horsley et al., 2010; Smith, 2001). Knowing that our external funding would eventually end, we wondered throughout the project how we might best support the sustainability of Project IMPACT. During the third year of the project, this wondering led us to consider lesson study.

Lesson study is a form of professional development that originated in Japan (Stigler & Hiebert, 1999) and has demonstrated its effectiveness as a professional development model in the U.S. (Lewis et al., 2009; Lewis et al., 2006). Typically, lesson study involves a group of teachers in developing a research lesson that addresses their selected goals for student learning. The group collaboratively plans the lesson and then engages in a process of teaching, revising, and reteaching the lesson based on their observations of student learning during the teaching of the lesson. This process of teaching, revising, and reteaching the lesson continues until the teachers feel comfortable with the lesson outcomes. Collectively, these steps represent what is referred to as a lesson study cycle (Lewis & Hurd, 2011).

Lesson study as a possible mechanism for sustaining the work of our professional development project appealed to us for three reasons. First, lesson study meets the general expectations of effective professional development (Darling-Hammond et al., 2009; Desimone, 2009; Lewis & Hurd, 2011). Second, we had utilized demonstration lessons (Loucks-Horsley et al., 2010) frequently in Project IMPACT, and lesson study seemed like a natural extension of these (see Gerstenschlager et al. (2021) for details regarding demonstration lessons and their connection to lesson study). Third, many of our project participants already had professional learning communities (PLCs) established in their schools, which would provide a natural place for lesson study to occur. For these three reasons, we aimed to train a small group of Project IMPACT teachers on the processes of lesson study with a goal of these teachers, who we referred to as emerging teacher leaders, conducting lesson study within their school contexts. In doing so, our hope was to support the sustainability of the project.

In this paper, our purpose is to share the reflections of this group of emerging teacher leaders following their implementations of lesson study in their school settings. In particular, our focus is on the self-identified challenges that this group met as leaders of lesson study in their school settings. By sharing these challenges, we aim to guide other mathematics education leaders who support emerging teacher leaders in facilitating the lesson study process.

How Literature Regarding Lesson Study Shaped Project IMPACT

Background on Lesson Study

Our own understanding of lesson study grew from a series of early works whose authors introduced the tenets of Japanese lesson study to a western audience (e.g., Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Like many U.S. mathematics education leaders, we first encountered the idea of lesson study in *The Teaching Gap* (Stigler & Hiebert, 1999). Here, Stigler and Hiebert argued that teaching is a cultural activity and most attempts at education reform and teacher professional development eventually erode due to a failure to integrate cultural change into activities of professional learning. As a potential solution to this problem, these authors introduced lesson study as an ingrained cultural practice focused on the continuous, incremental improvement of a specific *research lesson* over time (Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Lewis and Tsuchida (1998) further described the research lesson as not only "an actual classroom lesson, taught to one's own students" (p. 12), but also highly focused, collaboratively planned, observed by other teachers, recorded, and debriefed by a group including, at a minimum, the teachers involved in the lesson study.

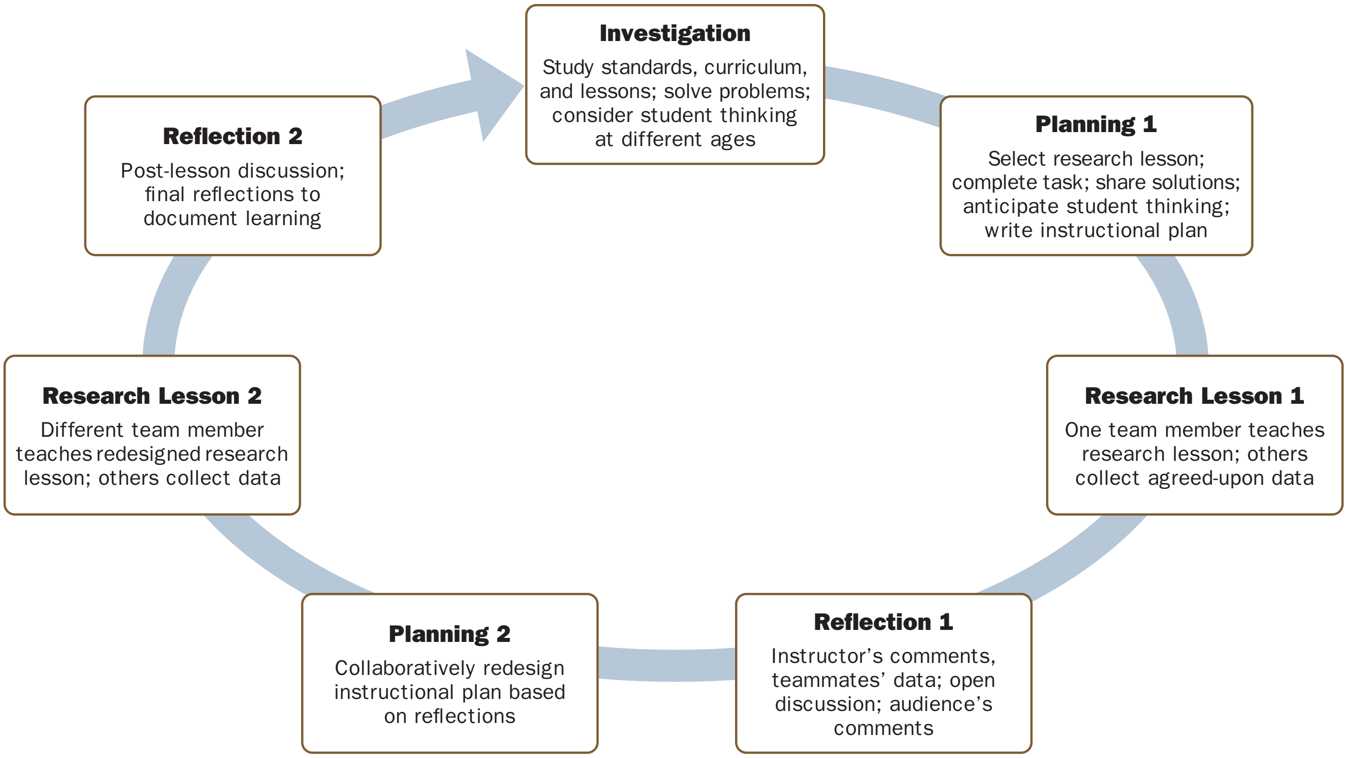
Given our emphasis on promoting sustainability in Project IMPACT, we were drawn to lesson study as the focus of professional development for our emerging teacher leaders and a possible impetus for affecting school culture. Research by Catherine Lewis, Clea Fernandez, and their colleagues confirmed this appeal in a variety of ways. In addition to providing rich descriptions of the nature of research lessons, their impact, and necessary supports (Lewis & Tsuchida, 1998), Lewis defined universal features of the lesson study cycle that influenced much of its western adoption. These features pervaded our own vision of lesson study and included an emphasis on shared long-term goals, important lesson content, careful study of students and student thinking, and live observations of lessons taught by lesson study participants (Lewis, 2002). Fernandez (2005) provided evidence that lesson study offers opportunities for both the development of mathematical content knowledge and the enactment of reform-oriented teaching, two of the fundamental goals of Project IMPACT.

Other writings helped us understand the transition of lesson study from its origins in Japan to the U.S. mathematics classroom (Fernandez, 2002; Lewis et al., 2006; Watanabe, 2002), provided theoretical lenses and experimental innovations through which to operationalize lesson study (Lewis et al., 2009; Takahashi & McDougal, 2016), and offered practical supports for implementing a lesson study cycle (Fernandez & Chokshi, 2002; Lewis & Hurd, 2011; Takahashi & McDougal, 2016). Many of the resources described here shaped not only the development of our own understandings of lesson study, but also influenced the handbook we selected, *Lesson Study Step by Step: How Teacher Learning Communities Improve Instruction* (Lewis & Hurd, 2011), to guide our emerging teacher leaders as they led their colleagues in a lesson study cycle. In the remainder of this section, we elaborate on what this body of literature suggests as a typical cycle of lesson study and its affordances.

A Typical Lesson Study Cycle

Although lesson study cycles may take on a variety of slightly different forms, the research lesson is integral to each of these, and a typical pattern of investigation, planning, teaching, and reflection regarding this lesson emerges across sources (e.g., Fernandez & Chokshi, 2002; Lewis et al., 2009; Stigler & Hiebert, 1999; Takahashi & McDougal, 2016). A typical lesson study cycle (see Figure 1, adapted from Lewis et al., 2009) begins with the lesson study team

FIGURE 1. *A Typical Lesson Study Cycle*



Note. This figure was developed from "Improving Mathematics Instruction through Lesson Study: A Theoretical Model and North American Case," by C. Lewis, R. Perry, and J. Hurd, 2009, *Journal of Mathematics Teacher Education, 12*, 285 – 304.

selecting a content topic for their investigation, examining learning standards which address the topic, solving related problems, sharing their solutions, and considering the resources available to them to support their teaching (e.g., textbooks, published lessons, or curricular guides). Once the team has selected the content and general structure for the research lesson, they move into a period of collaborative planning in which they choose tasks and sequencing for the lesson, generate and discuss exemplary responses, anticipate possible student thinking and solutions, and record their instructional plan in some agreed upon format. Soon after, the research lesson is taught by one of the participating teachers, with the remainder of the group observing and collecting previously agreed upon data generated directly from the lesson and students' accompanying thinking (e.g., students' conversations, solutions, models, understandings and misunderstandings).

Invited others, such as peer teachers, local experts, or members of educational leadership, may be present to observe the research lesson and participate in the reflective discussion that follows immediately after. These discussions tend to include comments by the teacher leading the lesson, presentations of data collected by the lesson study teachers, and a whole-group discussion of questions and features selected by the team. Based on feedback from this period of reflection, the lesson study teachers revise their written instructional plan, teach the modified lesson to a new group of students (generally with a different participant teacher), and host another round of reflective lesson debriefing. These second stages of planning, teaching, and reflection are similar in nature to their first stage counterparts, but this time with a goal of finalizing the lesson and documenting what was learned from the lesson study process in terms of incremental change.

Affordances of Lesson Study

A variety of positive attributions, including factors both internal and external to participating teachers, appear repeatedly in research related to lesson study. We categorize the recurring internal factors broadly as improvements in three areas: teacher knowledge and beliefs, instructional practices, and confidence and self-efficacy in teaching. Evidence for *improvements in teacher knowledge and beliefs* appears in the form of increased subject matter and pedagogical content knowledge (Fernandez, 2005; Lewis et al., 2004; Lewis et al., 2012), as well as beliefs regarding mathematics and the nature of teaching and learning mathematics (Lewis et al., 2009; Lewis et al., 2012). Enhancements to *instructional practices* include enriched classroom practices (Lewis & Tsuchida, 1998), learning how to reason mathematically during the enactment of a lesson and how to best support students in developing content knowledge (Fernandez, 2005), and a stronger ability to connect daily practices to long-term classroom goals (Lewis et al., 2004; Lewis & Tsuchida, 1998). Changes in c*onfidence and self-efficacy* present as increased professional confidence (Lewis et al., 2004; Rock & Wilson, 2005), a greater perception of one's ability to influence student learning (Lewis et al., 2012), and motivation to improve teaching (Lewis et al., 2004). As an offshoot of this motivation to improve teaching, the literature suggests that engaging in lesson study increases teachers' demand for high-quality professional development experiences (Lewis & Tsuchida, 1998), allows focused and sustained efforts to improve their growth, and extends their confidence in engaging in the processes involved in lesson study (Rock & Wilson, 2005).

Evolving with these internal developments, and as part of the sustained effort required by lesson study, are exterior features of the participating teachers' world related to student thinking, their professional networks, and the manner in which they utilize instructional resources. Lesson study encourages *focus on and insight into student thinking* by centering instruction on student work (Lewis et al., 2012) and improving teachers' ability to anticipate and observe student thinking (Lewis et al., 2004; Lewis et al., 2012; Perry & Lewis, 2009). Accompanying this shift in focus are gains in students' mathematical thinking measured by both classroom assessments and standardized testing (Lewis et al., 2012). This focus on student thinking, in addition to shared lesson planning, common teaching experiences, and lesson debriefings, is shown to *strengthen teachers' professional networks* (Lewis et al., 2004; Lewis et al., 2009) as well as increase the value they place on peer collaboration as part of their own professional growth (Rock & Wilson, 2005). Other improvements in *resource utilization* occur as teachers increasingly consult external sources such as professional literature and local experts (Lewis et al., 2012; Perry & Lewis, 2009; Rock & Wilson, 2005) and as they explore internal resources including each other's practice, their students' thinking (Lewis et al., 2012), and the protocols and tools they develop to facilitate their lesson study (Perry & Lewis, 2009). Additionally, the nature of lesson study leads to a higher quality of available lesson plans (Lewis et al., 2004; Lewis et al., 2009) and the sharing of new ideas regarding both content and teaching (Fernandez, 2005; Lewis & Tsuchida, 1998).

Project IMPACT's Plan for Emerging Teacher Leaders

Convinced that lesson study was the appropriate next step for Project IMPACT, we moved forward with identifying and working with emerging teacher leaders. At that time, Project IMPACT was in its third year of implementation and had 82 teachers representing kindergarten through sixth grade from eight different school districts in a southeastern state of the U.S. The components of Project IMPACT (i.e., demonstration lessons and content-intensive summer institutes) were designed to enable participating teachers to meet the standards set forth by the National Council of Teachers of Mathematics (NCTM, 2000, 2014) for teaching mathematics in ways that engaged learners in sense-making. A focus on learning through problem solving and sense-making of mathematical concepts through use of manipulatives and models was prevalent throughout activities. Participating teachers came to refer to lessons they experienced in the project as "Project IMPACT Lessons," noting these as a different way of teaching mathematics than that to which they were accustomed. By situating our work with emerging teacher leaders within Project IMPACT, the context of the project provided a pool of potential emerging teacher leaders that shared a common vision of effective mathematics instruction.

To identify participants from within Project IMPACT for the emerging teacher leader focus, we sent a general invitation to all project participants, inviting them to consider participating in the teacher leader training. We defined teacher leaders as individuals who provide instructional support to teachers. Likely, these individuals held titles such as mathematics coach, numeracy coach, or mathematics supervisor. In addition, we described emerging teacher leaders as teachers who perceived themselves as eventually moving into the role of a teacher leader or as teachers beginning to assume teacher leader roles with no official change in job title or job responsibility. In response

Table 1: Background on Emerging Teacher Leaders

Teachers by Grade Level Taught				
District	K-2	3-4	5-6	Teacher Leaders
A	2	5		
B	4	3	1	3
C		1	3	
D	1			1
E	1	1		
F			1	

to our invitation, 28 teachers from Project IMPACT agreed to participate, and 27 actually attended the initial training session. Table 1 provides information on the teacher leaders and emerging teacher leaders that participated in the training. For simplicity, we refer to all of these individuals as emerging teacher leaders.

The 27 emerging teacher leaders attended their first meeting in early September, near the beginning of the school year. At this meeting, they began by engaging in activities aimed at developing their understanding of working with adult learners and the stages of career development. Then, we turned our attention to lesson study.

A quick poll of the emerging teacher leaders demonstrated that none were familiar with lesson study. With this in mind, we provided an overview of the lesson study cycle and then engaged participants in a mock lesson study experience using video and tools associated with the selected lesson study handbook, *Lesson Study Step by Step: How Teacher Learning Communities Improve Instruction* (Lewis & Hurd, 2011). Specifically, we followed the professional development plan described in Chapter 2 of this book, in which the authors state, "The best way to learn about lesson study is to participate. In this chapter, we do the next best thing – participate vicariously" (p. 18). After completing the activities described in the handbook, the emerging teacher leaders were given their charge of returning to their school setting and leading their colleagues in a cycle of lesson study that followed the guidelines and expectations outlined in the handbook. Time was spent addressing the expectations and logistics of the charge, as well as the guidance our handbook offered for handling obstacles that might arise during the process. It is important to note that each emerging teacher leader received a resource bundle to support their work, which included the lesson study handbook by Lewis and Hurd (2011) and a video camera for recording the lessons and meetings associated with the lesson study.

In late February of that same school year, we met with the emerging teacher leaders for the purpose of debriefing their experiences as leaders of lesson study. This single-day meeting began with a discussion of the experience, during which we engaged them with the following eight prompts:

1. Looking back, I wish I had known . . .
2. The most challenging part of **leading** the lesson study was . . .
3. The big idea that I walked away with was . . .
4. The strength of our lesson that we implemented was . . .
5. The weakness of the lesson that we implemented was . . .
6. If I had this to do over again, one thing that I would do differently is . . .
7. Some of the obstacles I had for **conducting** the lesson study included . . .
8. If asked, I would/would not lead a lesson study again because . . .

Each prompt was written on a separate piece of large chart paper, with the pieces of chart paper rotated through the small groups of emerging teacher leaders. As they discussed their ideas related to each prompt, they recorded them on post-it notes and placed the post-it notes on the corresponding chart paper. After the chart papers had rotated through all of the groups, each group of emerging teacher leaders was assigned a poster and given the task of summarizing the ideas on the poster. This work was then presented to the whole group and used to launch follow-up discussions. This entire debriefing session was video recorded and lasted around three hours.

To analyze the data drawn from this single meeting, we began by recording the ideas from the post-it notes in a spreadsheet and then transcribed the portion of the debriefing session video that included group presentations of the chart papers and the ensuing whole group discussions. Next, two of the researchers analyzed this data with an eye on the challenges to leading lesson study that were identified by the emerging teacher leaders. Specifically, we used the process of open coding (Creswell, 2013) to categorize the data contained on the post-it notes that were written in response to the second prompt (i.e., The most challenging part of **leading** the lesson study was . . .). Next, these codes were grouped into themes (Creswell, 2013). Finally, we analyzed the transcripts, identifying passages that offered insight into the original codes drawn from the post-it notes. Throughout this process, the researchers oscillated between working individually and collaboratively, allowing for rich discussions of the data and resolution of any coding differences. In the next section, we elaborate on the results of this analysis.

Emerging Teacher Leader Reflections

The three themes that emerged from the analysis involved the logistics, culture, and coordination of lesson study. Within the logistics theme, the emerging teacher leaders identified the challenges of time for planning and scheduling issues. Although these logistical challenges were common to all emerging teacher leaders, the underlying issues that led to these challenges (e.g., weather-related school closures) were context dependent and, therefore, unique to each school. In contrast, the remaining two themes (i.e., culture and coordination) represented shared challenges resulting from similar issues faced as leaders of lesson study, which were the focus of our investigation. Therefore, in the following sections we will expand upon these two themes. Participant quotes, taken from small group presentations, will be included throughout these descriptions. However, these quotes will not be attributed to the individuals that spoke, as they were primarily given in the context of representing the perspective of either the presenting group or the whole group rather than the individual's perspective.

Culture

To begin the lesson study process, the emerging teacher leaders were tasked with identifying teachers in their schools who would potentially participate in the lesson study. For the most part, the potential teachers had not participated in Project IMPACT and, therefore, did not necessarily practice the student-centered instructional strategies that the emerging teacher leaders had learned through the project. Further, emerging teacher leaders reported that many of the teachers were not accustomed to working collaboratively with their colleagues in processes associated with the lesson study cycle. As a result, the school culture within which the emerging teacher leaders worked led to two challenges: teacher buy-in and teacher participation. These cultural challenges are described in the following sections.

TEACHER BUY-IN

In leading a cycle of lesson study in their schools, the emerging teacher leaders found it difficult to develop teacher buy-in so as to support meaningful engagement in the process. Initially, the difficulty stemmed from an inability to articulate the goals and purposes of lesson study.

> Getting teachers to understand what [lesson study] is — "What is it you're asking me to do, you know? I don't really know what a lesson study is." None of us knew what lesson study was before we came in September. So that was a challenge to explain [what lesson study is] without the [training] videos [from our book]. . . . I had to get [the potential teachers] to sit down and show them the video for them to be able to understand it. But they didn't want to do that because they didn't know what it was yet.

This difficulty was compounded by teachers' hesitation to "give up [their] limited time to do [lesson study]." In fact, "some teachers had to be bribed with [continuing education] hours that had to be cleared by central office. And that shocked me."

Once the emerging teacher leaders formed their lesson study groups, in some instances the emerging teacher leaders found it difficult to get "teachers to realize that lesson study is not my thing, but our thing. Since it's not my lesson plan." That is, teachers saw the lesson study as an assignment that the emerging teacher leaders had to complete as a result of their participation in Project IMPACT rather than a professional development opportunity for all involved. "We felt like they were - they were just helping us out. It was our thing. . . . So we did the bulk of the work."

In thinking about teachers lack of buy-in to the lesson study process, the emerging teacher leaders wondered if it was a result of the teachers not having participated in Project IMPACT. Emerging teacher leaders questioned whether or not the teachers approached the lesson study with the mentality of working together to improve their practice.

> What we sort of thought was maybe they're not [Project] IMPACT-trained teachers. You know? They don't have that mindset of: we're going in trying to make ourselves better. This isn't a degrading process. It's a learning process. I'm not here to say, "Oh, Mary, I would never do that. And what you did was horrible." It's, "Hey, we're a team. We're supposed to be making us better as a group." . . . It's really the group effort.

Closely related, other emerging teacher leaders wondered if the association of the lesson study with Project IMPACT led to a lack of buy-in.

> Maybe that's one of the things that's kind of shutting them down. If they've not – [if] they don't know anything about Project IMPACT. And if we just say, "Hey, we're going to do this Project IMPACT lesson or this IMPACT lesson study." Then they're like, "Look, what I've been doing is fine. So, you just need to leave me alone."

The emerging teacher leaders further reflected on this connection between the lesson studies and Project IMPACT as they considered the need for relationship building.

> So, I have to believe that most teachers want to improve. But if they say no, they have the right to say no. And I think, you know, the teachers that I talk to - the teachers I developed a relationship with - and it's teachers that I say, "I'm excited about this, you know, and I'd like to share this with you, you know." If I just go to somebody and I say, "I want to do this Project IMPACT thing with me," and they say, "no," and I'll say, "Fine." You know, you've got to have a relationship. And if you don't have a relationship, it won't work. . . . Just because we're learning this fabulous thing. We want them to be excited about it, because we're excited about it.

Based on their discussions, it appeared that Project IMPACT had developed a culture for collaborative professional growth and fostered a willingness among its participants (i.e., the emerging teacher leaders) to try new instructional strategies with the support of their peers. These same characteristics were not necessarily true of the school cultures in which the emerging teacher leaders worked, posing a challenge to them as leaders of lesson study.

TEACHER PARTICIPATION

As the groups of teachers moved through the lesson study process, the emerging teacher leaders faced a new challenge: teacher participation. Initially, "getting teachers to talk or share was a big deal. . . . What's the point of having this group discussion if you're the only one speaking?" The emerging teacher leaders noted that the presence of a video camera may have contributed to this hesitancy to participate.

> But if it was not recorded. . . . I don't care how many times I told them, "Y'all, I am not giving this to your principal." They didn't believe me, I don't think.

Compounding this issue of being recording was the idea that:

> Not many of the teachers, who have been teaching for a long time, have done anything like this [lesson study] and . . . maybe [having] their reactions to things that they've never heard of recorded [was] probably something they might have been scared of.

As teachers overcame their hesitancy to participate, the emerging teacher leaders noted additional challenges related to teacher participation. It "was hard to convince teachers that this was not an observation of their teaching. . . . We were there to observe the students learning." By focusing on the performance of the person teaching the lesson(s), the lesson debriefings focused on positive affirmation. "'Well, I thought it was great.' 'I thought it was good, too.' And they all just thought it was great." As the debriefings shifted to an analysis of student work, though,

> I was like, this was not a success. So how was it great? That one quote from the book, saying it was a great lesson, but the students just didn't get it. It's just like saying the surgery was a success, but the patient died. . . . But if during the lesson study, the goal is to actually look at and critique the learning of that lesson. If all we're doing is saying everything was great, wonderful. Well, all right, then what are we doing here?

From these discussions, it seemed that teacher participation was limited by teachers' inexperience with operating in a culture that values critical reflection about instruction.

Coordination

The impact of the cultural challenges described in the previous section extended beyond the lesson study groups' compositions and preparation to engage in the lesson study cycle. More specifically, these issues impacted the activities of the lesson study cycle led by the emerging teacher leaders, including co-planning, teaching, and debriefing the groups' research lessons. In this section, we will chronicle the observations made by the emerging teacher leaders regarding the challenges they faced as they led teachers in co-planning their lessons and dealt with misalignments in the groups' knowledge and expectations for their students.

CO-PLANNING

In their reflections regarding the most challenging aspects of leading the lesson study process, the emerging teacher leaders repeatedly cited managing their time efficiently and learning to plan collaboratively with their fellow teachers. Although the teacher leaders shared a variety of ideas during this discussion, their struggles with time management and co-planning were evident from the beginning:

> I think as we start out [discussing the most challenging part of leading the lesson study], we have the three main categories of time management, colleague buy-in, and planning. . . . I think it's a little misleading because the time management part was by far the most comments we actually had on there but it was the same thing: the time to plan and time to collaborate . . . is definitely a huge part of it.

However, as conversation around this idea evolved, the emerging teacher leaders framed these challenges as more than a simple issue of finding time to work together. Rather, they discussed internal conflicts, contrasting styles of planning, and issues regarding the scope and sequence of their lessons as obstacles to their planning process.

Supporting the notion that these issues were more than a simple logistical challenge, one participant elaborated on how an internal debate regarding teachers' autonomy led to struggles in co-planning.

> We work by ourselves really, almost all the time, and you can see, which it started to happen in our group, but it didn't end up happening, when it actually comes to suddenly you're collaborating and trying to get to that, it's really easy to have to try to stand your ground on saying, "I don't do it that way." I think it can be hard to kind of put it aside, and say, "Well I don't normally do it that way, but for this, I can go ahead and try to do it this way, and see what happens." You may like it, you may hate it, whatever. And, I think I'm preaching to the choir here, but that's the thing, everybody in here is willing to try, that's why you're here to start with. But, sometimes, that can be challenging in dealing with other teachers.

Building on this, another emerging teacher leader voiced an explanation of how contrasting styles of planning impacted collaboration between two emerging teacher leaders working together on lesson study.

> I work with Heather [another emerging teacher leader], and I plan a lot different than Heather does. I'm, you know, a big idea, kind of person, then I go with that, and Heather's more precise than me. She'll say exactly what she's going to say, and that, you know, stresses me out, because I know I'll never actually say what I wrote down. You know, it's just two different styles, not that one is more correct than another, but, you know, it's just different ways of doing it. So there's some translation going on there, but planning, as a whole, was a bit of a challenge.

Facilitating teachers' navigation of these types of interactions, within the already limited time available for lesson planning, proved to be a consistent challenge faced by the emerging teacher leaders as a whole.

A related concern arose as the emerging teacher leaders reflected on the challenges their lesson study groups encountered in planning instruction that could be modified for different grade levels and learners. A variety of cultural and logistical reasons led most of the emerging teacher leaders' lesson study groups to span a wide range of grades. One emerging teacher leader summarized:

> The last category that kind of came to light [was] . . . the brass tacks of us executing it. In terms of you have different teachers of different grade levels trying to figure out a topic. And, once you determine a topic, how that fits in the scope and sequence, four weeks from now, or days from now, when you're actually going to teach it, and trying to figure out how that all works together.

Another emerging teacher leader expanded on this idea:

> I had my lesson study in sixth and seventh grade, so meshing the expectations of the sixth- and seventh-grade standards and the student knowledge level was difficult for the group that I had. Just because of the way the mini schools were divvied up, or the type of students that were in each one, and then the standards we meshed at different times. That was hard for us.

Comments such as these illustrated the core challenges the emerging teacher leaders faced as they facilitated their groups' co-planning during their lesson study, and foreshadowed further difficulties that would arise in their teaching due to a lack of knowledge of the students with whom they worked.

KNOWLEDGE OF AND EXPECTATIONS FOR STUDENTS

In looking back across what they wished they had known prior to the start of their lesson study process, the emerging teacher leaders cited finding a more "efficient way to journal and collect lesson data" as a key concern. As with the other challenges of leading the lesson study process, discussion of this notion allowed the group to uncover deeper concerns, this time involving the importance of building on their knowledge of students and considering the ways in which students were prepared to learn mathematics. Initiating this conversation, one teacher leader noted:

> You have all this fancy stuff planned, but yet you never get to the point of what you're wanting to get across to the kids . . . then a lack of notetaking through the whole process [limits being] able to recall and go back to the lesson and see what the kids were getting and what they weren't.

One emerging teacher leader attributed part of this failing to not knowing "the learners individually, so you don't necessarily know what their learning styles are or their strengths or their weaknesses" and asserted that "you really need to have a knowledge of the students and where they're coming from" to be successful. Another emerging teacher leader supported this line of thinking, suggesting that knowing "more about the diversity of the student's knowledge and the grade level expectations with each of the classes for the lessons" would lead to more consistency across the research lessons, as "one class might have had this type of students where the lesson didn't go exactly the way that it did over here because of the students."

A specific example of this need for a better knowledge of students was cited repeatedly regarding students' preparedness to learn in a small-group, hands-on fashion. In their eagerness to design what they referred to as "Project IMPACT lessons," which involved extensive use of manipulatives for making sense of mathematics, the emerging teacher leaders found that "a lot of kids [from the non-IMPACT teachers' classes] weren't used to actually dealing with manipulatives and hands-on materials." One teacher leader noted a cultural barrier to this approach, as some of the non-IMPACT teachers felt the "whole idea of this lesson study bringing in manipulatives, heaven forbid, is viewed as we're playing," because "it's fun, and if it's fun, you're not really learning." Another voiced the reason that she suspected many students were not prepared to use this type of tool.

> The manipulatives will show the reasoning behind it, as opposed to just the straight scale of doing the [operation] . . . and the problem is, though, is that [not all non-IMPACT teachers are teaching this way]. Most or many students [from their classrooms] are not geared to work with manipulatives and come out with what you wanted them to come out with. It's more of, "Oh, this is great fun. I have something to play with right now," and let me play with it, and their focus goes away and it takes a lot more work on everybody's part.

Another teacher leader carried this reasoning even further, suggesting that some of the teachers they worked with did not understand how to use the tools themselves. He referenced a specific example that arose in planning for instruction.

> She actually said something [regarding] teaching the lesson plan. We were trying to use manipulatives to show why, like five tenths times five tenths is twenty-five hundredths, and she started that out, she says, "I can't use manipulatives so you're going to have to teach me how to do that." And, she could sit there and say, "I know this is the best way to teach using manipulatives, using those," but she's like, "it's, you know, all of our kids are not trained for them. So it's hard to use them [in research lessons]."

This combination of factors caused many teachers to doubt the effectiveness of their lessons, stating that they did not "feel like the class got out of it, what I wanted them to get out of it" or that they had to move to individual instruction with the manipulatives as "the group thing took their focus off of what our goal was." Collectively, these ideas regarding knowledge of, and expectations for, students represented challenges for the emerging teacher leaders as they facilitated the lesson study process.

Responding to Challenges: What Have We Learned?

As we reflected on the challenges identified by the emerging teacher leaders, we were struck by two realizations. First, as they engaged in the lesson study process, the emerging teacher leaders played the dual role of lesson study participant and lesson study facilitator. As they shared their ideas with us, they did not differentiate in these two roles. However, the challenges shared in the previous section were in response to the prompt of leading lesson study and, thus, represented challenges that the emerging teacher leaders felt compelled to overcome given their role as the facilitator of the lesson study process and their desire to make the experience productive for everyone involved. This led to our second realization: despite the challenges, these emerging teacher leaders pushed ahead and completed a cycle of lesson study within their school contexts. In doing so, they gained insights that would inform future opportunities to lead a lesson study. To this end, we share in this section the reflections of the emerging teacher leaders that they felt would likely address the earlier noted challenges. In addition, we feature what we learned as mathematics education leaders that would influence future work with emerging teacher leaders leading lesson study.

Emerging Teacher Leaders' Insights

In their first attempts to carry out a lesson study, the emerging teacher leaders found much of their effort directed towards establishing cultural norms and mechanics that would facilitate the process. In doing so, the emerging teacher leaders attributed much of their success to looking beyond the end result of their initial attempts and towards the processes and relationships involved in their work. One emerging teacher leader summarized this sentiment, noting the importance of reflection and persistence in developing the research lesson.

> This lesson thing that we're doing here, it's just a process. It's not the product. It's just like with math, it's the process. We're trying to develop a lesson that is effective and as we reflect on it, it becomes more effective for the students that we teach; and we reflect on it more, and it becomes a better and better lesson that involves our students and lets them become the leader of the lesson. And, you can't do that the first one out of the chute. I mean, if you put me on a rodeo horse today, I've got to tell you, I'm not lasting the eight seconds, and I may not last the eight seconds until it finally kills me. But, until I die, I'm going to be trying to improve.

In many ways, this process-oriented view of lesson study adopted by the emerging teacher leaders facilitated the product-oriented conception espoused in the literature (e.g., Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Without making arrangements to prepare for both the cultural and mechanical aspects of lesson study, the resulting research lesson was likely a mere shadow of its possibility. However, when a school's professional culture aligns with the lesson study process and is then focused through the lens of student learning, there is an opportunity for professional development and curriculum development to reinforce one another and create a whole that is greater than its parts. Recognizing this potential opportunity, in the remainder of this section we share insights the emerging teacher leaders gained as they responded to the challenges faced when implementing lesson study. These insights are related to encouraging teachers to fully participate in the lesson study cycle, time for planning and working together, and shifting their groups' professional focus to student thinking and learning. Whenever possible, the emerging teacher leaders' words are used to frame their ideas on these topics.

ENCOURAGE TEACHERS TO FULLY PARTICIPATE

The vast majority of advice offered by the emerging teacher leaders centered on encouraging those involved to engage in the process in meaningful ways. This facilitation occurred in three overlapping areas: encouraging broad participation in the project, setting norms and expectations for the various stages of the process, and building professional relationships with the team engaged in the lesson study. In this section, we will provide a brief summary of how each of these factors influenced the emerging teacher leaders' work.

Encouraging Broad Participation in the Project. The emerging teacher leaders suggested recruiting participants for the project from a broad group of teachers and administrators that would bring different ideas to bear. Although some groups selected their participants from Project IMPACT so as to have a shared vision of instruction, the emerging teacher leaders recommended intentionally choosing a broader range of participants to provide exposure to different ways of teaching. The group's rational was that "we're going to grow more if we get people that are different from us," and that this type of selection would allow the group to expose their peers to their new ideas about teaching and learning that arose from Project IMPACT. The teacher leaders also suggested inviting school-level administrators to participate in the lesson study so participating teachers could "know that [the administrators] are on our side and willing to learn these things, and because they need to be opened up to this way of teaching, too." Perhaps most importantly, the teacher leaders acknowledged their role in bringing lesson study to the attention of their peers. Their sentiment was that "this professional development activity, now that it's happened once in the building, those teachers may share some positive things with others" and that other teachers would be "more willing to participate in it again."

Setting Norms and Expectations. Although the emerging teacher leaders felt that, "Project IMPACT lessons are sort of our new norm," they recognized that this view of instruction develops over time. In response, they hoped that in future lesson studies they might have a way to "quickly introduce Project IMPACT philosophies to all members of the group." Additionally, almost everyone agreed that, during their reflection phases, "getting teachers to talk or share was a big deal," with an acknowledgment that, "we didn't do a good job of making clear what role they were supposed to take as observers." Thus, they acknowledged a need to delineate observational expectations.

The emerging teacher leaders also suggested being selective in terms of what components of the lesson study are video recorded, as some teachers may be reluctant to share their ideas freely when being recorded. They recognized that video recording during research lessons provided an opportunity to watch the lesson again at a different pace or with a different lens, but they felt it may make more sense to leave cameras off during planning and reflection. As summarized by the emerging teacher leaders, "If I could turn the video camera off, would they say more? Would they be more willing to actually share?"

Building Professional Relationships. Throughout the process, the emerging teacher leaders referenced developing strong professional relationships within their lesson study groups and leveraging these relationships to share good teaching practices. Once engaged in the project, trust between the research lesson teacher and those observing the lesson became paramount. As one teacher said, "Once you get into your lesson and you're comfortable with it, then [the feelings of nervousness] kind of disappeared." As a result, the emerging teacher leaders emphasized the importance of co-planning the research lesson and promoting shared ownership of the product as a way to build professional relationships. When they nurtured these relationships, the emerging teacher leaders found that lesson study was "a good way to introduce teachers to the IMPACT way of thinking, and helps examine good instruction and ways that we can show improvement."

TIME TO PLAN AND WORK TOGETHER AND FOCUS ON STUDENTS' THINKING AND LEARNING

The recommendations in the previous section dealt primarily with promoting a school culture that is conducive to the lesson study process. With this type of culture in place, each phase of the lesson study cycle runs more fluidly, and teacher leaders can more heavily emphasize the mechanics of planning, teaching, observing, and refining the research lesson. The emerging teacher leaders offered descriptions of these elements from their lesson studies as well, focusing on the importance of time for planning and reflection, and the need to highlight students' thinking and learning throughout the project's phases.

Across the board, the emerging teacher leaders referred to their own mismanagement of time, or contextual situations that limited their time together (e.g., weather, illness, outside commitments), as one of the more challenging aspects of their lesson study. They found that the initial phases of the process were "very time consuming and you weren't really sure if you were doing it right or how long it was going to take," and suggested "starting sooner" and generating "a more realistic timeline" as a key modification for future cycles. As the time spent "to plan, reflect, and focus was extremely beneficial," particularly in the second half of the cycle, starting early and using the initial stages of the project to gauge the time commitment needed for a full cycle may help others in their own implementations of lesson study. Supports, such as finding "better, more time efficient ways to journal and collect lesson data" or learning to "look at and critique the learning of the lesson" are also likely to arise from starting early and engaging in the initial phases, even clumsily, that will help improve the overall fidelity of the lesson study cycle. Other groups recommended looking towards platforms that are already used for planning and communication in their schools, and adapting these structures to assist with lesson study. Although an individual team's resources will vary, many organizational structures (e.g., professional learning communities, grade-level meetings) and technological platforms (i.e., Google Drive/Docs, Microsoft Teams, Slack) can be easily adapted for this purpose.

From a more pedagogically significant vantage, the emerging teacher leaders stressed the challenge of shifting their groups' focus, in all phases of the lesson study cycle, away from the individual teacher's actions and choices, and towards students' mathematical thinking and learning.

As one teacher leader described it, her stress in being observed teaching by her peers "went away" when the observing teachers told her, "I'm not watching you. I'm watching the kids." The emerging teacher leaders emphasized the idea that "we are not critiquing the teacher in her teaching," but rather critically examining the lesson and its influence on students' thinking and learning, and suggested that lesson study participants be repeatedly reminded of this central premise.

Mathematics Education Leaders' Insights

As mathematics education leaders, this was the first time we had engaged emerging teacher leaders in the process of leading a lesson study. As we reflected on the challenges they identified, we recognized aspects of our process that we would change in response, if we had the chance to do this again. These aspects primarily fell in two areas: utilization of demonstration lessons and conducting check-in meetings. We will discuss each of these in the following sections.

UTILIZATION OF DEMONSTRATION LESSONS

In hindsight, we wished that we had asked the emerging teacher leaders to go through the process of conducting/leading a demonstration lesson before we introduced them to lesson study. This would have allowed them to start with a professional development model with which they were familiar, as demonstration lessons represented a key component of Project IMPACT. Further, demonstration lessons, as described by Loucks-Horsley et al. (2010), include processes similar to that of lesson study (see Gerstenschlager et al. (2021) for a discussion of these similarities and differences). More importantly, though, the use of demonstration lessons could have potentially addressed three of the challenges identified by the emerging teacher leaders.

First, the time commitment for participating teachers in a demonstration lesson is only a few hours in a single day compared to many hours over possible weeks or months with a lesson study. This smaller time commitment likely would have helped with teacher buy-in, as teachers might have been more willing to commit to a smaller amount of time. Second, this small-scale opportunity could have potentially led to greater teacher participation and, thus, the opportunity to begin establishing the relationships and cultural norms needed for a successful lesson study. Third, the featured lesson in a demonstration lesson is developed by the individual who teaches the lesson. Therefore, the emerging teacher leaders would have gained experience in preparing a lesson far in advance that fits into the curriculum without the frustrations of having to do so collaboratively with a group of teachers.

Recognizing these affordances, if we have the opportunity to repeat this project, we feel that having the emerging teacher leaders carry out a demonstration lesson (or two) will set a strong foundation for later leading a lesson study cycle. In this way, we see leading a demonstration lesson as scaffolding the emerging teacher leaders towards leading a lesson study.

CONDUCTING CHECK-IN MEETINGS

As we considered the challenges identified by the emerging teacher leaders, we had a second realization: we should have scheduled meetings along the way to check-in with them. Although we encouraged communication through emails and established a Facebook group as a means of support, offering these opportunities for support from us (the Project IMPACT team) was insufficient, as the emerging teacher leaders did not take advantage of our invitations to consult with us. However, had we planned meetings along the way, we could have addressed several of the challenges that we only learned about afterwards. For example, several of the issues noted by the emerging teacher leaders were discussed in books that were a part of their resource bundle. A check-in meeting could have provided an opportunity to revisit these resources with an eye on the challenges they were facing. In addition, the challenges of, for example, engaging students who are not used to working with manipulatives is something with which we, as mathematics education leaders, have quite a bit of experience. A check-in meeting would have provided an opportunity for sharing insights and strategies for teaching in these situations. We also noted a perceived roadblock related to lack of knowledge of individual students. Had we engaged in check-in meetings and learned this earlier, we would have directed teachers to materials addressing learning trajectories and shifted focus away from individual students and toward strategies that engage all learners along the trajectory.

Discussion and Conclusion

In our work, we sought to support the sustainability of our externally funded, professional development project by developing emerging teacher leaders as facilitators of lesson study. In reflecting on this research study, one might think its results lack transferability to other contexts due to its close association with Project IMPACT. From our perspective, though, Project IMPACT simply provided a group of individuals who held a common vision of effective mathematics instruction as a result of their participation in the project. Therefore, we believe that our results serve to inform mathematics education leaders who are working with emerging teacher leaders regardless of the context. Unique to this report was the venture into lesson study led by individuals who had limited training and had not previously participated in lesson study. Although not ideal, with the enthusiasm that often surrounds lesson study, we hypothesize that similar grassroots efforts to learn from lesson study are being conducted, and our work serves to inform these efforts. With this in mind, we shared the reflections of the emerging teacher leaders following their implementations of lesson study in their school settings, with particular attention given to the self-identified challenges they faced as leaders of lesson study. These challenges were related to their school cultures (e.g., teacher buy-in and participation) and their efforts to coordinate the lesson study process (e.g., co-planning and knowledge/expectations for students). In response, we shared insights from the emerging teacher leaders, as well as our own, that should inform future opportunities for repeating this work.

By sharing our work, our goal was to support other mathematics education leaders in three key ways. First, we aimed to introduce lesson study to mathematics education leaders as a potential for sustaining professional development efforts. The literature has established the potential of lesson study for supporting teacher growth (e.g., Lewis et al., 2009; Lewis et al., 2006). It is, therefore, enticing to think of lesson study as a mechanism for extending the influence of professional development beyond the life of a project. However, using lesson study in this way requires utilizing classroom teachers (or emerging teacher leaders) as the leaders of lesson study. Consideration must be given to how to support emerging teacher leaders in the process of leading lesson study before we can examine lesson study's potential for supporting sustainability. Our results serve to inform these efforts.

Second, we saw working with emerging teacher leaders as a means for scaling up lesson study, thus allowing for broader participation. The literature tends to report on lesson studies led by lesson study experts or others who have strong experiences with lesson study (e.g., Rock & Wilson, 2005). Given the power of this professional development model, it is desirable to see more teachers provided with the opportunity to engage in lesson study, thus the need for scaling up lesson study. We hope that our work will inspire other mathematics education leaders to consider this possibility and that the narrative shared in this report will provide guidance for doing so.

Finally, the challenges identified in our work should be of particular interest to those working with teachers who are not accustomed to collaborative professional development efforts and/or who do not necessarily hold a common vision of effective mathematics instruction. Our reflections, as well as those of the emerging teacher leaders, provide specific insights into how to overcome those challenges.

By supporting mathematics education leaders in these three ways, our intent is to expand and enhance the opportunities for more teachers to participate in lesson study. Through participating, teachers will grow professionally and, in turn, positively impact mathematics achievement. ✪

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Leadership from Within the Classroom: Opportunities and Challenges for Elementary Mathematics Specialists

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Abstract

In this article, we describe the leadership opportunities and challenges experienced by eight Elementary Mathematics Specialists (EMS) who had all remained in their primary role as classroom teachers after obtaining their specialist certificates. Drawing on Gigante and Firestone (2008), we categorize the EMS' leadership tasks in terms of whether they supported colleagues in increasing their knowledge of teaching mathematics. After describing the leadership tasks and how they came about, we describe four challenges at least some participants faced in enacting leadership from their classroom teaching roles. We conclude with recommendations for ways different stakeholders can support EMS in taking on leadership tasks while remaining full time classroom teachers.

Introduction

Effective mathematics teaching at the elementary level requires specialized content knowledge and expertise that teachers do not often develop in their relatively limited undergraduate coursework (e.g., Ball, 2017; Wu, 2009). As a result, school and district leaders must find innovative strategies for supporting elementary teachers to continue to develop their pedagogical and specialized content knowledge over time. One way schools have addressed this need is through the hiring of teacher leaders, such as mathematics coaches or specialists, who can provide sustained professional development to teachers within a school or district (e.g., Ellington et al., 2017; McGatha et al., 2015). There is evidence that the use of full-time mathematics coaches/specialists can have a positive impact on student achievement (Harbour & Saclarides, 2020); however, this approach is cost-intensive and requires removing a teacher from the classroom. An alternative approach is to utilize teachers who have similar content-level expertise as mathematics coaches/specialists and are interested in engaging in leadership in addition to fulfilling their responsibilities as a classroom teacher. While prior research has studied the leadership tasks and challenges experienced by mathematics coaches serving in formal leadership roles (e.g., Campbell & Griffin, 2017), little is known about the leadership experiences of those who remain in the classroom full time.

In this paper, we describe the leadership experiences of eight 3rd – 5th grade public school teachers in their first year after completing an Elementary Mathematics Specialist (EMS) program, which aimed to strengthen teachers' pedagogical and specialized content knowledge through the lens of teacher leadership. Through focusing specifically on the leadership experiences of EMS classroom teachers, we aim to raise awareness about the potential role they can have in their school contexts, particularly in terms of increasing their colleagues' knowledge for teaching mathematics. Based on the teachers' experiences and challenges in enacting leadership tasks, we conclude by describing ways that different stakeholders (district and school leaders, mathematics coaches, teacher leaders, and EMS programs) can support EMS to successfully engage in meaningful mathematics leadership while remaining in the classroom. By focusing on the perspective of eight EMS classroom teachers, we shed light on an under-researched group of teacher leaders whose experiences can serve to motivate future research.

Elementary Mathematics Specialists

EMS programs situate the study of mathematics content and pedagogy within a framework of mathematics leadership with the goal of impacting not only the teaching of graduates but also the schools and districts in which they work (Association of Mathematics Teacher Educators, 2013). Courses dedicated to developing leadership capacity provide opportunities to interpret state assessment data, work with administrators, lead professional development opportunities, design community outreach programs, and analyze curriculum resources. Because of these experiences, EMS professionals are uniquely poised to take on a variety of leadership roles, such as specialized mathematics teaching assignments, coaching and mentoring, curriculum leadership, and community outreach (de Araujo et al., 2017). Studies have found that EMS programs have a positive impact on both knowledge and beliefs of participating teachers (Campbell & Malkus, 2014; Gibbons, 2017; Swars et al., 2018; Webel et al., 2018). The use of this expertise in school settings has been most often studied in terms of formal leadership roles, like math coaches, who support teachers to improve their math instruction (Gibbons, 2017; Hubbard & Livy, 2021). Our work adds to this literature by exploring the leadership experiences of EMS who are full-time classroom teachers, sometimes in "departmentalized" roles where they teach mathematics to multiple classes of students (see Markworth, 2017; Webel et al., 2017).

Teacher Leadership

Drawing upon York-Barr and Duke (2004), we define teacher leadership broadly as individual or collective teacher actions that "influence colleagues, principals, and other members of school communities to improve teaching and learning practices with the aim of increased student learning and achievement" (p. 288). When defining the work of teacher leadership, a distinction is often made between formal role assignments and informal work that emerges more organically. This can be an important differentiation for teachers who are trying to establish their legitimacy as a teacher leader (Berg & Zoellick, 2019) and receive recognition for their own agency in improving student learning and achievement (Muijs & Harris, 2007; Sinha et al., 2012; Wenner & Campbell, 2018; York-Barr & Duke, 2004). *Formal leadership* is generally defined as those acts connected to a role formally recognized by an administrative leader in the building, such as serving on a committee or facilitating a professional development session. Informal leadership includes acts not directly connected to a role assigned by an administrator, such as helping a colleague plan a lesson or implement a new math routine. It is common to recognize leadership tasks performed by those who hold formal positions; however, teacher leaders can also play a significant role in enacting change at their school informally by, for example, presenting at faculty meetings, providing input during the decision-making process, and disseminating information (Whitaker, 1995). While designating leadership acts as formal or informal can be useful when describing the range of leadership that can occur, Berg and Zoellick (2019) caution that this distinction alone provides a superficial perspective on teacher leadership that is insufficient for the continued development of the field.

One way to add nuance to studies around teacher leadership is through evaluating the extent to which different leadership tasks have the potential to increase others' knowledge of teaching (Gigante & Firestone, 2008). These tasks, which are referred to as *developmental tasks*, include designing lessons, answering questions about mathematics teaching and learning, and facilitating professional development. In contrast, support tasks are those that support teachers' work without necessarily increasing their knowledge by, for such as managing materials, establishing pacing guidelines, or piloting curriculum. Evaluating leadership tasks through this lens can provide insight into the value they provide others. Specifically, while support tasks can make the work of teaching easier, developmental leadership tasks deepen teacher knowledge, an important factor for improving instructional practice (Darling-Hammond et al., 2009). Though Gigante and Firestone (2008) argue that teacher leaders should engage in developmental leadership tasks with the goal of deepening their colleagues' knowledge and skills in effective instructional practices, researchers have found that instructional coaches often spend large portions of their time instead on support tasks (Kane & Rosenquist, 2019; Knight, 2012). That said, teacher leaders are more likely to engage in developmental tasks when they have access to four resources: time to interact with colleagues; positive relationships with colleagues; opportunities to work on professional development; and administrative support and reinforcement of the teacher leader role (Gigante & Firestone, 2008).

Teacher leaders will likely encounter many challenges engaging in leadership tasks if there is not ongoing work at the school and district level to maintain a culture that acknowledges their legitimacy (York-Barr & Duke, 2004; Wenner & Campbell, 2017). While legitimacy can come from an assigned formal role or position, it can also emerge from recognition of the teacher leader's specialized knowledge and skill (Berg & Zoellick, 2019; Diamond & Spillane; 2016). Principals, in particular, can validate legitimacy by recognizing teacher leaders' expertise, and clearly communicating their roles and responsibilities for leadership (Smith et al., 2017; Wenner & Campbell, 2017). In mathematics, however, researchers have found that administrators may be less likely to view the people in their own buildings as a primary source for instructional leadership (Burch & Spillane, 2003; Spillane & Hopkins, 2013) and teachers more likely to seek support for mathematics instruction from formal leaders with math-specific positions than those without (Spillane & Kim, 2012; Spillane & Hopkins, 2013). On the other hand, there is promising evidence that a positive shift in productive collaboration can occur when more formal leadership is assigned to teachers recognized for their mathematics instructional expertise (Diamond & Spillane, 2016). Beyond challenges related to legitimacy, researchers have also acknowledged the constraining force of a lack of time and opportunities for leadership (Berg & Zoellick, 2019; Markworth, 2017; Wenner & Campbell, 2017; York-Barr & Duke, 2004). This challenge might be particularly pertinent for teacher leaders with full time classroom responsibilities as they are unlikely to have release time for leadership activities, especially ones that fall outside of their regular duties (Smith et al., 2017). While the range of challenges we have described have been applied in prior literature to teacher leaders generally —both with and without formal leadership roles—we suggest that they may be especially salient for those that continue as classroom teachers. Thus, the leadership challenges of teacher leaders with full time classroom responsibilities warrants attention, and our study explores this area.

The research questions guiding our work were:

1. What was the nature of the formal and informal tasks EMS-certified teachers engaged in during the first year after they graduated from the program?
2. What challenges did the EMS-certified teachers face when engaging in leadership while serving as full time classroom teachers?

Methods

We employed case study methodology (Yin, 2014) to investigate the experiences of EMS who remained in the classroom as 3rd through 5th grade teachers. Eight participants were selected as typical cases of elementary teaching assignments (Seawright & Gerring, 2008), with contextual variation between cases. Specifically, the teachers were typical or representative of the range of school size and demographics in our larger study and varied in terms of the types of curriculum used and whether the teachers were departmentalized (teaching mathematics to multiple groups of students) or self-contained (teaching all subjects to a single group of students). Table 1 (next page) provides background information on each of the case study teachers and their broader school contexts.

Case study participants completed a two-year program that was co-designed by faculty across five institutions and included 24 credits of graduate level coursework aligned with the AMTE Standards for Elementary Specialists (2013). The coursework led to EMS certification from the state department of education. The coalition of faculty continued to meet biannually to revise courses and discuss programmatic issues (recruitment, communications with state education administrators, etc.) for the next several years (Goodman et al., 2017). The courses themselves were blended, with online coursework combined with five face-to-face sessions each semester (20 total over the course of the program). There were five content courses, each focused on developing deep knowledge of elementary mathematics concepts, awareness of how children develop this knowledge, and engagement with the kinds of tasks, representations, and discourse that support mathematics learning. There were also two leadership courses, which addressed the history of mathematics education, the role of textbooks and curricular programs, general leadership, and specific mathematics leadership skills like coaching teachers, facilitating professional development, interpreting standardized testing data, co-teaching, conducting

Table 1: Background information about participants

Condition	Amy	Denise	Emma	Joni	Keri	Leah	Mary	Taylor
School Size	480 (K-4th)		450 (K-5th)	100 (K-5th)	650 (K-6th)	520 (3rd-5th)	450 (K-5th)	720 (PK-5th)
% FRL	100%		30%	35%	50%	65%	55%	30%
Math proficiency testing scores[1]	30%	45%	60%	70%	60%	45%	75%	50%
Grade Taught	3rd	4th	4th	5th	5th	4th	4th	5th
Years of Teaching Experience (at grade level)	9(4)	4(1)	4(3)	4(4)	4(1)	4(0)	4(3)	8(0)

Note: Demographic data was taken from the year they were interviewed. All numbers were rounded to preserve anonymity.

lesson studies, and negotiating duties with school administrators.

As a part of the broader study, we conducted five semi-structured interviews with each case study participant over the course of one school year, including two interviews that occurred after observing a math lesson. Two primary leadership questions we asked teachers during the interviews were "Do you feel like you have had opportunities to be a leader in your building or district? (This could be informal, like colleagues asking for math advice or the principal seeking input about a program)," and "If you have not had many leadership opportunities, are there any that you wish you had?" Data for this study included teachers' responses to the two leadership questions above as well as any instance when the teachers discussed leadership opportunities they were involved in while responding to other interview prompts about their school year.

Our analysis process began by reading through the transcripts to identify the sections where the EMS discussed leadership tasks in which they were currently engaged or where they responded to specific interview prompts listed above. After identifying the leadership task each EMS engaged in, we coded them as either *formal* or *informal* based on whether the leadership task was formally recognized by administration or part of a formal structure or routine (York-Barr & Duke, 2004) to gain insights into how each of the leadership opportunities came about. Next, we classified each leadership task as being either *support* or *developmenta*l (Gigante & Firestone, 2008), using the EMS' description of the leadership activity to support this determination. For example, when a teacher described being on a leadership committee, we asked follow-up questions about the goals and tasks of the committee. Committees that looked at student strategies for math content across the grades with the goal of sharing the information with teachers was classified as *developmental*, whereas committees to select new curriculum or analyze testing data were considered *support* since they did not directly help teachers grow in their knowledge of how to teach mathematics. This second classification allowed us to distinguish between instances where the EMS could utilize their expertise to help their colleagues increase their knowledge of teaching elementary mathematics (*developmental*) versus those that were more administrative in nature (*support*).

After categorizing the types of leadership tasks each participant carried out during their first year as an EMS graduate, we analyzed the challenges they encountered while trying to engage in leadership tasks as full-time classroom teachers based on the ones found in the literature (issues around legitimacy, time, opportunities, and administrative support). When analyzing the challenges EMS described, we also looked for ones that did not fall within the previous categories as well as nuance that was specific to the EMS' context as a classroom teacher. After coding the data for each teacher, we conducted cross-case synthesis (Yin, 2014) to look for patterns and variations across each of the participants' experiences. For example, one pattern we noticed across multiple cases were instances where teachers

[1] Standardized testing scores indicate the percent of students who scored either proficient or advanced on the state's end of year test at the teacher's grade in the year *prior* to the interviews.

downplayed the informal conversations they had with colleagues as examples of leadership. We also noticed variation in the types of leadership opportunities afforded to each teacher; for example, some teachers had extensive opportunities to provide leadership in mathematics, including multiple direct invitations from administrators, while in other cases it was unclear whether administrators were aware of the teacher's completion of the EMS program.

Results

Table 2 shows the leadership opportunities EMS case study teachers engaged in within their primary role as classroom teachers. As shown in the table, there was a significant overlap between formal versus informal and support versus developmental leadership tasks. Specifically, the majority of EMS' formal leadership tasks were supportive in nature, while nearly all of the informal leadership tasks were developmental. This finding highlights the potential value of informal activities in supporting teachers' knowledge for teaching mathematics and the need for more formal leadership activities that are developmental. Note that the classification of leadership tasks as being developmental or support reflect the specific contexts of the case study teachers and are not intended to imply that similar leadership tasks in other contexts would have the same function.

Formal Leadership

EMS served in a variety of formal leadership roles that involved both support and developmental leadership tasks. Three EMS (Amy, Denise, and Keri) did not engage in any formal leadership activities. Others, like Leah, had several formal leadership responsibilities, including serving on multiple committees and being selected specifically by administrators to assist with math-specific supports. Some formal assignments were long-term, such as serving on a multi-grade level math committee (Taylor) or facilitating a grade level professional learning team (Joni), while others were short term, such as designing and delivering a professional development session for other teachers (Mary).

A majority of the formal tasks EMS performed were supportive in nature, rather than developmental. For example, Emma described serving on a "vertical planning/PD" committee at the school level. The committee met weekly and discussed issues of content coverage ("this is the three things we have to teach, and then we'll go and day by day… you know, this is fourteen days, how is this going to look") and the use of materials ("combine and share resources"). These discussions did not appear to support teachers to learn how lessons would be enacted, how students might respond, what misconceptions would surface, etc. Leah also described helping her principal analyze data from the multiple standardized tests and end-of-chapter tests that students took throughout the year in order to develop "a data wall that will allow us to look at any kiddo in the school and see, you know, where they are." Similar to Emma's work on the planning committee, this leadership task was supportive in nature because its use was limited to information purposes and was not used to influence instruction or deepen teachers' understanding of students' knowledge of specific mathematics content.

Joni engaged in formal leadership by serving on a team to develop processes for Professional Learning Communities (PLCs) in her school. When describing the tasks of the team, she talked about defining "essential standards" that would provide guidance for teachers about what content they should prioritize in each grade (a support task). But she also seemed to recognize that this fell short of the developmental guidance she felt was needed, making statements like, "we're starting to prioritize our standards with the PLC process…but as far as taking [leadership] further, I haven't," and "we unfortunately start and then haven't always followed through with everything." She lamented that the current approach focused on identifying priority learning standards, saying "there's just so much that I feel we're missing out on." We interpreted these comments as Joni's recognition of the unrealized potential of the PLC work. She saw the identification of essential standards as helping teachers know what content to prioritize in their classroom, but failing to ultimately improve their understanding of the mathematics in the standards or how it could be effectively taught.

Some of the EMS did engage in formal leadership that involved developmental tasks. For example, Mary talked about facilitating a "math lab," a structure established by administrators, in which "teachers who feel comfortable in certain areas can host other teachers within the district" to observe and debrief a lesson highlighting a particular practice[1]. Her math lab addressed mathematical problem

[1] Unlike in math labs reported in literature (e.g., Gibbons et al., 2017), the ones in Mary's district were led by volunteer classroom teachers rather than math coaches or university faculty. Participants in the math lab observed and reflected on the lesson, but were not involved in the lesson planning process.

Table 2: EMS Participants' Leadership Opportunities

	Formal	Informal
Amy		• **Engaged in conversations with colleagues about implementing a math routine in class** • Advocated for team-approach to grade level subjects
Denise		
Emma	• Mentored student teacher • Served on vertical Planning/PD committee	• **Engaged in informal math conversations with colleagues** • Mapped out units with other grade level teachers
Joni	• Served on curriculum committee • Led formation of PLCs focused on math	• **Co-planned with another teacher** • Led math night for parents
Keri		• **Collaborated with spxsecial ed co-teacher** • **Engaged in informal math conversations with colleagues**
Leah	• Served as grade level chair • Served on building leadership team • Served on scheduling committee • Mentored new teachers • Assisted principal in analyzing standardized testing data	• **Engaged in informal math conversations with colleagues** • **Provided feedback on curriculum enactment in a colleague's classroom**
Mary	• **Facilitated course for EMS** • **Led embedded PD within district on a math practice (with peer observations)**	• **Engaged in informal mentoring of colleagues** • **Planned and led discussions around math lessons for 4th grade team**
Taylor	• Served on 3rd – 5th grade math leadership team at school • **Served on math committee in district**	• **Co-planned with another teacher**

Note: Bolded leadership activities were developmental tasks, while the remaining activities were support tasks.

solving; she used the opportunity to challenge the idea that problem solving was synonymous with "doing word problems." She discussed the need to "shift our paradigm here a little bit from what we in the past have thought of problem solving," and described follow up conversations with teachers about specific teaching challenges. In contrast to EMS who described formal leadership opportunities as primarily focused on the scope and sequence of lessons, Mary was able to use her session to promote teacher learning about a specific mathematical practice. Similarly, Taylor had a formal leadership role as a member of a district mathematics curriculum committee. The task force "spent some time looking at how different strategies that students use to add, subtract, multiply and divide across the grade levels" and described "building a foundation for repeated addition or multiplication." This type of document had the potential to help her colleagues anticipate possible strategies that students might use and how they might connect these strategies to support students in building on their prior knowledge. While multiple EMS were able to engage in leadership tasks focused on mathematics, only the tasks completed by Mary and Taylor appeared to contribute to improving their colleagues' knowledge of teaching mathematics.

Informal Leadership

In contrast to the formal leadership tasks that were primarily focused on support, nearly all of the teachers' reported opportunities for informal leadership were developmental in nature. These opportunities included engaging in hallway conversations about teaching particular mathematics topics and working with teachers after school to expand the strategies students used during instruction. For example, Amy talked about helping other second-grade teachers strengthen their classroom discussion during the *Rocket Math* (a math fluency program) portion of the lesson.

> I get the problems I'm gonna use from the top of my *Rocket Math* sheets, but I get my strategies from the Van de Walle book….The 2nd grade team, they all came up one day after school. We spent like an hour and a half in my room, going through what we do in *Rocket Math*, and they've started doing it, and they're like 'it's amazing, you should see what they're doing.'[2]

This informal opportunity to support colleagues in improving their mathematics instruction emerged from other teachers recognizing and seeking out Amy for her mathematical expertise. Instead of focusing on procedures and skills as intended by the *Rocket Math* program, Amy supported other teachers to develop students' flexibility with different computational strategies.

> Another form of informal leadership common among the EMS teachers was sharing resources and co-planning with others. Keri shared that, like Amy, others had sought her out for information about teaching particular content.

There is a fourth-grade teacher, she went to the Guided Math [professional development program] with me, and she went back and she is like "I am trying to teach angles and I can't find, they are not getting it, do you have any suggestions for lessons?" So I gave her some stuff that I had found or that I thought would be good for them to understand it.

Through this informal leadership, Keri used her mathematics expertise to offer another teacher access to information and advice about mathematics teaching. Similarly, Leah described teachers stopping her in the hallway to ask for advice about how to teach specific content. "Another teacher asked me, I'm struggling with getting my kids to learn long division, like can you offer me some suggestions? So a lot of things [informal leadership] like that have been, oh we know you're the math person so…." Leah referenced the fact that both her administration and colleagues saw her as "the math person in the building" due to her expertise as an EMS multiple times when describing the varied leadership tasks she completed throughout the school year.

Looking across the case study participants, we found that the formal leadership opportunities EMS engaged in tended to be support tasks while nearly all of the informal leadership opportunities were developmental in nature. This pattern highlights a strong willingness among EMS to utilize their expertise to engage in developmental tasks, or as Helterbran (2010) put it, to take ownership of instructional problems and to engage collaboratively with colleagues to lead innovative efforts to improve instruction. It also affirms teachers with content and pedagogical expertise as effective interpreters of the instructional functions of teacher leadership tasks (Spillane 2000).

Challenges EMS Faced in Engaging in Leadership

Although nearly all of our case study teachers performed some leadership tasks, many EMS also described challenges they faced in engaging in leadership while maintaining their primary role as a classroom teacher. In this next section, we describe four categories of challenges faced by some of the teachers: a lack of opportunities to engage in leadership; a mismatch between the EMS and administration or other teachers' expectations for the leadership tasks; a lack of clarity regarding formal and informal leadership roles; and, in the case of Denise, the administration not recognizing her mathematics expertise.

A Lack of Time and Opportunities to Engage in Leadership

Multiple EMS referenced not being able to take on more leadership tasks due to time constraints or being the only math teacher at their grade. For example, Joni described not being able to exercise leadership "as much as I would like to" because of time limitations. "Honestly it comes back to time and I feel like as a school, we're just spread thin as far like what we're trying to do with our time and energy and things like that." Joni's expression of feeling stretched thin was common among the teachers. Leah, our case study teacher who engaged in the most leadership tasks, also expressed being overwhelmed by the amount

[2] This is a reference to *Elementary and Middle School Mathematics* by Van de Walle, Karp, & Bay-Williams (2013), a book used in several of the courses in the EMS program.

of time she spent on multiple support and developmental leadership tasks on top of her regular duties as a classroom teacher.

> It ended up being kind of frustrating because I was giving up so much of my time to [leadership tasks], and I didn't mind doing that because I knew it was benefitting other teachers, it was benefitting those kids, but then still having my same workload as everybody else and having you know almost no plan time got to be very frustrating by the end of the year.

While Leah was interested and willing to engage in a variety of leadership tasks to help support her colleagues and school, the workload quickly became unsustainable due to the leadership tasks being unpaid labor that had to be either squeezed into her regular teaching day or performed outside of traditional school hours.

Another issue that surfaced for Denise and Joni was a lack of opportunities to engage in leadership due to being the only math teacher at their grade level. In Denise's case, her school had a "departmentalized" structure where she taught math to the entire fourth grade in three separate sections while the other 4th grade teachers taught the remaining subjects. As a result, she described having limited opportunities for collaboration across content areas: "we've done unit stuff, like, 'what can we do that relates?' But not collaborative." Elsewhere she commented, "We're all departmentalized so [we are] not sitting and lesson planning together." Without such opportunities, Denise struggled to position herself as an expert to whom colleagues could turn for advice.

Like Denise, Joni did not have any opportunities to collaborate with colleagues due to being the only 5th grade mathematics teacher at her school. Despite this, Joni found ways to collaborate electronically with a 5th grade teacher at a different school in her district who sought her expertise. While she found this collaboration to be beneficial, she reflected that "it's not as collaborative as if we were teaching in the same building or even if, I mean, it's just not the same. It can't be, it isn't, and the dynamics of the school system and whatnot." These challenges described by Leah, Denise, and Joni seem to be immediately related to their role as EMS classroom teachers. That is, as continuing classroom teachers, EMS' leadership practice must necessarily fit within the constraints of full-time teaching responsibilities including sometimes being the sole grade-level mathematics teacher without colleagues to collaborate with.

Mismatched Expectations for Leadership Tasks

Some EMS also expressed challenges that highlighted a mismatch between their goal for leadership tasks to be developmental with others' goals for it to stay at the support level. For example, Leah described having "mixed feelings" about her work with the principal in analyzing standardized testing data, specifically around the lack of plans for how to use the analysis to inform instruction.

> It's okay to look at data, but then I think we need to be doing something with that data and that data needs to be driving something and it's really not at this point…. I don't see us like—okay so our kids are low in math, we're seeing a trend, so what are we doing to change that? Nothing…. It's like you know we're assessing our kids to death, just for data points for what? I don't even know. Um, just to prove that I guess we're in here teaching.

Although Leah had the knowledge and skills needed to interpret the standardized data and identify ways it could be used to inform instruction, she was not able to enact this vision because it conflicted with what she understood to be her principal's goal of using the data for tracking purposes.

Similarly, Taylor experienced conflicts between her vision for supporting mathematics instruction and the expectations of colleagues. Within her role as a member of the district's math curriculum committee, Taylor worked with colleagues to create a document describing student strategies for whole number operations across the grade bands. She recognized the potential for this document to help other teachers make informed decisions in their classes, but found that her fellow committee members seemed disinterested in extending their work beyond the committee meetings.

> It's difficult, it's, in the sense of, people are happy to be on the committee and to talk about math, but only so much is going to be solved when you meet for 20 minutes once a month. […] The struggle has been now that we have this information of different strategies that students use, what are we going to do with it, and that's where I see people saying, we have the document, we're happy with that versus what are we going to do

> with it. So that's been the struggle of bridging it from now we have this information, how can we make sense of it and use it to benefit our teaching and our instruction and our students' learning.

Taylor's expertise enabled her to recognize the value in the committee's work and see the potential for it to support teachers' learning of student strategies for whole number operations; however, she desired for their work to more directly connect to teachers' practice. At the time of our interview, Taylor was still trying to navigate her role on the committee and find opportunities to extend the work they did to support the instruction of other classroom teachers.

Lack of Clarity Regarding Formal and Informal Leadership Roles

One of our case study teachers, Amy, was in the unique position of having spent ten years as a math and literacy coach before returning to be a full-time classroom teacher. She taught in a school district that had a formal math coach; this math coach had completed the EMS program alongside Amy and Denise. Given her prior experiences, Amy expressed being initially cautious when trying to engage in leadership tasks within her role as classroom teacher. "I had to kinda feel things out and find where I fit, a little bit, before people ever started asking me about math." By the end of the school year, Amy reported that she had begun to informally "coach afar" when colleagues asked her questions; however, she still seemed to be figuring out how to engage in leadership tasks in a way that did not clash with the work being done by the math coach. "I'm slowly getting out into the school, but...I also don't want to overstep my bounds. Because I was a coach, and I don't want my coach in my building thinking that I'm trying to overstep her, you know." Amy's reflections highlight a tension that can arise between those who hold formal leadership positions and those who seek to engage in leadership tasks in a more informal capacity from the classroom. While it is certainly natural to look to those in formal leadership positions for guidance, the presence of a math coach or specialist should not preclude others from also engaging in leadership tasks. Based on our observations and interviews with the classroom teachers, we believe that Amy and Denise would have benefitted from additional clarity around the formal and informal leadership roles and a broader discussion around ways they could support the math coach in engaging in mathematics leadership at their school. As evident from other cases in this study, full time classroom teachers can engage in meaningful leadership tasks despite not having a formal leadership position.

Administration Not Recognizing the EMS' Mathematics Expertise

Unlike the other EMS, Denise struggled to engage in any leadership task, formal or informal, at her school. Her difficulties in engaging in leadership appeared to be due to two main factors: first, the way that fourth graders were divided into classes at her school limited her opportunities for collaboration; second, her administration and colleagues did not recognize her mathematics expertise. Instead of automatically assigning all students to multiple fourth grade classrooms, Denise's school also had a mixed-age class (2nd-4th graders) that selected a small group of students to attend based on their application. Even though Denise taught nearly all of the fourth graders, including special education students and English Language learners, her principal would compare her scores with those of the students who were in the mixed-age class.

> Being honest, I feel I'm in the underdog position, 'cause I'm always compared. I have the majority of the fourth grade compared to our mixed age class. And they only have like five percent…And so, they always compare those scores to my scores. And they're always wanting me to ask her what is she doing differently.

Although Denise had developed specialized content knowledge and leadership capacity through the EMS program, she was positioned by administration as a teacher needing additional support because her students had lower test scores than those fourth graders in the mixed-age class. Denise characterized her principal's view of expertise and legitimacy as coursing primarily through the lens of standardized test scores. Given the inappropriate comparison across two demographically different groups of students, she felt her expertise went largely unacknowledged. That administration did not recognize Denise's expertise played a significant role in her leadership opportunities given her role as an EMS classroom teacher who did not have authority and legitimacy afforded to her through a formal leadership position.

Discussion

Through completing EMS programs, teachers deepen their knowledge of elementary content and reinforce principles of high quality mathematics instruction across the elementary curriculum (Campbell & Malkus, 2014; Swars et al., 2018). While developing their content knowledge for teaching is a large component of EMS programs, graduates also gain leadership skills that allow them to engage in tasks not typically assigned to classroom teachers (e.g., analyzing standardized testing data, leading professional development, and coaching or mentoring other teachers) (McGatha & Rigelman, 2017). To date, the majority of the EMS research focuses on formal leaders, like mathematics coaches (de Araujo, 2015). The experiences of our case study teachers (Table 2) contribute to this literature by highlighting some of the varied ways that EMS can engage in leadership while remaining in their primary role as classroom teachers. Although both support and developmental leadership tasks can assist teachers in fulfilling their teaching responsibilities, only developmental tasks increase human capital and can lead to long-term gain in teachers' knowledge (Gigante & Firestone, 2008). We extend this literature by showing that the informal leadership tasks our case study participants engaged in provided them with more opportunities to increase their colleagues' knowledge of teaching mathematics (developmental) than the formal leadership tasks. These findings also provide additional support to findings that coaches with formal leadership positions often spend large portions of their time on logistical and administrative activities (i.e., support tasks; Knight, 2012; Kane & Rosenquist, 2019) rather than developmental tasks.

Collectively, our case study teachers had the expertise and willingness to engage in leadership tasks but had varied opportunities to do so within their primary role as a classroom teacher. We identified four types of challenges some of them faced as they navigated their role as a teacher leader: a lack of opportunities to engage in leadership due to limited time or few colleagues who taught mathematics; a mismatch between the EMS and administration or other teachers' expectations for the leadership tasks; a lack of clarity regarding formal and informal leadership roles; and the administration not recognizing the EMS' expertise. Our findings support earlier research on the various challenges teacher leaders—those with and without formal positions—face (Wenner & Campbell, 2017; York-Barr & Duke, 2004) and extends it by describing how such challenges might be particularly salient for those that continue as classroom teachers (Smith et al., 2017). For example, some EMS teachers (Denise and Amy) struggled to establish legitimacy or clarity around their leadership without a formal position, while others (like Joni) were constrained by a lack of time stemming from their full time classroom responsibilities.

Given the importance of developmental tasks (Gigante & Firestone, 2008), and the legitimacy that comes with formal roles (Berg & Zoellick, 2019), we believe there is a need for more structures in place within schools and districts that supports teacher leaders to engage in formal leadership tasks that are developmental in focus. As EMS take on such leadership tasks, however, it is important that administrators and other stakeholders understand and recognize the challenges they may face and seek ways to empower and support these instructional experts in their improvement efforts. In the following sections, we draw on the experiences of the case study participants to provide recommendations for ways that different stakeholders can support EMS in engaging in leadership while maintaining their role as a classroom teacher.

Implications for Practice

District and School Administrators

A common theme across the experiences of our case study participants was the important role that their principals played (or did not play) in helping them share their expertise and engage in leadership. In particular, the cases of Leah and Denise demonstrate the importance of administration recognizing the mathematics expertise teachers gain through EMS programs and publicly legitimizing them as a leader and resource for other teachers. Alongside publicly acknowledging the EMS' expertise, principals can also support the teachers in expanding their view of what counts as being a teacher leader to encompass both formal and informal leadership tasks and giving them more space to shape the nature of their tasks, including freedom to make them more focused on teacher learning (developmental). Like most of the teachers in our study, EMS may already have colleagues who hold them in regard as leaders because of their expertise and interactive styles. However, this study corroborates conclusions from Spillane & colleagues over multiple studies (Burch & Spillane, 2003; Diamond & Spillane, 2016; Spillane et al., 2003) that teacher leaders still depend upon administrators to legitimize their leadership, to develop a shared instructional vision, and to provide supportive structural and organizational arrangements, such as schedules that support collaboration and the creation of subject-specific formal leadership positions. Administrators can also provide more space for specialists to shape the nature of their formal tasks, including freedom to make them more focused on teacher learning (i.e., developmental) and less on logistics or program support. Finally, administrators can help to minimize hesitancy some EMS might experience through engaging in regular conversations with them around the types of leadership opportunities they are interested in pursuing and supporting them as they navigate those spaces.

Smaller districts in particular can utilize the expertise of EMS through providing and supporting them to engage in varied leadership at their school while still retaining their primary responsibility as a classroom teacher. The example from Mary of having colleagues observe her teaching and then participate in discussions before and after the lesson is one way that EMS can share their expertise with colleagues without a significant time commitment. For leadership tasks that require a significant time commitment, schools should consider ways to support the EMS by giving them additional time in the school day for leadership tasks (e.g., through not having to teach a particular subject or receiving a release from non-academic supervision duties) and/or providing additional pay for the increased duties. While achieving this would likely require creativity on the part of the administration, it would provide increased opportunities for support and professional development for other teachers without having to fund a full-time mathematics coach or specialist.

Mathematics Coaches

Educators who are employed formally as mathematics instructional coaches have the opportunity to leverage the expertise of EMS in their district to form a network of mathematics instructional leadership. Mathematics coaches can serve as advocates for teacher leadership as they highlight pockets of expertise among the teachers with whom they work, spurring elementary administrators to view *inside* expertise as the most important factor for shifting mathematics instruction and achievement in their building. Mathematics coaches should push to engage a high percentage of developmental work (e.g. collaborative task/lesson design, modeling or team teaching, facilitating professional development with teachers) for two reasons – these have been shown to have greater impact on increasing teacher skill and knowledge and these kinds of tasks put coaches in-the-know about which teachers have developed or are developing the expertise to lead innovative mathematics instruction in their building. Mathematics coaches can also put promising teachers in touch with resources for developing further expertise, such as EMS training and certification programs.

Teacher Leaders

We also encourage teacher leaders, including EMS, to expand their views of leadership beyond formal opportunities such as facilitating professional development and serving on committees. Sharing their knowledge and expertise about mathematics teaching in informal settings, like hallway conversations, are important forms of leadership as these were often developmental in nature and contributed to the learning and practice of colleagues. These forms of leadership may be particularly effective by EMS that continue as classroom teachers since others may view them as more credible sources than formal leaders who are no longer teachers (Spillane & Kim, 2012). And, because teacher leadership is rooted in how others perceive expertise and legitimacy (Berg & Zoellick, 2019), we also encourage teacher leaders to publicly share with colleagues and administrators their emerging knowledge and forms of expertise and how these may support the school community.

When seeking out leadership opportunities, we encourage EMS to be selective when taking on leadership roles and to specifically look for tasks that allow opportunities for sharing and developing content knowledge expertise. This will maximize the limited time they have for leadership activities and minimize burnout. For example, Leah's desire to be a teacher leader at her school resulted in her taking on multiple roles, including ones that were not specific to mathematics. By the end of the school year, she expressed frustration with the amount of unpaid leadership tasks she was completing each week and was left feeling like they were negatively impacting both her professional and personal life. We argue that spending time working closely with colleagues to advance their knowledge of teaching will result in greater change in the long run than serving in formal leadership roles that are focused primarily on logistics or programmatic concerns.

In addition, our results suggest that teacher leaders might advocate for more say regarding the nature of their formal tasks. For example, Joni expressed disappointment about the task of assigning priority standards for each grade level, seeming to recognize the limitations of the task in influence teacher knowledge or practice. She had ideas for more productive activities for Professional Learning Communities, but apparently was not able to advocate for these ideas. We hope that this article provides some support for teacher leaders to use in negotiations with administrators as they lobby for more attention to developmental work, and for more power in deciding the kinds of activities that they are charged with carrying out.

Elementary Mathematics Specialists Programs

In addition to providing opportunities for teachers to develop leadership skills in formal roles, our findings suggest that they should also specifically prepare graduates to 1) advocate for increased attention to development tasks within those roles, as noted in the previous point, and 2) engage in developmental activity through informal means (e.g., hallway conversations with colleagues, team planning meetings). Joni and Taylor in particular articulated frustrations with the progress being made in their schools, lamenting that there was not more attention to issues of teaching and learning within the formal structures for support for mathematics teaching. These issues could be the focus of conversations and problem solving within EMS programs. In addition, EMS programs could engage in advocacy for their graduates by communicating with school administrators and providing suggestions for leadership tasks and roles that might empower EMS in order to support teacher learning.

Areas for Future Research

One of the key findings from this study was the tendency for informal leadership tasks to be developmental and formal leadership tasks to be supportive in nature. Future research could further explore this potential relationship in order to better understand the contextual features that resulted in only some of the leadership tasks lending itself to increasing colleagues' knowledge for teaching mathematics as well as explore whether the pattern of both developmental and support tasks occur in other contexts. For example, to what extent can the presence of developmental leadership tasks be attributed to the specialized content knowledge teacher leaders gained through the EMS program?

While this study focused on the teachers' perspective, future research could provide a more nuanced understanding of the impact EMS have on the knowledge of their colleagues through incorporating the voices of those who interacted with them while they were engaging in leadership acts. For example, to what extent do colleagues report an increase in their knowledge of teaching mathematics as a result of the EMS' formal or informal leadership acts? In what ways, if at all, do these leadership acts result in increased collaboration among teachers? Administrators also played an important role in terms of expanding (in the case of Leah) or constraining (in the case of Denise) the EMS' opportunities for leadership. Future research could incorporate administrator's voices to better understand the extent to which they were aware of the EMS' interest in engaging in leadership and their perspectives in how to address the challenges EMS faced in their leadership endeavors.

Final Remarks

EMS programs were developed with the goal of improving elementary mathematics instruction by equipping teachers with specialized content knowledge, teaching practices, and leadership skills (AMTE, 2013). While some graduates of EMS programs go on to formal leadership roles (e.g., as a mathematics coach or instructional specialist), others choose to stay in their positions as full time classroom teachers. EMS-certified classroom teachers are well-positioned to support sustained reform in mathematics instruction because of their regular, informal interactions with colleagues and their ability to maintain status as a peer and collaborator (Spillane & Kim, 2012). As illustrated by our case study participants, EMS can share their expertise with colleagues through engaging in both formal and informal leadership tasks such as participating in math-related committees, leading professional development, and informally sharing resources and teaching strategies. Navigating the role of teacher leader is not without its challenges. Various stakeholders can support EMS in overcoming these challenges by publicly recognizing their expertise and positioning them as a leader, providing time within the regular school day to engage in leadership tasks, seeking out ways for EMS to engage in developmental rather than support leadership tasks, and providing clarity in leadership expectations in situations where there is someone in a formal leadership position. ✪

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Inclusion and Intervention: Understanding "Disability" in the Mathematics Classroom

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Abstract

All students' learning—including students with learning and intellectual disabilities—is deepened when students with multiple ability levels engage in teamwork on high cognitive demand tasks. Yet, we know little about supporting teachers in inclusive mathematics classrooms. This knowledge void presents challenges for mathematics education leaders who wish to foster inclusion. Synthesizing a small but growing body of mathematics education research, this manuscript is a resource for leaders supporting teachers in inclusive standards-based classrooms. In particular, this manuscript articulates (1) why productive struggle is essential for students with disabilities, (2) progressive definitions of disability and inclusion, and (3) conceptual descriptions of pedagogy in inclusive mathematics classrooms. It is followed by an appendix filled with tangible strategies that mathematics education leaders can adopt and adapt in their own contexts.

Introduction

Mathematics education leaders work tirelessly to help teachers learn how to best support their students. Yet, little is known about how to support students with learning and intellectual disabilities to be successful in inclusive general education classrooms that feature productive struggle on high cognitive-demand tasks. This manuscript synthesizes what is known about supporting students with learning and intellectual disabilities to meaningfully participate and learn in classrooms based on the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School, 2010). Most research on this topic is qualitative and conducted from a sociocultural perspective. Qualitative research is not designed to make claims about causality. Thus, this paper does not make claims about "what works" at scale. Instead, it informs how and why particular methods support students' learning. This manuscript is not an exhaustive review of existing research. It draws on literature purposefully to answer the following question:

> *How can mathematics educators support students with learning and intellectual disabilities to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks in inclusive classrooms?*

Thus, this manuscript will help mathematics education leaders better support teachers to practice equitable ambitious instruction for all students in inclusive classroom settings.

The first section describes why productive struggle is essential for doing mathematics, including for students diagnosed with disabilities. The second section draws on multiple research paradigms to discuss definitions of disability and the differential impact of these definitions on students. Mathematics education leaders can use these sections to help expand views on who inquiry learning is appropriate for and what it means to teach students with disabilities. After providing this conceptual underpinning, the third section explores what research says about supporting students with disabilities to learn mathematics through instruction involving authentic problem-solving. Mathematics education leaders can use this section's synthesis of research to help teachers expand their teaching practice to create inclusive learning environments. This manuscript's appendix contains six tables summarizing the evidence-informed strategies from the third section so that mathematics education leaders can try them out with teachers in their unique contexts. Overall, this manuscript provides conceptual and practical resources for reframing disability and providing quality instruction that gives all students access to collaboration with their peers, productive struggle, and cognitively demanding tasks.

A Moral Imperative: Fostering Productive Struggle for All Students

Much research in mathematics education is grounded in the idea that learning mathematics requires problem-solving. Mathematical problem-solving is not simply completing a task, it is doing something non-routine, perplexing, and difficult for the doer (Schoenfeld, 1988). In other words, mathematical problems are only problems insomuch as they create a sense of struggle for the problem solver. Struggle on mathematical tasks happens when students put effort into making sense of mathematics. This struggle is productive because it produces deep learning. Warshauer (2014) describes tasks' affordances for fostering productive struggle as related to cognitive demand, with the following types of tasks moving from having the least cognitive demand to the most cognitive demand: tasks that require primarily memorization, tasks that require using procedures without requiring conceptual understanding, tasks that require using procedures with conceptual understanding, and tasks that require doing math, engaging in high-level tasks that are worthwhile and not-straightforward. Thus, tasks requiring (a) conceptual understanding and (b) doing math most reliably foster productive struggle.

Mathematical tasks should be designed to foster productive struggle for all students, including students with learning disabilities and most intellectual disabilities. For example, the tasks in CPM Educational Program's materials support problem-based learning; they have a low floor and high ceiling. Low floor tasks allow students who have not mastered all relevant prior mathematical knowledge to get started on tasks. High ceiling tasks have task extensions that challenge students with rich relevant prior knowledge.

During work on such tasks, equitable mathematics learning is made possible through collaboration in heterogeneous student teams (Cohen & Lotan, 2014). When students of multiple ability levels work together on tasks whose solution strategy is not readily apparent, learning is deepened for typically achieving students, gifted students, and students who have been diagnosed with learning and intellectual disabilities, to name a few. Denying any student the inherent struggle of mathematical problem-solving simultaneously denies them much more: It denies them meaningful mathematics learning, opportunities to develop critical thinking, and the joy of aha moments. In the words of educational researchers Akyuz and Stephan (2020):

> Critical thinking, problem-solving, and modeling is necessary for twenty-first-century employment… and to withhold inquiry mathematics instruction to students with disabilities is immoral. … [And while] it is clear that direct instruction increases achievement [for students with learning disabilities] … there is no evidence that this is the only type of effective instruction for students with disabilities (pp. 2-3).

It is the moral responsibility of mathematics education leaders to establish inclusive teaching environments and support teachers with mathematics-specific teaching practices that make inquiry instruction through standards-based curricula available and accessible to students with disabilities.

Appropriateness of standards-based curricula

Students with identified exceptionalities can learn through standards-based mathematics curricula (Lambert & Sugita, 2016). In other words, students with identified exceptionalities can meaningfully learn from curricula in which:

1. mathematics is encountered through problem-solving,
2. mathematics is embedded in contexts such that mathematical strategies and topics are connected to real-world applications,
3. mathematics emerges through collaborative teamwork and with mathematical tools (e.g., algebra tiles, calculators), and
4. mathematics begins with student-invented strategies rather than standard algorithms (Jitendra, 2013).

Because standards-based mathematics curricula can be made accessible to students with identified exceptionalities, they also should be made accessible to these students. While some argue that working on grade-level content in standards-based curricula steals time from learning skills useful for life after school, Courtade et al. (2012) provide seven substantive reasons for why standards-based curricula are appropriate for students with intellectual disabilities:

1. Standards-based curricula are part of all students' right to a full educational opportunity.
2. Standards-based curricula are relevant for students with disabilities.
3. Students with disabilities seem to learn standards-based content and use it in their lives in unique ways.
4. Functional skills are not a prerequisite to academic skills.
5. Standards-based curricula are not a replacement for functional curriculum.
6. Individualized curricula are limited when they are the only curricula.
7. Students with disabilities show that they want to learn with their peers and succeed.

Beyond these practical justifications for inclusion, the *Individuals With Disabilities Education Improvement Act* (IDEA regulations, 2016) states that students with identified exceptionalities should be "To the maximum extent appropriate … educated with children who are nondisabled" except if "education in regular classes with the use of supplementary aids and services cannot be achieved satisfactorily" (IDEA Sec. 300.114). In other words, all students have the right to inclusive education in the "least restrictive environment" with their peers. Unfortunately, even though IDEA has essentially existed by other names since 1975, almost 50% of students receiving special education services still spend over 60% of their day in segregated (or "dedicated") special education interventions (Wehmeyer et al., 2021). More recently, the Supreme Court ruling Endrew F. v. Douglas County School District created the precedent that the word "appropriate" in IDEA means appropriately ambitious, in that students with identified exceptionalities have the right to meet challenging objectives and fulfill their potential for growth (Wehmeyer, 2019; Wehmeyer et al., 2021). Unfortunately, students are given increasingly fewer opportunities for participation in general education classrooms as they progress through the system (Cook & Cook, 2020).

The overrepresentation of Black and Brown bodies in special education

The problem of funneling students with identified exceptionalities out of general education classrooms is exacerbated for Black and Brown students. These students are disproportionately excluded from inclusive settings (Cook & Cook, 2020, p. 137, citing Skiba et al., 2006; Sullivan, 2011). The achievement gap, or, more appropriately, the opportunity gap and overrepresentation in special education is due to systemic racism through structures such as educational resource allocation, inappropriate curriculum and pedagogy, and inadequate teacher preparation (Annamma et al., 2013; Blanchett, 2006). In other words, critical scholars argue that the overrepresentation of Black and Brown bodies labeled as disabled is the result of (a) carceral pedagogies that emphasize compliance and memorization over critical thinking and problem-solving and (b) lack of access to standards-based curriculum (Annamma, 2017, 2018). Studies that counter these claims self-admittingly do not account for critically important moderators such as:

1. geographic location (e.g., that U.S. states with histories of slavery, de jure or de facto housing segregation, and discrimination are more likely to overidentify otherwise similar minority children than states without such histories),
2. disability type (e.g., that minorities are overidentified for more stigmatizing disability conditions, including intellectual disabilities and behavioral disorders), and
3. how the disproportionality may fluctuate in regard to context and setting (e.g., that minority children with disabilities are more likely to be placed in segregated or restrictive settings than otherwise similar White children with disabilities) (Morgan et al., 2017; Morgan et al., 2018, p. 12).

In general, educators need increased awareness that white children receive better educational healthcare when educators are more responsive to white parents' concerns. They also need increased awareness that minoritized students' parents may be more hesitant to seek out or accept a disability diagnosis due to the historical and experienced marginalization and criminalization of Black and Brown bodies in school buildings (Gregory et al., 2010; Guerrero et al., 2011; Morgan et al., 2013).

Mitigating the consequences of these potential tendencies will require that mathematics education leaders provide teachers with meaningful professional learning opportunities to examine implicit bias and critique institutional procedures that may need revising. The following section describes disability and related concepts in ways meant to help mathematics education leaders and the teachers they support to examine their implicit biases towards students diagnosed as disabled. Examining educators' implicit biases is essential in shifting students' educational experiences.

What is disability?

What is disability? The answer to this question is ideological and thus contested, with implications for appropriate designs for learning. There are three primary answers to this question (Thurber et al., 2018):

1. ***Medical model:*** A disability is a deficiency or abnormality.
2. ***Social model:*** A disability is a difference.
3. ***Cultural model:*** A disability is a valuable form of human diversity.

These different perspectives on disability are rooted in distinct models of disability. These latter two models of disability are closely related to each other and build on the work of disabled disability activists (Union of Physically Impaired Against Segregation & The Disability Alliance, 1975). These models argue that associating negative labels with neurological differences is problematic. The cultural model of disability goes further than the social model: it argues that neurological differences are valuable contributions to human diversity (i.e., neurodiversity; Armstrong, 2012) rather than a neutral difference. Thus, the cultural model flips the medical model's difference-as-deficiency framing to difference-as-exceptionality.

The majority of research on learning disabilities has been conducted under a behaviorist learning theory, which aligns with the medical model of disability (Lambert & Tan, 2016). Like the medical model of illness, this perspective of disability locates the cause of disabilities within individuals and aims to diagnose (i.e., identify error patterns in mathematical problem-solving), diminish, and correct perceived deficits through remediation (Lambert & Tan, 2016; Thurber et al., 2018).

Unsurprisingly, this has resulted in a plethora of research investigating the effectiveness of interventions that break down tasks into small chunks in hopes of remediating through simplifying. This approach can effectively support students to perform better on discrete tasks. However, it largely fails to support conceptual understanding because it tracks students away from high cognitive demand tasks (Bannister, 2016; Tan et al., 2019; Woodward & Montague, 2002). This segregation is unfortunate, as students with neurological differences can learn conceptually complex mathematics well enough to major in mathematics, even when those neurological differences cause them to struggle with early mathematics such as number sense (e.g., dyscalculic tendencies with compensatory aspects; Lewis & Lynn, 2018).

Following the cultural model, this paper locates the source of disability within social institutions and processes, including the physical and social environment, rather than within individuals. Ability and disability are not inherent nor static; they are socially constructed in particular teaching contexts (Lambert, 2015). Thus, instead of remediating children, mathematics education leaders must remediate learning environments to increase accessibility.

Reframing neurological differences as valuable diversity

Here, learning and intellectual disabilities are defined as neurological differences resulting in different needs than typical neurological development. Neurodiversity encapsulates a wide range of neurological differences, including dyscalculia, ADHD, and autism, to name a few. These diagnosed differences all occur on a spectrum. Acknowledging that there is a great deal of diversity within disability, speaking broadly, in this paper the term exceptionality encompasses learning and intellectual disabilities. Some papers cited herein focus on specific disabilities, yet, the instructional practices described are beneficial for all students, those with and without disabilities. When studies focus on specific diagnoses, those terms (e.g., autism, dyslexia) are used.

In regards to labels, individuals have preferences about person-first (i.e., a person with a disability) or identity-first (i.e., a disabled person) language, and these preferences should be honored in personal interactions. The remainder of this paper uses the term "students with identified exceptionalities" for its qualities of putting the person rather than the difference first ("students … with"), for qualifying that identity-labels are socially constructed and not inherently important ("identified ..."), and for pointing to difference rather than deficit ("exceptionalities"). This new label does not romanticize the real challenges faced by students with identified exceptionalities. It reframes and relocates the source of these challenges. Of note, some disabled disability activists and others dislike terms other than the medical diagnosis terms such as learning disability and intellectual disability; they find terms like "students with identified exceptionalities" to be infantilizing. To challenge historical and commonplace views of disability which can constrain students' access to meaningful learning opportunities, we use this phrase without intent to infantilize and hope that this explicit acknowledgment makes our intent match impact.

In this paper, the phrase "students with identified exceptionalities" encapsulates diagnoses of learning disabilities (e.g., dyslexia), mathematical disabilities (specifically mathematics learning disabilities such as dyscalculia), and mathematical difficulties, and intellectual disabilities (e.g., autism). Non-sensory physical disabilities are not addressed in this paper because we take it as common sense that students with physical disabilities should receive adequate supports to engage in productive struggle on cognitively demanding tasks, including work with materials required to engage in such tasks.

Sensory disabilities, such as visual or hearing impairments, are included in this paper. Equitable classrooms are not classrooms where students learn simply through reading a textbook (inequitable for those with visual impairments) or through listening to lectures (inequitable for those with hearing impairments) but are places of multifaceted pedagogy and opportunities to learn. It follows that simply including students with identified exceptionalities in the classroom is not sufficient. Inclusion is not only about keeping student bodies in the classroom; it is not about place. Inclusion requires the general education classroom to be a place where all students experience effective instruction. This is a matter of equity in education; it requires transformational instruction that creates equitable learning opportunities for learners with a wide variety of needs.

Rehumanizing mathematics education for students with identified exceptionalities

Students with identified exceptionalities have experiences of being disabled due to the design of society to favor the needs of the average or neurotypical person. Although the education system is designed to highlight these children's and youths' differences, they are children and youth first. It is the job of mathematics education leaders to rehumanize the educational experience of students with identified exceptionalities. Many of these students have similar learning preferences as their neurotypical peers. For example, Klingner and Vaughn (1999) found that students labeled with learning disabilities tend to prefer the same activities, homework, books, grading, and grouping as their peers without similar labels (Rexroat-Frazier & Chamberlin, 2019). They also found that these same students valued clear explanations, experiencing content in multiple ways, and responsive lesson pacing. Arguably, these are features of teaching and learning that all students might value, with or without identified exceptionalities. Listening to the voices of students with identified exceptionalities is a start to rehumanizing their experiences in mathematics classrooms.

To have conversations where students with identified exceptionalities can reflect on and share their experiences of meaningful learning, we must lift the veil of secrecy in diagnosis. Too often students often experience secrecy around their diagnosis (Lambert et al., 2019; Rexroat-Frazier & Chamberlin, 2019; Vaughn & Klingner, 1998). This secrecy is detrimental because it creates fixed, shame-ridden mindsets and obfuscates students' ability to advocate for themselves around their specific learning needs and goals. In a study of individuals' self-perceptions of the nature of their learning disability diagnoses, Lambert et al. (2019) found many who echoed the sentiment poignantly shared by Lynn Pelkey in her essay in *Learning Disabilities and Life Stories*:

> I do not know when I was labeled as learning disabled. It was not until junior high and maybe into high school that the term LD started to surface with frequency. For years, my fellow LDers and I wondered what LD meant. No one ever told us. We did know that it set us apart from others and that we were different. Being LD was not something that we received awards for. It was secretive and suspicious. It was something talked about in hushed tones. It was discussed at secret parent/teacher meetings. It was the reason that I had to go to summer school. Is it any surprise then, before I knew what LD meant, I felt ashamed about being LD? (Rodis et al., 2001 p.19, as cited in Lambert et al., 2019, p. 7).

If secrecy fosters such shame, perhaps it is time for mathematics education leaders to create policies and practices that open communication with students about their diagnoses. Such communication empowers students and gives them opportunities to advocate for themselves.

Of course, these conversations should include not only students' deficits, but also a meaningful conversation about what disability, or exceptionality, is, as well as clear highlighting of individuals' strengths. In the Lambert et al. (2019) study, participants described what they called gifts of their exceptionalities, namely, creativity and conceptual thinking, multimodal thinking, persistence, and motivation. Indeed, individuals with identified exceptionalities lament the endemic educational emphasis on their deficits, arguing that attempts to remediate them to the average learner happened at the cost of fostering their strengths (e.g., Lewis & Lynn, 2018; Robinson, 2016; Roy, 2015). Education can foster these strengths. The same Lynn who described feelings of shame about her LDness described her experiences with conceptual learning in mathematics general education as "magical":

> As I sat in that class, something magical happened to me. I could understand what he was teaching. I was learning. I even started participating in the class, raising my hand, and answering questions. I was LD. But then again I wasn't. I still couldn't multiply or divide very well, and I had to use elaborate ways to come up with the answer. But I wasn't memorizing, I was thinking, and I was figuring out the answer. I was learning. This was one of the experiences that shot a pinhole in the bubble that trapped me in my LDness. (Rodis et al., 2001 p. 21, as cited in Lambert et al., 2019, pp. 14-15).

Thus, conceptual learning can be empowering for students with identified exceptionalities. With her statement "I wasn't memorizing, I was thinking … [and that] shot a pinhole in the bubble that trapped me in my LDness," Lynn articulated that opportunities to engage in conceptual learning opened up possibilities for her and helped her see herself as capable despite the challenges her neurological differences led her to encounter with memorization.

Stories such as Lynn's are not typically elevated in special education research although they are in the budding area of mathematics education research on disability. The lack of overlap between special education research and mathematics education research (Garderen et al., 2009; Lambert & Tan, 2016, 2019) reflects these fields' distinct differences in terms of subscribing to medical, social, or cultural models of disability. Special education's leading theory of learning aligns with the medical model of disability. This leading theory is behaviorism, a theory that generally defines learning as a change in behavior. Behaviorism leads scholars to design for learning by focusing on individual instruction with instructional sequences that move from simple to more complex actions, with many opportunities for practice. In contrast, mathematics education's theories of learning align with the social and cultural models of disability. These theories are sociocultural (goals of enculturation) and sociopolitical (goals of emancipation, Gutierrez, 2013). These theories generally define learning as a change in participation, which is different from a change in behavior because participation includes a different orientation towards activity as a social and cultural endeavor. Sociocultural and sociopolitical studies take the stance that all students are sensible, competent mathematical doers and thinkers and thus offer an alternative narrative from the hegemony of behaviorist, quantitative, special education studies that make up the bulk of research on exceptionalities (Connor et al., 2011).

Research paradigms and their implications for educational design

Mathematics education leaders need to be aware of important differences between special education and mathematics education research so that they can use discernment when making research-based decisions. For example, special education research tends towards domain neutral interventions such as using mnemonic techniques while mathematics education research attends to the structure of mathematics as fundamental to intervention designs (Garderen et al., 2009), thus making instructional practices rooted in mathematics education research more likely to support students' learning of mathematics. Special education and mathematics education research also differ in their research settings, with the former occurring in one-to-one, team, or specialized settings and the latter occurring in inclusive whole-class settings (Garderen et al., 2009). If mathematics education leaders want to foster inclusive mathematics classrooms, then drawing on research that occurs in such classrooms has tremendous value. In addition, most research on MLDs in special education, about 75% of it, is conducted in elementary settings, perhaps due to flawed perceptions of mathematics as a hierarchical subject in which basic skills must be mastered before conceptual learning is possible (Garderen et al., 2009; Lewis & Fisher, 2017).

If mathematics education leaders want to create environments where teachers can support students with identified exceptionalities in inclusive classrooms then they will need the underused tool of qualitative research to understand how these students do learn. While the behaviorist quantitative research of special education helps identify the persistent errors students make, it falls short of explaining why students make particular errors and why errors persist despite instruction (Lewis, 2016, p. 100). Unfortunately, and as an indictment to mathematics education research, very little of such research on exceptionalities has yet been published in mathematics education journals (Lambert & Tan, 2019). Still, there is a small but growing body of qualitative research that is beginning to fill this gap.

When examining student thinking, qualitative research conducted from sociocultural and sociopolitical theories focuses on the ways that students do understand mathematical concepts and representations instead of only the ways that they do not understand them (i.e., their error patterns). For example, Lewis (2016) identified the persistent understandings of two girls with identified exceptionalities during fraction comparison tasks (e.g., Which is bigger, 2/8 or 5/8?). Lewis found that the errors made by the two girls were the result of three persistent understandings in which the girls (a) used fraction complements[1] instead of the fraction itself, (b) had a single factor understanding[2] of fractions, and (c) understood 1/2 as an action of halving rather than as a quantity. By analyzing what sense the students were making rather than only what errors they made, Lewis' analysis not only portrays the students as sensible doers and thinkers of mathematics but also provides a foundation to build on to move students beyond their current understanding of fractions.

Of course, this has implications for assessment. There is a pressing need for research-based assessments that capture the conceptual understandings of students with identified exceptionalities in content areas such as algebra and geometry (Garderen et al., 2009). Existing research focuses on how to support and assess learning of basic facts and procedural skills, an outcome of the lack of mathematics education research on the learning of students with identified exceptionalities. Assessments designed to capture conceptual understanding may reveal mathematical competence even when students have trouble with symbolic and non-symbolic processing. For example, as noted earlier, Lewis and Lynn (2018) documented how a student successfully majored in statistics despite mathematics learning disabilities with number sense and automaticity (dyscalculia). Qualitative research has shown that standards-based mathematics curricula can be made accessible to students with identified exceptionalities (Lambert & Sugita, 2016), and so the development of research-based assessments for middle and high school mathematics concepts is urgently needed. Without new forms of assessment, efforts to rehumanize the mathematics education experiences of students with identified exceptionalities will be constrained. Mathematics education leaders can begin this work and seek relationships with scholars who can support them and their teams.

The findings of qualitative research are an important balance to quantitative studies of efficacy. In a review of the 50 highest impact research reports on inclusive classrooms, Cook and Cook (2020) found that quantitative research on the efficacy of inclusive education was highly inconclusive, with both positive and negative effect sizes. For this reason, they suggest that educators engage in evidence-informed practice by taking both research and practical matters (such as families' values, etc.) into account when making decisions for and with students with identified exceptionalities. The next section overviews qualitative research on students with identified exceptionalities so that mathematics education leaders can make such evidence-informed practice possible for the teachers and staff they support.

How can we support all students in inclusive classrooms?

To understand how to create inclusive environments where students with identified exceptionalities can thrive in their mathematics learning, a definition is needed of what it means to do mathematics. Going through an a priori set of steps to complete a mathematical task is not doing mathematics. Doing mathematics requires struggle. Just ask a mathematician. For this reason, all students need to engage in struggle in mathematics classrooms. According to Warshauer (2014), common sources of struggle in mathematics learning for typically achieving students include:

1. getting started, for example, because of confusion about what the task is asking, forgetting the type of a problem, resigning due to uncertainty, or not putting any work onto paper;

[1] A fraction complement is the unshaded portion of a fraction. So, for example, the girls compared three pieces for 5/8 to six pieces for 2/8.

[2] A single factor understanding of fractions refers to focusing either on the size of pieces (e.g., fifths are smaller than halves, so 3/5 is smaller than 1/2) or on the number of pieces in the whole (e.g., 5 is more than 2, so fifths are larger than halves).

2. carrying out a process, for example, due to being unable to implement a process from a representation or due to its algebraic nature, or being unable to remember a fact or formula;
3. experiencing uncertainty in explaining and sense-making, for example, because of uncertainty in the reasons for their strategy choices or being unable to make sense of their work; and
4. expressing misconception and errors related to content (p. 385).

These struggles are also experienced by students with identified exceptionalities, and they become unproductive if students struggle without making progress towards task goals or give up.

Thus, finding instructional strategies to support all students to make progress, without lowering the cognitive demand of tasks, is essential in standards-based classrooms, especially inclusive ones. Students with identified exceptionalities are systematically denied a sense of intellectual authority, or "the belief that one has the responsibility for making sense of problematic situations rather than relying on someone else" (Akyuz & Stephan, 2020, citing Kamii, 1982). In her book *Culturally Responsive Teaching and the Brain*, Dr. Zaretta Hammond describes such authority as essential for independent learning, as contrasted to dependent learning in which students heavily rely on the teacher when they experience even small moments of uncertainty (Hammond, 2014).

Lynch and colleagues (2018) warn about authority-reducing instructional pitfalls that teachers commonly make when trying to support students with identified exceptionalities to engage in standards-based curricula.

Pitfall 1: Hinting. Hinting typically reduces the cognitive demand of the task and thereby removes the struggle and the learning. Hinting reduces the cognitive demand of tasks by narrowing students' focus. One way that teachers may hint is through funneling, where the teacher leads the student towards a correct answer by asking a series of questions that require short, fill-in-the-blank type answers from students (Wood, 1999). This prevents students from making connections and often redirects student thinking altogether. In this way, hinting leads students towards solutions built on the teacher's thinking rather than on the students' thinking, which in turn diminishes students' mathematical authority.

Pitfall 2: Backgrounding Problem Context. Backgrounding a problem's context removes sensemaking resources for students to draw on as they enter mathematical tasks. For example, consider a task that involves characterizing the rate of a redwood tree's growth (see Dietiker et al., 2015). Launching this task by focusing on how to represent growth or plot points on a coordinate plane, rather than focusing on students' informal expectations for how the rate of tree growth might be measured, backgrounds the problem's context, resulting in an overemphasis on procedures.

Backgrounding a problem's context also reduces the need to have a collaborative discussion. For example, pre-teaching mathematical concepts or skills relevant to a lesson can short circuit students' opportunities to learn from their peers. Pre-teaching requires making assumptions about which supports students will need, but allowing students to explore the problem context allows for more responsive scaffolds for student learning through just-in-time instruction. Backgrounding problem context and pre-teaching strip problems of meaning and learners of engagement and curiosity.

Pitfall 3: Providing Formulas. Providing formulas removes students' opportunity to engage in authentic mathematics. Because these pitfalls remove productive struggle, they are (unfortunately) some of the core strategies in special education's behaviorist interventions. In other words, because special education's theory of learning focuses on how individuals can come to successfully perform (not understand) a task and veils cognition, the roles of social interaction, and culture in learning, special education's designs for learning typically reduce tasks to a series of steps. Each of the pitfalls described above is key in transforming a cognitively demanding task into a series of steps. Instead of helping students with identified exceptionalities meaningfully engage in content, it steals their aha moments and their collaboration with their peers.

Adherents of the behaviorist theoretical perspective have two assumptions about standards-based curricula that lead them to believe that hinting, backgrounding problem context, and providing formulas are supportive of learning (Lambert & Sugita, 2016). The first assumption is that students with identified exceptionalities need expert help to construct problem-solving strategies. However, research has shown that students with identified exceptionalities can construct effective strategies without intensive scaffolding (see Lambert & Sugita, 2016, p. 352 for a list of such studies). The second assumption is that teachers in standards-based classrooms never make specific content or strategies explicit. However, a quick look through the renowned *5 Practices* book (Smith & Stein, 2018) indicates quite the opposite. Teaching a standards-based curriculum is complex and requires multiple modes of instruction. For example, mathematical discussions make mathematics explicit as students verbalize connections to prior content and collectively work to formalize mathematical concepts with canonical vocabulary. This is especially important during lesson launch and closure as students with identified exceptionalities can struggle to get started and put it all together.

Many argue that students with identified exceptionalities cannot engage in the productive struggle that is part and parcel of cognitively-demanding problem-solving tasks and teamwork. This ableism is so entrenched in education culture that it can seem like common sense. Yet, the common-sense appeal of such ableism is flawed and contributes to a lack of opportunities for students with identified exceptionalities to engage in the productive struggle that supports meaningful mathematics learning.

In the remainder of this section, describe how to increase opportunities for students with identified exceptionalities to engage in the productive struggle that supports meaningful mathematics learning. First, we identify instructional strategies that mathematics education research indicates may benefit students with identified exceptionalities. We then dig more deeply into how to support students with identified exceptionalities to engage in the text-heavy mathematics problems (i.e., context-rich word problems) characteristic of standards-based curricula (for example, see the sample problems available at https://cpm.org/lessons and https://cpm.org/try-this). Next, we address the dilemma of social and academic status in inclusive classrooms before summarizing research on co-teaching and introducing the framework of Universal Design for Learning.

Instructional strategies for broadening access

This section describes what research tells us about how to include students with identified exceptionalities in standards-based curriculum and instruction rather than only in discrete skills-based tasks. Participation in teamwork, problem-solving, and whole-class mathematical discussions is necessary to promote equitable learning and to foster 21st Century Skills. Citing the scholarship of Jo Boaler and others, Lambert and Sugita (2016) claim that "when students are engaged in problem-solving and mathematical discussion rather than memorization, they become equally efficient in calculation and better prepared to transfer knowledge and problem-solve" (p. 348, citing Boaler 1997; Boaler & Staples, 2008; Silver & Stein, 1996). Of course, some students with identified exceptionalities may require scaffolds to participate in these ways.

Mathematics education research is beginning to identify such scaffolds. A study by Lambert et al. (2020) identified culture-building teacher moves that supported the engagement of a student with autism in standards-based mathematics (appendix Table 1). As the result of a literature review on qualitative studies set in standards-based curriculum contexts, Lambert and Sugita (2016) found several strategies that hold promise for supporting students with identified exceptionalities (specifically, learning disabilities) in problem-solving and mathematical discussions (appendix Table 1). Browder et al. (2012) found that the use of graphic organizers helped students diagnosed with moderate intellectual disabilities (verbal and non-verbal) to engage in grade-level, standards-based content, including word problems. In Browder et al.'s study using graphic organizers as a scaffold, 11–13-year-olds were able to engage in the following mathematics:

1. ***Algebra:*** Solve simple one-step equations that relate to stories about daily events.
2. ***Geometry:*** Identify and describe the intersection of figures in a plane. Draw line segments and a coordinate plane to demonstrate spatial sense for familiar contexts like grocery stores.
3. ***Measurement:*** Develop numbers sense for real numbers. Develop flexibility in solving mathematical problems by selecting strategies and using appropriate technology. Use the next dollar strategy to solve problems related to everyday transactions.
4. ***Data Analysis:*** Collect, organize and display data to solve problems from familiar events. (Browder et al., 2012, p. 378)

The strategies for supporting engagement in standards-based curricula identified in these studies (see the appendix) have been shown to support meaningful learning because they increase participation (Lambert & Sugita, 2016) and thus also lead to identities as mathematical thinkers and doers. For example, students who have never offered more than one-word responses during whole group instruction have been shown to shift participation by the end of the year to have equal rates of engagement to their nondisabled peers after ongoing participation in particular mathematics routines (Lambert & Sugita, 2016, p. 359, citing Foote & Lambert 2011).

Importantly for mathematics education leaders, research indicates that teachers struggle to learn how to provide some types of scaffolds more than others. Pfister et al. (2015) found that curricular materials were able to support teachers to engage in important scaffolds such as using manipulatives and helping students to focus on the important aspects of the lesson. However, more interactional micro-scaffolds such as stimulating discourse, cognitive activation (e.g., What do you notice? What did you have to do so that …?), and handling errors productively were much harder for teachers because they required in-the-moment decisions responsive to student needs. Examples of what these five scaffolds look like when they are done well (and not) can be found in the rubric in Table 2 (see the appendix). Mathematics education leaders must create ongoing, embedded professional learning opportunities for teachers to support them in learning to provide scaffolds for students with identified exceptionalities while still providing opportunities for productive struggle in tasks with high cognitive demand.

Strategies for starting word problems

One of the most challenging instructional demands on teachers in inclusive, standards-based classrooms is supporting students to interpret word problems sufficiently enough that they are able to access the mathematics. In other words, teachers in such contexts must find ways to support students with literacy such that they are able to get started on mathematical tasks. Unfortunately, there is extremely thin research on how to specifically support students with identified exceptionalities with literacy challenges encountered in word problems in ways that do not cut-off inquiry (Lambert & Tan, 2016). This paucity of research is partially a result of research taking different approaches to studying neurotypical and neurodiverse students' struggles with cognitively demanding tasks. Lambert and Tan (2016) found that research on neurotypical students tends to focus on problem-solving while research on students with identified exceptionalities tends to focus on word problems. Research that focuses on the former aligns with standards-based instruction characterized by inquiry while research that focuses on the latter aligns with schema-based instruction characterized by an explicit, teacher-mediated approach (Jitendra et al., 2013; Lambert & Tan, 2016). It is important to question why this distinct difference in methodologies for students with and without diagnosed disabilities exists and whether the exclusion of studying the problem-solving of students with identified exceptionalities has negative consequences for those students (Lambert & Tan, 2016, p. 1061).

This disparity is unsurprising because most research on students with identified exceptionalities occurs in special education and not in mathematics education, not because it is impossible to do. Research in special education supports students with word problems through schema-based instruction, which involves unpacking the problem's structure (e.g., the pieces of a linear equation) before the student explores the problem (Browder et al., 2018; Jitendra et al., 2015). This, in essence, removes the inquiry. Providing students with any kind of scaffold that tells them what to do with a task's mathematical problem inherently lowers the cognitive demand and short circuits the productive struggle that supports meaningful mathematics learning. Recent modified versions of schema-based instruction are more complex but similarly inhibit opportunities for conceptual learning, for example by providing students with graphic organizers that are specific to a problem-type, explicit instructions, "rules taught as chants with hand motions representing the underlying problem structures," and more (Browder et al, 2018, p. 223). Attending to the schema of a word problem is important, but to support inquiry, it should occur as discussion about different problem-types after students have engaged in problem-solving with different problem types. Allowing students to collaboratively create graphic organizers and other visual representations may support the learning of students with and without identified exceptionalities.

One exploratory study (Moscardini, 2010) indicates that a potentially productive way to mitigate literacy challenges is through scaffolding the beginning process of problem-solving in ways that do not kill inquiry. This can be done by restating word problems (while retaining the cognitive demand) and re-reading word problems in small chunks that students model step-by-step. According to Lambert and Sugita (2016), "this reduces students' difficulties with language but does not reduce the cognitive demand of the mathematics task" (p. 358).

In a manual designed for teachers, Cole et al. (2000) suggest strategies such as making oral recordings of the text so that students can listen to and rewind the written material instead of reading it, providing students with the word problem text or recording in advance so that they can become familiar with the context of the problem, and discussing the problem context in teams (Aiquraini & Gut, 2012).

Another strategy that supports all students, including those with identified exceptionalities and those learning English as a second language (emerging bilinguals), is fully unpacking a problem's context in the lesson launch without pre-teaching. For example, in a problem about compound interest, teachers could elicit students' prior knowledge about what compound interest means without discussing how to calculate it. This is a matter of ambitious, equitable instruction in all classrooms (Jackson & Cobb, 2010).

Strategies from this section on supporting students with literacy such that they are able to begin problem-solving with word problems are summarized in the appendix in Table 1.

Mitigating status issues in heterogeneous, inclusive classrooms

Because teamwork is a key component of standards-based curriculum and instruction, teachers must carefully attend to issues of status so that students with identified exceptionalities are not marginalized and stigmatized by their peers. For example, students diagnosed with learning disabilities are excluded from making mathematical decisions when they are delegated non-mathematical responsibilities in teamwork (e.g., material management; Baxter et al., 2001). Exclusion from meaningful work on the mathematical aspects of the task is likely to negatively stigmatize mathematical competence.

This exclusion can be mitigated by giving students explicit instruction on how to work together so that students with higher proficiency levels do not take over the mathematical thinking for students with identified exceptionalities (Bottge et al., 2002; Cohen & Lotan, 2014; Horn, 2012). For a list of such instructional strategies, see Table 1 in the appendix.

One important and ongoing role of mathematics education leaders will be to help teachers become aware of their own biases about the abilities of their students; indeed, mathematics education leaders may need to examine their own biases about the abilities of students with identified exceptionalities. Teachers' attitudes and values around ability and mathematical competence influence the way they teach and thus also influence their students' attitudes and values, which can lead to unequal learning outcomes and exclusionary identity development for students with identified exceptionalities (Thurber et al., 2018 citing Davis, 2010, p. 58). As an example of how teacher biases can impact students with identified exceptionalities, these students can be called on fewer times than students who have not been diagnosed with a disability (Bottge et al., 2002). This may be one of the greatest challenges for mathematics education leaders, to create environments where teachers feel confident in their students' potential for learning and in their own ability to teach in heterogeneous classrooms. Creating strong co-teaching environments in which teachers collaboratively examine and hold each other accountable for their assumptions about and treatment of students may be one way that mathematics education leaders can begin this work.

Philosophical alignment and interdependence in co-teaching relationships

While co-teaching is not possible in all contexts, many conjecture that co-teaching is a productive way to support equitable instruction for students with identified exceptionalities in inclusive classrooms. Unfortunately, we know little about what productive co-teaching looks like and the kinds of student outcomes it supports (Friend et al., 2010; Rexroat-Frazier & Chamberlin, 2019). This is partially due to the fact that there is no shared definition of co-teaching in educational research (Rexroat-Frazier & Chamberlin, 2019). Despite this lack of coherence in the research on co-teaching, we do know a little about when co-teaching is productive or harmful and what kinds of collaboration between general and special education teachers might be promising for student learning in inclusive classrooms.

First, a promising definition of co-teaching was proposed by Sileo and van Garderen in 2010: "an instructional delivery model applicable to teaching students with disabilities in least restrictive integrated classroom settings in which general and special educators share responsibility for planning, delivering, and evaluating instructional practices for all students" (p. 14, as cited in Rexroat-Frazier & Chamberlin, 2019; p. 173). In this definition, teachers work together in an inclusive classroom by collaborating on multiple dimensions of instruction, including planning, teaching, and assessing for all students, not just students who qualify for special services. Not only might such co-teaching help mitigate status issues, but it can also allow for coherent, integrated support for students with identified exceptionalities.

This definition of co-teaching can be operationalized in the classroom in many different ways, with some more productively leveraging special education teachers' strengths than others. For example, a few possible co-teaching approaches are, "one teaching, one observing; one teaching, one circulating; team teaching; parallel teaching/split class and team pull out." The dominant co-teaching approach is one teach, one observe. In this approach, the general education teacher leads and the special education teacher supports (Rexroat-Frazier & Chamberlin, 2019, p. 175). This is not ideal. Ideally, co-teaching more fully leverages the strengths of special education teachers such as through team teaching. Of course, both general education and special education teachers will need to engage in professional learning to support this kind of work and will need time outside of the classroom to work together towards identifying their mutually beneficial goals.

Akyuz and Stephan (2020) identified planning and instructional practices for co-teachers that can help students with identified exceptionalities build autonomy. First, they described ways that co-teachers facilitate students' learning of the lesson objective. Three co-planning practices for supporting the learning goal are (1) creating mathematical tasks rich in imagery, (2) unpacking the learning goals, and (3) reflecting on the learning goals to guide further discussion. A related co-instructional practice is strategically selecting students to share their ideas based on how their solutions contribute to the learning goals. Second, Akyuz and Stephan described ways to support students to persevere through cognitively challenging points of the lesson. A co-planning practice is reflecting on students' progress on the learning goal, and a co-instructional practice is supporting social and sociomathematical norms, where social norms are the ways students expect and are expected to interact with each other and sociomathematical norms are the ways students expect and are expected to create and justify their mathematical work (Yackel & Cobb, 1996). Finally, Akyuz and Stephan identified two ways that co-teachers can modify their instruction to support students to deal with cognitive challenges that might arise: (1) by supporting visualization with gestures and tools, and (2) by restating students' words in clearer language.

Even when co-teaching is an option, there are many barriers to being able to implement the co-teaching practices described above. First and foremost, often neither general education nor special education teachers receive pre-service or in-service education on how to co-teach. Unfortunately, as mentioned earlier, research on the necessary knowledge and skills that teachers need to collaborate in inclusive classrooms is also underdeveloped. Other barriers to co-teaching include co-planning time, scheduling, caseload, administrative support, academic content knowledge, high-stakes testing, and co-teacher compatibility (Cook & Cook, 2020, p. 144).

This last barrier is especially important to overcome. Co-teaching should not be forced, as teacher attitudes in co-teaching situations can impact the tone for classrooms and impact student learning (Rexroat-Frazier & Chamberlin, 2019, p. 178, citing Sakiz, Pape, and Hoy, 2012). Desirable attitudes include mutual caring, interest, concern, encouragement, and high expectations. In addition, co-teachers should have relatively equal professional standing, co-teaching is not an opportunity for mentorship, and should negotiate their respective roles so that expectations for distributions of labor and responsibility are explicit (Rexroat-Frazier & Chamberlin, 2019, citing Walther et al., 1996). Finally, co-teachers should be relatively aligned with their philosophy of education, meaning how they view the purpose of their profession (Rexroat-Frazier & Chamberlin, 2019 citing Magiera et al., 2005). In sum, conditions are right for co-teaching to be productive when both teachers are willing and eager to work together to plan, teach, and assess; are equals in professional standing; have clearly defined each teacher's responsibilities in the classroom; and similarly orient to the purpose of teaching.

This definition does not apply to paraeducator aides. In classrooms fortunate enough to have paraeducators, it is important to ensure that their presence does not interfere with the classroom teacher's sense of instructional responsibility for students with identified exceptionalities or that the paraeducator's proximity to students interferes with peer-to-peer relationships or foster dependence (Cook & Cook, 2020). In a National Science Foundation-funded study of paraeducator professional development for supporting learning in mathematics classrooms, Storeygard et al. (2018) found that paraeducators benefit from having (1) a safe and encouraging learning environment where they can explore a lesson's mathematics before they encounter it with students, (2) access to the mathematics curriculum and opportunities to understand the curricular goals and learning philosophy, and (3) opportunities to engage in problems conceptually and reflect on how conceptual approaches differ from procedural approaches. With such support, paraeducators can contribute much more than routine monitoring and clerical tasks; they can fill in where teachers are spread thin by providing additional support for students with identified exceptionalities. For a summary of this section's takeaway strategies, see Table 3 in the appendix.

Designing for disability-first rather than differentiation

Historically, curriculum and instruction have been designed for the average learner. However, this average learner is an ideal type, it is imaginary (Rose, 2016). Because the average learner is imaginary, the reality is that fair instructional practices are not the same instructional practices for each student. Thus, curricula must be designed to support instruction that flexibly meets students' unique needs, especially in the case of students with multiple exceptionalities (Hartmann, 2015).

This is unique from differentiation, which some argue leads to within classroom tracking, and thus also to exclusion and stigmatization (Bannister, 2016). Many argue that differentiation through increased direct instruction and practice and through supporting students in their preferred learning style is beneficial to students. Yet, learning-styles instruction has no consensus in research (Bannister, 2016; Pashler et al., 2008). What research has shown is that interventions should focus on increasing participation in collaborative mathematics problem-solving (Lambert & Sugita, 2016).

Instead of differentiating in the traditional sense, differentiation can be front-loaded in curricula through flexible designs that support multiple-ability engagement. This is called Universal Design for Learning, or UDL. Consider this description from Thurber et al. (2018):

UDL is an educational framework that emphasizes the use of flexible goals, methods, materials, and assessments in order to provide effective instruction to a diversity of learners. Rather than approaching accessibility as an afterthought or only on a case-by-case basis, UDL principles help instructors to design courses that address the needs of diverse learners from the start so that all students may benefit. (Thurber et al., 2018, emphasis added)

Thus, UDL is meant to increase the accessibility of participation and content through the design of curricula. Examples of UDL include designing for opportunities for multi-modal engagement with mathematical concepts (e.g., algebra tiles) and for participation structures that distribute labor in ways that support students with identified exceptionalities to engage in cognitively demanding tasks (e.g., team roles).

While it may not be appropriate for UDL to replace all case-by-case accommodations and modifications, UDL makes learning more accessible to all students through design. For example, UDL supports students to learn through collaboration. Collaboration is more challenging for some students with identified exceptionalities, still, it has both short- and long-term benefits. In the short term, collaboration can increase student motivation. This in turn has long-term effects, since increased participation supports student learning. In addition, in the 21st century, collaboration is an essential skill that can contribute to learning both content and social strengths such as learning to work effectively with others. In particular, collaboration can support the learning of students with identified exceptionalities through increasing opportunities for one-on-one support through the mentoring of peers (CAST, 2018). Of course, collaboration needs to be carefully structured, such as through implementation of team roles (e.g., see the Team Support Guidebook at https://cpm.org/teamsupport) in order to mitigate status issues, as previously mentioned, such as through sentence starters that support students to ask each other for help, team roles that foster multiple-ability participation, and teamwork norms and rubrics (CAST, 2018). Adopting a curriculum designed in alignment with UDL is a great way to begin supporting teachers to make changes to their instruction because it makes teaching for inclusion less subversive. In other words, instead of swimming upstream as they work around curricula designed to require differentiation in the traditional sense, teachers using UDL aligned curricula can focus on expanding their visions and enactment of equitable, ambitious instruction in their inclusive classrooms.

Summary

So, how can mathematics education leaders and educators support students with identified exceptionalities to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks?

First, mindsets must shift from thinking about disability as a kind of diagnosable brokenness and lack (the medical model) to focusing on each student's existing capacity for learning, their special abilities, and their potential and how environment designs and culture may foster learners' ability to meet their potential or squander it (the cultural model). As Susan Robinson, a distinguished alum of Penn State University and CEO/founder of Global Health AspirAction, and a person with a genetic visual impairment, says in her Ted Talk, "The term 'disability' detonates a mindset of less than" (Robinson, 2016), a clear conflict with growth mindsets shown to be important for mathematical achievement (Boaler & Dweck, 2016; Bostwick et al., 2017). Redressing this is especially important for students of color as they are systematically disenfranchised, stigmatized, and underserved by the education system. By refocusing on students' strengths and building on their current understandings, teachers can build a classroom culture that fosters growth mindsets in support of all students' learning.

Second, students with identified exceptionalities can experience productive struggle during cognitively-demanding tasks when they are supported through ambitious instructional strategies with aligned assessments that highlight what students do know, some of which are identified in Tables 1 and 2 in the appendix.

Third, access to word-heavy mathematics tasks can be increased with small modifications that may benefit all students, such as by recording a read-aloud of the problem that students can rewind and relisten to in their groups. More strategies can be found in Table 1.

Fourth, it is critical to attend to status issues in inclusive classrooms that use teamwork to engage students in cognitively demanding tasks. Table 1 provides multiple research-bsed strategies for mitigating status issues.

Fifth, co-teaching may support all students' learning in inclusive classrooms by re-distributing the labor of teaching across two teachers with differing, complementary expertise. Research indicates that co-teaching may be most productive when both teachers are willing and eager to work together to plan, teach, and assess; are equals in professional standing; have clearly defined each teacher's responsibilities in the classroom; and similarly orient to the purpose of teaching (i.e., their philosophy of education). A summary can be found in Table 3 in the appendix.

Finally, mathematical tasks should be designed to have multiple entry points and engage students through multiple modalities with their peers in cognitively-demanding problem-solving tasks. This approach is in line with UDL (Lambert, 2020). UDL is generated by designing from the margins first; designing for disability first is a way to benefit all learners. This flips the traditional approach to designing for the imaginary average learner upside down, and it may have promising results. A proponent of this approach, Elise Roy, has given lectures on this design stance at leading design firms such as Microsoft, NASA, AIGA, and the U.S. Institute for Peace (Roy, 2015). Surely if innovators such as NASA see value in designs that re-able those who have previously been dis-abled, there may be value in exploring disability-first designs for learning as well.

By maintaining asset-based perspectives of and high expectations for students with identified exceptionalities, including those diagnosed with severe disabilities, mathematics education leaders and the educators they support can expunge barriers to conceptual learning and foster scaffolds for meaningful engagement for all students. ✪

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APPENDIX

Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks

Support Area	Description
Problem-solving*	
Multi-modal curriculum design	Provide students with choices about what materials are used to solve problems (equations, drawings, algebra tiles, connecting cubes, base-ten blocks, etc.)
Consistent routine	For example, in the case of CPM Educational Program's lessons, a consistent routine is (1) teacher-led lesson launch, (2) individual or team problem-solving, and then (3) a whole-class discussion in which students present their strategies and solutions.
Teacher scaffolds for problem-solving	Scaffold the starting problem-solving by restating word problems (while retaining the problem type) and re-reading word problems in small chunks that students model step-by-step. This reduces students' difficulties with language but does not reduce the cognitive demand of the mathematics task.
Equitable teamwork	Mitigate marginalization by providing additional support to teams (this often requires teacher professional development)
Mathematical discussion*	
Student rehearsal of strategy shares	For example, allow students to rehearse the strategy they will share out in a discussion by providing them with a paraeducator, allow students to use FlipGrid to record a strategy, etc.
Access to manipulatives and notebooks	Allow students to use their notebooks as a record of their problem-solving for as long as they need in order to support their participation in discussion; allow students to use manipulatives such as algebra tiles rather than equations to model their mathematical thinking during discussions.
Teacher questioning	Hold students accountable for explanations of their strategies by asking multiple follow-up questions.
Participation**	
Begin with relationships	Establish strong relationships from the beginning of the year, especially through finding shared interests.
Strengths-based views of exceptionality	Verbally notice both mathematical and social strengths when talking to and about students. Do so using non-medical language (e.g., "shy" instead of "non-verbal"). Ask questions to elicit thinking and then help students build from their current understandings. Pay attention to specifically what is challenging for students, such as verbal participation. Consider asking students' permission before sharing their thinking in front of a team or the whole class.
Scaffolded discussion with peers	Intervene in teamwork to support students to share out. Direct students to work with specific peers and physically move their notebooks or papers to be next to each other, then check in on their progress. Hold peers accountable even if students are quiet talkers and thus hard to hear.
Collaborative shares	Have students share out in pairs so that students who do not prefer verbal interaction can still participate.

Table continued on next page

Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks (continued)

Support Area	Description
Participation *(continued)***	
Make norms of mathematical discussion very explicit	Have the class define and describe what discussion looks and sounds like and create a durable, visible record of this discussion. For example, have the class work collectively to create a chart with an eye on one side ("looks like") and an ear on the other ("sounds like") with each side filled in. Students may generate ideas such as: • ***Mathematical discussions are:*** when you talk about math and what it can do, talking about how we use strategies, when two or more people have different answers, sharing ideas with others • ***Looks like:*** Notebooks out, eyes on the speaker, showing work to each other, taking notes on what other people are saying, agree on an "agree symbol" • ***Sounds like:*** "I agree with you because…," "I respectfully disagree with you because…," "This is how I did my work," "I don't understand the strategy," "Can you repeat that?," "I'd like to add on to what you said," "My strategy has a connection with yours," and "Can you explain more?" (p. 505, direct quotation from a student conversation poster image)
Notice students' participation	Some students may participate differently than others. For example, instead of raising their hand high, a student may raise just one finger slightly. Be sure to notice and respond to such participation as quickly as possible.
Get started on word problems	
Explore problem context	Create an engaging launch that fully explores the problem context, but does not lower the cognitive demand (Jackson & Cobb, 2010); consider exploring the problem context in teams (Cole et al., 2000)
Rephrase	Rephrase word problems without lowering the cognitive demand (Lambert & Sugita, 2016)
Read in small chunks	Re-read word problems in small chunks that students model step-by-step (Lambert & Sugita, 2016)
Oral recordings	Make oral recordings of the text so that students can listen to and rewind the written material instead of reading it (Cole et al., 2000)
Provide problems in advance	Provide students with the word problem text or recording in advance so that they can become familiar with the context of the problem (Cole et al., 2000)
Work in pairs	Allow students to work in pairs with a supportive peer (Lambert et al., 2020)
Teams create visual representations	Require teams to collaboratively create graphic organizers and other visual representations of the problem (modified from Browder et al., 2018)
Equal-status interactions in teamwork***	
Value rough-draft thinking	Create a classroom culture that values mistakes and rough-draft thinking (Nasir et al., 2014)
Teamwork accountability	Use accountability structures that hold each team member accountable for the group's shared work (Nasir et al., 2014)

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Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks (continued)

Support Area	Description
Equal-status interactions in teamwork* *(continued)***	
Visibly random teams	Use random assignment of team roles (Nasir et al., 2014)
High press questioning	Press all students for high levels of justification (Nasir et al., 2014)
Group worthy tasks	Use ''group worthy'' tasks (Cabana et al., 2014; Cohen & Lotan, 2014) by: • focusing on the big ideas of a lesson. • providing tasks that afford multiple solution pathways and/or require multiple representations. • providing tasks that require multiple intellectual abilities—finding information, problem-solving, basic skills, or material organization—such that no single individual can possess all of them.
Multiple-abilities framing	List out the intellectual abilities the task requires to students and then say something like, "None of us has all of these abilities that are required for this task. Everyone has some of these abilities, and so everyone will have something important to contribute to our shared work today. Listen carefully to one another, as you will all be important resources for your group." (Bannister, 2016, p. 342)
Assign competence	Make public, positive, evaluative statements that recognize specific intellectual contributions that students with identified exceptionalities make during teamwork (Horn, 2012). This can be done for other low-status students as well, such as students who are marginalized along lines of gender, race, social class, physical attractiveness, and prior academic performance (Bannister, 2014; Cohen & Lotan, 2014).

* Strategies are near direct quotations from Lambert and Sugita (2016, p. 357-358)
** Strategies from Lambert et al. (2020, p. 508-509)
*** Strategies summarized in Bannister (2016)

Table 2: Rubric for scaffolds. (Pfister et al. 2015).

Scaffolds	**Teacher Actions Rubric Examples**			
Scaffolding Questions	**Goals**	**(0)**	**(1)**	**(2)**
Micro-Scaffolds:				
Cognitive Activation				
Compare! What do you notice? What did you have to do so that …?	Poses clear, content-related, meaningful, challenging questions and problems, provides stimulation for describing or substantiating facts, observations, etc. Enables the establishment of relationships between contents Stimulating discourse	Set tasks with small steps Told the students which actions they have to carry out Posed questions that require a one-number answer Carried out actions with the manipulatives	Carried out actions with the manipulatives himself Often told students solution steps Sometimes requested observation, description, or substantiation of facts and findings Sometimes requested a comparison of solution strategies	Constantly requested students to verbalize and substantiate their solution steps Allowed problems (even correctly solved ones) to be discussed Invited the formulation of insights and observations
Handling Errors Productively				
Where are you stuck? What are you considering? How did you find it out? How can we find out whether that's correct?	Recognizes the learning potential or difficulty of a situation Intervenes in the learning processes in a supportive manner Endeavors to understand the students' solution strategies or reflections Supports students in tackling problems independently Checks the students' understanding following the intervention	Demanded that certain procedures be carried out When students were uncertain, he told them how to continue Rubbed out mistakes and wrote down the solution himself Pointed to what was written on the blackboard	Provided hints for using the structure of the Dienes blocks Requested the students to try the problem again with help of the manipulatives Demanded more careful work (not specifically mathematical)	Requested verbalization of the procedure Requested substantiation and proof Provided feedback on systematic procedures Let insights from a mistake be explicitly formulated, or the mistake to be "named" Established connections with other solved problems or problems that have not yet been solved

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Table 2: Rubric for scaffolds. (Pfister et al. 2015). *(continued)*

Scaffolds	Teacher Actions Rubric Examples			
Scaffolding Questions	**Goals**	**(0)**	**(1)**	**(2)**
Micro-Scaffolds *(continued)*:				
Stimulating Discourse				
Describe what you have done! Can you explain that in more detail? Can we solve/ write it differently?	Invites the students to comment on contributions or actions of others Responds to students' contributions Initiates reflections on solution strategies	Asked for numbers, results Let the students finish sentences he has started Spoke most of the time	Formulated central findings for students Primarily asked for results or subsequent steps Let the students finish teacher sentences Sometimes let students determine next steps Rarely included student ideas into discussions	Asked for reflections Let thought processes and insights be presented Mostly does not interrupt the students' contributions
Macro-Scaffolds:				
Using Manipulatives				
Can you show that with manipulatives? Can we place/ do it differently?	Employs manipulatives to support the learning process Allows facts to be represented actively using manipulatives Emphasizes the understanding of structural relationships or the systematic use of manipulatives	Let students name the units of the Dienes blocks Mostly manipulated the Dienes blocks himself Told the students what they should do with the Dienes blocks Mostly wrote down the problem solution by himself	Encouraged students to use the Dienes blocks in a structured manner Let the structure of the Dienes blocks be used clearly for the grouping or de-grouping process and for recording (interim) results In part, he established the relationship between manipulatives, representations, and notations Addressed the difference between an empty number line and a number line	Let notation forms and arithmetic steps be compared Worked out the characteristics or differences of the manipulatives, representations, and notations clearly on several occasions (e.g., difference between an empty number line vs. and a number line) Let different presentation forms be used for individual solution strategies
Target Orientation				
Describe the rule/pattern! Why does it have to be done like that?	Focuses on core content elements Demonstrates what is important, points out conventions Summarizes important findings, recapitulates these findings in his/her own words	Focused on carrying out the procedure correctly	Formulated central findings Always pointed out important things Recapitulated insights or relevant things	Summarized the students' thoughts "Translated" student contributions Let insights be formulated and summarized Worked out key characteristics and procedures

Table 3: Summary of reported strategies for supporting successful co-teaching relationships

Co-teaching supports	Description
Share responsibility	Support general and special educators to share responsibility for planning, delivering, and evaluating instructional practices for all students, not just students who qualify for special services (Rexroat-Frazier & Chamberlin, 2019; p. 173, citing Sileo & van Garderen in 2010 p. 14).
Co-plan	Support teachers to co-plan by (1) creating mathematical tasks rich in imagery, (2) unpacking the learning goals, and (3) reflecting on the learning goals to guide further discussion (Akyuz & Stephan, 2020).
Team-teach	Support teachers to team-teach to more fully leverage the strengths of special education teachers, including by both teachers (1) supporting visualization with gestures and tools and (2) restating students' words in clearer language (Akyuz & Stephan, 2020; Rexroat-Frazier & Chamberlin, 2019).
Reduce barriers	Reduce barriers to co-teaching: provide in-service education on how to co-teach and to gain academic content knowledge, provide co-planning time administrative support, reduce case-load, schedule appropriately, mitigate high-stakes testing, and ensure co-teacher compatibility (Cook & Cook, 2020, p. 144).
Teacher pairing	Ensure co-teachers have relatively equal professional standing and provide opportunities for them to negotiate their respective roles so that expectations for distributions of labor and responsibility are explicit (Rexroat-Frazier & Chamberlin, 2019, p. 175 citing Walther-Thomas, Bryant and Land, 1996).
Joint reflection on philosophies of education	Create time and space for co-teachers to examine, compare, and work towards alignment of their philosophies of education (Rexroat-Frazier & Chamberlin, 2019 citing Magiera et al., 2005).

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2. Manuscripts should be consistent with the purpose of the journal.

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