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- Case Studies
- Research Report and Interpretation
- Commentary on Critical Issues in Mathematics Education
- Professional Development Strategies

Note: The last two categories are intended for short pieces of 2 to 3 pages in length

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# Comments from the Editor 

Mark Driscoll<br>Education Development Center, Newton, MA•mdriscoll@edc.org

In my role as editor of Mathematics Education Leadership, I am pleased to have the opportunity to help members of NCSM engage with each other around issues of leadership aimed at improving mathematics education in North America. It seems particularly timely for us to be engaged with these issues, because leadership in education has become the focus of a rapidly increasing number of books and articles. Some are research-focused (e.g., Spillane, Halverson, \& Diamond (2001)); some are practice-focused (e.g., Kaser, Mundry, Stiles, \& Loucks-Horsley (2002)); and some combine the two, linking leadership practice to a particular body of research (e.g., West \& Staub (2003)).

Most of these efforts reflect a shift in thinking over the past decade. Leadership no longer is equated with authority, nor with qualities of people who are "born to lead." Instead, a great deal of effort is devoted to unpacking the practice of leadership and understanding how and why leaders do what they do. Further, writers on leadership express a growing recognition that the practice of leadership often is distributed throughout a community. "Leadership practice (both thinking and activity) emerges in and through the interaction of leaders, followers, and situation." (Spillane, Halverson, \& Diamond, p.27)

Each of the three articles in this issue addresses a challenging area of leadership practice in mathematics educa-tion-curriculum implementation, teacher learning, and instructional improvement. In each, readers should be able to discern pointers toward improving shared, collaborative leadership practice.

The first article describes a university-based professional development program which, over nearly two decades, has evolved into a deeply rooted university-district collaboration. By using ongoing feedback from teachers and admin-
istrators, organizers have shaped a comprehensive program aimed at fostering professionalism and creating a network of teachers who have extensive knowledge of both mathematical content and pedagogy.

The second article describes a district's multi-stakeholder approach to evaluating a new mathematics curriculum, an approach distinguished by the multiple sources of data gathered and employed to inform leadership decisions. Leadership of the initiative has been distributed across a cross-section of stakeholders, including a team of teacher leaders who became, in effect, the managers of a new district vision for mathematics instruction embodied in the new curriculum.

The third article reports on and interprets a study that looked inside nearly 200 mathematics classrooms in order to gauge the national status of quality mathematics instruction and to determine the components of lessons that seem likely to promote student understanding. For professional developers in mathematics education, the study's findings shed light on aspects of effective instruction that should be emphasized in work with teachers. For all mathematics education leaders, the report provides a way to consider more than test scores in order to gauge quality, and also provides an evidence-based model for discussing what is important in mathematics education.

I want to make one final comment about the shift of attention away from viewing leadership as a quality invested in special people, which I mentioned in the first paragraph. The existence of this shift does not deny the existence in our field of leaders with very special qualities. In the first article of this issue, you will see a photograph of Iris Carl, a leader with extraordinary qualities of character, who will be deeply missed by all of us who had the privilege to know her.

## References

Kaser, J., Mundry, S., Stiles, K.E., \& Loucks-Horsley, S. (2002) Leading every day: 124 actions for effective leadership.

Spillane, J.P., Halverson, R., \& Diamond, J.B. (2001) Investigating school leadership practice: A distributed perspective. Educational Researcher 30 (3), 23-28

West, L. \& Staub, F.C. (2003) Content-focused coaching: Transforming mathematics lessons. Portsmouth, NH: Heinemann.

# Professional Development to Support the NCTM Standards: Lessons from the Rice University School Mathematics Project's Summer Campus Program 

Richard Parr, Anne Papakonstantinou, Heidi Schweingruber, \& Pablo Cruz<br>Rice University School Mathematics Project

In the high school class, small groups of teachers are investigating the geometries on a variety of surfaces - balloons they had inflated, polyhedra they had constructed, and fruits and vegetables that were arranged at different centers. Down the hall, in one of the middle school classes, groups of teachers with stopwatches, meter sticks, marbles, toy cars, and ramps are collecting data to determine distance/time relationships for different scenarios. In one of the elementary classes, teachers are participating in a courtroom drama defending the impact of Standards-based instruction on students' understanding of mathematics concepts. Outside the building, pairs of teachers armed with digital cameras are walking around campus photographing different structures to illustrate their definitions of mathematical terms for a poster.

Above is a snapshot of activities that typically occur throughout the four-week Rice University School Mathematics Project (RUSMP) Summer Campus Program. The Summer Campus Program, held each June since 1987, creates communities of learning that increase PreK-12 teachers' mathematical knowledge while assisting them in the development of the pedagogical skills necessary to ensure that their increased understanding is transferred to student mathematical learning.

Providing professional development that encourages teachers to examine their beliefs and practice while providing support in mathematics content and pedagogy is an on-going challenge for programs designed to promote implementation of the National Council of Teachers of Mathematics (NCTM) Standards (1989, 1991, 1995, 2000).

Some teacher educators and researchers have suggested that in order to meet these goals traditional professional development activities must be restructured (Darling-Hammond \& McLaughlin, 1995; McLaughlin \& Oberman, 1997; Gray, 2001; Lewis, 2002). This restructuring must move away from top-down teacher training strategies that emphasize acquisition of new skills or knowledge. Rather, professional development must provide occasions for teachers to reflect critically on their practice, to fashion new knowledge and beliefs about content, pedagogy, and learners, and to build collaborative, professional relationships. Furthermore, a successful professional development program cannot be prescriptive, but must be adjusted to the context in which it operates (Darling-Hammond \& McGlaughlin, 1995).

One such program is RUSMP's (http://rusmp.rice.edu) Summer Campus Program, an annual professional development program that provides opportunities for PreK-12 teachers to enhance their mathematical knowledge, to develop more effective teaching practices that promote greater student involvement, and to develop skills in critical reflection through collaboration with peers. From its inception in 1987 as a single class for 48 middle and high school teachers, the Summer Campus Program today has expanded to five different classes for PreK-12 teachers engaging approximately 120 teachers each summer.

All RUSMP programs are guided by the fundamental belief that sustaining wide-scale instructional reform can only be accomplished through the development of the skills and knowledge of individual teachers. These efforts are framed in terms of developing professionalism among teachers. International studies of teachers' roles reveal that teachers
in European and Asian countries have many more opportunities to develop professionalism through on-going training, collaboration with peers, and participation in administrative decision-making than their United States counterparts (Darling-Hammond, 1996; Kinney, 1998; National Institute on Student Achievement, Curriculum, and Assessment, Office of Educational Research and Improvement, \& U.S. Department of Education, 1998; Stevenson, Lee, \& Nerison-Low, 1998; Stevenson \& Stigler, 1992). Through opportunities such as these, teachers develop skills and expertise that allow them to make informed decisions about their practice and enhance their teaching.

The Summer Campus Program is designed to improve teachers' content knowledge in mathematics, in conjunction with an examination of the teaching methods embodied in the NCTM Standards. Fostering professionalism and creating a network of teachers who have extensive knowledge of both mathematical content and pedagogy is essential for supporting sustained instructional change (Nease, 1999; Papakonstantinou, 1995; Schweingruber, 1999). RUSMP activities are designed to support the development of teachers' professionalism by focusing on three major areas: (1) solid knowledge of mathematics, including key concepts that students must master; (2) awareness of a variety of approaches to instruction and their appropriate use; and (3) the ability to plan and reflect on instruction together with other teachers. The overarching goal of RUSMP is to improve each teacher's mathematical knowledge and teaching methodology in order to boost teacher effectiveness. This goal is especially urgent in light of the scarcity of mathematics teachers, which is resulting in more novice teachers (Alternative Certification Program, substitute, and first-year) and teachers with less training entering the profession and teaching out of their field. It is essential for them to have strong content knowledge and teaching skills.

The RUSMP approach rests on the assumption that professionalism among mathematics teachers must include: a solid knowledge of mathematics, including the key concepts students must master; awareness of a variety of approaches to instruction and their appropriate use; and the ability to plan and reflect on instruction together with other teachers. RUSMP has developed key mechanisms for achieving these goals.

While the Summer Campus Program focuses on mathematics content and pedagogy, an equally important goal is to raise the level of professionalism among in-service


Former National Council of Supervisors of Mathematics President Iris Carl participating in RUSMP activities.
teachers. The Summer Campus Program has received state and national recognition (Cannon, Parr, \& Webb, 2003; Killion, 1999; Toenjes \& Garst, 2001; Killion, 2002a; Killion, 2002b) for its positive impact on teachers' understanding of mathematics, their classroom practices, their students' achievement on standardized tests, and their expanded contributions to their school districts. Lessons learned over its eighteen years of operation provide valuable insight for teachers, principals and district level administrators interested in supporting quality Standards-based mathematics instruction. A discussion of the current operations of the Summer Campus Program, the curriculum developed for the program, and the RUSMP Learning Plan, a graphic organizer that serves as a tool to allow teachers to translate their program experiences into the classroom, is intended to catalyze discussion and provide guidance to those interested in establishing similar programs.

## Operation of the Summer Campus Program

RUSMP was jointly conceptualized by Rice University mathematics faculty and Houston-area school district personnel. With an initial grant from the National Science Foundation, RUSMP was established in 1987 to serve as a bridge between the Rice University mathematics research community and Houston-area mathematics teachers. In addition to the original grant, RUSMP has received generous funding under the Dwight D. Eisenhower Higher Education and Teacher Quality Grants Programs and from corporations, foundations, and local school districts.

The growth of RUSMP owes much to its unique relationship with Houston-area schools and school districts. Throughout its history, RUSMP has striven to be responsive to the needs expressed by teachers, principals, mathematics directors, and superintendents in area schools. This responsiveness has resulted in constant changes and improvements in RUSMP and has led to its continued expansion. Though university based, RUSMP has an intimate knowledge of the schools in the Houston area and seeks to nurture a long-term, collaborative relationship with them. As a result, over the eighteen years of operation, several additional components have been added under the umbrella of RUSMP. These programs are described on the RUSMP web site (http://rusmp.rice.edu) and in other papers (Eaves, 2000; Killion, 2002c; Papakonstantinou, Berger, Wells, \& Austin, 1996).

The Summer Campus Program remains the centerpiece of RUSMP. It is founded upon the principle that teachers learn best from their fellow teachers. In keeping with the view that successful professional development must take seriously the need to develop teachers themselves as experts, the Summer Campus Program incorporates Master Teachers (Austin, Herbert \& Wells, 1990; Cruz, Turner, \& Papakonstantinou, 2003) who have demonstrated sustained success with innovative instructional practices in their own classrooms. Master Teachers, under the direction of RUSMP's Directors and university mathematics faculty, are responsible for planning the content of the Summer Campus Program.

A team of two Master Teachers works together to provide instruction for teachers who are grouped by grade level. The two Master Teachers are assigned such that one has experience teaching at the designated grade level and the other has experience teaching in the grades above that level. The intent is to provide participating teachers with instruction relevant to their grade level, but also to give them exposure to material beyond that grade level. Using the RUSMP curriculum as a guide, Master Teachers identify the key mathematical concepts that will be developed, discuss activities that will be provided, and select the materials to be used. The Master Teachers' extensive knowledge of current practices in education ensures that the teachers they are instructing receive information that is relevant to them. Master Teachers serve as role models for how teachers can effectively perform in the classroom. They provide teachers with implicit examples of how a lesson can be developed and taught, how to involve students
in discussions, how to work with other educators in the planning and implementation of a lesson, etc., through the way they lead classes in the Summer Campus Program.

In recent years, five class levels (PreK-2, 3-4, 5-6, 7-Algebra I, and Geometry and Above) have been offered to teachers. Enrollment is limited to approximately 120 teachers across the grade bands. The four-week program runs Mondays through Thursdays (8:30 a.m. - 3:30 p.m.) during the month of June. Each day before classes begin, breakfast is served to the entire group to promote a collegial atmosphere that builds relationships among teachers, Master Teachers, university faculty, and RUSMP staff. To foster the RUSMP's philosophy "teachers teaching teachers," classes begin with thirty-minute share sessions during which teachers make brief presentations of exemplary activities or share teaching tips. This forum provides opportunities for veteran teachers to share successful classroom practices with novice teachers and for teachers from different schools to share ideas.

During the rest of the day, teachers engage in carefully planned, conceptually-based instructional activities. RUSMP has developed a content/process framework that supports student creativity and active learning. This curriculum rests on an underlying philosophy of how children learn mathematics and is coherent with guidelines developed by NCTM. Since RUSMP believes that mathematics development is a social activity, collaboration is a hallmark of almost all Summer Campus Program activities. The purpose of instructional activities is two-fold. Teachers are provided with meaningful collaborative activities that they can modify for use in their classrooms, but more importantly, they also develop a deeper understanding of mathematics and mathematics teaching through indepth dialogue that accompanies each activity. This dialogue is meant to help teachers see the activity not as an isolated event but as an important piece in the process of developing mathematical thinking in their students.

Master Teachers develop concepts over several grade levels and discuss the vertical alignment of instruction with participants. As a result, teachers see not only what mathematics should have preceded an activity but also what mathematics connections will be made later. They keep journals with daily entries explaining how they felt about the day's lesson and what they learned that day. These writing experiences enhance their mathematical understanding of the concepts presented. The journals are read and responded to weekly by the Master Teachers.

Teachers use manipulatives and technology as tools: (1) to address various learning styles, (2) to model or represent mathematical concepts, (3) to abstract from the concrete manipulative to symbolic representation, and (4) to generate authentic data. Teachers receive training in the use of the latest graphing technology, data collection devices, and computer software, as well as in the use of the Internet and its application to mathematics instruction. Technology instruction is conducted by Master Teachers together with RUSMP's Director of Educational Technology and Secondary Education. A computer lab is open before and after classes and during lunch for teachers to complete assignments, check email, and email daily reflections to Master Teachers. A Rice University graduate student staffs the computer lab to assist teachers.

The curriculum also includes classroom-based assessments that aim to improve instructional decision making, as well as student learning. Teachers are encouraged to explore a wide range of assessment strategies - student writing, performance tasks, student self-assessment, observations, interviews - and to develop assessment activities that are natural outgrowths of classroom work. Master Teachers use a variety of assessment techniques to evaluate teachers' work in the program including discussions, work on longrange problems and open-ended questions, projects, dramatizations, homework, journals, essays, and portfolios. Use of computers, calculators, and manipulatives are included in assessments.

Teachers have a variety of opportunities to collaborate with colleagues, including opportunities to plan instructional activities for particular mathematical concepts. Teachers plan concept-based instruction focusing on the Texas Essential Knowledge and Skills (TEKS) using RUSMP's Learning Plan. RUSMP's Directors and Master Teachers assist teachers in the writing of the plans. Time for teachers to collaborate and create learning plans is provided weekly during class time. Teachers work together to create learning plans to use in their classrooms during the academic year. During the last week of the program, teachers present their learning plans to their peers.

During lunch, teachers participate in small group discussions on topics of interest or need, such as assessment strategies, classroom management, motivating students to learn mathematics, or they view selected videos appropriate for classroom use. RUSMP's Director leads these sessions. These informal sessions provide further opportunities for
teachers to learn from each other and build more personal and lasting connections to RUSMP.

Each Wednesday morning, the groups meet jointly for a one-hour colloquium talk presented by university mathematics faculty, post-docs, or other national leaders in mathematics and mathematics education on mathematics and its applications, curriculum, school reform, and minority and gender issues in mathematics education. The colloquia speakers serve as a bridge between the research and teaching communities. Lunch is provided for all to promote discussion of the colloquium topic of the day. Last summer's colloquia topics were "Area, Angle, and Curvature," "The Many Hats of a Mathematics Teacher," "The Language of Mathematics," and "NCTM Principles: The 'Character' of School Mathematics."

On the third Wednesday of the program, RUSMP hosts an Administrators' Day, a meeting for school and districtlevel administrators and business partners. Guests learn about the latest research in teaching and learning mathematics, participate in round-table discussions, visit classes with their teachers, preview learning plans and centers that their teachers are developing, and make plans with their teachers on how to improve the mathematics programs at their schools.

An important goal of the Summer Campus Program is to produce teacher-leaders who will make an impact in their school districts, statewide, and nationally. To encourage this, teachers receive assistance in preparing and making presentations in schools or at conferences. Each year several RUSMP participating teachers share their renewed excitement for teaching by presenting at the Texas Conference for the Advancement of Mathematics Teaching (CAMT). In addition RUSMP hosts Fall and Spring Networking Conferences for all past participants. At these networking conferences, after a keynote address by a university faculty member, Summer Campus Program Master Teachers and teachers make presentations to share new resources and teaching ideas.

For their work during the Summer Campus Program, teachers receive four hours of university graduate credit in education and 30 clock hours of credit toward state gifted and talented certification. In addition, teachers receive stipends, travel money to CAMT, materials, books, and technology, as well as follow-up support from the RUSMP Directors. The university waives the tuition and fees for
Rice University School Mathematics Project Summer Campus Program 2003

| MATHEMATICAL CONCEPTS |  |  |  |  |  | PROCESSES |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number \& Operations | Patterns, Functions, \& Algebra | Geometry \& Spatial Sense | Measurement | Data Analysis \& Statistics Probability | Concept Sequencing <br> Problem Solving <br> Reasoning \& Proof <br> Communicating <br> Connecting <br> Representing |
| Pre K-2 | - Whole number concepts \& operations <br> - Numeration <br> - Place value | - Balance \& equalities | - Shapes \& their properties | - Standard \& nonstandard systems <br> - Time \& temperature | - Chance |  |
| 3-4 | - Whole number concepts \& operations <br> - Fraction concepts \& operations | - Factors \& multiples <br> - Patterns | - Plane figures <br> - Congruence, similarity <br> - Transformations | - Measurement systems <br> - Perimeter, area | - Simple probability <br> - Interpretative data |  |
| 5-6 | - Fractions, decimals, percents, concepts \& operations <br> - Integer concepts \& operations | - Variable <br> - Patterns | - Polygons <br> - Transformations <br> - Spatial geometry | - Perimeter, area, volume <br> - Measurement systems | - Central tendency <br> - Theoretical \& experimental probability |  |
| 7-Algebra I | - Ratio \& proportion <br> - Integer concepts \& operations | - Polynomials <br> - Slope <br> - Linear \& non-linear functions | - Area, surface area, perimeter, volume <br> - Logic <br> - Nets <br> - Transformations <br> - Pythagorean Theorem | - Area, surface area, perimeter, volume <br> - Pythagorean Theorem | - Statistics <br> - Theoretical \& experimental probability |  |
| Geometry and Above | - Limits <br> - Direct \& inverse variation <br> - Proportionality | - Parent functions <br> - Transformations <br> - Rate of change <br> - Function development \& application <br> - Proportionality | - Proportionality <br> - Area <br> - Pythagorean Theorem <br> - Logic | - Perimeter, area, volume <br> - Circumference <br> - Precision <br> - Indirect measurement | - Mathematical models <br> - Regression analysis <br> - Residual analysis |  |

teachers as the university's cost-sharing for the federal grants that help support the activities of the Summer Campus Program. Major funding for the program currently comes from Teacher Quality Grants Program under the No Child Left Behind Act of 2001, with additional support from schools, school districts, corporations, and foundations.

## Summer Campus Program Curriculum

In the current efforts to align instruction with the NCTM Standards, the focus is often on practices, such as cooperative grouping or use of manipulatives, without providing a framework or rationale for selecting a particular activity. Simon (1998) notes that there is a need to attend to the key ideas in mathematics and to organize instruction to help students grapple with these ideas. At RUSMP, all programs are conducted with the primary assumption that successful mathematics instruction will occur only when teachers and students are engaged in meaningful discussion and exploration of essential mathematics concepts. In order to structure their classes in this way, teachers must have a thorough knowledge of mathematics that will enable them to identify the key concepts and how they are linked.

RUSMP Directors, other university faculty, and Master Teachers have developed a curriculum framework around which instruction is organized. The Curriculum Matrix identifies five major strands for mathematics instruction in grades PreK-12: number, measurement, geometry, statistics and probability, and patterns and functions.


RUSMP teachers prepare a presentation poster.

Within each strand, the key concepts to be covered at each grade level are identified. This provides a basic framework for Master Teachers to work with as they plan instruction. The Curriculum Matrix for the 2003 Summer Campus Program appears on page 7. (Other curriculum matrices may be found on the RUSMP web site.)

## The RUSMP Learning Plan

To support teachers in planning instruction, RUSMP has also developed a Learning Plan template, which aids in organizing daily instruction around central mathematical concepts. The plan guides teachers to design activities that are in keeping with the NCTM Standards and the philosophy of RUSMP. An individual plan is intended to focus on a single concept and elaborate on how this concept may be presented in the classroom. The Learning Plan template is divided into eight main sections: the concept to be focused on; materials and resources needed; exploratory activities; activities to develop the concept further; basic facts and standard algorithms connected to the concept; student products to demonstrate understanding of the concept; assessment; and alignment to school and district curricular objectives. The curriculum and the Learning Plan together serve as an anchor point for the coherence of all RUSMP programs and have allowed RUSMP to maintain focus as the number of programs has increased or grown in scope. The Learning Plan is intended to formalize a lesson blueprint and timeline for instruction. (For an in-depth description of the Learning Plan as well as completed learning plans, go to http://rusmp.rice.edu/curriculum/learning_plan.htm.) The Learning Plan asks teachers to begin with an important concept, find a challenging and interesting introduction to this concept, gather a set of activities that will deepen students' understanding of the concept, and develop assessments and student products (oral, written, and visual) that can aid in the assessment of students' understanding. This is all accomplished with the required skills and knowledge related to the concept as prescribed by the TEKS in mind. The annotated Learning Plan appears on page 9.

## Evaluation and Impact

Every year the RUSMP Summer Campus Program undergoes rigorous assessment of the impact it has on participating teachers. All teachers are administered surveys at the beginning and the end of their participation, with questions that assess their confidence in several areas of mathematics instruction and their beliefs about teaching and learning mathematics. Teachers are given tests of their content knowledge, geared for their grade level, at the

## ANNOTATED LEARNING PLAN

## Exploratory Activities

Introductory "hands-on" activities that introduce students to a concept, e.g. a two-team mathematical Tic-Tac-Toe game that leads students to graph ordered pairs. These activities need to provide thinking and are preferably not of the textbook or worksheet variety.

## Concept Development Activities

Activities/problems aimed at providing students with experiences to explore and think about the concept in many situations so that formal learning and understanding can take place.

## Basic Facts and Standard Algorithms Formalized

Taken from the TEKS, the basic facts and standard algorithms are the computational strand of the instructional unit. Once students have a foundation of interesting experiences and explorations with a concept, then the basic facts and standard algorithms can be formalized - with greater success, one hopes. Textbook exercises and sets of concept-related problems are needed here.

## Assessment

Teacher-made tests and alternative assessments (i.e. observations, student writing, portfolios, student self-evaluations, interviews, demonstration tasks) provide information about student learning and thinking, as well as, information upon which to base instructional decisions.

## Concept

An idea important in the main body of mathematics, e.g. multiplication, linear equations, area, slope. Concepts are used to organize instructional units. Concept-based organization encourages broad, rich units with connections among concepts.

## Materials and Resources

Examples: Algebra tiles, geoboards, Cuisenaire rods, etc., as well as, any necessary printed materials needed for the entire unit.

## Originality and Creativity

## Student Products

| Written | Encourage the development <br> of products - written articles, <br> etc. - that |
| :--- | :--- |
| Verbal | have students organize what <br> they have learned in new ways <br> that make sense |
| Kinesthetic | to them. Providing opportunity <br> for creativity in the classroom <br> tends to <br> increase interest and motivation. |
| Visual |  |

## Related TEKS

These are the Texas Essential Knowledge and Skills objectives covered by teaching this concept.
beginning and end of the program as well. They are also asked to evaluate the design and structure of the program itself in the post-survey. In the academic year following the program, RUSMP personnel observe a random sample of the participating teachers in their classrooms. Data collected from the 2002 program indicated that, upon completion of the program, over $90 \%$ of the teachers reported feeling fairly well prepared or very well prepared in using cooperative learning groups, using hands-on activities, using a variety of methods to assess students' mathematical knowledge, presenting applications of concepts, taking into account students' prior conceptions about mathematics, managing a class using manipulatives, and using technology. Paired samples t-tests performed on the available data indicated that teachers' sense of preparedness in all these areas had increased significantly ( $\mathrm{p}<.001$ ) over the course of the program, and scores on the tests of content knowledge also significantly increased from the beginning to the end of the program across grade levels. It also appeared that teachers' beliefs about teaching and learning mathematics had become more in line with the ideas promoted by the NCTM Standards, as they agreed more
strongly after completion of the program that students should write about how they solve math problems ( $\mathrm{p}<$ .001), that it is important to begin with a concrete example ( $\mathrm{p}<.001$ ), that teachers should let students figure things out for themselves ( $\mathrm{p}<.001$ ), and that students learn best when they study mathematics in the context of everyday situations ( $\mathrm{p}<.05$ ). Teachers were less likely to agree, however, that students need to master basic computational skills before they can engage effectively in mathematical problem solving ( $\mathrm{p}<.05$ ) and that a great deal of practice is necessary for students to get better in mathematics ( $\mathrm{p}<.001$ ). These results are typical of the data obtained annually from the Summer Campus Program.

RUSMP's eighteen-year partnership with Houston-area school districts to improve mathematics instruction affords RUSMP the experience and qualification to develop an effective module that meets the needs of current and future teachers. As noted by RUSMP's external evaluators, "increased cooperation between local school districts and RUSMP has resulted in greater compatibility between RUSMP programs and curricula and school districts' pro-
grams and curricula. Through this kind of collaboration with schools and the school districts, RUSMP's impact has moved beyond the individual classroom teacher to improvement of mathematics programs at the school and district level." (See Austin, Wells, \& Herbert, 1990; Cannon, Parr, \& Webb, 2003; Eaves, 2000; Killion, 2002a; Killion, 2002b; Killion, 2002c, Killion, 1999; Nease, 1999;
Papakonstantinou, Berger, Wells, \& Austin, 1996; Schweingruber, 1999; Toenjes and Garst, 2001.)

## Reflections and Conclusion

The Summer Campus Program supports RUSMP's efforts to raise the level of teachers' professionalism, thereby improving mathematics instruction in the Houston area. It is important to stress that the development of the Summer Campus Program has evolved out of RUSMP's experiences with teachers and schools. As the Summer Campus Program has evolved, so has RUSMP's role in the development of mathematics teachers in the Houston area has grown. Other successful endeavors such as the RUSMP/Houston Independent School District Algebra Initiative, the RUSMP Urban Program, and the RUSMP academic-year courses: Algebra for Elementary Teachers,

Geometry for Elementary Teachers, Algebra for Middle School Teachers, Geometry for Middle School Teachers, Advanced Topics for Middle and High School Math Teachers, Calculus for High School Teachers, and Technology Institutes for Middle School, Algebra I, and Calculus teachers have strengthened and improved Houston-area mathematics teaching. As in any successful partnership (Miller \& O'Shea, 1996), in order to be successful and for work to stay relevant, one needs to be responsive to the needs of collaborating partners - teachers, principals, district administrators, and students. The current configuration of the Summer Campus Program is effective and has been nationally recognized. However, RUSMP remains open to the possibility that programs may need to be altered in order to adapt to changes in collaborating school-district partners.

As the Summer Campus Program approaches twenty years of providing successful professional development and support to PreK-12 teachers in the Houston area, perhaps RUSMP's experience and successes can inform other organizations desiring to create and present similar mathematics professional development programs.

## References

Austin, J. D., Herbert, E., \& Wells, R. O. (1990). Master teachers as teacher role models. Mathematicians and Education Reform, 1, 189-196.

Ball, D. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. Phi Delta Kappan, March, 500-508.

Cannon, R., Parr, R., \& Webb, A. (2003). Advanced mathematics educational support: Support, recommendations, and resources for facilitating collaboration, between higher education mathematics faculty and Texas public high schools. Austin, TX: The University of Texas, Charles A. Dana Center.

Crawford, C. M. (2000). Impacting learning environments from prekindergarten through graduate school: Technologically appropriate professional development and classroom integration opportunities for educators. In Proceedings of SITE 2001: Society for Information Technology \& Teacher Education International Conference.

Cruz, P., Turner, S., \& Papakonstantinou, A. (2003). Building confidence in the classroom: The role of the master teacher. Manuscript in preparation, Rice University.

Darling-Hammond, L. (1996). The quiet revolution: Rethinking teacher development. Educational Leadership, March, 4-10.

Darling-Hammond, L., \& McLaughlin, M. (1995). Policies that support professional development in an era of reform. Phi Delta Kappan, April, 597-604.

Eaves, E. (2000). Progress Report and Next Steps: North District Houston Independent School District. In F. Curcio (Ed.), Diversity, Equity, and Standards: An Urban Agenda in Mathematics Education. New York: New York University.

Gray, K. C. (2001). Changing classrooms by treating teachers as active learners. Middle School Journal, 32, 15-19.

Killion, J. (1999). The Rice University School Mathematics Project. What works in the middle: Results-based staff development. Oxford, OH: National Staff Development Council.

Killion, J. (2002a). Rice University School Mathematics Project Summer Campus Program. What Works in the Elementary School: Results-Based Staff Development. National Staff Development Council.

Killion, J. (2002b). The Rice University School Mathematics Project Summer Campus Program. What Works in the High School: Results-Based Staff Development. National Staff Development Council.

Killion, J. (2002c). Algebra Initiative. What Works in the High School: Results-Based Staff Development. National Staff Development Council.

Kinney, C. (1998). Building an excellent teacher corps: How Japan does it. American Educator, Winter, 16-24.

Kutilek, L. M. \& Earnest, G. W. (2001). Supporting professional growth through mentoring and coaching. Journal of Extension, 39.

Lewis, A. C. (2002). School reform and professional development. Phi Delta Kappa, 83, 488-489.

McLaughlin, M., \& Oberman, I. (Eds.). (1997). Teacher learning: New policies, new practices. New York: Teachers College Press.

Miller, L., \& O'Shea, C. (1996). School-university partnership: Getting broader, getting deeper. In M. McLaughlin \& I. Oberman (Eds), Teacher learning: New policies, new practices. (pp. 161-181) .New York: Teachers College Press.

National Council of Teachers of Mathematics. (1995). Assessment standards for school mathematics. Reston, VA: NCTM.
National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

National Institute on Student Achievement, Curriculum, and Assessment, Office of Educational Research and Improvement, \& U.S. Department of Education. (1998, June). The Educational System in Japan: Case Study Findings. Retrieved January 2, 2003, from United States Department of Education: http://www.ed.gov/pubs/JapanCaseStudy/title.html

National Research Council. (2001). Educating teachers of science, mathematics, and technology: New practices for the new millennium. Washington, DC: National Academy of Sciences. National Research Council.

Nease, A. (1999). Do motives matter? An examination of reasons for attending training and their influence on training effectiveness. Unpublished doctoral dissertation, Rice University, Houston.

Papakonstantinou, A. (1995). The Rice University School Mathematics Project. Centerpiece, publication of the Rice University Center for Education.

Papakonstantinou, A., Berger, S., Wells, R. O., \& Austin, J. D. (1996, Nov/Dec). The Marshall Plan: Rice University mathematics affiliates program. Schools in the Middle, 4, 39-46.

Schweingruber, H.A. (1999). The Rice University School Mathematics Project. The Mathematics Teacher, 92, 644.

Schweingruber, H.A., Papakonstantinou, A., Herbert, B., \& Rohr, M. (1998). University/School District Collaboration for Change: Houston's Algebra Initiative. RUSMP Report 98-01.

Simon, M. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema \& B. Scott Nelson (Eds.), Mathematics teachers in transition. (pp. 55-86). Mahwah, NJ: Lawrence Erlbaum Associates.

Stevenson, H. W., Lee, S.-Y., \& Nerison-Low, R. (Eds.). (1998). Contemporary Research in the United States, Germany, and Japan on Five Education Issues: Structure of the Education System, Standards in Education, the Role of School in Adolescents' Lives, Individual Differences Among Students, and Teachers' Lives. Retrieved January 2, 2003, from United States Department of Education: http://www.ed.gov/pubs/Research5/title.html

Stevenson, H. W., \& Stigler, J. (1992). The Learning Gap: Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education. New York: Summit Books.

Toenjes, L., \& Garst, J. (2001). Identifying high performing Texas schools and their methods of success in middle school math and Algebra I End-of-Course performance. Texas Education Agency.

# Evaluating a New Mathematics Curriculum: A District's Multi-Stakeholder Approach 

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#### Abstract

: Selecting a new curriculum and determining whether it will be an effective addition to the district's instructional efforts can be one of the most challenging leadership tasks facing the district mathematics supervisor. This article describes a structured curriculum adoption and evaluation process undertaken by the Westside Community Schools in Omaha, Nebraska, in collaboration with the University of Nebraska at Omaha. The curriculum evaluation process reviewed a new mathematics program being undertaken within the district that incorporated direct feedback from students, teachers, and parents. The evaluation strategies included a field test process involving three distinct field test groupings, with three matched control groups, to examine standardized test scores from 425 students. Surveys from 132 teachers, 596 parents, and 2,172 students were used within the comprehensive review process. The evaluation process appeared to work well for examining the impact of the new program and results confirmed that a full curriculum implementation was warranted in the 2003-2004 school year.


$\infty$upervisors of mathematics are often involved in leading the adoption of a new mathematics curriculum and then evaluating the effectiveness of that curriculum. Determining whether a new curriculum is an effective addition to the district's instructional efforts can be one of the most challenging leadership tasks facing the district mathematics supervisor. Balancing the input of parents, teachers, administrators,
textbook companies and even the community at large is often difficult since all of these participants in the decision making process may have strong opinions related to the adoption process and its potential outcomes. Many supervisors of mathematics find that an open, careful, and datadriven pilot testing strategy is critical in such a context and helpful for later support of the new program as it is fully implemented.

Since the National Council of Teachers of Mathematics (NCTM) standards were first released in 1989 (NCTM), and with the more recently published Principles and Standards for School Mathematics document released in 2000 (NCTM, 2000), many schools and districts have carefully reviewed and attempted to reform their mathematics curriculum. The vision for such reform is founded upon the ideas that mathematics instruction should be dynamic, interesting, and relevant to students (Romberg, 1998; Royer, 2003; Schoenfeld, 2002;).

As school districts have sought to revise their mathematics programs to better meet the NCTM vision, they have struggled to find curriculum resources and materials that can truly meet their individual needs. This is rarely an easy task for a district. In fact, unfocused and poorly planned district curricula have been theorized by studies within the 1990's, such as the Third International Mathematics and Science Study, to be an important reason why American schools sometimes lag behind our international peers at some grade levels (McLeod, 1995; Sawada, 1997; Valverde and Schmidt, 1998). With these studies as a context, many curriculum initiatives (such as several funded through the

National Science Foundation) have sought to better meet district needs and produce instructional materials that are more in line with the national reform efforts in mathematics education. Districts around the country have often tried to directly link their mathematics reform process to new curriculum materials. However, these adopted programs are rarely evaluated after their initial implementation, and thus their actual effectiveness for districts is not well understood.

The lack of formal curriculum evaluation is not surprising given the challenge of conducting a thorough evaluation process within a school setting. Such evaluation efforts are typically quite difficult because they need to consider the complexity of the classroom where a wide range of extraneous variables can be attributed to encouraging temporary rather than lasting effects (such as the novelty of a new curriculum, etc.). Careful curriculum evaluation designs usually take considerable work and careful planning and do best when targeting a variety of stakeholders, including teachers, students, and parents (Goldsmith, Mark, Kantrov, 2000).

This article examines the systematic curriculum evaluation process used by one district, the Westside Community Schools in Omaha, Nebraska, as it carefully adopted and reviewed a new elementary mathematics program. The curriculum evaluation process was facilitated within the context of a strong leadership effort undertaken by a district lead teacher, a district curriculum supervisor, and a university professor, working collectively to involve all important stakeholders in the process.

## Adopting a New Mathematics Program

The Westside Community School District is an urban school district of approximately 5,200 students, 1,200 of whom are not residents of the district, but rather attend through Nebraska's school choice program. The District has a K12 curriculum with ten elementary schools (grades K-6), one middle school (grades 7-8), and one high school (grades 9-12). The elementary schools where the new mathematics program was adopted and examined ranged in size from 133 to 412 students. The previous mathematics program used by the district was Math in Our World from Harcourt, Brace, and Jovanovich (1996).

The program adopted by the district was called Everyday Mathematics which is published by the Everyday Learning Corporation (2002). This program appears to be both dynamic and challenging, with hands-on elements, inte-
grated problem solving strategies, and numerous extension activities. The company website describes the program as a K-6 enriched mathematics curriculum, developed by the University of Chicago School Mathematics Project, that empowers students and teachers to understand mathematical content far beyond arithmetic. Its reputation across the midwestern states is relatively well established, although there have been differing perceptions of the curriculum and its utility for various districts and ability groups of students. The national press has reported on various communities who have struggled with a range of differing local perceptions of the program. Given the importance of having good instructional resources in their mathematics classrooms, Westside decided to undertake a formal evaluation of the new curriculum in a limited number of classrooms before full implementation of the program within the 2003-2004 school year.

## YEAR 1

The district adoption of the new program, Everyday Mathematics, was actually a two-year process. It began with the selection of a district "Curriculum Review Committee" which was empowered to examine potential new mathematics programs. This committee was composed of elementary, middle, and high school teachers, and representatives from gifted education, early childhood, and special education programs, along with several administrators and parents. In all, about 25 people routinely attended the committee meetings. Other contributing personnel included a university mathematics education professor and a mathematics specialist from a local educational service center.

During the first year the Curriculum Review Committee met one day a month and initial activities (of the adoption committee) included a review of current educational mathematics publications, and the NCTM's Principles and Standards for School Mathematics (2000). In addition, an extensive packet of research articles describing the best practices in mathematics instruction was distributed to the committee. The time together was spent discussing the material to create a common understanding of its meaning. It helped to define a clearer vision for the committee of philosophy and beliefs for mathematics education. The committee also examined existing data of test scores to review the district's current level of performance in elementary mathematics. A survey of all elementary classroom teachers and students was designed to help determine current practices and student perceptions relative to mathematics.

By the end of the first year, the committee had developed a personalized rubric that they used to evaluate potential new curriculum programs and represent their philosophy of good instruction as reflected by NCTM's Principles and Standards for School Mathematics (2000). The committee examined numerous textbook series and supporting materials. The process was both invigorating and draining, as the committee met frequently, within long, systematic, and spirited review sessions. By the end of the year, based on their examination of various commercial curricula, and their review of current mathematics research and practices, Everyday Mathematics seemed to best fit the expectations of the committee members. A plan for piloting this program to further review it was then initiated for year 2 of the adoption process.

## YEAR 2

The second year of the adoption process was devoted to the formal pilot testing of the Everyday Mathematics materials. A total of 24 classrooms, representing all district schools and all grade levels, were selected to use and evaluate the curriculum. The teachers on the Curriculum and Review Committee made up the majority of these piloting classrooms. Throughout the year the Curriculum and Review Committee continued to meet and reflect upon anecdotal observations.

This pilot testing process was essentially an "impact analysis" that was found to be common for the review of new curriculum programs. In such evaluation studies, impact analysis can be defined as "determining the extent to which one set of directed human activities affected the state of some objects or phenomena, and . . . determining why the effects were as large or small as they turned out to be" (Mohr, 1992, p.1). The study examined the consistency of several sources of data in what is often called a triangulation of information process. The field test used three sets of matched classes of students and also examined achievement test scores; student, teacher, and parent surveys; and teacher focus groups. This field testing process is fairly useful in the careful evaluation of curriculum programs and has been used successfully by other organizations (Adams, 1999; Kulm, 1999), and is similar to curriculum evaluation strategies recommended by various researchers (Manouchehri \& Goodman, 1998; McNeely, 1997).

Throughout the evaluation and pilot testing process the Curriculum and Review Committee teachers played a key role and continued to meet. The committee was chaired by
an elementary mathematics specialist (lead teacher) who had been released for two years from classroom teaching responsibilities to devote full-time to this leadership role. The responsibility of the committee actually went beyond the selection of new mathematics materials. Through this program adoption, they were in charge of reforming the mathematics education program. Their ongoing involvement allowed them to grow in the areas of mathematics education and pedagogy. Over the two years' time, their responsibilities and interests typically evolved into leadership roles in mathematics curriculum and instruction. They helped to develop tentative plans for implementation, provided training for teachers, and developed surveys to help get teacher and parent perceptions. Before the completion of the two years, the teachers of the committee had refined their long-term goals for the mathematics curriculum. They essentially became the managers of a new district vision for mathematics instruction to be represented by the new curriculum.

## Looking at Student Achievement

In order to realize Westside's vision for mathematics instruction as represented by the new curriculum, it was felt by district administrators that standardized test scores had to be a part of how the curriculum was evaluated. In today's educational environment educators and the community at large are quite interested in standardized achievement scores and how those scores appear to be impacted by different educational strategies. The district was thus very interested in having their standardized tests scores (those related to mathematics achievement) be included as a focused component of the overall data examined. In this field test, several standardized test scores were available for examination through their traditional use in the district, and included the Stanford Achievement Test (9th edition), and the Otis-Lennon School Ability Test (OLSAT). The Stanford Achievement Test measures mathematics problem solving and mathematics procedures in two different subtests at six different elementary levels. The Otis-Lennon School Ability Test strives to measure a student's general thinking skills as well as help identify some relative strengths and weaknesses in their reasoning strategies. Both standardized instruments were considered to be good operational measures of the mathematics-related achievement targeted by Westside when adopting a new mathematics program. Together, these tests could address both basic skills and higher order thinking. Scores for the 1999 and 2000 school year were used as a baseline measure (before program initiation), and scores for the 2001-2002
school year (after one or two years of program use), were collected to examine potential differences. Classes of students who had received the Everyday Mathematics program for two years and for one year were compared to students who had not been exposed to the program.

The field test groups were carefully selected to provide groups as equivalent as possible for the overall data analysis. Criteria included free and reduced price lunch participation and gender. Three groups were eventually selected.

- Comparison Group 1: Students from two schools who received the program as third graders ( $\mathrm{n}=26$ ) were compared to a random sample of third grade students from similar schools who had not had the program ( $n=63$ ).
- Comparison Group 2: Students who experienced the program for two years, in grades four and five ( $\mathrm{n}=51$ ) were compared with fourth and fifth grade students from a similar school who had not yet had any exposure to the new mathematics program ( $\mathrm{n}=37$ ).
- Comparison Group 3: Students from five schools who had the program as fifth graders $(\mathrm{n}=137)$ were compared with a similar group of students from four schools in which the program was not used in grade five ( $\mathrm{n}=131$ ).

For each of the three comparison groups, three dependent variables were investigated: the SAT 9, including the Total Math percentile rank; the Problem Solving Subtest percentile rank; and the Procedural Mathematics subtest percentile rank score. These statistical runs used a variety of parametric and non-parametric techniques, including Analysis of Variance (ANOVA) procedures, with baseline SAT 9 scores and the Otis-Lennon test scores used as covariates.

The resultant analyses were generally supportive of the Everyday Mathematics program with achievement relatively higher in the Grade 3, and Grade $4 / 5$ pilot groups. Analysis of the pilot groups for Grade 5 was within the margin of error for the test, not statistically significant, and was considered as relatively equivalent. The analyses also showed that prior SAT 9 scores, and the Otis-Lennon test were appropriate covariates for the analyses. Overall, the district was encouraged by the relatively supportive results for the mathematics program on these standardized test scores.

The natural limitations of a curriculum evaluation process that might emphasize standardized test scores were an important concern to the district. Could any increased achievement be simply a novelty effect of the new curriculum as teachers tried harder to do something new? Was the new curriculum really mapping to student outcomes in a way that could even be reflected on the standardized tests? In order to feel more confident that the new curriculum was indeed playing a role in these observed differences in test scores, other sources of data needed to be examined.

## The Voice of Stakeholders: Examining Survey Feedback

Beyond the students themselves, the district recognized that a new curriculum has other stakeholders associated with it. Teachers try to facilitate learning within its structure and parents try to encourage their child's success within it. Each of these two stakeholder groups can have a different perspective on the curriculum, and individuals within these groups may have varying opinions on its relative success. It had actually been a long-standing practice within the Westside Schools to informally survey students, parents, and teachers regarding their opinions relative to any new curriculum adoptions and this practice was extended into a more rigorous and comprehensive survey process. This particular evaluation-related process also resulted in a unique opportunity to be able to compare the survey responses of students, parents, and teachers who were involved in the implementation of the program with those who were not. In addition to the surveys, focus groups of teachers were held with those who had used the program and those who had not to obtain a more thorough examination of the program's strengths and weaknesses.

## TEACHER SURVEY

The teacher survey included 85 questions about mathematics instructional practices, program content, teachers' opinions about their students' attitudes toward mathematics, and the adequacy of the program they were currently using in relation to district mathematics standards. To allow for comparison of the training and opinions of teachers using the new program with those who were not using it, teachers were asked to indicate whether they were currently using the new program, and if they were, whether they had used it for one or two years. Teachers who were using the new program were asked to evaluate the quality of the materials and the adequacy of training. Since the teachers in the field test group had received additional training for the new curriculum, this was a useful
way to determine the teachers' perceptions of the effectiveness of that training. A total of 132 teachers responded, representing essentially all of the district's elementary teachers. A few sample questions follow.

## SOME SAMPLE TEACHER SURVEY QUESTIONS

Overall, my students' attitudes toward mathematics this year have been:
A. Very Positive
B. Somewhat Positive
C. Somewhat Negative
D. Very Negative

Overall, the rigor of the mathematics curriculum I used this year was $\qquad$ for my students.
A. Too difficult
B. About right
C. Too easy

Parents' concerns relative to their child's performance in mathematics this year have been:
A. Less than most years
B. About the same as most years
C. Greater than most years

Note: For electronic copies of the full survey send an e-mail to bjackson@westside66.org

A teacher can typically spend a considerable amount of time using supplemental resources for their classroom. Teachers in both groups in the district (new vs. traditional programs) were asked about how much they had used supplemental resources in particular areas during the last year. Three differences surfaced between the new Everyday Mathematics program and the traditional program. Feedback from the survey suggested that the new curriculum group used basic worksheets, routine games, and drill and practice strategies less frequently than their colleagues in the traditional curriculum classrooms. This feedback was seen as consistent with the higher level of interactivity associated with the new program.

It is important to note that teachers in the newer curriculum group had received more training than their colleagues, and had been prepared to deliver the Everyday Mathematics curriculum as effectively as possible. The training seemed well embraced by the teachers. Feedback from the survey suggested that teachers within the newer program felt that they needed less additional training in several different topics. Nine areas surfaced as feedback differences, with
the teachers involved in the newer curriculum seeing less need for additional training. These training areas included reasoning, connecting ideas, algebra, communication, algorithms, transitions, self-guided learning stations, best practices, and manipulatives. It was interesting to note that teachers in both programs commonly desired more training within most topical areas. However it was apparent that the new program had a significantly less perceived "need" by teachers for these nine training areas.

Perhaps the most interesting difference between the perceptions of teachers within both instructional groups was a survey question that simply asked teachers how well they felt mathematics instruction was going this year. Teachers within the new program thought it was indeed going better and had a higher percentage of positive responses on a Likert scale question that asked teachers to reflect on their students' learning in mathematics as "less than most years," "about the same as most years," or "greater than most years." Responses also suggested that Everyday Mathematics teachers felt there was a slightly better attitude in those classrooms.

## TEACHER FOCUS GROUPS

Survey responses can only help confirm opinions that are already well identified on the instrument itself. If the survey developer does not anticipate particular questions, it is hard to have those questions surface automatically within the data retrieved by the survey. To provide more of a deeper look at what teachers really felt about the program, two focus groups of teachers were formally conducted. Each group consisted of 11 to 12 teachers who had used the program for at least one year. The facilitator of the focus groups inquired about overall reactions to the program; its impact on teachers, students, and parents; and the need for additional training and support. Focus groups were audio taped, and data were summarized from typed transcripts.

Several general themes emerged from the district focus groups, and provide useful interpretation information for the program evaluation. These themes were generally supportive of the new program, but suggested that it was more difficult and time consuming to implement. Briefly, these themes included the following:

1) Teachers generally perceived a greater time need for overall lesson preparation in this program as compared to the earlier program.
2) Teachers perceived a need to devote more class time to mathematics instruction than was typically necessary with the previous program.
3) Teachers perceived a stronger integration by this program with other content areas than had been achieved with the previous program.
4) Teachers generally perceived a greater application to "real-world" situations in this program.
5) Teachers perceived that parents were often having more difficulty in helping their children with the mathematics homework of this program.
6) Teachers believed that the program was generally accessing a higher level of mathematics content at each grade level.
7) Teachers believed that students generally enjoyed the program.
8) Teachers were generally enthusiastic and supportive regarding this program.

Particularly noteworthy within the focus group themes were the teachers' perceptions regarding the higher level content and overall students' enjoyment of the program. One teacher commented, "They really like math. They look forward to it. As soon as we get there in the morning we're starting." Another said, "Morning after morning, I look around and they've all come in, picked up a paper and are all working quietly without being told because they like doing it." Another teacher attributed the students' enjoyment of the content, in part, to its variety. "You're not teaching just one thing the whole time. You're doing all these different things with that lesson so it really isn't just an hour of adding. It's doing a lot of different things."

Regarding the higher level of the content, a second grade teacher said, "I can honestly say to my second graders, "Well, this is the first time I've ever taught this to a second grader. I've taught it to fifth graders, but now we're going to do it in second grade." Some teachers anticipated that the standardized test scores would go up as a result of the Everyday Mathematics program. A third grade teacher commented, "In other programs they just get into a pattern. They do 20 multiplication problems so there's not a lot of thinking involved. I watched the children take the Stanford [SAT 9]. The problems are varied on the Stanford, they asked them to do different things. In the
past, they had a problem with that. Our kids were in a pattern of just doing the same thing over and over. Here I watched my kids take each [test item] and really attack each one. I think it will show up in our scores." A fifth grade teacher in the other focus group said, "The kids came out of the test going, 'Well, that was easy.' I think they felt more comfortable. They came out going, 'Well, that's nothing.'" Such responses within the focus group data suggested that teachers were generally supportive and relatively impressed with the new curriculum.

## STUDENT SURVEYS

Teacher surveys and focus groups are helpful in examining the potential effectiveness of any new program. However, the students themselves are really the key target audience and direct beneficiaries of any curriculum. Two student surveys were distributed to help get the opinions of students directly, one survey with 9 questions for first and second grade students and another survey with 21 questions for students in grades three through six. Survey questions focused on students' perceptions of their competence in mathematics and the degree to which they enjoyed various aspects of mathematics curriculum content. Students' schools and teachers were identified so the opinions of students who had not been exposed to the new program could be compared with those who had received the new program for one or two years. Some sample questions follow.

SOME SAMPLE STUDENT SURVEY QUESTIONS GRADES 3-6

I am good at math.
A. Agree
B. Disagree
C. Not sure

I enjoy talking with others about math.
A. Agree
B. Disagree
C. Not sure

I sometimes use math in other subjects.
A. Agree
B. Disagree
C. Not sure

SOME SAMPLE STUDENT SURVEY QUESTIONS GRADES 1 AND 2

Math is fun.


I can solve math problems.


Note: For electronic copies of the full survey send an e-mail to bjackson@westside66.org

For the primary survey, 693 students participated and selected pictures of faces to help give their response on the survey (from happy to sad) as a means of relating their agreement or disagreement with a question. The surveys were read out loud by the teacher to aid in student comprehension. The students in the Everyday Mathematics group and the students in the traditional instructional group differed on three variables. These included responses to the following items "I am good at math," "I like to use objects to help me figure out problems," and "I can solve math problems." A higher score represented greater agreement. Each of the comparisons was generally supportive of the Everyday Mathematics program.

For the district's Intermediate Survey 1,479 students participated and selected Likert responses to represent their level of agreement or disagreement with each item. There were four variables that differed between the Everyday Mathematics and Traditional Instructional groups. These included: "I like doing projects in math," "I like using the computer to work on math," "I like doing math at home," and "I enjoy solving math problems." Each of these responses was generally more supportive of the new program.

## PARENT SURVEY

Although student and teacher support is indeed key for the success of any new curriculum, the district recognized that parents need to play a role in its success. Thus, the evaluation process for this curriculum adoption effort also dealt with parents. A 39 question survey was mailed to all elementary parents (2061), with a return rate of twentynine percent (29\%) for 596 parents responding. Some sample questions follow.

Families with more than one elementary student were asked to answer the questions relative to the child whose birthday comes first in the calendar year. So as not to focus parents' attention solely on limited elements of the mathematics program, the survey included similar questions about other curricular areas, and the perceived effectiveness of their student's schooling. Specific to this study, questions focused on their child's mathematics performance, students' enjoyment of the subject, and opinions about the homework associated with mathematics. Parents were asked to identify their child's school and grade so opinions of parents of students in the various groups described above could be compared.

## SOME SAMPLE PARENT SURVEY QUESTIONS

My child enjoys math.
A. Strongly Agree
B. Agree
C. Disagree
D. Strongly Disagree
E. Don't know

I have a good understanding of the mathematics program in my child's school.
A. Strongly Agree
B. Agree
C. Disagree
D. Strongly Disagree
E. Don't know

I think my child is appropriately challenged in mathematics.
A. Strongly Agree
B. Agree
C. Disagree
D. Strongly Disagree
E. Don't know

Note: For electronic copies of the full survey send an e-mail to bjackson@westside66.org

Generally, for the curriculum evaluation itself, there were no notable differences between the Everyday Mathematics and Traditional groups on any variable on the parent survey. There were only slight differences in a few of the parents' responses on one variable related to mathematics (but not significant). The parents in the control group agreed slightly more strongly with "I feel confident in helping with mathematics homework."

## Building on What Has Been Learned

One important aspect of good curriculum evaluations is that such evaluations should eventually help lead to an enhanced learning experience for students. Within the context of this particular curriculum evaluation, the district was trying to examine if its initial promise for enhancing student achievement was indeed becoming a reality in the classroom using this program. The results of the evaluation were generally supportive of the new program and will now help the district further embrace the program. However, the district also realized that the strong administrative support provided for the program, such as the consistent teacher in-service process and willingness to formally evaluate the program was probably a significant factor in the overall success of the program. Strong leadership was also a key factor and each of the three leadership
team members (lead teacher, district administrator, and university supervisor) found that they needed to be continually involved with all aspects of the program and its evaluation process.

By the end of the two years of evaluation effort, the committee was ready to make a recommendation to the Board of Education suggesting that Everyday Mathematics be considered as the formal curriculum for their elementary students. After board approval, the committee was also empowered to develop a further implementation plan. The key focus of the implementation plan was to support teachers in ongoing professional development. A full-time math facilitator was requested to be available to teachers to support them in their classroom. Monthly grade level meetings were planned to work with teachers in program related topics such as using materials, pacing, lesson focus, management of program components, grading, with designated grade level leaders. The middle and high school math department teachers were asked to assist with the program, and also to be on call to help explain any content questions that surfaced from teachers.

Thus, after the two year evaluation process, the curriculum was essentially underway. Some of what was learned within the adoption process for this curriculum related to the general group dynamics of facilitating change. In actuality, this curriculum adoption effort was perhaps the most carefully planned adoption effort ever undertaken by the district. By acknowledging that a careful pilot study process was being built into the adoption timeline right from the start, it appeared that the participating teachers, administrators, and parents were all the more willing to assist with the additional work needed for the adoption process to succeed. The use of a full time facilitator, in this case a released master teacher, was an important lesson learned in its own right. Having a full time, knowledgeable and available advocate for the curriculum adoption process was often recognized as critical for ensuring the strong participation of all stakeholders.

A good curriculum evaluation program should look to the future needs of the district. The results of this particular evaluation will be used by Westside to further address identified teacher, parent, and student related insights and to further enhance teacher training. Teacher training is critical to the implementation of any new program but
particularly so in relation to any new mathematics program. Mathematics instruction required by the Everyday Mathematics Program, and similar curricula, often involve an approach that is considerably different from more traditional mathematics instruction. The recent evaluation process will aid in future planning related to keeping an effective mathematics curriculum in the classrooms of the district.

The ongoing effort of an effective curriculum implementation process related to mathematics instruction continues at Westside. As teachers become more experienced with the new curriculum, more professional development is planned by the district to help them become increasingly efficient and effective with the new materials. It may well be that some of the most important professional development will occur as teachers become more experienced with the curriculum and its new approaches.

An effective curriculum in today's fast paced learning environment is one that is interesting, dynamic, and well supported by the various stakeholders involved. Within this context, students need to be achieving, teachers need to be engaged, and parents need to be supportive. Such an embraced curriculum then needs to be accountable to such stakeholders, who deserve to know if it is working as expected. This ongoing accountability requires good curriculum evaluation, with ongoing and periodic feedback, and strong leadership. The Westside Community schools were pleased that they had planned for such accountability and leadership right from the beginning of the new mathematics program using a systematic and inclusive evaluation process.

Strong formative evaluation can often be an important "glue" to helping build and maintain a cohesive curriculum for a district. An evaluation itself can thus aid achievement. When everyone is aware of both the successes and challenges of a planned curriculum, they are more likely to undertake these new learning activities with realistic expectations, sustained enthusiasm and a better understanding of student needs. Most importantly, when all stakeholders participate in helping examine whether a program is truly effective, they are expressing an active interest in both the program's general success and the related academic success of their students.

## References

Adams, A.A. (1999). An evaluation of a district-developed NCTM standards-based elementary school mathematics curriculum. Paper presented at the Annual Meeting of the Mid-South Educational Research Association, Point Clear, AL. ED 435759.

Everyday Learning Corporation (2002). Everyday Mathematics. A University of Chicago School Mathematics curriculum available through McGraw Hill Companies: Desoto, Texas.

Goldsmith, L.T., Mark, J., \& Kantrov. I. (2000). Choosing a standards-based mathematics curriculum. Portsmouth, NH: Heinemann.

Harcourt, Brace, Jovanovich Publishing. (1996). Math in our World. San Diego, California.

Kulm, G. (1999). Making sure that your mathematics curriculum meets standards. Mathematics Teaching in the Middle School, 4 (8), 536-41.

McLeod, D.B. (1995). International influences on the NCTM standards: A case study of educational change. Paper presented at the annual meeting of the North American Chapter of the 1995 International Group for the Psychology of Mathematics Education, Columbus, Ohio.

McNeely, M. E. (1997). Guidebook to examine school curricula: TIMSS as a starting point to examine curricula. Associated with the Third International Mathematics and Science Study resource materials. Eric Document ED 410128.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, Virginia: NCTM.

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Reston, Virginia: NCTM.

Manouchri, A. \& Goodman, T. (1998). Mathematics curriculum reform and teachers understanding the connections. Journal of Educational Research, 92 (1), 27-41.

Mohr, L.B. (1992). Impact Analysis for Program Evaluation. Newbury Park, California: Sage Publications ISBN \#0-8039-4981-2.

Romberg, T.A. (1998). Comments: NCTM's Curriculum and Evaluation Standards. Teachers College Record, 100 (1), 8-21.
Royer, J.A. (2003). Mathematical Cognition: A Volume in Current Perspectives on Cognition, Learning, and Instruction. Greenwich: Connecticut: Information Age Publishing.

Sawada, D. (1997). NCTM's standards in Japanese elementary schools. Teaching Children Mathematics, 4 (1), 20-23.
Schoenfeld, Alan H. (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. Educational Researcher, 31 (1),13-25.

Valverde, G.A. \&Schmidt, W.H. (1998) Refocusing U.S. Math and Science Education. Issues in Science and Technology Education, 14 (2), 60-66.

# Looking Inside the Classroom: Mathematics Teaching in the United States 

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Mathematics education has been in the spotlight for some time now. Over the past fifteen years, out of concern that an overemphasis on computation and algorithms had led to a misrepresentation of the discipline of mathematics, the National Council of Teachers of Mathematics (NCTM) has produced a series of national standards documents (NCTM, 1989, 1991, 1995, 2000). These documents make a case for more attention to problem solving and conceptual understanding as hallmarks of quality mathematics instruction. However, there continue to be differences of opinion about the extent to which mathematics instruction should be directed by the teacher and/or instructional materials, with some mathematics educators viewing guided discovery as appropriate, and others defining problem solving as only those instances in which students are engaged with open-ended questions for which they devise their own approaches.

Given the time required for instruction based on openended problem solving, some mathematics educators worry that students will not have opportunities to learn many important mathematics ideas. In some cases, use of hands-on activities, manipulatives, calculators, and realworld contexts has been equated with problem solving. Critics argue that using manipulatives or technology without rigor is far from mathematical; and that much of the problem solving that takes place in the discipline of mathematics remains a mental exercise, often without specific
applications to real-world situations. In addition to these disagreements, some mathematicians, educators, and parents favor more direct instruction focused on explication of procedures and concepts followed by considerable practice on skills and applications. Within these differing stances regarding the best instructional approaches, there is a broader consensus that mathematics instruction is best when it aims at student understanding, not only understanding of mathematics disciplinary content, but also understanding the essential role of problem solving in mathematics as a discipline.

Very little information was available, until recently, about the extent to which teaching for understanding characterizes instruction in the nation's mathematics classrooms. Much of the information that exists on classroom practice comes from large-scale survey data. A strength of surveys is their capacity to provide information on the extent to which a variety of instructional strategies are being utilized, but they lack the capacity to describe the quality of instruction (Burstein et al., 1995; Mayer, 1999; Porter et al., 1993; Spillane and Zeuli, 1999).

A quarter century ago, the Case Studies in Science Education (Stake and Easley, 1978), a national observation study involving a cross-section of 11 U.S. school districts, described the conditions and needs of science, mathematics, and social studies education. The researchers reported that the mathematics instruction students experienced was quite varied in quality; while some of the observed mathe-

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matics classes stressed important concepts and were described as interesting to students, most overemphasized facts and memorization and were not seen as relevant to the students. Mathematics education observation studies since that time have generally either been quite small, or have been conducted in the context of the evaluation of a reform initiative, in both cases limiting the generalizability of the results.

The Inside the Classroom study provides new insight into the extent to which teaching for understanding is occurring in our nation's mathematics classrooms, complementing the self-report data on teacher preparedness and frequency of various instructional strategies, e.g., lecture, available from the 2000 National Survey of Science and Mathematics Education (Weiss et al., 2001). The study included observations of 184 mathematics lessons in 90 schools, selected to be representative of lessons nationally, as well as interviews with the teachers of those lessons. This article shares findings about the national status of quality mathematics instruction and the components of lessons that seem likely to promote student understanding.

## Methodology

The study design for Inside the Classroom drew upon the nationally representative sample of schools that had been selected for the 2000 National Survey of Science and Mathematics Education. A subset of middle schools from the schools that participated in the 2000 National Survey was selected. To ensure that these sites would be as representative of the nation as possible, systematic sampling with implicit stratification was used. When a middle school agreed to participate, the elementary schools and high school(s) in the same feeder pattern were identified and one of each was randomly selected. Two mathematics teachers were then randomly selected from each school for classroom observations.

Observations were conducted by experienced mathematics educators trained in the use of the "Inside the Classroom Observation and Analytic Protocol." Researchers were asked to take detailed field notes during the observation, including describing what the teacher and students were doing throughout the lesson, and recording the time spent on various activities. Following the observation, the researcher interviewed the teacher about the lesson, focusing on why the particular content and instructional strategies had been selected.

Researchers completed the protocol using the data collected during the observation and interview, documenting the nature and quality of the observed lessons in a number of different areas, including the accuracy and developmental appropriateness of the mathematics content and the extent to which the classroom culture facilitated learning. The lessons were ultimately assessed on the extent to which they were likely to impact student understanding in mathematics and develop their capacity to "do" mathematics successfully.

The completed protocols were reviewed for clarity, comprehensiveness, and consistency by a senior Horizon Research, Inc. mathematics education researcher, and revised by the observer as needed. Data from the analytic protocols were weighted in order to yield unbiased estimates for all mathematics lessons in the nation. The weighted estimates of the frequency of classroom practices based on Inside the Classroom data are generally equivalent to those based on the 2000 National Survey sample, suggesting that estimates of lesson quality based on the observation data are an accurate depiction of what happens in the nation's mathematics classes.

## The Quality of Mathematics Lessons Nationally

Inside the Classroom researchers rated the observed lessons on individual indicators in a number of areas, e.g., the quality of teacher questioning. Following the rating of individual components of the lesson, researchers were asked to provide an overall rating of the lesson. The scale observers used is divided into the following levels:

Level 1: Ineffective instruction
a. passive "learning"
b. "activity for activity's sake"

Level 2: Elements of effective instruction
Level 3: Beginning stages of effective instruction (low, solid, high)
Level 4: Accomplished, effective instruction
Level 5: Exemplary instruction

Lessons judged to be low in quality (those rated $1 \mathrm{a}, 1 \mathrm{~b}$, and 2) are unlikely to enhance students' understanding of important mathematics content or their capacity to do mathematics successfully. While low quality lessons fell down in numerous areas, their overarching downfall tended to be the students' lack of engagement with important mathematics. Examples of low quality lessons included:


Figure 1

- A mathematics class where students spent most of the time playing a mathematics-related game with no attention to the mathematics concepts implicit in the game; and
- A mathematics lesson in which the primary purpose was to learn algorithms without concern for the meaning of the concepts represented by the algorithms.

At the other end of the scale, high quality lessons (those rated high 3,4 , and 5) were designed and implemented to engage students with important mathematics concepts; they were very likely to enhance their understanding of these concepts and to develop their ability to engage successfully in the processes of mathematics. Regardless of the pedagogy (e.g., investigations, teacher presentations, reading, discussions with each other or the teacher), high quality lessons provided opportunities for students to interact purposefully with mathematics content and were focused on the overall learning goals of the concept. Examples of high quality lessons included:

- A 3rd grade class where students worked individually on mathematics problems, with the teacher circulating and asking challenging questions to help them articulate their thinking.
- A middle school mathematics lesson where small groups of students developed strategies to find the volume of irregularly shaped objects and shared them with the rest
of the class; and
- A lecture in an advanced placement calculus class, where the teacher derived the general exponential growth and decay formula and provided examples of how the formula was applied in the growth of bacteria populations.

Other lessons were purposeful and included some elements of effective practice, but also had substantial weaknesses that limited the potential impact on students. The specific areas where "middle quality" lessons fell down varied. Examples included:

- A lesson where the teacher spent a substantial amount of time describing the context of a problem, leaving too little time for the students to engage with the rich mathematics in it;
- A lesson where the teacher posed good questions, but moved ahead as soon as any student gave a correct answer, without checking if others were understanding; and
- A discussion that involved high-quality ideas, but was too fast-paced for many of the students.
Data from the Inside the Classroom study indicate that most mathematics lessons in the United States are low in quality, with a general lack of teaching for understanding. As can be seen in Figure 1, based on observers' judgments, only 15 percent of $\mathrm{K}-12$ mathematics lessons in the United States would be considered high in quality, 29 percent medium in quality, and 56 percent low in quality. In the high quality lessons, students were fully and purposefully engaged in deepening their understanding of important mathematics content. Some of these lessons were "traditional" in nature, including lectures and worksheets; others were "reform" in nature, involving students in more open inquiries. In contrast, in the low quality lessons, which included both traditional and reform-oriented lessons, learning important mathematics would have been difficult, if not impossible.

Detailed analyses were conducted in order to learn more about the characteristics that distinguished lessons that seemed to promote student understanding from those that did not. A number of factors emerged, including the extent to which the lesson was able to engage students with the mathematics content; create an environment conducive to learning; ensure access for all students; use questioning to monitor and promote understanding; and help students make sense of the mathematics content.

## Effective Lessons Provide Students with Opportunities to Grapple with Important Mathematics Content in Meaningful Ways

Certainly one of the most important aspects of effective mathematics lessons is that they address content that is both significant and worthwhile. Lessons using a multitude of innovative instructional strategies would not be productive unless they were implemented in the service of teaching students important content. Based on the lessons observed in this study, mathematics lessons in the United States are relatively strong in this area, with 69 percent of lessons judged to include significant and worthwhile content. (See Figure 2.)

Lessons Receiving High Ratings on Selected Indicators


Figure 2
It is important to note that while the majority of mathematics lessons in the United States included important content, most lessons were nevertheless rated low. Clearly, while the inclusion of important content is necessary for high quality mathematics education, it is not sufficient.

Effective lessons include meaningful experiences that engage students intellectually with mathematics content. These lessons make use of various strategies to interest and engage students and to build on their previous knowledge. Effective lessons often provide multiple pathways that are likely to facilitate learning and include opportunities for sense-making. Unfortunately, students are not often intellectually engaged with important mathematics content, with only 20 percent of lessons rated highly in this area.

## Lessons Should "Invite" Students to Engage Purposefully with Content

It is clear that teachers need a thorough understanding of the purpose of the lesson in order to guide student learning. It has also been argued that students need to see a purpose to the instruction, not necessarily the disciplinary learning goals the teacher has in mind, but some purpose that will motivate their engagement (Kesidou and Roseman, 2002). In the ideal, lessons will "hook" students by addressing something they have wondered about, or can be induced to wonder about, possibly but not necessarily in a realworld context. Many observed lessons failed to incorporate strategies to gain student interest and motivation; in many cases, lessons "just started," often with a warm-up problem that was unrelated to the rest of the lesson, or by the teacher handing out worksheets for the students to complete.

Teachers who succeeded at engaging students intellectually with mathematics content had various strategies for doing so. Some lessons that "invited the learners in" did so by engaging students in first-hand experiences with the concepts. For example, in a 7th grade lesson on fractions and percents, one student measured the height and arm spread of a second student, and the class was asked to use these numbers to express the relationship both as a ratio and as a percent. Other lessons invited the students in by using real-world examples to illustrate the concept vividly. Still others used stories, fictional contexts, or games to engage students with the content of the lessons. The following are examples of lessons that were particularly successful at motivating student interest and engagement:

A teacher of a 3rd grade mathematics class worked to develop an understanding of how parentheses may be used to direct order of operations in number sentences by involving students in writing number models for different ways a basketball team might score 15 points.

In a high school Algebra I lesson, the teacher presented three line graphs showing data about two fictitious companies regarding productivity (intersecting lines), production cost (parallel lines), and sales (equivalent lines). She discussed each graph with the class and then asked the class to vote for the company they would hire based on the graphs.

## Lessons Should Foster Students' Understanding of Mathematics as an Investigative Process

How mathematics is portrayed is key to student understanding of the discipline. Lessons can engage students with concepts so they come away with the understanding that mathematics is a dynamic body of knowledge, generated and enriched by investigation. Alternatively, lessons can portray mathematics as a set of algorithms to be memorized. Based on Inside the Classroom observations, only 15 percent of mathematics lessons nationally provide experiences for students that clearly depict mathematics as investigative in nature (rated 4 or 5 on a five-point scale). The following lesson is illustrative of those that highlighted the investigative nature of mathematics:

A 7th grade pre-algebra lesson began with the teacher introducing a new word problem. The purpose was to help reinforce the need for careful reading of problems, justification of strategies used and solutions presented, and the concept that there are multiple ways to approach solving a single problem. The students and teacher were engaged for a considerable time in a whole class discussion about strategies used to solve this single word problem with students presenting their solutions. The teacher stressed that there was "not a right way or a wrong way" to solve a problem, but "many ways to get into an investigation." Throughout the lesson, the teacher made statements like "I think it would be a good idea to make sure you can verify your answer with others in your group." and "I need you to convince me it's the right answer."

In contrast, many lessons presented mathematics as algorithmic in nature. The following example is typical:

According to the observer, "success in this 6th grade mathematics class hinged on students learning algorithms. Students were to learn rules and procedures, not the concepts behind them. Although the teacher had told them at the beginning of the lesson that moving the decimal place in both the divisor and dividend the same number of places was essentially the same as multiplying them both by the same power of 10 , the message he gave students throughout the lesson was, essentially, "Just do it." When students pushed him for the reason
they had to move the decimal, more than once the teacher responded: "The divisor must be a whole number."

In some cases, high stakes accountability may help explain why lessons tend to focus on a procedural view of mathematics. Based on Inside the Classroom observations, an estimated 18 percent of mathematics lessons include review/ practice to prepare students for externally mandated tests. On rare occasions, teachers were able to integrate test preparation fairly seamlessly into instruction that was geared toward learning of mathematics, as the following example illustrates.

The teacher passed out two worksheets to the students in an 8th grade pre-algebra class. The first one contained the mango problem, in which members of a family each take $1 / 3$ or $1 / 5$ of the mangoes in a basket until finally there are only three left. The task for students was to determine how many mangoes were originally in the basket. The second worksheet was for students to use to write down their solution to the problem; it included prompts such as "what I know," "strategy," and "steps."

The students worked independently; the teacher moved around the room and looked over shoulders, but said little. His questions encouraged students to think about what they were doing, and challenged them to articulate their ideas with more than a one-word answer.

The teacher noted that he was trying to continue with the planned curriculum while getting students ready for an upcoming benchmarks exam. The observer indicated that the lesson in fact provided a nice combination of test-preparation and a review of problem-solving strategies.

More often, the test preparation piece had the feel of an "add-on," and in some cases the entire lesson was focused on having students perform well on a high stakes test without also focusing on student understanding. The following example is typical:

The teacher of an 8th grade mathematics class reminded students that, "When you take the test, they might not give a specific unit, but all the units will be cubic." The teacher then turned to the topic of inequalities. She asked: "What's the opposite of an inequality?" Students responded: "An equality." The teacher said: "Okay, we're going to refer to these as inequalities. This is important because you can use inequalities to represent everyday situations. Why should you learn them? Because they're on the test."

## Lessons Should Take Students from Where <br> They Are and Move Them Forward

Although it is unlikely students are learning if they are not engaged, engagement is not enough; to develop student mathematical understanding, lessons need to be at the appropriate level, taking into account what students already know and can do, and challenging them to learn more. Approximately half of all mathematics lessons were rated high for the extent to which the content was appropriate for the developmental level of the students in the class. The estimated 17 percent of lessons nationally that were judged to be at the low end of the scale on developmental appropriateness were only occasionally too difficult, where it appeared that students lacked the prerequisite knowledge/skills, and the content seemed inaccessible to them. More often lessons were pitched at too low a level for some or all of the students. The following examples are typical:

According to the observer, "Some of the students in a 2nd grade mathematics class appeared to find the lesson too easy, and were handed worksheet after worksheet to keep them busy."

The content of an 8th grade mathematics lesson seemed to be at too low a level for the students. Said the observer, "There were no instances in which the students seemed really stuck, when the process of moving to a deeper understanding of the content could occur. They were introduced to a new concept, they made sense of the definition, they applied it to different situations, but they didn't take the next step and see how this concept might be further explored."

Some lessons used multiple representations of concepts to facilitate learning, providing greater access to students with varying experiences and prior knowledge, and helping reinforce emerging understanding. One such lesson was observed in a 7th grade mathematics class:

The teacher introduced the concept of symmetry by first demonstrating the concept with examples. The concept development unfolded by engaging students in (a) exploring the concept, (b) investigating its application to familiar cases, (c) making connections to meaningful contexts, and (d) expanding it in a more challenging activity. Students were asked to write the alphabet in capital letters and find which letters have a line of symmetry. The teacher drew examples on the chalkboard $A, B, C, D, E$, to explain, demonstrate, and discuss possible lines of symmetry. Students then worked on
their own for a few minutes, investigating the symmetrical properties of each letter, expressing some puzzlement about letters like $N, Z$, and $H$.

A discussion about symmetry in real world and familiar examples followed. The teacher presented examples that helped students make connections between symmetry and familiar contexts. Then she continued soliciting students' input of their own examples. The teacher welcomed their ideas and expanded the discussion around each example. In the last 15 minutes of the lesson, students worked on a hands-on activity designed to apply the concept of symmetry. Students were to draw the left side of a Christmas tree (on graph paper), add decorations of their choice, (e.g., half of a star), then exchange with their neighbor and draw the other half of their neighbor's tree.

## Effective Lessons Create an Environment Conducive to Learning

Based on the observations in this study, a classroom culture conducive to learning is one that is both rigorous and respectful. Nearly half of mathematics lessons nationally received high ratings for having a climate of respect for students' ideas, questions and contributions. Ratings for rigor were much lower, with only 14 percent of mathematics lessons nationally judged to have a climate of intellectual rigor, including constructive criticism and the challenging of ideas. Table 1 shows a cross tabulation of the two variables; note that only 14 percent of mathematics lessons nationally are strong in both respect and rigor (with all of the lessons that were judged high in rigor also judged to be respectful to students), and 26 percent of lessons judged low in both areas.

Nineteen percent of mathematics lessons were categorized as respectful but lacking in rigor. Inside the Classroom observers used phrases like "pleasant, but not challenging" to describe such lessons. The following example is typical:

An observer described a 4th grade mathematics lesson where "the teacher was very enthusiastic, and encouraged her students to be the same. She gave lots of verbal encouragement to students as they worked... The culture suffered from a lack of focus on the intellectual content, however. The teacher appeared more intent on the students having a positive experience with mathematics through completing the task than really engaging with the concepts. The classroom was a welcoming environment for students, and there was a focus on 'learning,' but the level of learning expected seemed rather low."

Table 1
CROSS TABULATION OF CLIMATE OF RESPECT AND INTELLECTUAL RIGOR

|  |  | Percent of Lessons |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Intellectual Rigor, Constructive Criticism, and Challenging of Ideas Are Evident |  |  |
|  |  | Low | Medium | High |
| Climate of Respect for Students' Ideas, Questions, and Contributions | Low | 26 | 2 | 0 |
|  | Medium | 23 | 4 | 0 |
|  | High | 19 | 11 | 14 |

## Effective Lessons Help Students Make Sense of the Mathematics Content

Focusing on important mathematics content; engaging students; and having an appropriate, accessible learning environment set the stage for learning, but they do not guarantee it. It is up to the teacher to help students develop understanding of the mathematics they are studying. The teacher's effectiveness in asking questions, providing explanations, and otherwise helping to push student thinking forward as the lesson unfolds often appeared to determine students' opportunity to learn.

Researchers observed some extremely skillful questioning, where the teacher was able to use questions to assess where students were in their understanding, and to get them to think more deeply about the mathematics content. There were many more instances where the teacher asked a series of low level questions, with the focus primarily on the correct answer, rather than on understanding. Questioning was among the weakest elements of mathematics instruction, with only 15 percent of lessons nationally incorporating questioning that seemed likely to move student understanding forward. Lessons that were otherwise well-designed and well-implemented often fell down in this area.

When effective questioning was observed, the teachers used questions both to find out what students already knew and to provoke deeper thinking in helping them make sense of mathematics ideas. For example:

The observer reported that an 8 th grade mathematics class was a very nice illustration of an interactive lecture, where the instructor asked for examples and justifications from the students as a means of assessing their understanding. "For example, when generating examples of tessellations around the room one student proposed the border of the bulletin board that was made of circles.

Student: 'How about the border?'
Students: 'No... that won't work.' (several students talk at once and reject this contribution) Teacher: 'Why won't it work? Can the circle ever work?'

The discussion became focused on why the circle did not create a pattern that fit the definition of a tessellation. While the student who suggested the circle had been focusing more on patterns, the disagreement helped him redirect his analysis back to the definition of tessellations presented earlier."

More often observers noted that the teachers moved quickly through the lessons, without checking to make sure that the students were "getting it." As soon as one or two of the most verbal students indicated some level of understanding, the teacher went on, leaving other students' understanding uncertain.

By far, the most prevalent pattern in mathematics lessons was one of low-level "fill-in-the-blank" questions, asked in rapid-fire, staccato fashion, with an emphasis on getting the right answer and moving on, rather than helping the students make sense of the mathematics concepts. The following example illustrates this pattern as it played out in a high school mathematics lesson:

The observer reported that questions asked of students tended to be low-level and leading. The students were given the following system of equations:

$$
\begin{aligned}
& 6 x+5 y=-2 \\
& 5 x-4 y=31
\end{aligned}
$$

The following "discussion" occurred:
Teacher: "What do we want?"
Students: " $x$ and $y$ "

Teacher: "What do I need to do to get $x$ and $y$ ?"

Students: "Get rid of the first matrix."
Teacher: "What do I need to do to get rid of it?" Students: "Multiply by the inverse."

Said the observer, "discussions during this lesson were much more about identifying steps to do than about justifying the steps by considering conceptual underpinnings."

Interestingly, observers reported that some teachers asked good questions, but were so intent on getting the right answer that they supplied the answers themselves, in effect short-circuiting student thinking. The following example is typical:

Said the observer of a high school calculus lesson, "When the teacher put a problem on the board and asked students to solve it, which they did in silence at their seats, the teacher often solved the problem on the board as they were working through the problem, or else waited about one minute and asked a student for input. On one problem the teacher asked for a student's input as to the next step toward the solution, but then disregarded the student's suggestion (which was one correct way to proceed) and went with his own strategy, saying: 'Yes, we can do that. But let's....' So the teacher solved the problem his way, even though he had asked for a student's strategy."

Teacher questioning is one way, but not the only way to help students understand the mathematics. The important consideration is that lessons engage students in doing the intellectual work, with the teacher helping to ensure that they are in fact making sense of the key concepts being addressed. The following example is illustrative of lessons that included appropriate "sense-making":

The purpose of a 2 nd grade mathematics lesson was to allow students to demonstrate understanding of place value-ones, tens, and hundreds, and to practice with thousands place. The lesson emphasized numbers containing a zero, since this was something students found difficult. The lesson began with students working in groups of four. Each student in the group had a group member number. The teacher would give a digit for all the \#1s to write on their marker board, then a digit for all the \#2s, \#3s, and \#4s. The teacher would then give a number using all the digits and the students in the group would line up with their digits in the proper order to build the number. Students would look at each group's response
and indicate their agreement with thumbs up or down.

The teacher encouraged students to question each other if there was an answer they didn't understand or didn't agree with. If a group did not represent the number correctly, the teacher would probe with questions to see if they could identify their error. She also asked students to respond to discrepancies that appeared among the groups' solutions. The class did several examples like this and then the students worked individually on more examples. After that the teacher had the students put their marker boards away, then wrapped up the lesson by asking, "What did we learn in math today?" Students gave responses like, "If there's a zero, you have to count it" after which the teacher asked for more explanation. She emphasized, "When we write numbers, the digits have to be in the right spot. Remember that the zeros are important, too. This will get easier as we go along."

Although researchers observed some lessons where students were helped to make sense of the mathematics content as the lesson progressed and/or at its conclusion, most lessons lacked adequate "sense-making;" only 18 percent of lessons received high ratings in this area. Many teachers seemed to assume that the students would be able on their own to distinguish the big ideas from the supporting details in their lectures, and to understand the mathematics ideas underlying their explorations. The following lesson descriptions illustrate inadequate sense-making in mathematics lessons.

Students in a 6th grade mathematics class were asked to complete a practice worksheet, which involved their measuring nine angles and identifying each as acute, right, obtuse, or straight. Said the observer, "Instead of students being encouraged to make sense of mathematics, students were to follow directions. Students were not asked to explain their thinking either during the whole-class discussion or on the assessment. Mathematics was presented as a set of rules and procedures."

The student in this Algebra class who put the equation $6 x+$ $7=-14 y$ into standard form on the board explained that she first subtracted $6 x$ from both sides getting $7=-14 y-6 x$, which in standard form is: $-6 x-14 y=7$. Some students seemed confused, and asked the teacher if that was right. The teacher said it was, then solved it a different way, by first moving the $y$-term, getting the answer $6 x+14 y=-7$. As she began solving it this way, some students seemed fixed on first moving the $6 x$-they didn't understand that either way was correct. The teacher concluded "So you can have two different answers."

The observer noted that the teacher never mentioned that these two answers are mathematically equivalent.

In summary, while the aim of instruction in all cases needs to be understanding, based on the Inside the Classroom observations, there appear to be multiple approaches for achieving this goal. Observers saw lessons that were welldesigned and well-implemented using lectures, manipulatives, or paper and pencil tasks to help develop student understanding of important mathematics concepts. Observers saw other lessons using each of these strategies that seemed unlikely to lead to student conceptual understanding. Factors that seem more instrumental than instructional strategies in promoting student opportunity for learning include the extent to which lessons engage students with important mathematics concepts; create an environment that is both respectful and rigorous; use questioning effectively; and help students make sense of the mathematics concepts being addressed.

## Discussion and Recommendations

Teaching for understanding, most mathematics educators would agree, requires teachers who have a command of the important mathematics concepts being addressed, and who have the requisite knowledge and skills to help students develop their understanding of these mathematics concepts. Rather than focusing primary attention on which instructional strategies teachers use, student understanding would more likely be enhanced by ensuring first that instruction, regardless of instructional strategy, is purposeful; accessible; engaging to students; both respectful and rigorous; and maintains a clear and consistent focus on student learning of important mathematics concepts.

To the extent that teachers teach as they have been taught, they must experience teaching for understanding if they can be reasonably expected to teach for understanding. Similar logic certainly underlies calls for undergraduate mathematics courses to use cooperative learning and other "reform-oriented" strategies, but the findings from the Inside the Classroom study suggest that the key to instruction aimed at meaningful learning is not the particular strategies that are used, but rather engaging prospective teachers as learners with instruction that develops their conceptual understanding of mathematics.

Any instructional strategy can be implemented well, or implemented poorly. Working on open-ended problems that never lead to conceptual understanding is no more
beneficial to learners than is sitting through inaccessible, uninteresting lectures. Of course, lectures do not have to be boring demonstrations of the use of algorithms or derivations of formulas. A well-conceived and well-delivered lecture can provide learners thoughtful explorations of important ideas. In theory, at least, a good lecture can engage learners in mathematical investigation by setting up an accessible yet challenging problem situation; identifying important questions that have been asked about the situation; discussing how they have been investigated, and which methods turned out to be useful pathways, and which were dead ends; and concluding with an explanation of how we now know what we know, as well as what we still do not know. If prospective teachers were to experience a variety of well-implemented instructional strategies in their pursuit of mathematics content understanding, and if their mathematics education courses attended explicitly to what constitutes high quality use of each strategy, they would likely be better prepared to implement high quality instruction in the mathematics lessons they will teach.

Even with excellent initial preparation, teachers need ongoing opportunities for continuing education, just as all other professionals do. Providers of teacher professional development can help teachers explore and enhance their vision of, and understandings about, effective mathematics instruction; and they can help teachers consider how to use their enhanced understanding to improve the design and implementation of their classroom lessons.

In addition, with the advantage of knowing which grades the in-service teachers are teaching, and often which student instructional materials are being used, professional development can be designed to provide very targeted assistance for teachers-clearly identifying the key concepts being developed in particular activities; sharing the research on student thinking in the specific content area; suggesting questions that teachers can use to diagnose student thinking and monitor student understanding; and outlining the key points that should be emphasized to help students make sense of the mathematics concepts. Teacher professional development activities, in turn, need to reflect the elements of high quality instruction with clear, explicit objectives; a supportive but challenging learning environment; and means to ensure that teachers are developing understanding. Modeling teaching for understanding and making its characteristic elements explicit in professional development will provide teachers additional opportunities to learn how to improve their own practice.

Professional development for mathematics teachers often focuses on, and advocates, particular instructional strategies, such as the use of manipulatives or cooperative learning groups. Instructional strategies, however, did not appear to determine the quality of the mathematics lessons observed in this study. We recommend, consequently, that professional development for mathematics teachers focus on
aspects of effective instruction that cut across instructional strategies: learning goals that are both important and developmentally appropriate; examples and activities that capture students' attention and interest; an intellectual climate that both nurtures and challenges students; and, critically important, tasks, questioning strategies, and explanations that explicitly help students make sense of the concepts they are studying.

## References

Burstein, L., McDonnell, L., Van Winkle, J., Ormseth, T., Mirocha, J., \& Guiton, G. (1995). Validating national curriculum indicators. Santa Monica, CA: RAND.

Kesidou, S. \& Roseman, J.E. (2002). How well do middle school science programs measure up? Findings from Project 2061's Curriculum Review. Journal of Research in Science Teaching, 39(6), 522-549.

Mayer, D.P. (1999). Measuring instructional practice: Can policymakers trust survey data? Educational Evaluation and Policy Analysis, 21(1), 29-45.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1995). Assessment standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Porter, A.C., Kirst, M.W., Osthoff, E.J., Smithson, J.S., \& Schneider, S.A. (1993). Reform up close: An analysis of high school mathematics and science classrooms (Final Report to the National Science Foundation on Grant No. SPA-8953446 to the Consortium for Policy Research in Education). Madison, WI: University of Wisconsin-Madison, Wisconsin Center for Education Research.

Spillane, J.P. \& Zeuli , J.S. (1999). Reform and teaching: Exploring patterns of practice in the context of national and state mathematics reforms. Educational Evaluation and Policy Analysis. 21(1), p. 1-27.

Stake, R.E. \& Easley, J.A. (1978). Case studies in science education: Volume I and Volume II. Urbana, IL: Center for Instructional Research and Curriculum Evaluation, University of Illinois at Urbana-Champaign.

Weiss, I.R., Banilower, E.R., McMahon, K.C. \& Smith, P.S. (2001). Report of the 2000 national survey of science and mathematics education. Chapel Hill, NC: Horizon Research, Inc.

Weiss, I.R., Pasley, J.D., Smith, P.S., Banilower, E.R., \& Heck, D.J. (2003). Looking inside the classroom: A study of k-12 mathematics and science education in the United States. Chapel Hill, NC: Horizon Research, Inc.

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