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Call for Manuscripts

The editors of the ***NCSM Journal of Mathematics Education Leadership (JMEL)*** are interested in manuscripts!

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM. Categories for submissions include:

- **Case studies and lessons learned** from mathematics education leadership in schools, districts, states, regions, or provinces
- **Research reports** with implications for mathematics education leaders
- **Professional development** efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice
- Other categories that support the NCSM vision will also be considered.

Submission Procedures

Each manuscript will be reviewed by two volunteer reviewers and a member of the editorial panel. Manuscripts should be submitted to the NCSM website at: **https://www.mathedleadership.org/call-for-journal-manuscripts**.

Submissions should follow the most current edition of APA style and include:

1. A Word file (.docx) with author information (name, title, institution, address, phone, email) and an abstract (maximum of 120 words) followed by the body of the manuscript (maximum of 12,000 words)
2. A blinded Word file (.docx) as above but with author information and all references to authors removed.

***Note:** Information for manuscript reviewers can be found at the back of this publication.

NCSM

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NCSM Vision

NCSM is the premiere mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

Purpose Statement

The purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of NCSM by:

- *Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education*
- *Fostering inquiry into key challenges of mathematics education leadership*
- *Raising awareness about key challenges of mathematics education leadership in order to influence research, programs, policy, and practice.*

Comments from the Editors

Paula Jakopovic, *University of Nebraska at Omaha*
Sean Nank, *California State University San Marcos*

"The secret of change is to focus all of your energy not on fighting the old, but on building the new" — Socrates

With new seasons come change, and we at the *Journal of Mathematics Education Leadership* are excited to present a new submission category for upcoming issues of the journal. We will now be accepting manuscripts in the category of "core practices," which can include program descriptions of leadership focused initiatives that are founded in the current literature and research. In Fall 2021, we featured the first manuscript in this category, a piece written by Jasien and Hayes called, "Inclusion and Intervention: Understanding 'Disability' in the Mathematics Classroom," and in our Summer 2022 issue, we featured an article by Zimmerman and Wilson, titled, "Beyond Right or Wrong: Supporting Teachers in Strengths-Based Approaches to Examining Student Work." Each of these pieces are situated within the current literature, and highlight examples and models of effective leadership strategies, tools, and initiatives. We encourage our readers, as well as potential authors and reviewers, to check out these articles, as well as the journal webpage if you are interested in learning more about this new addition to the journal: https://www.mathedleadership.org/pubtype/journal/

We are also humbled and elated to announce that the *Journal of Mathematics Education Leadership* was awarded the National Council of Teachers of Mathematics "2022 Publication Award for Affiliate Journals." This award is presented annually to NCTM affiliate organizations and journals for work that promotes outreach and engagement initiatives that positively impact the mathematics education community. It is truly thanks to the hard work of our authors, reviewers, and editorial team that we received such an honor- thank you for your support!

In our latest issue of *JMEL*, we present two articles focused on helping mathematics leaders attend to salient features of their interactions with one another and with classroom teachers. In "Critical Colleagueship Development Amongst Elementary Instructional Leaders: A Comparative Analysis of Process and Outcomes," Donaldson examines the interactions among teams of mathematics coaches to determine the effects that professional learning opportunities and structured collaborations have on the development of critical colleagueship. "Critical colleagueship" (Lord, 1994; van Es, 2011) requires members to objectively and constructively reflect on their personal impacts on student learning to facilitate open dialogue that support the improvement of teaching practices. In this study, Donaldson identifies several takeaways for mathematics leaders, including the importance of creating and maintaining structures for productive teamwork, along with identifying and striving toward common, measurable goals.

Our second article, "How Understanding Mathematical Discourses Shapes Principal Noticing," presents findings from a study by Rhodes et al., who developed a framework for examining how principals and mathematics leaders take up and "notice" important moments in lessons centered around students' mathematical thinking. Grounded in the literature on professional noticing (Goodwin, 1994; Jacobs et al., 2010; van Es, 2011), pedagogical content knowledge development (Hauk et al., 2014; Schulman, 1987), and leadership content knowledge (Stein & Nelson, 2003), the *Pedagogical Content Knowledge for Leadership (PCKL) Framework for Noticing Content and Teaching* offers a tool for mathematics leaders and administrators to categorize a leader's ability to critically attend to powerful moments during mathematics instruction and provide feedback to classroom teachers.

These articles examine structures that we can use to support the development of mathematics leaders, both those who see themselves as having specialized content knowledge and those who do not, in their efforts to support our classroom teachers. Both recognize the need for us to break down barriers to the critically important work that we do in service of teachers and students. In order for our efforts to be successful, they must be done with thoughtfulness and intentionality. We encourage you to consider the ways in which these two articles can support your own critical colleagueship and focus on professional noticing. ✪

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Critical Colleagueship Development Amongst Elementary Instructional Leaders: A Comparative Analysis of Process and Outcomes

Sara Donaldson, *Wheaton College (MA)*

Abstract

Mathematics coaches, as school-based instructional leaders, are well situated to promote instructional effectiveness and student learning. This ability is enhanced through ongoing professional learning opportunities that position them alongside other instructional leaders actively developing the skills, knowledge, and dispositions necessary for facilitating change. This study, conducted in an urban school setting, draws on social learning theories to examine the influence of professional learning processes on mathematics instructional leaders' critical colleagueship development and collaborative inquiry engagement. Findings indicate increasing attention on establishing collaboration agreements, shared transformational learning goals, and consistent meeting structures promotes critical colleagueship and team functioning in support of coaches' own professional learning and their facilitation of learning for the teachers whom they support.

Keywords: critical colleagueship, mathematics instructional leaders, elementary school, teacher knowledge, social learning theory, transformational professional learning.

A social learning perspective defines knowledge development as both an intellectual and a social endeavor requiring active learner engagement as understanding is socially negotiated through interaction and experience (Akyol et al., 2009; Bandura, 1986). Transformational professional learning (PL), that which promotes the rethinking of current ways of knowing and doing and leads to changed practice in support of student academic achievement, goes well beyond the simple transfer of knowledge by centering social learning as productive professional dialogue around divergent, practice-based ideas (Mezirow, 1978; Steyn, 2017). Promoting this productive discourse requires opportunities for educators to take an inquiry stance to critically examine their practice in "ongoing, reflective, collaborative, and inclusive ways" (Voelkel, 2022, p. 346). This type of peer interaction supports transformative PL by encouraging ongoing knowledge and efficacy development. When skillfully facilitated, this critical dialogue enables individuals to openly exchange ideas, develop collective understanding, discuss their craft, self-reflect, and support implementation of new practices that support student achievement (Benoliel & Schechter, 2018; Donaldson & Karp, 2023; Vavasseur & MacGregor, 2008).

Literature Review

Existing research indicates transformational PL is promoted through social learning that includes: (a) job-embedded collaborative learning opportunities, (b) critical discourse centered on shared examination of the impact of personal and collective practice on student learning outcomes, and (c) supportive facilitation by instructional coaches with strong content and pedagogical knowledge and the ability to lead sustained learning that is collaborative and grounded in reflective practice (Desimone & Pak, 2017; Myers et al., 2021). Each of these features is taken up in this section.

Job-Embedded Collaborative Learning

Transformational PL differs from traditional professional development workshops where educators are passive recipients of content determined by administrators in response to student performance data and associated perceptions of gaps in professional knowledge (Jensen et al., 2016; Wei et al., 2009). This traditional top-down, deficit focused approach to PL can diminish educators' perceptions of their individual and collective ability to influence student mathematics learning outcomes (Chetty et al., 2014; Goddard et al., 2000). As a result, educators often attribute students' failures to external factors, such as prior achievement or demographic characteristics, feeling they have limited power to influence student outcomes, thus decreasing their motivation to engage in change efforts (Abrami et al., 2011; Linder et al., 2013).

This sense of helplessness can be combatted by ongoing, job-embedded opportunities for collaborative PL that seek to deprivatize practice and build capacity from educators' strengths, as opposed to filling gaps or pointing to deficits (McCrory et al., 2012). This sustained, shared PL, supported by critical discourse around daily practice, also promotes positive perceptions of collective mathematics efficacy for teaching (C-MEFT), defined as an individual's beliefs about the collaborative ability of the group to achieve mathematics teaching and learning goals. This occurs as individuals raise moments of cognitive disequilibrium, reconsider existing ideas and practices, and develop a shared vision for how to collectively promote positive change (Benoliel & Schechter, 2018; Goddard et al., 2000). This positive impact is important as educators' C-MEFT perceptions have been found to better predict student achievement than other commonly identified factors such as students' socio-economic status or prior mathematics performance (Goddard et al., 2000; Hattie, 2012; Visible Learning, n.d.). Hattie (2012) proposes that positive perceptions of collective efficacy for teaching support transformative PL and student achievement by promoting educators' willingness to honestly analyze their collective impact on student learning. As educators openly engage in conversations with colleagues around critique of personal practice they come to believe transforming their practice will promote students' academic success regardless of factors outside their scope of influence.

Critical Colleagueship Development

When this collaborative, inquiry-oriented learning is structured to strengthen social connections and to promote a shared sense of vision and commitment, educators are more willing to critically analyze and take ownership for their personal impact on student achievement through open engagement in conversations around personal and shared practice (Curry & Killion, 2009; Hattie, 2012), or what has been termed critical colleagueship (Lord, 1994; van Es, 2012). Critical colleagueship promotes transformed learning and sustains change efforts because it moves PL discussions beyond the collegial sharing of experiences and perceptions to involve active probing of and elaboration on diverse ideas and perspectives (Donohoo & Katz, 2017; van Es, 2012). As a result, educators can work together to create a PL culture that values the deprivatization of practice, critical reflection around the relationship between teaching and student learning, and the development of a shared vision of the values and components necessary to promote high-quality mathematics teaching and learning (Nelson, 2008).

Critical colleagueship development facilitates collective reflection and intellectual discourse around authentic, practice-based problems and amplifies this change effect (Lord, 1994; van Es, 2012) by drawing out divergent thinking around shared challenges and opportunities (Geijsel et al., 2003; Kintz, et al., 2015; van Es, 2012). The degree of critical colleagueship evident in group interactions is determined by three discussion characteristics: (a) collaborative interactions (the extent to which discussion joins ideas and perspectives from multiple individuals); (b) participation and discourse norms (the extent to which discussion examines multiple, evidence supported perspectives and ideas); and (c) focus of activity and discussion (the extent to which discussion references specific instructional artifacts and practices; van Es, 2012; see Appendix A for more detailed descriptions and examples). Regular engagement in practice-based conversations characterized by critical colleagueship strengthens cohesion resulting in a positive sense of collective efficacy for growth and change (Kintz, et al., 2015; Minckler, 2014; van Es, 2012) as individuals are able to see moments of cognitive disequilibrium as opportunities to bring new awareness and understanding to their practice (Anderson, 2008; Benoliel

& Schechter, 2018; Donaldson & Karp, 2023; Schechter & Ganon-Shilon, 2015).

Integrating opportunities for critical colleagueship development into organizational structures by focusing them on building collaborative cultures and shared pedagogical approaches further enables development of positive C-MEFT perceptions by including opportunities for educators to engage in school/district-wide consensus building, thus fostering an atmosphere of shared concern, respect, and empowerment (Donohoo, 2017; Dufour, 2004). However, without strategic facilitation, educators may fall back on traditional norms of polite talk and isolationism as many educators doubt their ability to collaborate and have limited experience critically examining and explaining their instructional decisions and actions (Males et al., 2010; Nelson, 2008; Russo & Beyerbach, 2001).

Mathematics Coaches' Role and Development

Full time, school-based mathematics coaches (referred to throughout as 'coaches') are instructional leaders responsible for supporting classroom teachers' ongoing learning and instructional effectiveness (McGatha & Rigelman, 2017). Their close daily contact with teachers and students places them in a particularly powerful leadership position as change agents who can promote critical colleagueship among classroom teachers as a source of personalized and collaborative professional learning that draws on individual and shared strengths and classroom contexts (AMTE et al., 2022; Campbell & Malkus, 2010; Coburn & Woulfin, 2012). Unfortunately, coaches may be limited in their ability to facilitate the work of identifying and addressing teaching and learning challenges alongside classroom teachers without ongoing opportunities to actively engage in PL with other instructional leaders (e.g., coaches and principals) to build the complex set of knowledge, skills, and dispositions coaching requires (AMTE et al., 2022; Desimone & Pak, 2017; Saderholm et al., 2016).

Federal, state, and district policies (Coburn & Woulfin, 2012) and professional organization position statements (AMTE et al., 2022) often call for the use of instructional coaches as part of teacher professional development. Many mathematics coaches are hired because they have been effective mathematics teachers but "become novices all over again…with a different set of challenges related to their role as a building or school district math leader" (Fennell et al., 2013, p. 173). Shifting from being an effective classroom teacher to becoming an effective mathematics instructional leader requires ongoing professional learning that includes opportunities to engage in the same types of collaborative, transformational learning experiences they will be using with teachers (Desimone & Pak, 2017; Voelkel et al., 2021). Regular engagement in active, practice-based learning alongside other coaches enables ongoing development of pedagogical and leadership expertise and promotes professional relationships that support critical colleagueship development and new models of thinking about what constitutes effective PL (Voelkel et al., 2021). Positioning coaches as active, collaborative learners also helps them to move beyond the evaluative, transmission style coaching they likely experienced as teachers as they gain appreciation of the power of structured opportunities for critical colleagueship development through reflective inquiry around their own daily practice (Elfarargy et al., 2022; Hoffman et al., 2015; Lammert et al., 2020).

Using a convergent mixed-methods approach, this manuscript examines the relationship between the differential use of collaborative PL structures and processes and development of critical colleagueship across four teams of elementary mathematics coaches from a single school district. The findings add to existing knowledge of how to effectively structure job-embedded learning for mathematics coaches in support of their instructional leadership capacity within transformative social learning communities.

Research Context and Methods

Libertyville (a pseudonym), the New England school district on which this study is focused, is an example of an urban district where the elementary mathematics coaches had limited opportunities to engage in critical conversations around daily practice with their peers or to take ownership of their own professional learning, as their monthly meeting agendas were set by district administrators and centered on transfer of information as opposed to examination of practice and development of shared knowledge and vision (Donaldson, 2018). The goal-based intervention (Newcomer et al., 2010) from the larger study on which this data is drawn, was designed to build upon the professional skills, knowledge, and relationships of the districts' elementary mathematics coaches and empower them to use a structured process to promote their own ongoing, job-embedded professional learning. Coaches were provided an opportunity to engage in structured collaborative inquiry into their own coaching practice to help them see themselves as change agents capable of using critical colleagueship to drive active development of a cohesive vision for mathematics instructional effectiveness and learning success throughout the school district (Donaldson, 2018; Donaldson, 2019-2020).

Participants

The 20 school-based, elementary mathematics coaches in the Libertyville Public School District, each of whom worked full time at one of the district elementary schools, all participated in the intervention. As members of their school's instructional leadership team, these educators served as what McGatha and Rigelman (2017) define as mathematics coaches because they provided instructional support to classroom teachers through grade level planning meetings and individual coaching sessions. They also served as a link between the district's mathematics leadership team and their school administrators, keeping school principals abreast of district level mathematics initiatives and overseeing mathematics assessment administration and data analysis. For the duration of the six-month study, each coach worked as a member of one of four collaborative inquiry teams (CITs; n=4 or n=6) determined by existing district established inter-school cohorts that were based on the geographic locations of the school buildings.

As can be seen in Table 1, most coaches were females (n=16) with each team having one male member and three or five female members. All teams had a mixture of individuals who worked as elementary classroom teachers prior to becoming coaches (n=12) and individuals who previously worked as middle school (n= 7) or high school (n=1) mathematics teachers. Participants' coaching experience ranged from less than one year up to 17 years (M=6.78) and all but two of the coaches had worked in the district for most or all of their careers. With the exception of Team Y, all teams were comprised of individuals who had worked as coaches in the district for over a year prior to engaging in this collaborative inquiry work. One member of Team Y was new to the district but had experience as both an elementary classroom teacher and mathematics coach in a different state. And another Team Y member was a first-year mathematics coach, but a long-term district employee having been an elementary classroom teacher for more than 20 years.

Intervention Design: Collaborative Inquiry Team Development Structures

Engaging Libertyville's elementary mathematics coaches in inter-school CITs supported their ongoing PL and provided opportunities to develop critical colleagueship around self-identified goals aligned to the district's vision for mathematics instruction, professional growth, and student achievement. However, critical colleagueship does not develop on its own, but instead requires clear articulation of individual accountability as well as established agreements for communication and collaboration (Cosner, 2009; Thompson & MacDonald, 2005). Although 18 of the 20 participants had worked as mathematics coaches in the district for at least one year prior to the start of the study, their work together in the past had been limited to scheduled time at their monthly elementary mathematics coaches' meetings where they worked to complete tasks assigned by district administrators. As a result, even though they knew each other, each coach worked in isolation at their individual school, not as a part of a cohesive or coordinated team focused on district-wide capacity building. Increasing opportunities to interact in structured, purposeful teams can support the development of collaborative relationships, a critical organizational resource for capacity building and change enactment (Cosner, 2009; Desimone & Pak, 2017; Smith et al., 2005). The researcher designed and facilitated the intervention to promote effective teamwork by (a) providing opportunities

Table 1: Participant Characteristics

Team	Gender	Coaching Experience (years)	Prior Teaching Experience
W	f=3; m=1	M=7.0 (min=3; max=17)	2 ES, 1 MS, 1 HS
X	f=5; m=1	M=5.2 (min<1; max=12)	4 ES, 2 MS
Y	f=5; m=1	M=8.7 (min=3; max=17)	4 ES, 2 MS
Z	f=3; m=1	M=6.3 (min=2; max=12)	2 ES, 2 MS

Note: f = female; m = male; ES = elementary school; MS = middle school; HS = high school for collaborative engagement within coaches' schedule without direct supervision from the researcher or district administrators; (b) establishing structures and expectations for honest communication, collective decision making, and conflict resolution; and (c) supporting the identification and development of shared, data-based inquiry goals connected to their daily work. The researcher collaborated with district administrators to put the collaboration structures and processes in place (e.g., bi-monthly meeting time, an online space for asynchronous work, team charter templates, and action planning templates) and introduced them to the coaches but did not facilitate meetings or provide feedback around ongoing collaborative work.

Opportunities for Collaborative Engagement

Time for collaboration was built into the coaches' normal workday over a six-month period. Each CIT was expected to meet bi-monthly for 60 to 90 minutes, once in person as part of a full day mathematics coaches' meeting and once virtually at a time that worked within team members' schedules. Each CIT was also asked to use the district's G-Suite for Education platform and email to collaborate asynchronously between meetings through ongoing communication and the exchange of resources in support of their shared inquiry work.

Charter Development: Establishing Collaborative Norms and Expectations

Establishing clear agreements and expectations for collaboration, mutual support, and accountability, as well as establishing a specific and measurable work focus, supports successful development of interdependence (Cosner, 2009; DuFour, 2016). Therefore, the CITs began their shared inquiry work by using a template to create a team charter for collaborative norms and responsibilities and determining their collaborative inquiry focus (see Appendix B). Team charter development provided an opportunity for each CIT to establish specific collaboration agreements for in-person meetings, virtual meetings, asynchronous work, and communication. For instance, as each CIT completed their team charter templates, they talked through perceived personal strengths and challenges related to teamwork, potential teamwork barriers (e.g., lack of attendance or unequally distributed workloads), and collaboration agreements and roles to be used at subsequent meetings.

Action Plan: Establishing Measurable and Actionable Goals

After establishing initial guidelines and expectations for their collaborative work, CITs examined existing student achievement and shared instructional rounds data and identified a specific, shared, practice-based problem, which they saw as worthy of collaborative inquiry and as aligned to district professional development priorities for mathematics teaching and learning (e.g., standards-based instruction, supporting multilingual learners, or student-centered instruction). With their chosen foci in mind, each team used a template to develop a collaborative action plan that helped them break down their larger goal into *small-wins*, specific individual and collective action steps or sub-goals, and a timeline for the work (see Appendix C).

Research Design

This convergent mixed-methods study centers a comparative analysis (Bray et al., 2014) of quantitative and qualitative data drawn from the larger intervention study (Donaldson, 2018) to examine relationships between CIT functioning and outcomes. Quantitative data from attendance records, monthly survey data, online platform use records, and collaborative inquiry planning documents (e.g., the team charter and action planning templates) were used to examine the extent to which each CIT used different intervention structures and processes. Qualitative data from conversation analysis (Clayman & Gill, 2012) of verbatim meeting transcripts were used to explore how critical colleagueship developed within each CIT. The two sets of data were then merged to gain greater insight into the following question: How did differences in the use of the collaborative inquiry process structures influence critical colleagueship development and team functioning amongst this group of mathematics instructional leaders?

Data Collection and Analysis

Engagement in Collaborative Inquiry Teamwork

Individuals' active engagement in the collaborative inquiry process was a key indicator of intervention fidelity. Without authentic engagement, critical colleagueship likely would not develop. Quantitative data related to the use of intervention structures were disaggregated by team and month and then merged with critical colleagueship data from the analysis of meeting transcripts to examine how differing levels of engagement influenced critical colleagueship development within each team.

Specifically, levels of CIT participation and engagement were examined through frequency tabulations of team meeting transcripts (the number of times each individual spoke at each meeting), meeting attendance records (the number of member absences), and asynchronous use of collaboration tools (the number of times each individual used the team's G-Suite for Education space or email to collaborate between meetings). The researcher also used descriptive statistics (mean, maximum, minimum) to analyze monthly feedback survey responses related to individual and team participation, attendance at team meetings, and tabulations of contributions to asynchronous interactions and synchronous meeting discussions to determine individual engagement (see Appendix D for the survey questions).

Development and Use of Team Planning and Process Tools

As described earlier, each team was provided a charter template (Appendix B) and an action planning template (Appendix C) at the beginning of the intervention to support successful development of interdependence. These tools enabled teams to establish clear norms and expectations for collaboration and accountability, as well as establish a specific goal and plan for their work together and for tracking progress. The development and use of these two templates were examined through both the documents themselves and the discussion of the documents during team meetings. Analysis of the documents focused on the extent to which the templates had been completed. This included examining whether all sections of documents had been filled in as well as the quality of the information, for instance whether goals were specific and measurable (e.g. "By February 2018, we will see an increase in the percentage of students at the engagement level during mathematics instruction based on Schlechty's Levels of Classroom Engagement," Team X goal) versus broad and ill-defined ("More students will be engaged in math class"). Meeting transcripts were analyzed to determine the frequency of references to and use of these two templates. Team use of the planning and process tools (charter and action plan) was examined through meeting transcripts (how many times the documents were discussed and referenced) and the documents themselves (the degree of completion and revisions and additions made over the six month time period).

Development of Critical Colleagueship

Development of critical colleagueship was examine through analysis of meeting discussions. CIT discussions from both in-person and virtual meetings were audio recorded and then transcribed verbatim for analysis. All participant names were removed the transcripts prior to analysis with each team member being assigned a label based on their team and the order in which individuals spoke at the first meeting. For instance, the first person who spoke from team X was identified throughout the study as X1. All recordings and transcripts were stored on the researchers' password protected device.

Using each meeting transcript as the unit of analysis, conversation analysis was employed to examine the naturally occurring discourse and consensual meaning making (Clayman & Gill, 2012) within the CITs' meeting discussions of practice-based work. Trustworthiness of this qualitative analysis was supported by grounding the hierarchical deductive coding process within a priori themes from extant literature and the inclusion of raw data within the manuscript (Creswell & Plano Clark, 2018; Nowell et al., 2017). As a first step to this deductive analytical noticing, each meeting transcript was analyzed using *a priori* codes based on van Es' (2012) three-stage rubric for community development in terms of three discussion elements:

- *collaborative interactions* (the degree to which discussions involved multiple individuals, the use of joint versus individual pronouns, and the joining of ideas and perspectives about mathematics teaching and learning);
- *discourse norms* (the degree to which discussions contained multiple ideas and perspectives about mathematics teaching and learning); and
- *focus of discussion* (the degree to which discussions referenced specific mathematics teaching and learning artifacts or incidents from classrooms).

Using the three-stage rubric, each element was coded (1) beginning if the exchange primarily involved exploration of single perspectives and general ideas by one or two members, (2) intermediate if the exchange involved limited sharing and probing of different perspectives and practice-based connections among some team members, or (3) high-functioning if the exchange involved active probing and elaboration of multiple, practice-based perspectives by all team members (see Appendix A for examples and further explanation).

The rubric was deemed reliable in a study of a professional learning community structured around the examination of classroom mathematics instruction videos. Van Es and colleagues (2012) had several individuals code a subset of transcripts, resulting in 85% inter-rater reliability. They also examined coded discussions for confirming and disconfirming evidence of the three elements from early versus later sessions of their intervention to further validate the coding rubric. Knowing that the rubric requires subjective judgments about the presence of defined criteria and that data reliability refers not to the reliability of the scale itself, but instead to consistency of scores obtained from that scale (Barry et al., 2014), the first round of in-person meeting transcripts for this study (one for each CIT) were analyzed independently by the researcher and by a second individual (a professor of mathematics education with a background in discussion analysis) and then the scoring was compared. There was 83% inter-rater reliability across the four meeting transcripts which is greater than the 70% threshold percent agreement statistic deemed acceptable in the social sciences (Stemler, 2011). Because there was an acceptable level of consensus, the researcher's scores were used for all subsequent meeting transcripts. At the conclusion of the study, this qualitative data was quantified using the three-point rubric (van Es, 2012) and then graphed to support insight into changes to critical colleagueship functioning and development over time. Scores for exchanges within conversations were averaged across each full meeting transcript, resulting in whole (1, 2, 3) and half scores (1.5 and 2.5).

Seeking to better understand the nature and content of CIT discussions in terms of both participation and discourse norms and focus of activity and discussion, a second round of conversation analysis and coding of meeting transcripts was done using *a priori* codes developed by Ke and Xie (2009) for analysis of learning interactions in terms of the knowledge construction and regulation of team functioning and learning. In their model, exchanges involving knowledge construction represent a four stage progression: (K1) individualistic sharing of information and ideas, (K2) egocentric elaboration on ideas, (K3) comparing and synthesizing multiple perspectives, and (K4) planning future, school-based application of new ideas. Exchanges involving regulation of learning and team functioning consist of: (R1) teamwork planning and coordination, (R2) self-evaluation and regulation, and (R3) technical issue management (see Appendix D for definitions and illustrative examples). This second round of conversation analysis data was then converged with both the critical colleagueship development data and the quantitative data related to engagement and tools/structures usage data to provide understanding of the complex relationships between process and outcomes for the four different CITs.

Comparative Analysis: Team Processes and Outcomes

The following comparative analysis examines connections between individual and collective engagement in collaborative inquiry, goal achievement in terms of each CIT's development and use of both their team charter and action plan, and the development of critical colleagueship within each team.

Charter Development and Team Engagement

Engagement

Active engagement in the collaborative inquiry process by all participants was a key indicator of implementation fidelity. Levels of engagement were defined by the following factors: number of meetings held, member attendance at meetings, number of times each member contributed ideas during meeting discussions, and asynchronous interactions within the G-Suite for Education space and email between meetings. High engagement was defined as (a) 100% of meetings being held, (b) no member missing more than one meeting, (c) all team members actively participating in all meeting discussions, and (d) all members contributing asynchronously online at least one time per month. Engagement was considered low if (a) more than one meeting was cancelled, (b) more than one individual from a team missed more than two meetings, (c) less than 90% of team members were actively involved in meeting discussions, and/or (d) more than one individual from any given team did not contribute asynchronously at least one time per month.

Overall, Team W had low engagement cancelling two of their five virtual meetings and one of their six in-person meetings. Additionally, only one team member was present at all team meetings, with two of the other members each missing one meeting, and the fourth member missing four meetings. When team members were in attendance, they all actively contributed to discussions. However, only one team member shared resources and communicated asynchronously online between meetings.

Team X members had high engagement throughout the study. The team held all meetings, and never had more than one of their six members absent from a meeting, with three team members each missing one meeting. All team members communicated and shared resources online between meetings and actively participated in team discussions.

Team Y also had relatively high engagement. Team Y held all of its meetings and all but one of its members actively participated in team discussions and collaborated asynchronously online between meetings. Three of its team members were present at all meetings, while the other three members each missed one or two meetings.

Team Z had relatively high engagement. They cancelled one virtual meeting and had one member absent for one other meeting. All team members actively participated in team discussions, however they had very little asynchronous interaction with each other between meetings with only one member adding a few resources to their shared G-Suite for Education space after their second meeting.

Charter Development and Use

After the researcher introduced the purpose and structure of team charters at the initial intervention meeting, CITs were given time to collaboratively complete the charter template (see Appendix B). Successful completion entailed (a) all members adding their names and contact information, (b) all members adding and verbally sharing their teamwork strengths and weaknesses, and (c) all members actively working to come to consensus around the eight teamwork functioning questions. Teams were then encouraged to reference and adjust their charters over the course of the intervention as needs and tensions arose. The number of times each team mentioned their charter in meeting discussions and the number of changes made to the document itself after the initial meeting were used to gauge the extent to which each team used their charter. Participants' self-assessment of adherence to the charter within the monthly feedback survey provided data around its perceived usefulness for team functioning.

Charter Completion. With the exception of Team Y, all teams successfully completed their charters during their initial meeting. Instead of collaboratively working on their charter during their first meeting, Team Y chose to spend most of its time defining team member roles (e.g., facilitator, note taker) and creating a rotating schedule for these roles for the course of the intervention. Instead of discussing individuals' perceived strengths and challenges related to the work, they asked each person to fill in this information independently before the next meeting. They also stated they did not feel it was necessary to discuss potential teamwork process issues, such as acceptable or unacceptable excuses for missing a meeting or expectations regarding team interactions and accountability, with one member stating, "We're all adults and professionals and expect everyone will behave that way" (Y6).

Not engaging in collaborative discussion and disclosure at the initial meeting appears to have impeded their progress as they were unable to agree upon a focus and plan for their inquiry work at their next meeting. Team members left this second meeting expressing negative perceptions of their collaborative abilities, with one member approaching the researcher and the district supervisor at the end of the day stating:

> I don't know how this is possibly going to work. No one is listening to each other and there are two people taking over the conversation who do not seem to respect each other's ideas. Every time I tried to talk I was interrupted. I was really excited to get to work with other coaches, but right now I'm feeling like I just want to work alone at my school (Y1).

Two underlying factors may explain these initial concerns: (a) Team Y did not collaboratively talk through potential process issues, setting clear expectations for collaboration, or sharing personal strengths and potential challenges at their first meeting and (b) Team Y had two members who were new coaches, including one who was new to the district. As a result, these two individuals had not worked with the other coaches before and had not been part of previous discussions of school improvement plans across the different schools. Both factors, not having talked through potential collaboration issues and having new team members, can negatively influence trust, or the belief that individuals will make good-faith efforts, honor commitments, and not take advantage of others (Cosner, 2009). Trust is a precondition for cooperative behavior, interdependence, and group effectiveness as it is vital for team members to balance individual and collective needs and ideas and to support collaborative and productive discourse (Anderson, 2008; Tschannen-Moran & Gareis, 2015), two essential characteristics of critical colleagueship (van Es, 2012).

To help promote productive collaboration moving forward, the district mathematics supervisor met with team members individually prior to their third meeting, providing them each an opportunity to voice concerns and to reflect on the influence their own actions have on the development of a CIT. The district supervisor also actively facilitated their third meeting, with a goal of reviewing their team charter and developing specific collaboration agreements and structures to support their future teamwork. This additional focus on collaborative functioning appears to have positively influenced subsequent work (e.g., "We are starting to rely on each other more. Through texts and our [virtual meetings], we are starting to feel like a team," Y4, Month 3 feedback survey) and are evidence of Anderson's (2008) proposition that

> …the biggest problem for any team is the assumption that you can put people together to work on a task, and they will automatically become a team and know how to work together... The trick is to put the effort into the process side of teaming (p. 468).

Charter Use and Perceptions of Adherence. Team W and Team Z did not reference their charters at any of their meetings or make any changes to them after initial development. However, monthly feedback indicated members of both Team W and Team Z consistently felt they were mostly adhering to their charters even though there was evidence, such as member absences, meeting cancellations, and tasks not being completed, that indicated otherwise. For instance, at their November in-person meeting, members of Team W had the following exchange:

> W1: Right, so when we are together, I think we do a great job of collaborating, we just didn't have time the other day to do our hangout [virtual meeting].
>
> W2: Right, when we're together we get lots done, but it's just hard with all the other responsibilities we have. Like the other day, my principal needed me to do something else when we were supposed to be meeting, so that makes it really hard. And then we have to try to align our schedules again to make up that time.
>
> W3: Yeah, we have so many people asking things of us, that it's hard to get together.
>
> W2: But as far as adhering to our charter, I personally don't feel like any of us is letting the other people down.

This exchange indicates that even though they did not always meet the responsibilities laid out in their charter and action plan, team members felt they were not letting each other down and placed the onus for their lack of regular meetings on external factors, including schedules and principal expectations.

Members of Team X and Team Y also indicated through monthly feedback surveys that they strongly agreed that they were adhering to their charters throughout the study, both at and between meetings. Team X and Team Y's use of their charters in support of their collaborative work was also evident within their meeting transcripts, with exchanges involving teamwork coordination (R1: Ke & Xie, 2009) being evident in almost all of their meeting discussions. This teamwork coordination included establishing and referencing meeting agendas, coordinating meetings and school visits, clarifying team goals, and ensuring all team members agreed with next steps and expected deliverables. One example is this exchange from Team Y's month three in-person meeting where they developed the agenda for their next meeting and coordinated the work individuals would do prior to that meeting:

> Y3: Okay, So, I put that into the agenda. And we're moving onto the December roles, so Y2 you're the facilitator and Y5 you're the resource manager. Should we create an agenda now for the Google hangout meeting?
>
> Y2: Sure, that would help me out a lot.
>
> Y4: Great, Y1 [absent from the meeting] might want to change that lesson [a shared lesson they were going to all examine] though because I think she chose it because it was something that she was going to be teaching. So, that might need to change.
>
> Y3: Another thing to consider connected to what we are doing is that we could do a consultancy for someone. So, like someone could bring a dilemma that they're having around this [CIT] work and we could do the consultancy protocol around it where we ask them questions and they explain it. That might help us keep it connected to the classroom.
>
> Y4: Sure. But will that really benefit all of us?
>
> Y3: I think so. With a consultancy I think everyone takes away something because by talking about the dilemma there's always something that you can connect to and talking through it will help you in your context too. I don't know, we don't have to do that.
>
> Y4: Oh, no. I think it's a good idea.

Y3: I'll link the protocol in here [on the agenda] so you can look at it.

Y2: Okay, so we have the consultancy. What else?

Y3: Are we taking out this lesson for now?

Y2: No, I don't think so. Y1 can teach it between now and then so we can talk about it.

Y4: Do you think I should try to teach that lesson too with my second grade?

Y2: I think anyone that wants to should try to teach it and then we can talk about it and compare what we did and how it went.

Y4: We could talk about how we changed it to make it more open-ended or how we included more student discourse.

Y2: There you go. That might be kind of cool. Everyone could try to do it, if they want to. So, for the consultancy can anyone just bring up a problem?

Y3: No, so one person would come prepared with a dilemma.

Y2: Okay, so who wants to bring a dilemma?

Y5: Do we want to wait and see who has a dilemma? It seems kind of silly to just make up a dilemma.

Y3: Sure, that makes sense.

Y2: Okay, did we make a decision? What are our goals?

Y4: So, we're all going to try teaching the lesson and add in more discourse and make it more open-ended.

Y5: Yeah, I told the teacher I'm going to let her do the teaching checklist on me and critique me.

Y6: Oh, that's a great idea.

Y2: Okay, and one of us is going to come with a dilemma. So, at our next meeting we'll have 20 minutes to talk about the lesson and 20 minutes for the consultancy which will leave us 20 minutes to plan forward and to just check in.

This type of attention to teamwork coordination appears to have supported team productivity as throughout the study monthly survey feedback indicated members of Team X and Team Y felt they were productive in all aspects of their CIT work and the teams consistently completed their monthly action plan goals.

Action Plan Development and Goal Achievement

At their second meeting, the researcher introduced teams to the action planning template (Appendix C) and then teams were provided time to determine a focus and long-term goal for their work and an initial plan for beginning their collaborative inquiry. Successful goals were expected to (a) align to a shared issue across team members' school improvement plans and (b) have the characteristics of a SMART goal, being specific, measurable, attainable, relevant, and timebound.

Action Plan Development

As discussed earlier, Team Y did not successfully develop a long-term goal or action plan at their second meeting. With their district supervisor's support, the team was able to successfully identify a goal and an initial action plan at their third meeting (see Table 2). Teams W, X, and Z established a long-term goal (see Table 2) at their initial meeting that met most of the SMART goal criteria and put forth ideas for initial tasks for team members to accomplish between meetings. In subsequent meetings all of the teams refined their goals, making them more specific and developing small-wins (sub-goals) and collective and individual action steps. Only Team X established a plan for making their goal measurable by defining specific success criteria.

Goal Completion

At the final meeting (month six of the intervention), all teams shared their goals, summarized the inquiry work they had completed, and identified next steps for their CIT work. Teams X, Y, and Z all reported having accomplished all or most of their small-wins and having established next steps to continue progress toward their long-term goals over the second half of the school year. Team X reported having accomplished its three small-wins: (a) developing a look-for tool to observe levels of student-to-student discourse in classrooms, (b) revising the tool after piloting it, and (c) using the revised tool to collect data and determining its value for engaging teachers in discussions around improving classroom practice. And Team Y reported having made some progress on its three small-wins: (a) guiding teachers in choosing high quality tasks to promote student discourse, (b) providing modeling and support for teacher use of strategies that support student reasoning and discussion, and (c) helping teachers to create a student-centered environment for mathematics learning. Although Team Z members reported having achieved most of their small-wins, these were not shared

Table 2: Areas of Focus and SMART Goals for Collaborative Inquiry Written on CIT Action Plans

	Initial Area of Focus	Long-Term Goal
Team W	Supporting teachers' ability to increase student-to-student discourse, authentic student engagement, and cognitive demand within a blended learning model [station rotation involving computer, small group, and independent practice].	By February 2018, based on weekly observation notes, we will see an increase in collaborative groups engaged in authentic mathematics conversations, during blended learning in the classrooms we support.
Team X	Supporting student-to-student engagement in problem solving contexts by helping teachers plan for productive struggle and perseverance.	By February 2018, we will see an increase in the percentage of students at the engagement level during mathematics class based on Schlechty's Levels of Classroom Engagement (www.schlechtycenter.org).
Team Y	Supporting teachers' incorporation of higher level DOK questioning and tasks into instruction.	By February 2018, we will see an increase of students communicating reasoning and responding to others around rigorous tasks in the classrooms we support.
Team Z	Using data and standards to purposely support mathematics instructional planning.	By February 2018, there will be an increase in opportunities for student engagement during the math block in the classrooms we support.

goals, but instead were unique sub-goals and actions steps each individual member was enacting at their respective schools, related to the team's shared long term goal. The team explained that although they valued the opportunity to regularly meet and "bounce ideas off of each other" (Z2, culminating session presentation), they had not come to consensus around shared action steps due to principal demands and school-based priorities. As a result, members were not working interdependently, a key criterion for working as a team (DuFour, 2016) and developing critical colleagueship (van Es, 2012). This lack of interdependence is discussed below as part of the critical colleagueship development analysis.

Team W reported that due to external factors (member illness and principal requests) it had not completed any of its small-wins, but that progress had been made on two goals not identified in their action plan: (a) developing a bank of high-quality tasks for teacher use and (b) creating and using classroom instruction videos to promote discussions among grade level teaching teams. The team also shared they had spent a lot of time calibrating their definition and vision of student-to-student discourse, work they felt would enable them to determine the data needed to identify how to best support high quality mathematics discussions in classrooms at their individual schools moving forward. Team W members also stated that "building rapport and collaboration between our cohort members was the most beneficial aspect of our cohort work" (as written on their presentation slide at the culminating CIT meeting).

Overall, there were differences among the four CITs in terms of member engagement in and use of the intervention processes and structures. Three of the teams (Teams X, Y, and Z) had high or relatively high engagement overall with most members present at all meetings and actively collaborating both during and between meetings. All four teams collaboratively completed their team charters, with Team Y needing some structured support as described above. However, even though all four teams regularly reported within their monthly feedback surveys that they had adhered to their charters, only Team X and Team Y actively used their charter throughout their work together to coordinate their work and to hold each other accountable. And finally, although all four teams created a shared long term goal that was time bound, specific, and relevant to both their daily work and district priorities for mathematics teaching and learning there were some differences. Only Team X determined success criteria to make their goal measurable and they were the only team to have achieved their sub-goals by the end of the six-month intervention. And Team Z was unique in that the members established a shared long-term goal, but each individual developed their own, unique small-wins and action plans as opposed to having a shared plan for collaborative inquiry. In the next section consideration will be given to the relationship between these differences and the development of critical colleagueship across the four CITs.

Critical Colleagueship Development

This section will first summarize the development of critical colleagueship within the four CITs across the six-month period. This will be followed by discussion of illustrative examples of discussion excerpts connected to three themes that emerged as key differences within the discussions of teams that progressed to high-functioning levels of critical colleagueship and those who did not.

Summary of Change Over Time

As described earlier, critical colleagueship within professional learning contexts is defined as the promotion of cognitive disequilibrium through the critical analysis of existing instructional practices and beliefs (Lord, 1994; van Es, 2012). Quantification of the qualitative conversation analysis using van Es' (2012) three-stage rubric indicated that critical colleagueship across the three characteristics of collaborative interactions, participation and discourse norms, and focus of activity and discussion did not develop in a linear manner for any of the teams. However, except for Team Z, there was an upward trend, with all three characteristics (interaction, discussion, and focus) ending at a higher level of development for teams W, X, and Y (see Figure 1). This development of critical colleagueship indicates discussions from teams W, X, and Y progressed, to different extents, toward active analysis of diverse perspectives and the development of shared understanding.

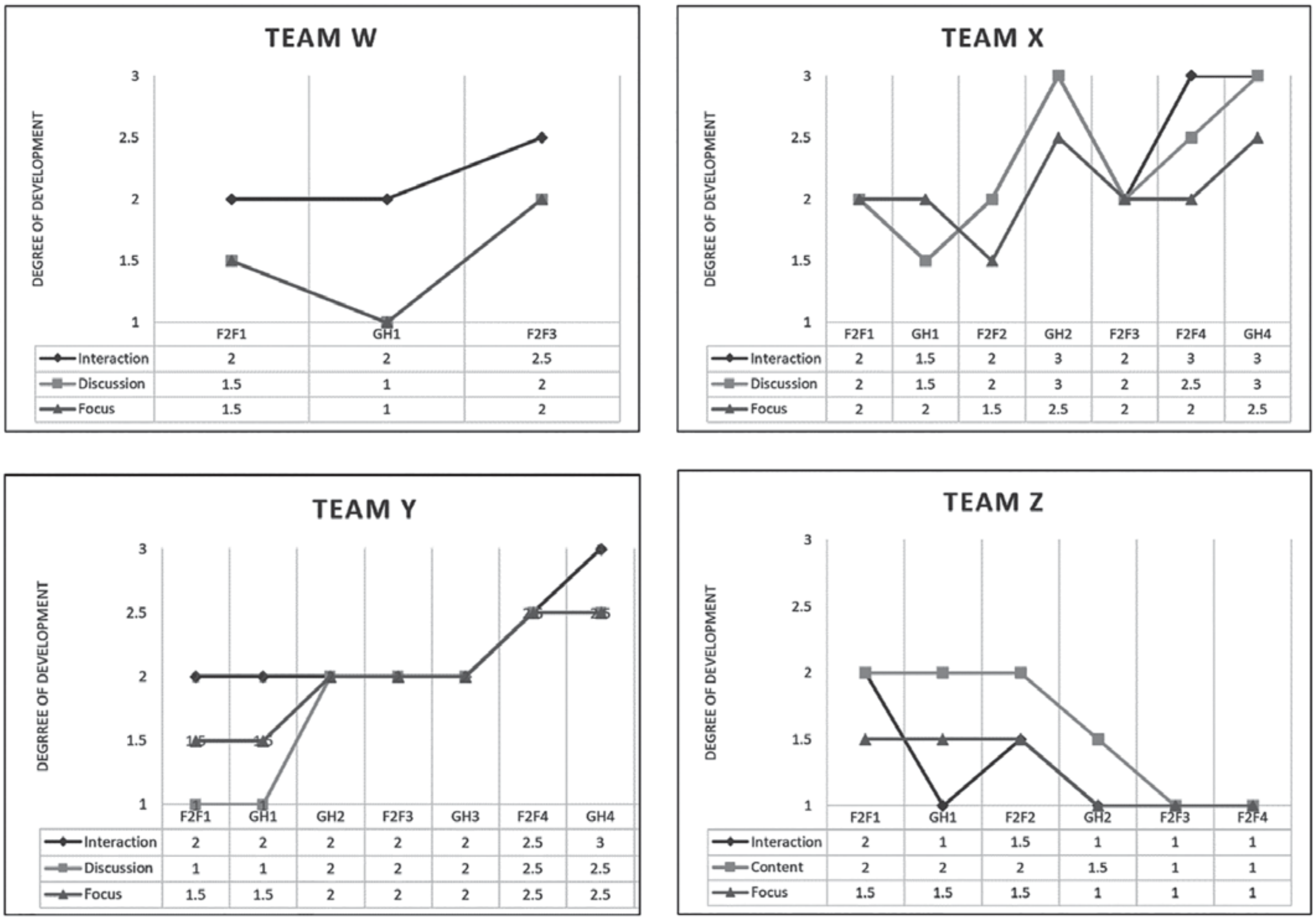
As was discussed earlier, Team W did not meet regularly, cancelling all but their first virtual meeting and canceling one in-person meeting because only one team member was in attendance. Despite having low engagement in the collaborative inquiry process, conversation analysis using the Van Es (2012) rubric indicates that critical colleagueship did still develop in Team W with elements of their final in-person meeting representing an intermediate stage of team development for all three aspects of critical colleagueship (Figure 1).

Team X showed critical colleagueship development across all three characteristics, progressing from an intermediate to a high-functioning stage of development, with all team members engaging in interactions that built upon and connected multiple, context specific incidents and ideas (Figure 1). Discussion content also included multiple interactions Ke and Xie (2009) would label allocentric elaboration (K3), with individuals synthesizing, probing, and challenging the ideas of others in pursuit of deeper, shared understanding and broadened perspective (see Appendix D for coding descriptions). The team's monthly survey feedback consistently indicated positive perceptions of collaborative capacity and interdependence (e.g., "It is great to work on a team with different members who move forward to the lead or step back based on the needs of the group" X1, Month 1, monthly feedback survey comment) that are consistent with characteristics of high-functioning critical colleagueship as well as positive collective efficacy perceptions (Goddard et al., 2000; van Es, 2012).

Team Y also showed critical colleagueship development across all three discussion characteristics, progressing from a beginning to an intermediate stage in both participation and discourse norms and focus of activity and discussion and from an intermediate to a high-functioning stage in collaborative interactions (van Es, 2012; Figure 1). This indicates team discussions progressed from focusing on individual interests and general ideas to including active analysis of diverse perspectives and the development of shared understanding (van Es, 2012).

Team Z was the one team that did not develop critical colleagueship according to analysis of meeting transcripts (see Figure 1). The discussion at team Z's initial in-person meeting was the strongest in terms of critical colleagueship discussion characteristics, with both collaborative interactions and participation and discourse norms being at an intermediate stage on the three-stage rubric (van Es, 2012), and with all members contributing to the conversation and using their own experiences to build upon each other's ideas, exchanges Ke and Xie (2009) would label egocentric elaboration (K2). The focus of their initial discussion was between a beginning and an intermediate stage in that it centered on the team's goal but was not grounded in specific mathematics teaching and learning events in district schools. Subsequent meetings, both in-person and virtual, lacked both a focus on the team's goal of using data-based planning to increase student engagement and equitable involvement by all team members. One team member (Z4) rarely spoke during meetings (accounting for only 8.3% of tabulated discussion contributions across all meetings). The other three members

FIGURE 1.
Longitudinal Critical Colleagueship Development by Team



Note. *F2F connotates an in-person meeting. GH connotates a virtual meeting. Scores are only listed for meetings that were held and successfully recorded. Beginning = 1; Intermediate = 2; High-functioning = 3. Scores for both discussion and focus for Team W overlap for all meetings as represented in the lower line. See Appendix A for definitions and examples of the coding of each element.*

contributed relatively equally to discussions (Z1=22.6%, Z3=32.2%, Z2=37.3%), but primarily talked collegially about technologies being used at their respective schools and other district initiatives in which they were involved. Overall, almost half of their discussion time, across all meetings, involved personal information and idea sharing (K1; Ke & Xie, 2009) with minimal probing or connecting of ideas, indicating a lack of critical colleagueship development. Team Z completed the study at the beginning stage for all three indicators of critical colleagueship development, lower than where they started (see Figure 1).

Colleagueship versus Congeniality

As described earlier, the characteristics of critical colleagueship that promote transformational PL include shared sense making centered on daily practice, productive disequilibrium and collaborative self-reflection as problem solving tools, and a desire to unveil diverse perspectives in the critical analysis process (Donaldson & Karp, 2023; Lund, 2020; Males et al., 2010; van Es, 2012). This differs from congenial interactions that are more typical in U.S. education settings where a culture of politeness and a lack of collegial trust often leads to courteous exchanges and individuals feeling a need to protect versus critically examine their practice (Males et al., 2010; Marshall et al., 2004). Three themes related to this difference between critical colleagueship and congeniality arose within the conversation analysis of the four teams in terms of how team members: (a) responded when practice related issues were raised, (b) addressed concerns related to external barriers to CIT work, and (c) talked about the value of their time together.

Sharing and Grappling with Issues. Although Team W members were actively involved in discussion, when they met, almost half of their exchanges involved what Ke and Xie (2009) label information sharing (K1), an indicator of beginning stages of critical colleagueship, with team members sharing individual work, as opposed to allocentric elaboration (K3), or discourse around diverse perspectives that involves the elaboration, probing, and pressing of ideas and experiences which van Es (2012) proposes are behaviors indicative of high-functioning critical colleagueship and supportive of transformative PL.

The absence of high-functioning critical colleagueship was evident in the following exchange from their third in-person meeting where an issue was raised and acknowledged (e.g., "That's so maddening."), but not grappled with by team members (e.g., "I don't know how you make that happen."):

> W4: So, you know it's the same old discussion. Some of us coaches are placed in rooms with undesirable teachers.
>
> W3: Right, I have these teachers who have these big binders and they're like this is what I taught last year at this time and so this is what I'm going to teach at this time this year.
>
> W4: That's so maddening.
>
> W3: Right. And I have so many teachers with no flexibility. And they're supposed to be working in a team because we have the dual language program where students have one teacher for math half the week teaching in English and the other teacher for the other half of the week teaching in Spanish. And the two teachers will not collaborate… my question to you is how did you break through with your teachers? … I can't even get them to be talking about standards instead of strategies. There's just no common ground.
>
> W4: I don't know how to best make that happen in your situation because to be successful those teachers really have to collaborate and if they're not willing to compromise at all, I don't know how you make that happen.

Similarly, almost half of Team Z's discussion time, across all meetings, involved personal information and idea sharing (K1; Ke & Xie, 2009) with minimal probing or connecting of ideas, indicating a lack of critical colleagueship development. For instance, in the following excerpt from their fourth monthly in-person meeting, members of Team Z congenially discuss their work to help classroom teachers at their schools use more rigorous tasks during mathematics instruction. Although multiple individuals share their work, they do not take up or probe each other's ideas or struggles but instead just share related experiences and express their agreement (e.g., "Right,…", "Yeah…", and "Uh huh").

> Z2: I've just been spending time in K [-indergarten] and [grade] 1 shifting to working on how to choose tasks and help students really talk about them. I'm finding that a lot of my teachers struggle most with the management part of it. So, I need to just really look at what parts of this different teachers can handle.
>
> Z3: Right, I have one grade level that can implement new things pretty quickly, but there are other grade levels that really struggle with it. And I struggle with supporting the younger grades with management for the same reasons as you because I just don't have the background.
>
> Z2: I was just thinking that even just being in K-2 classrooms for us is a really good thing with all of us being intermediate people…
>
> Z3: Right.
>
> Z2: 'Cause I taught second grade for one year but everything else has been grades 5 or 6, so I find that I still have the knowledge base to support these teachers but just taking time to go in and hangout in the younger classrooms to see how they work and how the children think, I think is a really good thing.
>
> Z3: Yeah, seeing how the kids react to things.
>
> Z1: Right, how are we supposed to be able to develop really good tasks if we don't really know our audience. I think that makes a lot of sense. I need to know where these little heads are in this room and where they are in that room.
>
> Z3: Yeah, and in a lot of the classrooms the teachers don't have the language and the routines, so it's really hard to implement these types of tasks if they haven't been given the tools from their teacher for one reason or another it becomes very difficult to, umm, that process is not easy.
>
> Z2: Uh, huh.

The exchanges above from Team W and Team Z are evidence of a focus on congeniality versus critical colleagueship, as members are sharing information but not probing or challenging those ideas from different perspectives. In contrast, Team X regularly connected to and probed each other's ideas and brought issues to meetings explicitly asking for support and advice. This discussion from the team's third in-person meeting is one such example:

> X3: Just this year it seems like it's hard to find the time to do these observations… our group here we're working together well and everything's great, but we're finding it impossible to make any contact on the road and take that out and get it going.
>
> X6: If it's okay, I'm thinking about just using a couple of the teachers who really have this up and running and to use the observational tool with them and to have it just be me and them, just the self-observation. Because I'm thinking if I get a couple of people really on board with this, especially a couple of vocal, on-board people, then I feel it would grow out.
>
> X3: The early adopters.
>
> X6: Yes, the early adopters, thank you. Then I think I can really roll this out.
>
> X2: Thank you for that, that's a good perspective. I'm just feeling that I've been so frustrated not having time to role this all out…I feel so great after our meetings, but then I feel like when I get back to school…there's no intersection for me. …And no matter how well I had it all planned out I just feel like I'm falling further behind. I feel like I'm not really accomplishing anything.
>
> X1: Can I make a suggestion for framing this work?
>
> X2: Sure.
>
> X1: When you're talking about it with the teachers, they're going to focus in on an area that they are most attracted to whether it's by the questions we ask or something they're struggling with. We already know what the goals are and then it's like what are we going to do and then we pick this one little thing that we can already work on, right away. And then we just do that piece. Ideally then you'd be able to go in and do another piece. So, maybe it's just controlling the observation and making sure that it includes a debrief because without the debrief are we really doing anything? I don't know. You tell me.
>
> X2: So, I wanted to do two grade levels, but maybe I just start by working with two teachers instead.
>
> X3: Right to get something meaningful done, even if it's small.

This exchange differs from Team W's, in that in the earlier case W4's expression of frustration was met with collegial empathy but no offer of ideas or potential solutions (e.g., "I don't know how you make that happen") whereas in Team X's exchange critical colleagueship was evident in that multiple team members pressed X2 to consider their struggle from different perspectives in a solution focused manner (e.g., "Can I make a suggestion for framing this work?" and "…maybe I just start by working with two teachers instead"), indicating a positive sense of collective efficacy instead of falling back on external blame and powerlessness.

Similarly, Team Y members regularly connected to and probed each other's ideas and brought issues to team members explicitly asking for support and advice. When discussion centered on developing and discussing teaching and learning, Team Y members had multiple interactions involving allocentric elaboration (K3; Ke & Xie, 2009), with individuals synthesizing and challenging the ideas of others to deepen their shared understanding and broaden individual perspectives. These types of interactions are indicative of high-functioning critical colleagueship and support individual and collective transformative PL.

The following exchange from Team Y's month four in-person meeting illustrates that when critical colleagueship is present individuals willingly expose struggles and openly listen to alternative perspectives that may lead to improved teaching and learning:

> Y3: …that was the classroom I was talking about that doesn't do any whole group instruction. They only do small group because they have so many severe kids. You know, what would be the point of doing that whole class when they wouldn't be getting anything out of it. I'm not trying to be mean, but we have some really unique kids at our school that just need something else…But anyway.
>
> Y2: But I think you're asking the right question. What is the purpose? Just getting into what are their needs and how are you meeting them.

> Y3: Those kids are just so isolated in that classroom already, and if I went in there and was like we're going to do a number talk with everyone. I would just be setting things up for failure.
>
> Y4: So, why don't you just go in and set it up for the kids that it would be appropriate for, so the teachers can see how it works.
>
> Y2: Right and then they could think about how to adjust it to work for the other students.
>
> Y3: Oh yeah. That might work. I mean there are only a few students who are really severe…

Like the example from Team X, this discussion ends with a statement that indicates a sense of agency ("Oh yeah. That might work.") resulting from multiple team members offering diverse perspectives and probing the specifics of the conversations ("But I think you're asking the right questions" and "So, why don't you…"), as opposed to congenially offering empathy as was seen in Team W's discussion ("That's so maddening").

Discussion of External Barriers. Another discussion characteristic indicative of high-functioning critical colleagueship and positive C-MEFT perceptions was a solution focused approach to discussions of systemic and structural barriers, such as a lack of time, teacher buy-in, and principal support. Unlike some of the other teams, Team Y's members responded to the discussion of barriers by engaging in collaborative problem solving. For instance, during their second virtual meeting a team member brought up a struggle they were having with two barriers, finding time for CIT work and getting buy-in from her teachers:

> Y2: I wonder…I'm feeling like if there is something that we are doing for someone else how we can incorporate this into our daily work. I'm feeling like I'm having trouble getting traction into this work. Maybe it's because I'm new this year and I missed all of the work last year.
>
> Y3: So, I have a question. Do any of the small-wins help bring this [our CIT work] to your day-to-day?
>
> Y2: I think maybe. I feel good about it when I'm filling out this form, but then when I'm back at my school I'm having trouble seeing how this fits into what I'm trying to do.
>
> Y4: I'm wondering if you should try to find one teacher that you can start this work with and then build from there.
>
> Y2: Yes. Absolutely. That might work.

In this exchange, Y2 appears to have trusted their team members to support as opposed to judge their struggle to implement the team's action plan at her school. Instead of empathizing with them (as was seen in the discussion excerpt from Team W above, "I don't know how you make that happen"), team members (Y3 and Y4) provided suggestions for small steps Y2 could take to move forward with the CIT work within their own context (e.g., "I'm wondering if you should try…"). Professional exchanges where team members share struggles and offer suggestions for overcoming potential barriers promote positive C-MEFT perceptions by demonstrating collective capacity. This type of open exchange of problems and potential solutions has been found to also support critical colleagueship development through collective reflection on teaching and learning practices connected to specific practice-based incidents (Hamann et al., 2001; Males et al., 2010; van Es, 2012). This mutual trust and development of productive collaboration were evident in a team member's comment at the final session: "I was really hesitant at first … to work with my team. To be honest, I figured I'd just continue to do my own thing. But…we're really getting on as a team and helping each other get some really good work done" (Y1).

On the other hand, similar to Team W, when members of Team Z identified external factors (e.g., district initiatives) that appeared as barriers to their shared work and their ability to engage in instructional coaching work they were acknowledged as personal sharing and there was no effort by team members to probe the issues or work to overcome them. For example, in this discussion at their month three in-person meeting members brought about the same issue that Y2 brought up in the excerpt above, not having enough time for their CIT work, but instead of offering possible solutions ("I'm wondering if…"), here Z1's concern is met with agreement that these external demands are a shared problem ("Right…" and "That's frustrating.") similar to the example from Team W above:

> Z1: Right, and I feel like I don't have the time to really work with the teams at my school because there are so many other things on the agenda all the time.

> Z3: Right, and the next CPTs are going to have to focus on the interim assessments, so then what are you supposed to do. And then there'll be something else procedural that will come up.
>
> Z1: Right, all the paperwork and analysis of the assessment data eats up a lot of time.
>
> Z4: Even getting the kids to be able to complete the assessments on the computer takes a ton of time.
>
> Z3: Right, I find they can't even get into it sometimes. That's frustrating.

Team Z completed the study at the beginning stage for all three indicators of critical colleagueship development, lower than where they started (see Figure 1), which may indicate individuals lacked confidence in their collective capacity and had limited readiness to engage in transformative PL.

Appreciation of Time Together. Although most of Team Z's and Team W's discussions did not include the collaborative discourse indicative of high-functioning critical colleagueship, team members spoke positively of the opportunities for collaboration and the exchange of ideas the CIT work provided. For example, a member of Team W stated, "We are like-minded and enjoy sharing ideas with each other" (W3, month 2 feedback survey) and one Team Z member commented:

> … it's kind of nice at these meetings to just be able to talk and catch up with you guys about what is going on at our different buildings, because it's been so structured [in the past] that we hadn't really been able to debrief and talk (Z1, month 3 meeting transcript).

Based on Wenger and colleagues' (2011) framework for assessing value creation in professional learning communities, Booth and Kellogg (2015) propose participants find value in different ways, moving through a developmental cycle that begins with enjoyment of engagement with peers by discussing and sharing ideas. The types of statements above (e.g., "it's kind of nice…to just be able to talk and catch up…") as well as Team Z's final presentation statement, "bouncing ideas off each other was a great benefit," may indicate that members of Team X and Team Z were at this beginning stage of value creation. Over time, this initial stage, where members simply enjoy engaging with peers, provides opportunities for trust development and vicarious success experiences as members listen to peers' ideas. So, although Team X's and Team Z's discussions did not reflect the discourse and interdependence characteristic of high-functioning critical colleagueship, the idea exchange and trust development occurring during these meetings may still account for members' satisfaction with their CIT work, but their continued identification of external factors as barriers to their collaborative work and lack of willingness to critically examine issues and ideas brought up in the conversations may also indicate a lack of readiness to engage in a process of transformational PL.

Discussion

The key variable of interest in this study was the development of critical colleagueship within the four CITs, as it is engagement in public reflection and collaborative discourse around practice that promotes transformational PL (Hamann et al., 2001; van Es, 2012). Critical collegiality and discourse, when connected to daily practice, create the cognitive disequilibrium needed to promote meaningful change as educators consider multiple perspectives, examine new ideas, and debate their usefulness for achieving goals (Benoliel & Schechter, 2018; Puchner & Taylor, 2006; van Es, 2012).

Ernest and colleagues (2013) propose that collaborative learning goes beyond exchanging information (Ke and Xie's, 2009, level K1 and van Es', 2012, beginning stage), involving reciprocity and comparing points of view (Ke and Xie's, 2009, level K3 and van Es' 2012, high-functioning stage) to produce higher quality knowledge construction than could be developed individually. Critical colleagueship development, or the progression from a focus on individual interests and references to discourse around diverse perspectives to promote shared understanding (van Es, 2012) and positive beliefs in collective capacity, stems from active and collaborative participation. Thus, this comparative analysis focused on determining how differences in engagement in a structured collaborative inquiry process, including the development and use of team charters and action plans, influenced the development of critical colleagueship amongst these teams of instructional leaders over time. Based on an examination of this study's results and existing empirical literature, it appears that two features of the structured collaborative inquiry process were influential: (a) explicit attention to teamwork culture development and ongoing collaborative functioning and (b) development of a shared measurable goal and action plan.

Proactive and Ongoing Attention to Teamwork Functioning

Developing a team charter at the beginning of the inquiry process provided an opportunity for team members to determine agreements for social interaction and measures of accountability (Anderson, 2008; Servais et al., 2009). As Team Y found in this study, not engaging in collaborative development of a charter at the beginning of the CIT process can lead to individuals lacking a sense of how to move forward as an effective team (e.g., "I don't know how this is going to work…"). The detailed operating norms Team Y eventually developed were referred to at every subsequent meeting and promoted critical examination of ideas and practices as all members knew what was expected of themselves and others. This provides evidence for Servais and colleagues' (2009) proposition that "developing a set of operating norms is an important first step to guide the [collaborative inquiry] process and assure accountability to the team" (p. 8). These agreements support members as they develop relationships and build a foundation of trust. In addition to establishing these collegial expectations at the beginning of the process, both Team X and Team Y referenced the agreements within their charters regularly and spent time coordinating their work together at all meetings. This contrasts with Team W and Team Z who, although they established expectations within their charter, did not reference these agreements or spend time on teamwork coordination at subsequent meetings. As Team X and Team Y progressed to higher levels of both critical colleagueship and knowledge development and completed most of their action plan goals, while Team W and Team Z did not, it appears explicit attention to and documentation of team functioning and culture both at the beginning and throughout the collaborative inquiry process was an influential factor.

Shared Measurable Goals and Action Steps

Although all four teams established a long-term goal their use of the action plan template within their collaborative inquiry process differed. Team X and Team Y both established a shared, measurable goal and steps to achieve that goal and then used their action plan to set meeting agendas and to support active, collaborative engagement during and between meetings by all members in efforts to achieve their shared goal. Both teams progressed to high-functioning stages of critical colleagueship and had multiple exchanges involving allocentric elaboration (K3) and application (K4), Ke and Xie's (2009) highest levels of knowledge development. On the other hand, although they established a shared long-term goal, neither Team Z nor Team W collaboratively worked toward that goal through shared action steps. Members of Team Z each had different sub-goals and worked independently toward achieving their goals, as opposed to having shared action steps around which to collaborate. And, although Team W collaboratively accomplished work, the tasks they completed did not align to their established goal. Additionally, neither team used their action plan template to support their collaborative work, including not assigning tasks to members or developing meeting agendas. Neither Team W nor Team Z progressed past an intermediate stage of critical colleagueship and had few exchanges that progressed past knowledge sharing or egocentric elaboration. Evidence from this study supports and builds on findings from other empirical studies that, although agency around goal setting and meeting agendas promotes positive efficacy perceptions, having and purposefully using clear structures to focus and coordinate shared work appear necessary for promoting critical colleagueship, transformational PL, and team productivity.

Implications

This study examined the impact of engagement in structured collaborative inquiry on inter-school mathematics coaches' critical colleagueship development in their work to promote impactful and cohesive mathematics teaching and learning across district elementary schools. The findings around what structures, systems, and routines promoted successful team functioning and outcomes have implications for both district level support of coaches' ongoing PL as well as coaches' ability to facilitate transformational learning experiences for the teams of teachers whom they are tasked to support. This includes providing transformational PL opportunities for coaches and promoting the explicit development and use of structures to support effective facilitation of PL within regularly scheduled meetings to help promote and sustain transformational PL in support of students' academic achievement.

Mathematics coaches are well positioned to be change agents within schools. To achieve this potential, they need ongoing opportunities to engage in collaborative learning with other instructional leaders (e.g., coaches from other schools and principals) to build the repertoire of skills, knowledge, and dispositions needed to effectively lead PL within and across schools (AMTE et al., 2022; Desimone & Pak, 2017; Voelkel et al., 2021). Structuring these learning opportunities as collaborative inquiry into problems of practice arising from their daily work can enable coaches to see the value of this type of transformational PL. This practice-based inquiry can also help them rethink mental models of traditional top-down PL and coaching, thus building their change building capacity (Desimone, 2009; Voelkel et al., 2021). Being able to speak to specific challenges and benefits of these types of PL experiences also supports coaches' ability to help teachers engage in collaborative work and to share their needs and goals (Elfarargy et al., 2022; Fennell et al., 2013). Similar to new teachers who are apt to revert to teaching the way they were taught, as novices in the role of instructional leaders, mathematics coaches need to experience, firsthand, this type of transformational learning, not just be exposed to new ideas (Elfarargy et al., 2022).

We know that not all educators are self-directed learners willing to take initiative and persist in the learning process. This study's findings indicate that putting specific structures in place and supporting the coaches' use of team charters and action plans throughout the collaborative learning process promoted both critical colleagueship development and transformational PL. Specifically, the collaborative development of team charters enabled teams to get to know each other, to consider the diverse strengths each individual was bringing to the work, to proactively discuss common teamwork challenges, and to create clear expectations for collaboration and communication. Once in place, these documents also provided a tool to support and monitor teamwork functioning throughout the process.

Additionally, the collaborative development of an action plan with clear goals and success criteria as specific, incremental work to be done supported both individual and collective accountability. Regularly referring to these collaboration plans and charters appeared to have promoted goal accomplishment and team functioning. It also appears that facilitation by an instructional leader, whether that be a coach for a team of classroom teachers or a district level curriculum coordinator for a team of coaches would further support critical colleague development and transformational PL. This knowledgeable other can help create and sustain a safe learning environment, promote intentional discourse, help individuals overcome a sense of avoidance, and bring new ideas and perspectives to the surface (Elfarargy et al., 2022). In this study, this type of facilitation enabled Team Y to move past their initial teamwork roadblocks to develop high levels of critical colleagueship and it may have helped Team Z determine how to develop shared sub-goals and action steps and Team X to move their discussions past congenial exchanges of ideas and toward the critical discourse needed for transformational PL. Although some teams of educators are capable of effectively leading their own learning (as was the case for Team X), it is important for mathematics instructional leaders to recognize this is not always the case and that their active support in establishing time for educators (whether they be teachers or coaches) to meet regularly and attending to the explicit development and use of PL structures and processes will support success (Voelkel, 2022).

This study was limited in scope and leaves open opportunities for future research, including examining how instructional leaders can use structures such as team charters and action plans to facilitate collaborative inquiry opportunities for the teachers whom they support. Additionally, mathematics teacher educators in higher education, or mathematics coaches themselves, may use the structures and processes to create professional learning networks across districts where there are only one or two instructional leaders as a way to broaden perspectives and further change efforts. And finally, as was evident in Team W and Team Z in this study, educators can find value in working together even when not engaging in high levels of critical colleagueship or knowledge development. Examining how instructional leaders can help educators (both teachers and coaches) use this sense of satisfaction with collaborative PL to move toward a willingness to engage in critical colleagueship to sustain transformed knowledge and practice through honest reflection and constructive discourse is a potential area for future research. ✪

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Appendix A
Qualitative Coding Rubric for Examining Critical Colleagueship

	Beginning	Intermediate	High Functioning
Collaborative Interactions	Discussions dominated by 1-2 participants	Discussions involve some members but not distributed among the group	Discussion involves most participants
	Conversation initiated not taken up by the group.	Conversation initiated and taken up by only 1-2 group members.	Conversation initiated and taken up by most members of the group.
	Participant talk is one-sided and egocentric.	Participants begin to listen to each other and validate ideas; some support of each other and some one-sided talk.	Participants listen to each other and pursue each other's ideas; try to understand other's ideas and experiences; offer support to others.
	Participant talk is not connected. Participants shift focus.	Participants begin to have collaborative conversations but still redirect to own interests	Participants raise issues and pursue one another's issues and ideas
Participation and Discourse Norms	Only one perspective represented	Different ideas and perspectives are raised but not always discussed	Different ideas and perspectives are raised and consistently discussed
	Little or no references are made to shared referents or specific school/classroom incidents to build discussion	Some evidence from shared referents or specific school/classroom incidents is used to build discussion	Consistently use evidence from shared referents or specific school/classroom incidents to build discussion
	Little elaboration, probing, or pressing of ideas	Some elaboration, probing, and pressing of ideas	Elaborating, probing, and pressing on one another's ideas frequently occur
Focus of Activity and Discussion	Topics of discussion are peripheral to progress on team's goal, is general in nature and may address a variety of topics peripheral to mathematics teaching and learning.	Discussions focus on progress made on team's goal but are not necessarily connected to mathematics teaching and learning in district schools	Analysis of progress on team's goal is directly connected to mathematics teaching and learning in district schools
	Participants' talk is primarily sharing general experiences and not grounded in shared referents or specific school incidents	Ideas developed with both general experiences and shared referents or specific school incidents	Discussions are grounded in specific mathematics learning and teaching through shared referents or specific classroom incidents

Note. Adapted from van Es' (2012) framework for development of teacher learning community in a video club (Table 4, p. 186).

Appendix B
Collaborative Inquiry Team Charter Template

Team Member Names	Contact Information (email, cell, etc.)	Preferred Contact Method & Limitations (i.e.: no calls after…)
Team Member Names	**Strengths related to teamwork & the team's chosen POP**	**Weaknesses related to team-work and chosen POP**

1. **What roles will each member have during and between meetings?** (Consider both logistical tasks, such as arranging meetings, preparing agendas and meeting minutes, and keeping materials organized online; as well as team process roles, such as questioning, ensuring everyone's opinion is heard, etc.)

2. **When will your team hold its monthly Google Hangout meeting?** (Day and Time)

3. **What are your team's expectations regarding meeting attendance?** (Being on time, leaving early, missing meetings, etc.)

4. **What constitutes an acceptable excuse for missing a meeting or a deadline? What types of excuses are not considered acceptable?**

5. **What process will team members follow if they have an emergency and cannot attend a team meeting or complete their individual work/deliverable on time?**

6. **What are your team's expectations regarding the quality of team members' preparation for team meetings and the quality of the deliverables members bring to the team?**

7. **What are your team's expectations regarding team members' ideas, interactions with the team, cooperation, attitudes, and anything else regarding team-member contributions?**

8. **What methods will be used to keep the team on track?** (How will your team ensure that members contribute as expected to the team and that the team performs as expected? How will your team celebrate members who do well and manage members whose performance is below expectations?)

Adapted from CATME Smarter Teamwork tools http://info.catme.org/catme-tools/

Appendix C
Collaborative Inquiry Team Action Planning Template

Action Plan for ________________________________

Long Term Goal(s):	**Small-Win Strategy #1:** **Small-Win Strategy #2:** **Small-Win Strategy #3:**

Small-Win Strategy #1:

Planned Work, Activities, and Tasks	Resources Needed	Timeline	Person(s) Responsible	Evidence of Success

Appendix D
Monthly Feedback Survey

Indicate approximately how often you performed each action as part of your inquiry team work over the past month (never, once or twice, weekly, almost daily):

- I communicated with team members through email
- I shared resources with team members through email
- I created a new Google Doc or Sheet on my own
- I collaborated with teammates using a Google Doc or Sheet
- I added new resources to a Google Drive folder
- I used resources someone else had shared on Google Drive
- I created a new folder on Google Drive and shared it with colleagues
- I participated a Google Hangout meeting/discussion
- I accessed a link or notification through the communication Stream
- I posted a link or notification on the communication Stream

For each Google Classroom tool, indicate how valuable it was for supporting your team's inquiry work over the past month (it made it more difficult, it helped a little, it was somewhat helpful, it was vital to our success):

- Email
- Google Docs/Sheets
- Google Drive
- Google Hangout sessions
- Communication Stream

Do you have any other comments you would like to share related to the use and availability of tools within Google Classroom?

Respond to the following statements related to how your team worked together over the past month.

- For each work format, indicate how productive you feel your team was during the past month (not at all, somewhat, productive, very):
  - In-person meeting
  - Video meeting
  - Collaborating online
  - Working individually
- This month my team accomplished (choose one): all of its work goals, most of its work goals, some of its work goals, few of its work goals
- Because (choose all that apply): we collaborated and everyone completed their assigned responsibilities, one or two members completed all or most of the work, our work plan was not realistic, we could not come to consensus on critical decisions, other:
- This month my team members and I adhered to our team charter, in terms of expectations for communication and collaboration online between meetings: strongly disagree, disagree, agree, strongly agree
- This month my team members and I adhered to our team charter, in terms of expectations for communication and collaboration at both our video and in-person meetings: strongly disagree, disagree, agree, strongly agree

Do you have any other comments related to how your team worked together over the past month?

Final Thoughts:

Over the next month, I hope my team continues to:

One change I would like to see in our inquiry work is:

How Understanding Mathematical Discourses Shapes Principal Noticing

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Abstract

Principal leadership is a key factor in student achievement, but we are not yet sure how knowledge of content influences leadership. Teacher evaluation systems assume principals understand the pedagogical content knowledge (PCK) for multiple disciplines, a particularly challenging expectation for secondary leaders. This study presents a noticing framework of PCK for leadership that describes a progression through four levels, from content-neutral pedagogy to an interconnection between pedagogy and mathematical discourses. Using the framework, the study provides evidence that principals can learn to notice significant mathematical events but may struggle to respond to teachers about those events. The framework can serve as a tool for leaders to learn to notice the role of mathematics in classrooms during their work with teachers.

Keywords: pedagogical content knowledge for leadership, noticing, mathematical discourses, standards for mathematical practice.

Principal leadership is the second-most influential school-related factor impacting student achievement behind teaching (Branch et al., 2013; Leithwood et al., 2004). Although a principal must have a breadth of knowledge about many influences on student achievement, such as hiring quality teachers, aligning curriculum and assessment, and fostering partnerships with parents (Murphy, 2017), this paper examines the knowledge of mathematical content that principals use when they supervise and evaluate classroom teachers.

An important role for school administrators is evaluating teacher effectiveness related to student learning. Teacher evaluation protocols are now available to help principals develop knowledge about what to observe and how to provide feedback for improving instruction. Domain 1 in the Teacher Observation Protocol (Marzano, 2017), Domain 1a in The Framework for Teaching: Evaluation Instrument (Danielson Group, 2022), and four dimensions of The 5D+ Rubric for Instructional Growth and Teacher Evaluation (Center for Educational Leadership, 2016) all describe the importance of specialized content knowledge for teaching. These evaluation protocols also have rubric elements related to ways in which teachers consider disciplinary content in their planning and instructional decisions. These rubrics assume that observers have enough knowledge of the discipline being observed to notice the pedagogical content knowledge (PCK) that teachers draw upon. Middle school and high school principals face particular challenges as the disciplinary content knowledge increases at higher grades. Expecting secondary school administrators to have a sufficient depth of content knowledge to notice PCK and provide related feedback in every discipline they supervise is unrealistic.

The literature does not yet articulate how content knowledge directly influences instructional leadership (Lochmiller et al., 2012), or what principals may need to know about content to support teacher learning and change (Larbi-Cherif, 2016; Lochmiller & Acker-Hocevar, 2016; Steele et al., 2015). Thus, articulating for middle and high school instructional leaders some aspects of the discipline that are essential for students to learn mathematics and helping supervisors notice when those aspects are being enacted may position them to make important leadership decisions that can improve student learning of mathematics.

Purpose

This paper presents a Pedagogical Content Knowledge for Leadership (PCKL) framework that describes a progression from general to content-specific noticing. Using this framework, we describe what nine middle school principals or associate principals attended to during videos of mathematics lessons, and how they said they would respond to the teachers. In our analysis, we demonstrate how the PCKL framework can support principals who are learning to observe mathematical events in classrooms. We conclude with ways that instructional leaders and professional development providers can use the PCKL framework to develop a sharper vision of productive mathematics classrooms and learn to provide feedback targeting student mathematical engagement.

The PCKL framework describes how a leader uses content in a mathematics classroom observation along a continuum: 1) how a classroom event can be observed without considering the content, 2) how the content can be observed within a classroom event but not viewed as important to the event, 3) how instructional decisions can be observed as intersecting with mathematical content, and 4) how mathematical Discourses (Gee, 2011) can be observed as key to the classroom event. Using the PCKL framework, the authors coded the levels of noticing that principals with varying leadership and professional development experience demonstrated when observing math lessons. Our analysis indicates that principals can learn to notice important mathematical events during lessons, but even when they do, they may struggle to provide related feedback to teachers.

Theoretical Frameworks

Discourses and the Standards for Mathematical Practice

Mathematical Discourses (Gee, 2015a) describe the spoken, written, and visual forms of communication that students use as they develop an understanding of mathematics, a sort of disciplinary literacy for mathematics communities (Croce & McCormick, 2020). Big 'D' Discourse captures socially recognizable ways of 'being' within a group, the inextricable ways that members talk and interact, the objects or tools they use, and their values and beliefs. We can quickly discern tourists not only through their language and their cameras, but also through what they wear, how quickly they can pull out the necessary currency, how loudly they speak, or whether they make a faux pas over dinner because tourists have not learned the Discourses that shape the cultural identity of members of the host location. Similarly, we can discern if students have developed a mathematical identity by observing their interactions and behaviors in their classrooms. We act out socially recognizable identities when we use big 'D' Discourses.

The Standards for Mathematical Practice (SMPs) are the first standards listed in the Common Core State Standards for Mathematics (CCSS-M) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), defining ways in which students act out the socially recognizable identity of an emerging mathematician. By beginning each Standard for Mathematical Practice (SMP) with, "Mathematically proficient students …," the authors of the CCSS-M make clear that how student mathematicians engage with the discipline, the tools they use, the ways they interact, and the language they use matter. The SMPs describe mathematically essential behaviors such as making sense of problems, constructing arguments to justify conjectures, using mathematical tools appropriately, and looking for regularity in repeated reasoning. Whereas the Discourses encompass the plethora of ways that students enact a mathematical identity, the SMPs are descriptors of eight essential Discourses that teachers and leaders can observe. Schools that strive to provide a robust mathematics experience can use the SMPs as a guide for developing a culture of learning in their classrooms, a culture alive with mathematical Discourses.

Noticing

Although researchers have a variety of perspectives about teacher noticing (König et al., 2022), noticing has become an important tool in how researchers understand the work of mathematics teachers. Goodwin (1994), taking a sociocultural perspective, suggested that professional communities can negotiate a shared vision of a knowledge base that is of interest to members of the profession. This perspective of professional vision suggests that noticing is a sociocultural phenomenon, and that one's profession shapes the events attended to and the interpretation of those events.

Teacher noticing is broadly accepted as consisting of three interrelated components: attending to a salient incident, making sense of the incident, and identifying what is important and deciding how to respond (Jacobs et al., 2010; Kaiser et al., 2015; Sherin et al., 2011; van Es & Sherin, 2002). van Es (2011) drew on this understanding of noticing to propose a framework for learning to notice student thinking. The framework shows a trajectory of four levels of noticing student mathematical thinking from a baseline where teachers, "Attend to the whole class environment, behavior, and learning, and to teacher pedagogy," to an extended level where teachers, "Attend to the relationship between particular students' mathematical thinking and between teaching strategies and mathematical thinking." Teachers at the extended level are able to draw connections between instructional decisions and student learning. Moving along the trajectory affords opportunities to learn about the relationship between teacher practice and student understanding.

Researchers have examined how experts notice similarly or differently from novices (Bastian et al., 2022; Huang & Li, 2012; Scholten & Sprenger, 2020). Expert-novice studies found that, through intentional interventions, prospective teachers can learn to attend to salient events and interpret them like more experienced teachers (Jacobs et al., 2022; Miller, 2011; Roth McDuffie et al., 2013). However, teachers who notice student mathematical reasoning at a high level may need additional support in learning how to respond (Jacobs et al., 2010). Targeted professional learning experiences can help both novice and veteran teachers prepare effective responses to the events they deem salient (Jacobs et al., 2022; Jilk, 2016; Sherin & van Es, 2009).

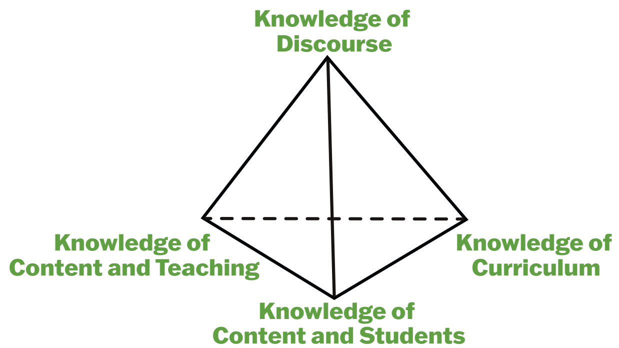
Pedagogical Content Knowledge

In an attempt at articulating standards for professional teachers, Shulman (1987) included PCK - "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8) - as important among the different bases of knowledge that teaching requires. He continued, "It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction." Hill et al. (2008) articulated PCK as comprised of three parts – knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

Knowledge of content and students refers to a teacher's understanding of how students learn mathematics, such as the common mistakes students will make, how they will respond to confusion, or how to draw out student reasoning. Knowledge of content and teaching refers to the teacher's relationship with mathematics, such as how teachers explain math concepts, how they use technology to support student learning, which ideas they deem worthy of further classroom consideration, or which mathematical representations they choose to illuminate the concepts that students are considering. Knowledge of curriculum refers to the teacher's understanding of available materials and when different choices are most appropriate, how to select or modify tasks based on student needs, or whether the trajectory of mathematics through the year and across grades is coherent.

While working with the ideas of PCK, Hauk et al. (2014) described an interplay among the original three-part PCK framework with mathematical Discourses, "the ways of combining and integrating language, actions, interactions, ways of thinking, believing, valuing, and using various symbols, tools and objects to enact a particular sort of socially recognizable identity" (Gee, 2011, p. 29). As shown in Figure 1, Hauk et al. (2014) visualize PCK as extended into a tetrahedron connecting knowledge of Discourse to each of its three components. They defined knowledge of Discourse as "knowledge about the culturally embedded nature of (big D) discourse, including inquiry and forms of communication in mathematics both in and out of educational settings" (p. 171).

FIGURE 1.
Extended Model of PCK



Note: Tetrahedron is used to visualize the relationship between knowledge of Discourse and the three components from the PCK side of the MKT framework. Adapted from "Developing a Model of Pedagogical Content Knowledge for Secondary and Post-Secondary Mathematics Instruction" by S. Hauk, A. Toney, B. Jackson, R. Nair, and J. J. Tsay(2014), *Dialogic Pedagogy, 2(28).*

Leadership Content Knowledge

While the idea of PCK has been understood for the work of teachers, the role of content in the work of school leaders is less clear. Just as teaching and content are inextricably intertwined, Stein and Nelson (2003) showed evidence that there is also a specialized content knowledge for educational leaders that they termed leadership content knowledge (LCK). Although no researcher claims that a principal needs to understand content in the same way that teachers do, some do suggest that the roles of supervision, evaluation, professional development, and resource allocation all intersect with disciplinary content. The disciplinary content knowledge a school leader holds may influence how they enact those roles (Overholt & Szabocsik, 2013; Printy et al., 2008; Stein & Spillane, 2005). Exactly how content influences instructional leadership remains less clear (Lochmiller et al., 2012) as does what they need to know about the disciplines they supervise (Larbi-Cherif, 2016; Lochmiller & Acker-Hocevar, 2016; Steele et al., 2015).

Studies have demonstrated that when administrators learned about characteristics of rich mathematics tasks, high-quality, inquiry-based instruction, or different mathematical representations, they were able to notice student thinking and the link to teacher practices (Boston et al., 2016; Steele et al., 2015). In one study wherein principals were shown the same classroom video in October and again in June, with supported professional development in between, their attention moved from such issues as classroom management, wait time, and which students were called on towards mathematical discourse and student thinking (Nelson & Sassi, 2000). The authors explain, "they need to understand that students' subject-matter thinking is central and that the administrator's 'eye' for classrooms needs to be tuned to following the student thinking in class rather than the teacher's behavior alone" (p. 576). As principals' understandings of what matters in the classroom expanded, what they attended to changed.

Methods

This qualitative study used structured interviews to investigate the professional noticing of mathematics leaders. The participants, methods of gathering data, and manner of coding of interviews are described below.

Participants

The nine participants in this study were chosen based on their role as a middle school principal or assistant principal with at least four years of experience as an administrator. We recognize that the roles of principals and assistant principals have significant differences, but as this study focuses only on the similar work of classroom observation, we refer to all participants as principals. Our participants' experience fell into three categories: (1) three had participated in at least one year of professional development (PD) about research-based instructional practices in mathematics designed for teachers plus a separate component for school leaders, (2) three had taught mathematics at the secondary level but had not had intentional PD about mathematics leadership, and (3) three had neither taught mathematics nor had PD for leading mathematics. One of the principals in the final group had participated in PD designed for mathematics teachers without the additional work on mathematics leadership. Their pseudonyms are listed in Table 1.

Blake, Bradley, and Henry worked in the same school district at different schools, and the other six participants worked for four other school districts in the same region. The school district that offered mathematics leadership PD, located in the Pacific Northwest, has less than 15% of children living below the poverty level and a slightly higher per capita income than the state. More than 80% of children live in English-only homes and 6% live in Spanish-speaking

Table 1: Pseudonyms of Participants by Training Experience.

PD and teaching experience	Pseudonym
PD designed for leaders	Blake Bradley Henry
Former mathematics teachers	Matthew Peter Stuart
PD designed for teachers	Sam
Neither former mathematics teachers nor related PD	Lindsey Warren

homes. Each of the three leaders had participated with their teachers in a three-day workshop about research-based mathematics teaching practices at the beginning of the school year and in five math Studios throughout the year. Studios entailed collaborative planning of a lesson, observing student actions and instructional decisions as the lesson was enacted in one of the teacher's classrooms, and debriefing the lesson (Lesseig, 2016a).

In addition to Studio, principals performed learning walks in each other's buildings. Learning walks involved a group of principals and instructional coaches walking through 3-5 middle school mathematics classrooms for 10-15 minutes each with a brief huddle between classes to discuss what they saw or heard. After the walk, the team of observers categorized any instances of conjecturing, generalizing, or justifying (Lesseig, 2016b) they noticed from students. These three reasoning processes are essential to mathematical sense-making, proof, and problem solving. Distinguishing between when students tentatively believe an idea might be true (making conjectures), see commonalities across cases (generalizing), or build arguments to demonstrate the truth of a statement (justifying) requires close attention to student mathematical reasoning.

The professional development experience used several other frameworks to focus principal attention on student thinking and behaviors. One framework was Weaver's five levels of student discourse (2007) that require increasing levels of cognitive demand: (1) answering, stating, or sharing; (2) explaining; (3) questioning or challenging; (4) relating, conjecturing, or predicting; and (5) justifying or generalizing. Another was student mathematical habits of mind, that in addition to reasoning practices (i.e., conjecturing, generalizing, justifying), include choosing mathematical representations and connecting them to one another and to everyday life; looking for repeated reasoning, patterns, and structure; transforming equations into other forms; and using precise vocabulary to describe complex mathematical ideas (Matsuura et al., 2013). A final framework was funneling or focusing questions (Hagenah et al., 2018), drawing observer's attention to whether the questions teachers asked funneled the students' thinking down the teacher's prescribed path to a desired outcome or focused the students' thinking on their own understanding of the concept. At the end of the session, participants collectively crafted a feedback statement for the principal of the school they were studying to share with the teachers.

Data Collection

During two interview sessions for this study, participants provided information about their professional experience and followed a video analysis protocol (see Appendix A). They observed three sets of videos of middle school classes, one in the first session and two others in the second. The first session included a video of a fifth-grade geometry lesson from the National Council of Teachers of Mathematics (NCTM) Principles to Actions website (NCTM, 2017a). The second interview session included two video clips from an eighth-grade algebra lesson from the NCTM Principles to Actions website (NCTM, 2017b) and three video clips from a sixth-grade probability lesson from the Baker Evaluation Research Consulting Group (BERC Group, 2013). At the end of each video clip, the principals were asked to describe everything they noticed about the lesson. Once they described what they noticed, principals were asked how they interpreted each event that they had discussed. After viewing all video clips from a lesson and discussing all of their observations and interpretations, the principals then described what they would highlight with the teacher in a follow-up conversation about the lesson. We used ATLAS.ti (Muhr, 2018), data analysis software for audio and video recording, to code the interviews and sort audio clips for related topics. We transcribed all salient audio clips for more intensive data analysis.

Data Analysis

We used an inductive emic approach (Tracy, 2013) to build theory about pedagogical content knowledge for leadership. Data analysis occurred in two stages. In the first stage, we used a thematic analysis (Braun & Clarke, 2023) to explore patterns in what participants attended to during mathematics lessons and how they would respond, which led to the development of the PCKL framework. In the second stage of analysis, we used qualitative content analysis methods (Schreier et al., 2020) to examine participant comments in relation to the PCKL framework and coded the level at which participants attended to classroom events or described how they would respond to teachers.

Initial Categorization: General Topics and the Presence of Mathematics

Our first pass of coding focused on what principals noticed. Nearly all of the literature that describes what principals notice focuses on content-neutral topics that could be observed in any classroom regardless of the discipline being taught. These topics often relate to classroom culture, equity, pedagogical practices, and student interactions (Humez, 2015; Johnson et al., 2011; Schoen, 2010; Weinberg, 2010). Thus, we started our coding using these four topics as well as a general topic on the discipline of mathematics. Assessment emerged as an additional common content-neutral topic during analysis. Classroom culture was later identified as related to both classroom management practices and student participation, and therefore separated into two topics. Table 2 shows the complete list of codes, with a description and sample comment from the participants for each code.

The majority of participant comments were related to pedagogical practices, leading to the creation of pedagogical practices sub-codes. Similarly, the category of mathematics was too broad to capture the variety of ways content emerged in principals' comments and was also divided into sub-codes. Tables 3 and 4 elucidate the different sub-codes for pedagogy and mathematics respectively.

Table 2: Codes Related to Principal Noticing, with Description and Example of Each.

Original Codes	Description	Example
Classroom culture: Engagement and participation	Related to how many students were actively doing work, participating in the discussion, or called on to share an idea	From a very short video, the kids are polite, but not many are participating. She did the rest of the teaching. The kiddos are kind and nice to each other, but I'm not sure how many are actively engaged in the learning.
Classroom culture: Management, rules, and routines	Related to how the classroom was managed and whether students followed routines or rules for appropriate behavior	Strong classroom expectations have been established and are followed. In terms of running a tight ship, this teacher has built a strong classroom environment.
Equity	Related to which students were asked to respond or were talked to	The two students who volunteered are the same kids who volunteered at the beginning of class. Those were two voices we've already heard from.
Pedagogical practices	Related to decisions teachers made or actions teachers took during the lesson	I'm curious about balancing the productive struggle and getting the right answer. How far do you let students go down an unproductive path before you bring them back? What is the balance the teacher is trying to strike?
Student interaction	Related to how students were grouped, how they worked together, and how they treated one another	There wasn't anyone interrupting. A kid would get to explain without anyone jumping on top of that.
Assessment	Related to whether and how teachers assessed what students understood during the lesson	When she responded to students with 'exactly right,' what about the students who didn't get it? There was a missed opportunity to assess where every student was at.
Mathematics	Related to any description where mathematics was evident	The teacher had a goal of what she wanted to walk away with — the formula for the area of a triangle. The teacher's goal superseded the importance of student growth in determining the meaning.

Table 3: Pedagogical Practices Sub-codes with a Description and Example of Each.

Pedagogical practices sub-codes	Description	Example
Interaction with students	Related to how the teacher interacted with students	She found a balance between having fun with students and moving them forward in the lesson.
Learning target	Related to the learning goal of the lesson or whether students understood the learning goal	The teacher was doing much of the talking. She was trying to help students make connections with the learning target, helping kids to move towards the learning target.
Lesson design	Related to the lesson plan	She talked about how the investigation was connected to both forms of probability. Everything was connected and she had the standards. The lesson was definitely planful.
Productive struggle	Related to whether and how students were encouraged to wrestle with the content	Students were doing the thinking and learning, not the teacher. The teacher wanted the kids to be doing the thinking, not just for getting an answer. Her questions were on the right track -- she'd add a question to have them think about it another way. They were still persisting with the same task.
Questioning	Related to how teachers questioned students and allowed for student responses, and to the types of questions asked	The teacher asked prompting questions to groups at their level. There was no right answer so it's fun for kids. What did you expect? Were you surprised by that?
Recording thoughts	Related to teacher creation of public records, or students writing down their ideas or referencing notes	The posters she was writing seemed random. I couldn't see them fully, but what was there didn't help me follow the conversation.
Task selection	Related to the tasks that students were asked to do and how the task required students to interact with content	Based on what I saw, she picked the perfect prompt. It was challenging enough to get them to think. They were talking with the teacher and with each other.
Teacher's role	Related to what a teacher does or should do	She's not just going through the motions — not just standing in front of the room giving answers. Don't lead them step by step through the work. Stop talking when you've given them enough. Let kids ask questions of themselves.
Time	Related to how time was used in the lesson, including the pacing of the lesson	I would question about the timing, giving think time, wait time, before going on to the next piece.
Use of language	Related to how precisely language was used, whether definitions were provided, and purposes for using language	Her focus on math language was appropriate for the lesson and she stretched kids who didn't want to articulate their meaning, not because of laziness. She helped kids complete their thoughts and sentences.

Some topics were addressed more frequently than others. Participants, for example, mentioned questioning more than 100 times but talked about the use of language only 24 times. Sometimes participants discussed questioning, use of language, learning targets, or tasks in a content-neutral way, and other times related these to mathematics. We coded noticing the learning target as content-neutral when a participant noted that a teacher had written a learning target on the board, because teachers in all disciplines do that. However, we coded noticing the learning target as mathematical when a participant connected the mathematics to the learning target, such as when one participant wondered whether the teacher really understood that the learning target required students to describe the area of a triangle.

Second Categorization: A Continuum of Noticing

After coding what participants noticed, we examined how participants discussed the different noticing topics. We began our analysis of how participants discussed different topics by analyzing comments coded as questioning. Participants described questioning as being a key tool that

Table 4: Mathematics Sub-codes with a Description and Example of Each.

Mathematics sub-codes	Description	Example
Learning target	Related to what the learning goal of the lesson was or whether students understood the learning goal	I don't know what the learning target was to begin with. If the learning target had something to do specifically with slopes of lines and writing equations, if the goal was the content piece, how would you know they achieved that? When will you know that?
Mathematical habits of mind	Related to teacher and student behaviors that support student learning	Students have time to be in their brains and struggle with the math and do not rely on someone else to struggle with the math for them.
Questioning	Related to how teachers questioned students and allowed for student responses, and to the types of questions asked	Clarifying questions that allowed kids to take numbers to context - this was really important. What does it mean, what does it mean for the company? Because she's pushing for the application, she's more intentionally helping kids make sense of the math.
Student understanding	Related to what students were understanding or not understanding about the mathematics or a teacher's response to student understanding	She did not use [the student]'s thinking to clean up the mistakes. The teacher didn't circle back to clarify the misconception, and what [the student] is saying doesn't make much sense either.
Task	Related to what students were asked to do and how the task enabled students to interact with mathematics	Students were able to access enough of the task to make some connections to their mathematical schema. The richness of the task warranted a deep dive into the mathematics.
Representations and tools	Related to the tools that students use to make sense of and solve problems and communicate reasoning, including different mathematical representations	The picture is a piece of supporting evidence to the algorithm. It's like students are learning the algorithm and then looking at the picture to connect it to the algorithm.
Use of mathematical language	Related to how precisely language was used, whether definitions were provided, and purposes for using language	She [the teacher] handled that interaction well. It would be easy for her to just gloss over [the student saying "point 12 cents" rather than "0.12" or "12 cents"] and say, I know what she meant. Maybe, maybe not that's what she meant, but if she tried to do the calculation as .12 cent, she is going to get a very different answer than if she used 12 cents.

teachers use for a variety of purposes, such as managing the classroom, furthering student understanding, pressing for justification, or encouraging sense-making. We were able to sort comments into four levels of noticing, which we summarize in Table 5. Participants interpreted questioning as serving classroom management and assessment purposes when they noticed teachers asked questions to prompt student engagement, provide formative assessment information, or keep the class on pace. We categorized such purposes as general pedagogy because teachers in all disciplines use questioning for these purposes. In general pedagogy, observers attend to the frequency and depth of questions teachers ask and to whom.

At times, participants noticed that questioning connected instructional decisions with student learning of content, but the particular content did not matter. When participants noticed that teachers drew right answers out of students or used questioning to fix a student's misconception, content was part of the observation, but observers could notice the same purposes for questioning in a science, history, or art class. Because these observations relate pedagogical decisions to content but are not discipline-specific, we labeled these quotations as *parallel content and pedagogy*. At this level, the observer attends to teacher questions and student responses.

A third approach to noticing focused on an intersection of content and pedagogical practices. Participants regularly described questioning as a means of encouraging students to move beyond right answers and to explain their thinking. Mathematics or mathematical topics were rarely explicitly mentioned when discussing questioning. However, when participants noticed that questions pushed students for further explanation, we interpreted this as demonstrating understanding that student explanation of reasoning is key to mathematical learning. We labeled these comments as an *intersection of content and pedagogy*. At the intersection of content and pedagogy, observers attend to whether the teachers' questions push students to explain their thinking.

Finally, participants noticed when teachers used questioning to push students to engage with the content as emerging mathematicians. Principals who noticed that the teacher questioned students to make sense of an equation in context, to make connections between different representations, or to generalize mathematical principles demonstrated an understanding that questions can press for the use of mathematical Discourses. We labeled these comments as *pedagogy and mathematical Discourses*. At this level, observers attended to whether questions pushed for student enactment of the SMPs such as sensemaking, modeling with mathematics, using structure, or connecting representations. Table 5 provides sample purposes and related quotations that participants gave for questioning at different levels.

Noting that participants could interpret purposes for questioning along a continuum, we then considered how participants noticed *representations and tools*. How participants described the purpose for classroom use of *representations and tools* fell along a similar continuum as questioning. We categorized observations related to using tools to support student engagement as part of "just good teaching," or *general pedagogy*. At this level, participants described manipulatives as answer-getting devices, engaging entry points into the task, or a way to assess student understanding. At the *parallel content and pedagogy* level, participants noticed that teachers used representations and tools to demonstrate key ideas. As with questioning, at this level, content was mentioned in relation to instructional decisions, but not in a disciplinary-specific way. At the *intersection of content and pedagogy* level, participants noticed how multiple representations supported students in developing an understanding of important concepts or visualizing mathematical relationships. Finally, participants interpreted the use of representations and tools as a means of providing opportunities for students to behave as emerging mathematicians by solving complex problems with different representations or generalizing mathematical principles at the *pedagogy and mathematical Discourses* level. See Appendix B for examples that show how principal noticing related to *representations and tools* fell along the same continuum we found with *questioning*.

We found that participant noticing of all seven mathematical topics followed the same pattern as *questioning* and *representations and tools*. Appendix C provides comments at different levels for two additional mathematics topics: *use of language and learning target*.

Table 5: Purposes of Questioning and Examples Coded at Different Levels.

Level of noticing	Purpose	Example
General pedagogy	Equitable engagement	There was not a diversity of students she was asking questions to. In the class of about 25, one girl spoke 3 times.
	Draw students into the conversation	Then the teacher asked another student, 'How do I write this?' Why did she ask that question. I think she was just trying to get someone else talking other than herself.
Parallel content and pedagogy	Provide hints to fix student understanding	When the student gave the answer, and her response was, 'Is it just a one?,' she was just giving them the answer that you did something wrong, fix it.
	Funnel student thinking	I noticed that the teacher had a hope for the students' activity and what they would conclude. She asked very leading questions as opposed to more open-ended questions.
Intersection of content and pedagogy	Encourage students to move beyond right answers	She's moving beyond right answers and into the thinking behind right answers. She's patient, giving kids time to explain their thinking.
	Describe why the answer is right or wrong	She said, 'Whether it works or not, tell me why," because we can learn from wrong answers as much as right answers.
Pedagogy and mathematical discourses	Make sense of problems	The teacher is asking questions like, 'Why is the equation working this way? Why does it end up telling us what it tells us?' The conversation is helping the students put the pieces of the puzzle together.
	Make use of structure	The questions were probing, pushing kids down a path of inquiry. She wants them to figure out the difference between [1/2 the (length times width)] and [(1/2 the length) times the width].

Results

Development of the PCKL Framework

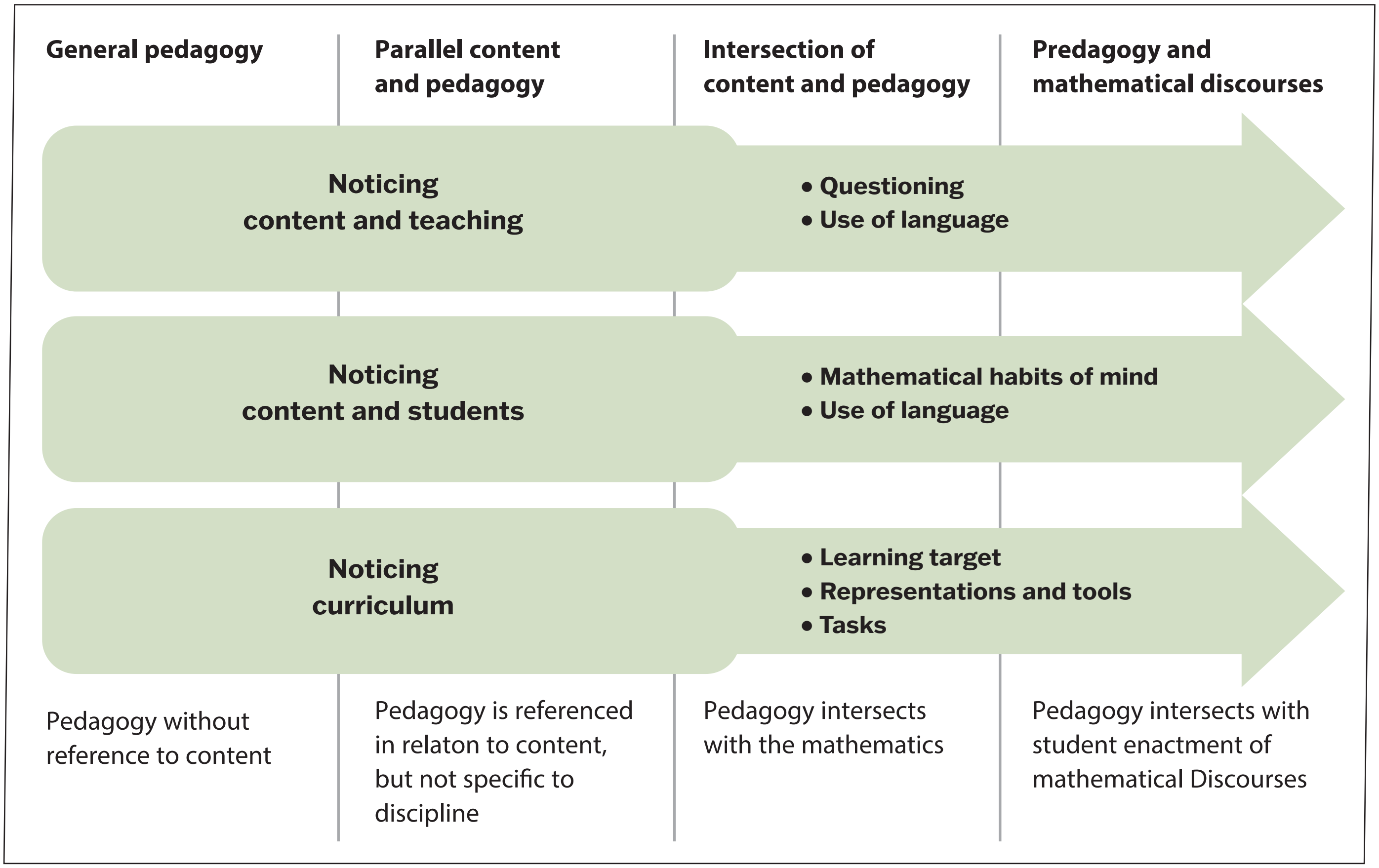
Observations for each mathematics topic fell along the continuum: general pedagogy, parallel content and pedagogy, intersection of content and pedagogy, and pedagogy and mathematical Discourses. The PCKL framework describes the different levels along the continuum for each topic. Figure 2 (see pg. 42) provides a visual overview of the PCKL framework which is presented with descriptions in Tables 6 - 8 (see pgs. 43-44).

Similar to van Es' framework for learning to notice student thinking (van Es, 2011), the PCKL framework considers *what* principals notice, as shown in the arrows, and *how* they notice, as shown in the columns. *What* they notice are aspects of the three categories of PCK (Hill et al., 2008) — knowledge of content and teaching, knowledge of content and students and knowledge of curriculum. *How* they notice builds along the continuum from general pedagogy towards an understanding of mathematical Discourses (Gee, 2015b; Hauk et al., 2014).

Leader Noticing of Mathematics

After building the PCKL framework, we returned to each of the participants' observations, interpretations, and responses, coding each for the level of noticing based on the framework. Because every comment had been coded as content-neutral or mathematical, the content-neutral comments were necessarily level 1. The other comments were coded as level 2, 3, or 4 based on the framework. The first two authors met regularly throughout the coding process to maintain clear definitions of each set of the noticing levels. The authors also used specific examples to monitor coding rules and adjust as needed. This analysis allowed us to compare the level of attention and response, providing insight

FIGURE 2.
Levels and Topics of Noticing in the PCKL Framework



into the connection between what participants noticed and what they would discuss with their teachers. Table 9 shows the highest level of attention (Att) and the highest level of response (Res) about each topic for each participant. The first group of participants had PD for leaders of mathematics, the second group taught mathematics, the third had PD for mathematics teachers, and the fourth had neither taught mathematics nor had mathematics-specific PD. Level 0 on the table indicates that participants did not address that topic during their interviews, and levels 1 – 4 represent the levels on the PCKL framework.

The data from Table 9 (see pg. 45) indicate that, just as novice teachers can be taught to observe important events like more experienced ones (Roth McDuffie et al., 2013), principals can learn to notice mathematical Discourses with intentional PD. Only the participants who had PD targeted at leaders of mathematics discussed several topics at level 4. All three noticed *habits of mind* and *representations and tools* at a high level, and Blake and Henry noticed *questioning*, *learning targets*, and *tasks* at level 4. During their interviews, all three of these participants specifically acknowledged the PD experiences where they learned about how high-quality instructional practices influence student learning. For example, Henry said,

> Through the [professional development], I really grew to understand how you ask a question, how you give students time to really process that, and how when answering the question, you allow for multiple pathways to get to that answer, that you have students show their work, explain, and describe why they do it, and not simply respond with, “Yes that’s correct,” and “No, that’s incorrect.”

This participant alluded to focusing on *questions* and student *habits of mind* as well as *tasks* that have multiple pathways to find solutions as a result of the PD experience. The learning experience helped him to understand how pedagogy is interconnected with student enactment of mathematical Discourses.

Table 6: Pedagogical Content Knowledge for Leadership (PCKL) Framework for Noticing Content and Teaching.

Topics	Level 1 General pedagogy	Level 2 Parallel content and pedagogy	Level 3 Intersection of content and pedagogy	Level 4 Pedagogy and mathematical Discourses
Questioning	Attends to the frequency and depth of questions teachers ask and to whom	Attends to teacher questions and to student responses	Attends to whether the teachers' questions push students to explain their ideas	Attends to whether questions push for student enactment of the Standards for Mathematical Practice (such as sensemaking, modeling with mathematics, and using structure or multiple representations)
Use of language	Attends to who is doing the talking and how students and teachers talk with one another	Attends to whether students and teachers use mathematical language Interprets focus on acquisition of vocabulary and definitions as a priority of mathematics instruction	Attends to teacher press for use of precise mathematical language Interprets precision of student language as an important mathematical learning outcome	Attends to student use of precise language to explain mathematical ideas Interprets the precision of mathematical language as an important tool for articulating reasoning

Table 7: Pedagogical Content Knowledge for Leadership (PCKL) Framework for Noticing Content and Students

Topics	Level 1 General pedagogy	Level 2 Parallel content and pedagogy	Level 3 Intersection of content and pedagogy	Level 4 Pedagogy and mathematical Discourses
Mathematical habits of mind	Attends to how the teacher interacts with students about learning: e.g., teacher asks questions or tells answers, allows students time to think on their own, provides immediate feedback, and praises students	Attends to student responses to teacher interactions: e.g., student demonstrates frustration or confusion about what teacher expects, feels empowered to support teammates, moves into productive or unproductive struggle	Attends to the role of the teacher in supporting student learning: e.g., teacher asks questions about how ideas are connected, pushes for higher levels of thinking and analysis, requires justification of ideas with evidence, prompts for metacognition, and withholds evaluation of student solutions	Attends to student behaviors that support learning: e.g., student explains thinking, makes hypotheses, justifies, generalizes, and articulates answers, uses math vocabulary, provides evidence to support reasoning, tries and abandons different ideas
Student understanding of mathematics	Attends to how many students appear engaged and which students speak	Attends to whether the class can explain the mathematics	Attends to individual students' mathematical thinking and explanations	Attends to the individual students' mathematical thinking and the connections between teaching strategies and student mathematical thinking, justification, generalizations of mathematical principles

Table 8: Pedagogical Content Knowledge for Leadership (PCKL) Framework for Noticing Curriculum.

Topics	Level 1 General pedagogy	Level 2 Parallel content and pedagogy	Level 3 Intersection of content and pedagogy	Level 4 Pedagogy and mathematical Discourses
Learning target	Attends to whether the teacher has a clearly articulated learning target for students and whether the teacher assesses for student proficiency of the learning target	Attends to alignment between the learning target and the assigned task or grade level	Attends to how the teacher's decisions led to student understanding of the learning target	Attends to how the Standards for Mathematical Practice (such as inquiry, justification, and generalization) are embedded in the learning goals
Representations and tools	Interprets the use of representations and tools as a means for students to find answers, as an entry point for engagement, or as a means of assessing student understanding	Interprets the use of representations and tools as a means of showing mathematical concepts to students	Interprets the use of representations or tools as a means of developing conceptual understanding or visualizing mathematical relationships	Interprets the use of representations or tools as a means of solving complex problems or generalizing mathematical principles
Tasks	Attends to whether the task engages students	Attends to whether students follow a prescribed pathway or can access the task through multiple solution pathways	Attends to whether the task encourages students to reason about mathematics and show connections between different representations	Attends to whether the task requires justification or generalizing of mathematical principles or incorporating the use of different representations including context

On the other hand, mathematics teaching experience did not appear to be as influential in developing this same lens. Matthew and Peter, who taught secondary mathematics for 17 and 12 years, respectively, noticed only one topic each at level 4. Like those with leadership PD, Matthew and Peter recognized *representations and tools* as important to learning math, as did Sam who had PD for teachers, but none described them as being interconnected with mathematical Discourses as Blake and Henry did. For example, Peter and Matthew described the visual representation as a means of understanding the mathematical concept and connecting an image to the rule. Peter said, "I want her to be able to talk about what individual students understood and what she would do to help students who aren't yet understanding. Students would be clear about the model and why it works and connect it with algorithmic language." Neither described the use of tools as a means of behaving like a mathematician. Blake, however, described representations and tools as important for making sense and justification, a means of acting like an emerging mathematician, when she said, "This [task] was a way to look at math. This was a way to look at shapes. This was a way to fold and count, and models for this unit are important as a way to justify their thinking." When describing that seeing how graphs and equations related to a company in context is really important, she added, "We keep the mathematics if it has meaning and we've made a connection to it." Unlike their peers, one former mathematics teacher never demonstrated level 4 noticing, however.

The data also show that principals who attend to mathematical Discourses are positioned to direct teacher attention to them during follow-up conversations. In one case a participant demonstrated how what she noticed during the lesson prompted her to respond to the teacher at level 4. Of her observation, Blake said,

> I ended up recording her questioning prompts. 'Where are you coming up with this?' 'Where did you get the ½?' 'Why?' 'I'm curious.' 'Is there another way?' 'Is that

Table 9: Levels at Which Participants Attended (Att) and Responded (Res) to Mathematical Topics.

Participants grouped by experience (PD or math teaching)	Question		Language		Habits of mind		Student under-standing		Learning target		Reps & tools		Tasks	
	Att	Res	Att	Res	Att	Res	Att	Res	Att	Res	Att	Res	Att	Res
Blake	4	2	2	0	4	4	3	0	4	4	4	1	4	0
Bradley	0	1	4	0	4	4	0	0	0	2	3	0	4	0
Henry	4	1	3	0	4	0	4	0	4	2	4	0	0	0
Matthew	4	4	3	0	3	3	0	0	1	2	3	0	2	0
Peter	3	1	0	0	4	4	2	0	2	2	3	3	0	1
Stuart	2	1	0	0	2	0	2	0	1	1	3	0	3	3
Sam	2	0	2	0	3	0	1	0	4	1	3	0	3	0
Lindsey	3	0	3	2	3	1	2	0	2	0	3	0	2	0
Warren	0	2	1	0	2	0	0	0	2	1	1	0	3	0

Note: This table shows the highest level that each participant attended to each topic (Att) and the highest level that each participant said they would respond to the teacher (Res). The first group had PD for mathematics leaders, the second group taught mathematics, the third had PD for mathematics teachers, and the final group had no mathematics PD nor math teaching experience.

> the same thing?' 'How do you know?' Clearly, she is looking for student knowledge, she's not leading them anywhere, she's not saying, 'Oh you're on the right track.' 'I appreciate your answer.' 'Let's build on that one.' It was very much an open-ended, 'What could this look like?' 'What is your mathematical thinking?' It wasn't until the very end that she even put numbers in there to check it. Her purpose was for students to just explain their thinking. 'How did you fold the paper?' 'How did you count?' 'Where did these numbers come from?' 'How did you make a formula?,' no matter how big the right triangle ended up being.

In this observation, the participant noticed questioning strategies that lead to mathematically productive habits of mind and using representations to make sense of the mathematical concepts. In describing how she would follow up with this teacher, she said she would address the teacher's questions with a level 4 understanding.

> Her question, 'Does this apply to all right triangles?' would be part of my follow up in terms of what does this mean for the rest of this unit. 'What other shapes have you done? Where are you? Where is this going? Do you have other ways to model this with different shapes? How are you using this formula?' Just, 'Where is this going?' because that's how she's leaving it. And how can kids generalize that math information?

This planned response to the teacher demonstrated an understanding that the mathematical representations she attended to in her observation were essential to generalizing mathematical relationships, not just for the right triangle that students were working on but for other shapes as well. The principal's reinforcement of that important idea during the follow-up would focus the teacher's reflection on how her pedagogy could support student enactment of mathematical Discourses.

Although it appeared that noticing mathematical Discourses enabled participants to direct teacher attention to the Discourses, it did not appear sufficient. Often when participants noticed mathematical Discourses being enacted, they would respond about content-neutral or parallel content and pedagogy. Bradley observed mathematical Discourses during the lesson:

> Most of [the students] were explaining their thoughts. The one little gal was explaining how to punch it into her calculator – the task was the opposite direction – but she was understanding how to manipulate the table… It was all the right stuff. They obviously had enough to mess around with, they were making hypotheses or postulates, saying, "Here's my answer," they were forced to justify that, they were forced to collaborate with their peers. And then she would also push by asking, "Is that the only way?" "Is there another equation that's there?" She still hasn't given them the answer. She let them mess with it for a little while.

When asked how he would respond to the teacher, he said he would ask how she would ensure that every student understood the mathematics because often with an inquiry-based approach, the teacher may be unaware of the students who do not understand. Even though the participant noticed the importance of conjecturing, generalizing, justifying, and manipulating different representations, he dropped to a level 2 response related to assessment of the content of the lesson much as he might in any other classroom. While no one would disagree that we want teachers to assess student learning, this participant missed the opportunity to also prompt further teacher reflection on the interplay between pedagogy and mathematical Discourse.

In no case did participants in this study indicate that they would ask a teacher to reflect on mathematical Discourses at a level 3 or 4 on the framework unless they had addressed the topic at that level when describing it during the attending phase of the protocol. Thus, data from this study indicates that attention to mathematical Discourses may be necessary but not sufficient for principals to include them in their responses.

Where those who had PD for leaders were more likely to attend to mathematical Discourses, the former mathematics teachers, Matthew, Peter, and Stuart, showed more willingness to respond at levels 3 or 4 on topics they had observed at that level. Where Matthew noticed 4 topics at level 3 or 4, he responded at level 4 twice; Peter responded at level 3 or 4 in two of the three topics he noticed at that level; and Stuart responded at level 3 on the only topic he noticed at level 3. Those with PD for leadership had a lower rate of response at higher levels, even when they noticed higher levels on more topics. Blake, who noticed at level 3 or 4 in six topics, only responded at level 3 or 4 on two of them; Bradley responded at level 4 on only one of the four topics he noticed at a higher level; and Henry never responded above level 2 even after noticing at levels 3 or 4 in six topics. Matthew explained why he would address the mathematics while others might shy away from it, stating, "I would focus on the math since I'm a math teacher. It's easier to focus on the math." He added that he may not discuss the content with a Language Arts teacher.

Henry, who regularly noticed mathematical Discourses, said he would address content-neutral topics in follow up conversations. After noticing two key inflection moments in the class where he wished the teacher would stop and question the students for deeper understanding, he reflected on how he would respond with, "I don't know if I'd discuss questioning and discussion or student engagement. Engagement is an easy one because teachers know when kids are paying attention or not. In each section, who's engaged in discussion? How can you tell?" Even armed with a clear understanding of key mathematical moments and related pedagogical opportunities, he dropped to discussing the student engagement with the teacher. He added an explanation of why he would raise content-neutral topics with teachers,

> I know what instructional practices look like across content areas. I know what questioning, discussion, and engaged learners look like. I do not necessarily know the math piece, so I'd use Kathy Norwood's approach of drawing the ideas out of the teacher.

Henry noticed five topics at level 4: *questioning, habits of mind, student understanding, learning targets*, and *representations and tools*. However, when asked how he would respond, he said he would ask teachers to reflect at lower levels related to student engagement or assessment. His attention to the mathematical Discourses was not sufficient for him to focus teacher attention on key mathematical moments.

Some evidence in this study, therefore, suggests that previous mathematics teaching experience may aid principals in providing feedback to teachers when they attend to key mathematical events. Others may need support building confidence and strategies for how to respond about pedagogical decisions that promote mathematical Discourses.

Discussion

Evaluation and supervision systems rest in an unstated assumption that principals can support teacher learning without the knowledge of discipline-specific instructional strategies or, in the case of mathematics, an understanding of mathematical Discourses. As Goodwin (1994) found, communities of practice socially construct shared ways of seeing. Teachers stand at the intersection of different communities of practice that have complementary but not fully aligned professional visions. Effective teachers rely on a shared vision of good instruction that can include such skills as focusing intentionally on content standards, choosing tasks that engage students, or asking questions that further student understanding, a vision that is clearly articulated in rubrics for teacher supervision. However, they must also rely on pedagogical content knowledge to push students to enact the practices that promote content-specific learning. The professional vision of mathematics teachers, therefore, must include more than a vision of "just good teaching" and more than just PCK. A mathematics teacher's professional vision must include a vision for what mathematical Discourses look and sound like in classrooms and what teachers do to elicit them.

During evaluation processes, principals observe classrooms to gather evidence that documents the effectiveness of instruction and work with teachers to compare that evidence to rubrics. The evaluation process typically uses gathered data, the rubric, and conversation to determine the teacher's strengths and areas for growth. Because principals frequently gather the data that are discussed, what they notice during the lesson shapes the conversation. If their data does not contain discipline-specific events, some key features of lessons may pass without critique and salient opportunities for improvement may be missed. As Bradley explained,

> What matters is the process that [students] took to get to that [answer] and then what their thinking is. And then having another student be able to come by and say, 'Yeah, I got to this answer which may or may not be the same, and I got to it in a completely different way using a completely different model.' And so, the idea of open questions and really allowing students to explore their own thinking and make that explicit in the classroom [matters].

The components of instruction that matter for teachers who are learning to enact high-quality instructional practices and for principals who supervise them include the big D discourses and the important mathematics that is embedded in them. Bradley's comment indicates an awareness of the power of mathematical Discourse that he developed during leadership PD. Principals who can support a classroom teacher in developing such a vision of effective instruction centered on student enactment of mathematical Discourses are in a strong position to support powerful mathematics instruction throughout their schools.

Experienced principals who have used instructional frameworks for teacher evaluation and supervision are well-versed in effective general pedagogy. The PCKL framework can further advance principals' abilities to notice important classroom events through content-specific awareness. An observer at level 1 on tasks "attends to whether the task engages students." At level 4, an observer "attends to whether the task requires justification or generalizing of mathematical principles or incorporating the use of different representations including context." Content-neutral pedagogical observations thus form a strong foundation that can be built upon for observations about mathematical Discourses. An observer who already attends to the assigned task can learn to attend to the mathematical characteristics of the task. Rather than asking principals to abandon what they know, the PCKL framework demonstrates how principals' current knowledge of general pedagogy is a valuable asset they can build upon to support the mathematics teachers they supervise.

Mirroring what researchers have found about teacher noticing (Sherin & van Es, 2009), this study provides evidence that principals can learn to attend to mathematical Discourses necessary to support teachers in strengthening their own vision of the Discourses. Professional development providers can support mathematics leaders' growth by focusing their attention on how it looks and sounds when students talk and behave as mathematicians. PD experiences for principals could draw attention to some of the mathematical topics found in the PCKL framework, such as characteristics of rich mathematics tasks, student use of mathematical tools and representations, or focused questioning to support learning. This study suggests that PD opportunities with an intentional focus on level 4 of the PCKL framework, particularly on student enactment of mathematical Discourses, may help principals attend to key mathematical events during classroom observations. School leaders who are learning to notice the mathematical Discourses at a high level will likely need further support to learn to focus a teacher's lens on significant mathematical events.

Using the PCKL Framework

The PCKL framework can guide the work of PD providers by articulating how mathematical topics are directly linked to the interconnected relationship of pedagogy and mathematical Discourses. For example, PD for leaders might highlight mathematical Discourses by including a video case (Johnson & Mawyer, 2019) that allows principals to observe students justifying their reasoning about a concept they generalized when doing a rich mathematics task. A reflective conversation about that video case could focus on both the construction of the mathematics task to promote Discourses and student behaviors that foster mathematical habits of mind. Similarly, a team of leaders might do a learning walk (Elmore et al., 2009; Fisher & Frey, 2014) in mathematics classrooms to observe how Standards for Mathematical Practice are embedded in the learning targets. During these PD experiences, facilitators could support principals in learning to respond to teachers by asking them what they would focus on during a post-observation conversation or collaboratively creating questions they would ask teachers based on what they observed. Using the PCKL framework as a guide in planning learning experiences for principals may support PD providers in furthering principal attention to important events.

Mathematical content knowledge may not be essential for learning to attend to mathematical Discourses, but principals who lack the mathematical background of former teachers may need additional support to develop strategies for responding. This finding is apparent from Henry who had a clear vision of mathematically productive mathematics classrooms but said he would use lower-level responses in follow-up conversations with teachers. As principals learn to attend to mathematical Discourses, PD providers may also consider how to intentionally support those who have not taught mathematics so they learn to respond to teachers about key mathematical moments when they notice them.

Even if they do not have access to external PD providers, principals can use the PCKL framework in their own work with teachers. If a principal feels particularly confident with a content-neutral topic or regularly observes parallel content and pedagogy, the framework can provide guidance for furthering what they look for in mathematics classrooms. For example, during observations, principals who regularly pay particular attention to who speaks and for what purpose can also listen for how those students use mathematical vocabulary. Is learning vocabulary treated as an important learning target, as an essential tool for making sense of the mathematics, or as necessary to effectively justify or discuss a generalization of key mathematical concepts? Once principals articulate for themselves how language is used, they may consider how to provide teacher feedback that further develops student mathematicians and supports teachers in becoming better than "just good teachers."

Principals can also enlist their teachers in developing a shared vision of a classroom alive with emerging mathematicians. The mathematics team and their supervisor could form a video club to watch lessons curated by the mathematics research community and negotiate what enacted Discourses look and sound like. They may choose just one topic and study it using multiple videos or watch the same video multiple times, changing the observation focus. A principal could also take mathematics teachers through classrooms and discuss when they observe student enactment of Discourses. Together, they could consider how teachers would like to receive feedback focused on higher levels of the PCKL framework.

Conclusion

The PCKL framework presented in this paper shows a progression of noticing along a continuum from general pedagogy to student enactment of mathematical Discourses in the three components of PCK (Hauk et al., 2014). We used the framework in this study to articulate levels of principal noticing. In so doing, we found that principals were able to learn to attend to key mathematical classroom events and that they may need additional support to know how to respond to teachers. We contend that the PCKL framework can provide much-needed guidance to PD providers for supervisors of mathematics teachers or to principals who strive to notice more during lessons.

The CCSS-M charges teachers with developing mathematically proficient students, and the rubrics in teacher evaluation documents require that principals support teachers in developing the necessary pedagogical content knowledge. This study identifies what principals need to know about the discipline of mathematics if they are to meet this challenge, and suggests ways that they might learn about mathematical Discourses in their work with teachers. Learning opportunities that move principals along the PCKL continuum of noticing may position them to play a key role in fostering powerful mathematics teaching and learning.

Because this study built the PCKL framework from what the participants noticed, the findings may be limited. There may exist other important elements of PCKL that this set of participants did not name. The videos were designed for use with teachers and therefore leaned towards teacher behaviors, so using videos focused solely on students may have drawn out other important ideas. Replicating this study with participants who have expertise in mathematics education may also add to topics in PCKL. The number of participants in the study is also a limitation in drawing generalizable conclusions about the importance of PD for supervisors of mathematics. Although the evidence in this study mirrors teacher noticing research indicating that PD focused on noticing at high levels appears necessary but not sufficient to know how to respond well, the number of participants and the relatively small data set indicate that further research would be beneficial.

This study does, however, lay the foundation for articulating the mathematical content knowledge that exists at the intersection of content, pedagogy, and leadership. As the province of school leaders who focus on improving student learning of mathematics, the PCKL framework narrows the scope of their knowledge base and focuses their lens on what really matters – the emerging mathematicians at their school and a culture of learning that supports their development. ✪

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Appendix A
Video Noticing Protocol

We are going to watch three video clips of teachers in math classrooms and I will ask you to record anything you notice as we watch. We'll then discuss what you noticed and your interpretation of the event. After we've discussed everything you noticed, I'll ask you how you would respond to the teacher as though she were on your staff, in her evaluation or in a follow-up conversation, or perhaps in how you might consider your work with your entire staff.

The first video is in a 5th classroom discussing the formula for area of a triangle. This teacher is working to support student engagement in and understanding of mathematics and to develop procedural fluency through conceptual development. The students have worked on a task and we enter as they are debriefing the task.

We'll watch the two videos of an 8th grade classroom in sequence. This teacher is working on supporting student engagement in and understanding of mathematics and posing intentional questions. The first video is taken during the work time of a lesson, and the second is a whole class discussion of what was learned during the lesson. I will ask you to tell me what you noticed and how you interpreted what you noticed about the first video before starting the second, but I won't ask you how you would respond to the teacher until after we've discussed what you noticed in the second video.

The final set of three videos is from a 6th grade classroom. The teacher is teaching about probability. The first video is the opening of the lesson and is a whole class discussion. The second video is taken during student exploration time and focuses on the teacher's interactions with small groups. The third video is the debrief of the lesson.

a. We'll watch the video of the lesson. As we watch, record anything you notice. You'll be able to watch the video or portions of the video as many times as you'd like. If there's a portion you would like to watch again, please record the time and we can go back to that part of the video.

b. (After first observation) Are there any parts you want to watch again or even the whole video?

c. Would you please share what you recorded? In what you noticed, who was involved and what were they doing?

d. How do you interpret what you recorded?

e. (After completing steps 1 a - d for all video clips of the same lesson). What norms appear to be present in the classroom we just observed?

f. How would you respond to this teacher if you were her supervisor, either in her evaluation or a follow-up conversation? How might you use what you learned from this lesson in your broader work with your staff?

(If not already discussed, follow up with) What did you notice about the mathematics that the students were engaged in? What did you notice about the way students were thinking about the mathematics?

Appendix B

Ways of Observing Representations and Tools with Examples

Level of noticing	Purpose	Example
General pedagogy	Engage students	The kids were rolling dice and they seemed to be having a good time. I mean, rolling dice is engaging.
	Help students find right answers	Every kid came to same conclusion with the manipulatives. There was likely a prescribed way to use them since they all got the same results.
Parallel content and pedagogy	Demonstrate a disciplinary concept to students	The teacher was trying to formulate a picture for students so they can see. It seems like the teacher was showing them.
Intersection of content and pedagogy	Support conceptual understanding	She asked the student to explain the equation, where the equation came from. If they were provided the equation then what does it mean? [Students need to understand] how their tool works, understand how it represents what's happening.
	Visualize mathematical understanding	The teacher has students make a visual representation of their thinking. They were able to use the manipulatives to show what they were thinking.
Pedagogy and mathematical Discourses	Generalize mathematical principles	What students would normally understand without the model is that [1/2(l·w)] is the same thing as [(1/2 l)·w]. But they're not the same thing. They are two different things that yield the same answer because they represent two different [ways of visualizing how the formula is constructed]. Without the model, I think that's really hard to visualize but when the students cut and then flipped them, then you can see why [the equations look different].
	Solve complex problems using different mathematical representations	They have created the algorithm and what they think that means, they've graphed it and they're all in the same spot, so they should be able to tell you what is going to happen in any of those equations [that intersect at the same point], which is what they all have in common within the context.

Appendix C
Examples of Comments at Different Levels

Use of Language

Level 1: General Pedagogy: "The kids were engaged and very on target with their language."

Level 2: Parallel Content & Pedagogy: "They are learning productive language from the beginning rather than saying, 'That number or that number.' They are learning the proper language from the start which will help them as they grow and mature, those pieces will be with them already. The teacher is doing a good job of teaching those basic expectations of teaching those labels of what things are."

Level 3: Intersection of Content & Pedagogy: "She consistently would ask the kids to dig a little bit deeper for that understanding and explanation piece to try to take the thoughts and use the correct vocabulary to say it out loud."

Level 4: Pedagogy & Mathematical Discourses: "Based on where you put those in your calculator, what are those things? The student's being forced to explain so it's not just what are those things really so it's not just what you punch in next. Explaining your thinking is, 'Why did you pick that?' That vocabulary to be able to explain the depth of understanding, the 'Why did you think that would work?'"

Learning Target

Level 1: General Pedagogy: "I want to know what her hunch is about the students' ability to demonstrate proficiency with the learning target for today."

Level 2: Parallel Content & Pedagogy: "I don't know whether learning about right triangles is a fifth-grade standard. That would be something I would have to look into before going in to observe a teacher."

Level 3: Intersection of Content & Pedagogy: "The teacher was doing a lot of the talking because she was trying to help students make connections with the learning target. She was moving students towards the learning target."

Level 4: Pedagogy & Mathematical Discourses: "The goal on this task is not the formula. If that was the case, you would just give it to them and write it down. Sometimes when teachers don't understand the distinction, they'll do this kind of lesson plan but they will not have that kind of patience. The goal is to understand the formula, to understand where that formula comes from."

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