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The ***NCSM Journal of Mathematics Education Leadership*** is published at least twice yearly, in the spring and fall.

The editors of the ***NCSM Journal of Mathematics Education Leadership*** are interested in manuscripts that address concerns of leadership in mathematics rather than those of content or delivery. Editors are interested in publishing articles from a broad spectrum of formal and informal leaders who practice at local, regional, national, and international levels. Categories for submittal include:

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Submittal of items for this publication must include three hard copies and a disk copy in text format. Do not put any author identification in the body of the item being submitted, but do include author information as you would like to see it in the ***Journal***. Items submitted for publication will be reviewed by two members of the NCSM Review Panel and one editor with comments and suggested revisions sent back to the author at least six weeks before publication. Final copy must be agreed to at least three weeks before publication.

On the cover:

This issue's cover was created by Bonnie Katz, reflecting conversations with the editor about the value of origami activities in fostering geometric thinking, and inspired by the work displayed at ttp://www.geocities.com/tp_kong/index.html.

National Council of Supervisors of Mathematics (NCSM)

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Comments from the Editor

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Advocacy for the importance of high quality, challenging mathematics in schools has always been a major activity of mathematics education leaders throughout North America. And going back at least as far as the launching of Sputnik in 1957, news about our falling behind in mathematics or science usually has served to open the minds of citizens and policy makers to that advocacy.

Lately, I have been wondering if public support is still reliable, even when the news is bad. Recently, the report of the 2003 Program for International Student Assessment (PISA) revealed the mediocre performance in mathematics of U.S. fifteen year-olds. In an article (McNeil (2004)) on the PISA ranking, one of the continent's premier newspapers held up the possibility that it just does not matter:

> In all but the most arcane specialties (like teaching math), the need for math has atrophied. Electronic scales can price 4.15 pounds of chicken at $3.79 a pound faster than any butcher. Artillerymen in Iraq don't use slide rules as their counterparts on Iwo Jima did. Cars announce how many miles each gallon gets. Some restaurant bills calculate suggested tips of 15, 18 or 20 percent. Architects and accountants now have spreadsheets for everything from wind stress to foreign tax shelters. The new math is plug-and-play. (p.3)

Shortly thereafter, the author asked: "So is it necessary that the average high-schooler spend years nailed to the axes of x and y?"(p.3). Although the question was followed by quotes from experts arguing "no" as well as experts arguing "yes," it was easy to discern a definite lean in opinion behind the article, perhaps even a bellwether of popular opinion. Let's face it: quite possibly, mathematics education leaders' advocacy will confront — more than ever before — the familiar "But when will I ever use it?" objection to school mathematics.

One of this JMEL issue's articles (by Lundin et al) raises possibilities that spending years being "nailed to those axes" may have benefits beyond transparent, job-related utility. The article raises questions, and invites further interaction, concerning the correlation between taking challenging mathematics courses in high school and achieving higher GPA in first year of college. One can argue that, even if this correlation holds up in larger studies, it merely puts a different twist on the utility test, and so risks clouding arguments for the study of mathematics tied to its discipline, beauty, and history. Perhaps. However, in this era, a mathematics leader's advocacy can use all the tools that can be mustered.

The other three articles in this issue all reflect the current burst of interest in the design and leadership of teacher professional development that extends teacher learning over a period of time, and tries to engage teachers in focused and structured learning experiences. This burgeoning and exciting facet of the world of mathematics education leadership raises questions about what is learned by teachers and what effect the learning has in the lives of teachers and students. (See, for example, Hill & Ball (2004)) I hope that potential writers will be inspired by the articles in this issue and share in their own writing ideas that accelerate progress in assessing teacher learning through quality professional development. Our profession badly needs those ideas.

This journal, and the other NCSM publications, can and should serve you as vehicles for such sharing and networking. Anyone who has attended the NCSM Annual Conference can attest to the electricity in the rooms as members interact, identify common leadership challenges, and push each other for ideas that can impact the challenges. The Council's several publications can serve as vehicles for such energetic networking. I hope that you will work with us editors to make it happen.

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Hill, Heather C. & Ball, Deborah Loewenberg (2004) Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education,* 35, 330-351.

McNeil, Donald G., Jr. (2004) The last time you used algebra was…. *The New York Times*, The Week in Review Section, December 12, 2004.

Using Scenario Tasks to Elicit Teachers' Algebraic Thinking: *A Recommendation for Professional Development*

Frances R. Curcio, Queens College of the City University of New York
Daniel Scher, Best Practices in Education
Sharon L. Weinberg, New York University

One of the greatest challenges for providers of professional development in mathematics is to determine the degree to which professional development experiences help teachers to improve their mathematical content knowledge and the pedagogical strategies they employ. Collecting such information may not only serve to evaluate the effectiveness of the professional development opportunities offered, but it may also inform the design and content of subsequent professional development sessions. Traditional paper-and-pencil mathematics content tests may seem to be efficient in assessing content knowledge, but they are limited in that they create a degree of anxiety among teachers and are viewed to be threatening. Furthermore, such measures test mathematical content knowledge often at the exclusion of pedagogical content knowledge. Other approaches may include personal interviews or classroom observations, but these, too, may be limited for similar reasons.

One way to elicit mathematical understandings and pedagogical strategies is to present teachers with a realistic classroom scenario in which student responses are plausible but problematic, and ask teachers how they would respond with respect to the correctness of students' ideas (i.e., elicit mathematical content knowledge) and how they would approach or resolve the conflicts or dilemmas (i.e., elicit pedagogical strategies).

In the course of conducting classroom observations for a National Science Foundation-funded Local Systemic Change Project[1], we began to collect "teachable moments" — capsule instances where an unexpected student response paved the way for a significant mathematical insight if further pursued. For a variety of reasons (e.g., lack of time, lack of confidence to follow a student's lead, lack of content knowledge, or lack of interest in the student's response), some teachers chose not to address the issues raised by these unanticipated responses. With a goal toward analyzing middle school teachers' algebraic thinking and the pedagogical strategies they employ, and to understand more fully why some teachers chose not to pursue their students' reasoning, we developed scenarios (Scher, Curcio, & Weinberg, 2004) based on these actual classroom observations as well as from *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), the adopted curriculum.

Because the development of algebraic reasoning is a critical component of the middle school mathematics curriculum, in this paper we present three algebra-related scenario tasks that may be useful in eliciting teachers' thinking related to algebra as well as their instructional strategies. As noted earlier, these scenarios are based on actual classroom situations. Accordingly, as providers of professional development elicit and analyze teachers' responses to the

Horizon Research is gratefully acknowledged for funding the design of the scenario tasks and the analyses of the responses from teachers.

A previous version of this paper was presented at the 36th Annual Conference of the National Council of Supervisors of Mathematics, Philadelphia, PA, on Monday, 19 April 2004.

[1] National Science Foundation Grant No. 9731424. The opinions expressed herein are those of the authors and do not necessarily reflect the view of the NSF.

scenarios, they may use the information to structure and design future professional development sessions.

THREE SCENARIO TASKS

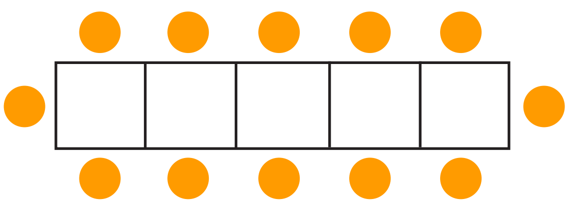
Scenario Task A: Seating Capacity

This task, observed in a grade 7 classroom, is a variation of a problem in *Covering and Surrounding* (Lappan et al. 1998b, p. 32).

SCENARIO TASK A: SEATING CAPACITY

Students in your 7th-grade algebra class are working in groups to answer the following question:

A square table can seat four people, one on each side. When 5 square tables are placed side by side, as shown below, 12 people can sit around them—5 on each side and 2 on the ends. How many people can sit around n square tables when they are placed side by side?



One group says: "n people can sit on each of the two long sides, and two people sit on the ends. So the total number of people is 2n +2."

Another group says: "If there's just one table, then 4 people can sit. Each time we add a table, that increases the number of people by 2. Thus the total number of people is 4 + 2n."

How would you help the groups in analyzing these two responses?

In particular, where is the error in the above work and how can it be corrected?

It should be noted that when the class came together to review the results of the small group work, the teacher was faced with two seemingly plausible lines of reasoning. Because one led to an incorrect algebraic answer, $4 + 2n$, the teacher dismissed it without considering the merits of its underlying reasoning, and how it could be amended. Could other teachers do better with this "teachable moment?" Scenario Task A is designed to find out.

Scenario Task B: Perimeter versus Area

In this lesson, we observed a grade 6 discussion of the relationship between the perimeter and area of a square. Scenario Task B is based on the response of one student who noticed an unexpected numerical pattern in the data. Is this pattern a mere curiosity or can it be related to algebraic thinking? Finding and articulating the algebraic connection is the object of this task.

SCENARIO TASK B: PERIMETER VS. AREA

Students in your 6th-grade algebra class are creating a table that lists the perimeter and area for squares of varying sidelengths:

Sidelength of square	Perimeter	Area
1	4	1
2	8	4
3	12	9
4	16	16
5	20	25
6	24	36
7	28	49

A student notices an interesting pattern in the table that she shares with the class:

A square with side length 5 and perimeter 20 has area 5 x 1 + 20 = 25.

A square with side length 6 and perimeter 24 has area 6 x 2 + 24 = 36.

Extending this pattern across the table, she finds:

Sidelength of square	Perimeter	Area
1 x –3	+ 4	= 1
2 x –2	+ 8	= 4
3 x –1	+ 12	= 9
4 x 0	+ 16	= 16
5 x 1	+ 20	= 25
6 x 2	+ 24	= 36
7 x 3	+ 28	= 49

How would you proceed with your class from here? Explain why this numerical relationship exists.

Scenario Task C: Binomial Expansion

In this lesson, we observed a discussion on binomial expansion. Although this topic was not part of the adopted NSF curriculum, the teacher of this advanced grade 8 class had chosen to include it in her course.

SCENARIO TASK C: BINOMIAL EXPANSION

Using algebra, you show your 8th-grade algebra class why $(a + b)^2 = a^2 + 2ab + b^2$. *On a test, however, many students write:* $(a + b)^2 = a^2 + b^2$. *How might you help your students to understand this identity?*

Please write a response that is detailed enough to allow another teacher to follow your ideas and use them as a basis for a lesson in his or her own class.

Categorizing Responses to the Three Tasks

We administered the three scenario tasks to approximately fifty mathematics teachers and mathematics coaches in a local New York City community school district, and graduate students in secondary mathematics education at a local university. We conducted this exploratory project to examine the degree to which responses varied and how the responses might be used to reveal teachers' algebraic thinking. We found that not only did responses to each scenario vary considerably from one another, but that they had distinguishing characteristics that revealed teachers' approaches that emphasized numerical examples (Response Type 1), using a table (Response Type 2), developing a generalization (Response Type 3), or making connections or extending the solution (Response Type 4).

All responses per scenario were read independently by each of the three authors of this article and classified into one of the four categories (i.e., Response Types 1, 2, 3, or 4). In creating these categories, we were guided by the belief that regardless of the quality of the curriculum materials or the type of reform effort implemented, teachers with an inadequate understanding of mathematics or a misunderstanding of mathematical concepts will compromise student learning and the goals of the reform. For example, innovative middle school curriculum materials highlight problem-solving strategies such as making a table when studying algebraic relationships (Lappan et al. 1998a; Romberg et al. 1999). To be effective, these approaches require teachers to understand the distinction between "making a table," as an end in and of itself, and constructing a proof of an algebraic relationship. Knowing how and when to utilize a table to demonstrate an algorithm is important, but one must be vigilant to avoid misleading middle school learners to believe that the construction of tables on a relatively small number of cases generalizes to all cases and, therefore, substitutes for "proof."

Response Type 1 is "categorized by misconceptions, limited understanding, or reliance upon concrete examples." Response Type 2 is "characterized as communicating a basic understanding of algebraic concept(s)." Response Type 3 is characterized as a movement toward generalization. Response Type 4 reveals "additional insight and alternative solutions" (Scher, Curcio, & Weinberg, 2004, p. 2).

The four categories obtained for each of the three scenarios are described in Tables 1, 2, and 3.

DISCUSSION

Scenario Task A: Seating Capacity

Response Type 1 in Table 1 considers two aspects of the seating problem: the algebra and its underlying reasoning. The teacher observes that the group who answered "$4 + 2n$" reasoned correctly (i.e., there are four people at the first table and the number of people increases by two for each additional table) but faltered when translating their counting strategy into an equivalent algebraic representation. Creating a table of n values, as suggested, has the potential to uncover the nature of the algebraic mistake (specifically, a column of $4 + 2n$ values contains the same numbers as $2n + 2$, shifted up one row). Yet, the response makes no attempt to unravel the algebraic inconsistency.

Both Response Type 2 and Response Type 3 reflect an understanding that the expression $4 + 2n$ overcounts the number of tables by one. Response Type 2 substitutes $n + 1$ in place of n to convert the correct $2n + 2$ answer into the incorrect $4 + 2n$. Response Type 3 operates in reverse — it replaces n by $n - 1$ to convert $4 + 2n$ into the correct $2n + 2$. While both methods have mathematical merit, Response Type 3 seems, on a pedagogical response type, more likely to aid the faltering group.

Response Type 4, in addition to explaining the algebra, offers an entirely different line of reasoning that leads to the same algebraic answer. Note that nowhere in our

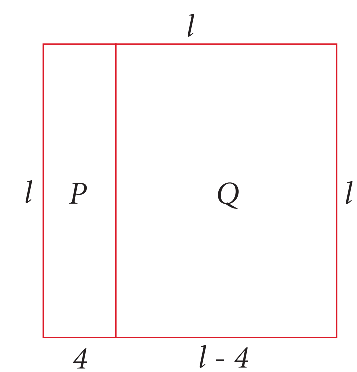
TABLE 1:

Response Types for Seating Capacity Scenario Task

RESPONSE TYPE 1	RESPONSE TYPE 2	RESPONSE TYPE 3	RESPONSE TYPE 4
It is helpful to create a table and see what type of patterns appear. You may want to "guess" at an answer beforehand (such as $2n+2$ or $4+2n$) and see if it makes sense based upon the results in the table. Does $2n + 2$ make sense? for $n = 1$, $2(1) + 2 = 4$ yes for $n = 2$, $2(2) + 2 = 6$ yes for $n = 3$, $2(3) + 2 = 8$ yes for $n = 4$, $2(4) + 2 = 10$ yes Seems to make sense. Does $4 + 2n$ make sense? For $n = 1$, $4 + 2(1) = 6$. Already doesn't work. This is a good way to use visuals and an actual simulation (i.e., draw n tables and actually count the number of people) in order to illustrate a property. The second group's reasoning is correct (i.e., there are 4 people at one table and it increases by 2 every time a table is added), but the conclusion doesn't reflect this reasoning. Going back to the table and checking against concrete numbers may help clear this up.	I would have them set up a table so it becomes easier to see a pattern and arrive at a formula. # of tables | total # of people that can be seated 1 | 4 2 | 6 3 | 8 4 | 10 n | $2n + 2$ To address the 2n + 4, show that when the number of tables is 1, 2n + 4 gives 6, and only 4 people can be seated at 1 table. The group arrived at this number by finding the next number after n on the table. E.g., # of tables | # of people seated n | $2n + 2$ $n + 1$ | $2(n + 1) + 2 = 2n + 4$	First look at how the answers are similar (both have $2n$ + something) and how they differ (the number added is 2 vs. 4). Also have groups describe what each piece of their formula stands for. The $2n + 2$ group was better at this: 2 —> number of long sides n —> number of people sitting at long side +2 —> 2 people sit on the end. If the 2nd group sees this done by the first group, they might discover their own error, which was that n represented additional tables. The challenge would then be how can the second group's formula be adapted if we go back to the original problem, which states that n is the number of tables, and how do we show it is equivalent to the first group's? $4 + 2(n - 1) = 2n + 2$ n = # of tables $n - 1$ = # of additional tables	This answer contained all elements of a Response Type 3 response as well as the following: The first group is making the right point by the way they analyze the problem. They can try to solve it another way like: If there are n tables with 4 people seated on each one, how many people will lose their seats if these tables are placed side by side? The class can then see whether the new answer matches their original answer. $4n - 2(n - 1) = 2n + 2$

TABLE 2:

Response Types for Perimeter versus Area Scenario Task

RESPONSE TYPE 1	RESPONSE TYPE 2	RESPONSE TYPE 3	RESPONSE TYPE 4
The pattern here is that the sum of the side length of the square and the perimeter equals the area. Let's say A = area P = perimeter S = side length of square The pattern proves that $A = P + S$. 1 = 4 + (1 x –3) 1 = 4 + –3 1 = 1 4 = 8 + (2 x –2) 4 = 8 + –4 4 = 4	I would explain why that relationship exists and explain that the student is not doing anything different than the rest of the class, only arranging the numbers differently. That is, generally area is found by squaring the side length. Here, we are doing that in a different way. Example: side length = 5 x 1 perimeter = 20 (= 5 x 4) area = 25 Generally, $5^2 = 25$. Here: (5 x 1) + (20) = (5 x 1) + (5 x 4) Factor out a 5 to get: $5(1 + 4) = 5 \times 5 = 5^2$, and actually the same as we are used to.	I would have students determine, given side s, the area and perimeter of a square algebraically. They would find that the area of a square is s^2 and the perimeter is $4s$ (if they do not previously know this). Then, $sx + 4s = s^2$ What must x be? Well, x must have an s so that the left side will have an s^2 term. Does $s \cdot s + 4s = s^2 + 4s = s^2$? No. So given that $s \cdot \text{something} + 4s = s^2$, and there must be at least an s, what can I multiply by s to get rid of $4s$? The answer is –4. So $s(s - 4) + 4s = s^2$ by the distributive property.	This answer contained all elements of a Response Type 3 response as well as the following: Let l = side length of the square. We can look at it in a geometrical way:  The area of P (= $4l$) is equal to the perimeter of the square. Good observation! The relationship does exist.

problem statement did we require an alternative strategy. But the inclusion of one in Response Type 4 demonstrates an algebraic flexibility.

Scenario Task B: Perimeter versus Area

Response Type 1 in Table 2 incorrectly states that area equals perimeter plus side length and claims that numerical data alone "proves" the algebraic relationship. Response Type 2 also focuses on numbers, but with a difference: here, the explanation skillfully manipulates the term (5 x 1) + 20 to show its equivalence to 52. The work is grounded in one specific example, but the manipulations show an understanding of numbers extending beyond calculation to more purposeful pattern finding. Only for Response Type 3 does the explanation deliver a generalized algebraic proof.

Response Type 4 includes a geometric interpretation of the underlying algebra. The work is notable, too, for including the short message, "Good observation!" While Response Type 2 states that the student's discovery is not "…anything different than the rest of the class," Response Type 4 displays a mathematical appreciation of the insight's uniqueness.

Scenario Task C: Binomial Expansion

Response Type 1 in Table 3 begins promisingly by proposing numerical substitution as a way to demonstrate the inequality of $(a + b)^2$ and $a^2 + b^2$. Nearly every response to this item, regardless of response type, included this concrete approach. It remained to establish the correct identity.

Response Type 1 offers the "FOIL" mnemonic, a rule-based method unlikely to promote conceptual understanding. Response Type 2 relates the expansion of $(a + b)^2$ to the process of multiplying two-digit numbers—a concrete link. It is unclear, however, whether teachers' knowledge of multiplication itself rises above an algorithmic understanding.

Response Type 3 stands apart from the previous answers by taking note of the context provided in our classroom scenario. Since the scenario states that an algebraic approach to the binomial expansion had not proven effective, the respondent gives a geometric representation of $(a + b)^2$ illustrating clearly the origins of the "$2ab$" term.

Most current algebra texts feature Response Type 3 ideas as a way to visualize $(a + b)^2$ (Bellman, Bragg, Chapin, Gardella, Hall, Handlin, & Manfre, 2001; Lappan et al., 1998c; McConnell, Brown, Usiskin, Senk, Widerski, Anderson, Eddins, Feldman, Flanders, Hackworth, Hirschhorn, Polonsky, Sachs, & Woodward, 1998). By contrast, Response Type 4 includes a mathematical connection that is, to the best of our knowledge, original. If $(a + b)^2$ did equal $a^2 + b^2$, then a, b, and $a + b$ may be viewed as the bases and hypotenuse of a right triangle. Yet in any triangle, the sum of two sides is always greater than the third. The equality cannot hold.

Recommendations for Professional Development

In all three scenarios, Response Type 1 relies almost exclusively on numerical data. The responses step back from algebra, using numerical substitution as a spot-check of a conjecture's viability. Response Type 1 in Table 1, for example, concludes from an inspection of tabular data that the algebraic statement $2n + 2$ "seems to make sense." Yet nowhere in the response does the teacher move beyond the suggestiveness of data to the conclusiveness of algebraic reasoning.

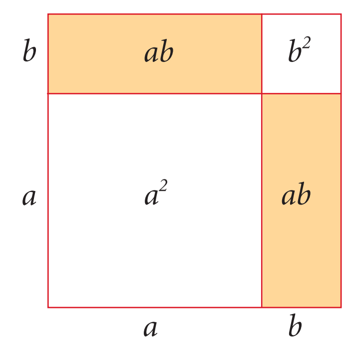
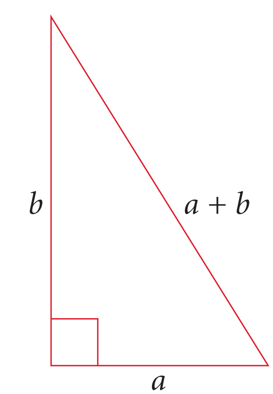
It may be possible that Response Type 1 teachers were thinking about their students when they responded to the task. As such, the teachers' reliance on numerical data may say more about the ability of their students than the teacher's own knowledge of algebra. It is suggested that teachers indicate and discuss the types of classes they teach and describe their students prior to completing the tasks, or during the completion of the tasks.

Many current algebra curricula feature tabular data (Lappan et al.1998; McConnell et al. 1998), but only as a first step towards greater generality. Response Type 1 teachers favor this approach because of its concreteness, but remain uncomfortable with the transition to algebraic representation. These teachers need help moving from concrete examples to generalizations—experiencing the "power" of algebra. As a start, teachers in a professional development workshop could answer any of our three scenario tasks and then examine the corresponding table of responses to discuss what seems to differentiate Response Type 1 from, say, Response Type 2, and then determine where each of their current thinking fits in relation to the given categories.

Describing the qualities of Response Types 3 and 4 requires some care. Certainly, these answers display a greater facility with algebraic symbolism. Facile symbol manipulation alone, though, does not guarantee algebraic

TABLE 3:

Response Types for Binomial Expansion Scenario Task

RESPONSE TYPE 1	RESPONSE TYPE 2	RESPONSE TYPE 3	RESPONSE TYPE 4
It is often useful to substitute numbers in place of letters to "check" your final answer. Explain the FOIL method of multiplying **F**irst numbers, **O**utside numbers, **I**nside numbers, and **L**ast numbers to get $a^2 + 2ab + b^2$. If some students still are not convinced that distributing the "squared" doesn't work, substitute in actual numbers: $(3 + 4)^2 \neq (32 + 42)$. Why? We can go through the FOIL process or simply add 3 + 4 to get 72 = 49 (which is concrete…kids won't argue that). Now distribute the square to get 32 + 42 = 9 + 16 = 25. $25 \neq 49$. Kids will clearly understand an actual numerical example even if they don't automatically think "FOIL."	The multiplication of two numbers involves more than a multiplication of the first terms and the second terms. It involves multiplying each digit in one number by all the digits in the other number. For example, in 45 x 10, we multiply the 0 by 5 and by the 4. A zero goes in as a placeholder, and then we multiply the 1 by the 5 and the 4. Doing this for $(a + b)^2$ we get: $a + b$ $*\ a + b$ $ab + b^2$ $a^2 + ab \quad 0$ $a^2 + 2ab + b^2$ If this gives students trouble, I'd like to break up my first example and then follow the same procedure. 45 x 10 = 40 + 5 * 5 + 5 200 + 25 200 + 25 + 0 200+ 225 + 25 = 450	Draw the following:  Show the side lengths as $a + b$. Ask what the area of the square would be (if necessary, ask how to find the area of a square before asking the area of THIS square). With students, fill in the dimensions and areas of the four pieces, possibly using different colors for the different-sized pieces. Add the pieces together to get the area —> $a^2 + 2ab + b^2$. If students are still unsure, or prefer working with numbers, have students choose numbers for a and b and test $(a + b)^2 = a^2 + b^2$. Perhaps challenge the class to find an *a* and *b* that will make this true. After a couple of tries, they may say it's impossible.	This answer contained all elements of a Response Type 3 response as well as the following: If $(a + b)^2 = a^2 + b^2$, then *a*, *b*, and $a + b$ are 3 sides of a right triangle:  Of course not! (Why not?)

maturity. The responses point to other, more subtle qualities that help to describe accomplished algebraic thinking. These are as follows:

1. Recognizing student work (at an arithmetic level) that makes unexpected connections to algebra;
2. Spotting correct reasoning among faulty algebra; and
3. Uncovering connections between algebra and geometry.

This list suggests those areas of professional development that may benefit teachers who are comfortable with algebraic manipulation, but are not facile in connecting their knowledge to the more roughly-hewn reasoning of their students. These teachers need practice in recognizing the germ of a good algebraic idea in approaches that are neither typical nor entirely accurate. Such teachers could also benefit from studying geometric arguments that illuminate the meaning behind algebraic statements. Studying the Response Types 3 and 4 in our tables is a first step in that direction.

Closing Comments

As this exploratory study makes clear, an approach that employs the use of scenario tasks based on actual classroom practice has the potential for eliciting a wide range of responses that may be systematically linked to the ways in which teachers think about and formulate their own approaches to presenting mathematical content in the classroom. By incorporating such tasks in professional development, and encouraging teachers in such sessions to reflect upon the type of responses they are most likely to produce in the classroom given a particular scenario task, we are providing them with an opportunity to evaluate their responses in comparison to others and to become more flexible in their mathematical thinking in a non-threatening and supportive setting. Furthermore, because scenario tasks are content specific (e.g., algebra-based), they are best suited for professional development sessions structured by mathematical topic and grade level.

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Classroom Assessment in Middle Grades and High School

George W. Bright, University of North Carolina at Greensboro
Jeane M. Joyner, Meredith College

The "bottom line" for mathematics instruction is to help students learn more. Professional development that helps teachers understand both mathematics and students' thinking more deeply is one strategy for ultimately improving students' learning. This is supported by a clear argument that is consistent with arguments made about professional development at the elementary level (Carpenter, & Fennema, 1999; Carpenter, Franke, & Levi, 2003; Fosnot & Dolk, 2001a, 2001b, 2002, Ma, 1999; Schifter, Bastable, & Russell, 1999; Seago, Mumme, & Branca, 2004).

- First, more learning is associated with better instruction.
- Second, better instruction happens when teachers align instruction with the needs of students.
- Third, aligning instruction is more likely to happen when teachers have clear understanding of what students know and can do.
- Fourth, clear understanding of students' thinking requires having accurate information about students' thinking and interpreting that information within frameworks of mathematics and student development.

The starting point, then, is gathering accurate information about students' thinking. In order for this information to be useful, however, it must be interpreted and those interpretations should be used to influence instructional decisions. We label this process as classroom assessment; that is, classroom assessment is the process of gathering information about students' mathematical thinking, making inferences from that evidence about what students know and can do, and designing instruction to account for the inferred levels of students' understanding. While there are many purposes for assessment, in general, the purpose of classroom assessment is to make better instructional decisions so that students learn more. Professional development on classroom assessment can provide teachers with the tools that they need to implement this process for the ultimate benefit of students.

There is considerable evidence that effective implementation of classroom assessment leads to greater student learning.

> Black and Wiliam (1998) conclude from an examination of 250 research studies on classroom assessment that "formative assessment does improve learning' — and that the achievement gains are "among the largest ever reported for educational interventions." The effect size of 0.7, on average, illustrates just how large these gains are…. In other words, if mathematics teachers were to focus their efforts on classroom assessment that is primarily formative in nature, students' learning gains would be impressive. These efforts would include gathering data through classroom questioning and discourse, using a variety of assessment tasks, and attending primarily to what students know and understand. (Wilson & Kenney, 2003, p. 55)

> Classroom-based formative assessment, when appropriately used, can positively affect learning. According to the results of this review, students learn more when they receive feedback about particular qualities of their work, along with advice on what they can do to improve. They also benefit from training in self-assessment, which helps them understand the main goals of the instruction and determine what they need to do to achieve. But these practices are rare, and classroom assessment is often weak. The development of good classroom assessments places significant demands on the teacher. Teachers must have tools and other supports if they are

> to implement high-quality assessments efficiently and use the resulting information effectively. (Pellegrino, Chudowsky, & Glaser, 2001, p. 38)

Classroom assessment does not always receive high priority, in spite of the research that supports its efficacy.

> U.S. society generally places greater value on large-scale than on classroom assessment… National standards in science and mathematics recognize this type of assessment [classroom assessment] as a fundamental part of teaching and learning… To guide instruction and monitor its effects, teachers need information intimately connected to what their students are studying, and they interpret this evidence in light of everything else they know about their students and their instruction. The power of classroom assessment resides in these connections. (Pellegrino, Chudowsky, & Glaser, 2001, p. 41)

Classroom assessment is likely to have its greatest impact directly on the learning that occurs in individual classrooms; this learning can in turn affect results of large-scale testing. However, teachers will not be able to use classroom assessment effectively unless they understand how to incorporate it into their everyday work. Professional development can help teachers learn to do this.

One side benefit of implementation of classroom assessment seems to be that teachers often develop a greater sense of satisfaction about their teaching. This seems to be because they are able to identify what students know, and they can better interpret the progress that students are making. Teachers can point to specific student responses and behaviors that document learning.

Key Elements of Professional Development on Classroom Assessment

There are several elements that professional development on classroom assessment needs to address. These are outlined below, with examples taken from *Dynamic Classroom Assessment (DCA)*, a program created with support from a National Science Foundation grant (#9819914). *DCA* helps middle grades and high school mathematics teachers learn to incorporate classroom assessment into their regular instructional planning. *DCA* consists of a core module (ten 3-hour sessions) and three extension modules (10 hours each), totally 60 hours of professional development.

First, classroom assessment involves setting clear learning targets and exploring how different assessment methods can be aligned with those learning targets. Learning targets — sometimes called learning goals or learning objectives — are specifications of what students are intended to learn. Teachers should also think about what kinds of evidence would be acceptable as indication of attainment of those targets; that is, what behavior or response or verbalization is acceptable as a clear indicator that the desired learning actually occurred. Different assessment methods — for example, multiple choice item, open-ended problem — have the potential to reveal different information about students' thinking, just as different approaches to solving mathematics problems may indicate different levels of sophistication of mathematical thinking. Thinking about the evidence that is related to a learning target can help teachers choose an assessment method that might reveal that kind of evidence.

In the last decade or so, there has been a lot of attention in professional development on "alternative assessments," so many teachers can identify different assessment methods and understand some of the advantages and disadvantages of each. In *DCA*, therefore, we point out that many of the incorrect answers that students give result from the application of a particular "logic;" incorrect answers are seldom completely random, though of course there is the possibility that they result from carelessness. One of the problems we discuss is a division problem:

What is 6 ÷ 2/3?

A. 9 B. 4 C. 1 D. 1/9

We challenge teachers to identify thinking that might generate each of these options. Choice A is the correct answer, and choice B could indicate multiplication of 6 and 2/3. Choice C is more of a challenge for many teachers to "see;" if students interpret the fraction bar as a division sign and apply order of operations, they would execute the two "divisions" from left to right, first computing 6 ÷ 2 (with 3 as the answer) and then computing 3 ÷ 3 (with 1 as the answer). Choice D would be generated if students "inverted" the wrong factor; that is, computing 1/6 x 2/3. Recognizing the need to look for the logic behind students' incorrect answers is an important first step for many teachers in being able to understand students' thinking.

Second, feedback to students will be more effective when teachers distinguish between errors in what students know and errors in the way that students show what they know. That is, errors that students make may be fairly accurate communication of a significant misunderstanding of mathematics (e.g., an error of substance) or miscommunication of what turns out to be fairly accurate understanding of mathematics (e.g., an error of presentation). For example, *DCA* provides two student responses to this problem: ***What is 2 more than 3 times 4?*** Two of the responses are given below:

Student A: (3 + 2) x 4 = 20
Student B: 3 x 4 = 12 + 2 = 14

Many teachers initially classify both response as "incorrect," though for different reasons. Often they say that "Student A does not understand order of operations;" that is, there is an error of substance, while "Student B has written a number sentence that makes no sense;" that is, there is an error of presentation. Debriefing of these initial thoughts often leads teachers to the view that Student A might have read the question with a pause after "3": What is 2 more than 3 (pause) times 4. If so, this students' answer is reasonable, even though it is not what most teachers desire. Most teachers agree that Student B probably has a correct understanding of the problem but has presented that thinking in a way that leads to incorrect symbolism. One teacher used the phrase "run-on equation" to describe Student B's response.

The terminology that we use to describe these underlying issues is "substance of an idea," which is the meaning that students have internalized, and "presentation of an idea," which is the way that this meaning is communicated. This terminology evolved from the work of Pimm (1987, 1995). Students' errors can typically be categorized as errors of substance or errors of presentation. Feedback to students will be more effective when teachers distinguish between these two kinds of errors and tailor their feedback accordingly. That is, when teachers can identify the nature of a student's error, they can provide feedback that helps that student understand whether the error reflects deep misunderstanding or mistakes in communicating understanding. "Teachers should give specific feedback on errors and strategies, with suggestions on how to improve, but should keep the focus on deep understanding rather than on superficial learning of procedures" (Wilson & Kenney, 2003, 59). The net result is improved learning for students and better self-monitoring of learning by students.

Third, skillful questioning is an important part of the way that teachers can gather information about students' thinking. There are several kinds of questions that teachers might ask, but the most important ones for revealing students' thinking are clarifying and probing questions. These questions help students clarify their own thinking and clarify that thinking for the teacher and other students. The main focus of clarifying and probing questions is to reveal more of the information that is inside students' heads, not to put more information into students' heads as a means of "fixing" perceived errors. Developing skill at creating specific questions takes practice and reflection.

Weiss and her colleagues, in a national study of mathematics and science instruction, found that the most common form of questioning in instruction is "low-level 'fill-in-the-blank' questions, asked in rapid-fire, staccato fashion, with an emphasis on getting the right answer and moving on, rather than helping the students make sense of the mathematics/science concepts" (Weiss, Pasley, Smith, Banilower, & Heck, 2003, p. 67). Overall, "questioning is among the weakest elements of mathematics and science instruction, with only 16 percent of lessons nationally incorporating questioning that is likely to move student understanding forward" (Weiss, et al., p. 65). Instruction seems to be oriented much more toward covering the curriculum and getting students to say the right things rather than helping students make sense of the underlying mathematical ideas.

The typical questioning strategies used by teachers can have the effect of limiting the amount of engagement of students with key mathematics ideas. The questions can also limit the amount of information that a teacher can get about how students are thinking about mathematics.

> If the teacher limits questions to a narrow band of procedural questions, the answers given may not be sufficient for the teacher to make informed inferences about the breadth or depth of students' understanding. That is, the teacher may take a series of correct answers by a student as evidence of understanding, when in fact it is very limited evidence merely of the student's ability to give the correct answers. (Wilson & Kenney, 2003, 56)

Rapid-fire, low-level questioning is not likely to reveal much about students' thinking, so in order for classroom assessment to be implemented effectively, teachers need to consider carefully the kinds of questions they ask and the purposes for those questions.

In *DCA* we propose a categorization of questions based on a teacher's purposes. There are three purposes:

a. ***Engaging questions:*** invite students into a discussion, keep them engaged in conversation, invite them to share their work, or get answers "on the table"

b. ***Refocusing questions:*** help students get back on track or move away from a dead-end strategy

c. ***Clarifying questions:*** help students explain their thinking or help the teacher understand their thinking

DCA offers teachers opportunities to think about questioning through reviewing (a) a transcript of a conversation between a teacher and students, (b) curriculum materials, and (c) a classroom vignette on videotape.

Fourth, information about students' thinking and inferences about what students understand are not useful unless they can inform instructional decision-making. It is through better instruction that students will learn more. Improving instructional planning happens when there are opportunities for a teacher to reflect, discuss options with colleagues, explore different instructional strategies, and consider possible ramifications on students' learning of use of these strategies.

Because classroom assessment helps teachers make instructional decisions that are better aligned with the needs of students, teachers who use classroom assessment effectively can be expected to deliver "stronger instruction" in the sense that students will more likely be engaged in significant learning. In a phrase, these classes can be described as having greater intellectual rigor. "Fewer than 1 in 5 mathematics and science lessons are strong in intellectual rigor; include teacher questioning that is likely to enhance student conceptual understanding; and provide sense-making appropriate for the needs of the students and the purposes of the lesson" (Weiss, Pasley, Smith, Banilower, & Heck, 2003, p. 103). Making sense of students' thinking is a key to effective implementation of classroom assessment.

The *DCA* materials offer opportunities for teachers to reflect on and improve their instructional decision-making. This process begins in the first session, but the emphasis on this important issue increases across the remainder of the program.

Shauna, a high school geometry teacher

DCA materials were field-tested in several different settings in North Carolina, South Carolina, and Virginia. A sample of teachers were interviewed and their instruction was observed multiple times during the delivery of the professional development. Here is a brief description of what happened to one teacher. (The quotes set off below are taken from the interviews.)

At the start of the professional development program, Shauna taught geometry in a block schedule; each class was 90 minutes long. She planned lessons carefully and followed through on those plans, but with little deviation from her plans. She was attentive to her students' understanding through observation of students' work; her questioning focused mainly on leading students through the material to get them to the answer. She explicitly encouraged students to talk about their mathematical ideas, but she attended mainly to the most vocal students. This seemed to "leave out" some students from engagement with mathematical ideas. Some students' inattention resulted in off-task behavior.

The structure of the initially observed lesson was a variation on a traditional high school lesson. It began with two "brain teasers" on content unrelated to the lesson. Then there was an introductory activity in which students computed the measure of angles formed by parallel lines cut by a transversal. Next, Shauna reviewed the homework. Then she asked students to study a textbook page and complete a worksheet; answers were shared informally among groups of students. Finally, students worked independently on new homework. The worksheet asked students to analyze work from four unnamed "students;" the work of three of these hypothetical students was incorrect. Shauna had participated in a similar activity in the professional development program.

> *"I had four different students' answers to the first question. I had my students critique each one of those. If they did it correctly, then they explained how they went about doing that. If the student didn't do it correctly, I wanted my students to tell me why. What was it that the student didn't do correctly? What was their mistake? Did they set it up wrong? Did they work out the problem wrong?"*

> *"That was one of the assessment methods [in the professional development sessions]. It's almost the approach that*

a teacher has to use. When we get their papers, we have to actually figure out did the students work the problem correctly. If not, what was incorrect. I had never had a lesson like that before with this class. I was really impressed with their discussions, because they are my lower class. It really was beautiful."

Shauna acknowledged that the sessions helped her recognize that students work problems in different ways, and in ways that are different from her strategies. However, her favorable reaction to the sessions appears to be due to the fact that she got new activities to use with students; she was not yet distinguishing much between classroom assessment and assessment in general.

"We need to continually vary our assessments. We need to continually vary our activities, especially with block scheduling. I tend to get in a rut. Let's check our homework, let's take notes, here's a few problems — the same old same old. It [the professional development] challenges me to continually think of different ways and more effective ways to assess and to teach. There's definitely more than one way to learn, and I have to keep my eyes open to that."

By the end of the professional development program, Shauna asked more clarifying questions. After she questioned a student, she encouraged other students to expand on the response. When a student's response seemed incomplete, she posed questions that focused clearly on what she thought was the point of confusion. Her students were engaged in the content, almost to the point of exuberance, so Shauna had to refocus students' attention repeatedly on the mathematics of the lesson. Shauna clearly used assessment strategies to try to understand her students' thinking.

The final observed lesson began with review problems that were worked individually and then debriefed with the class as a whole. Then students worked as teams to play a game (a variation of Wheel of Fortune) that lasted the rest of the period. In each round of the game, teams had to reach consensus on an answer. Shauna chose one student to give the answer for each group, and she asked clarifying questions as necessary to be sure the answer was understood both by the team and by the rest of the class.

Shauna's beliefs about the role of questioning in instruction seemed to have changed dramatically. She used more — and better — clarifying questions during instruction, and she thought about questions as she planned instruction. She realized that having students share thinking was helpful to them. It was less clear how she used information about students' thinking in adapting lessons, since she never commented on how it affected her planning.

"I got the idea [having students reach consensus in their teams] from a graduate course I took 4 or 5 years ago. But the workshop [the classroom assessment sessions] really made me concentrate on how I question my kids. I have changed a lot of my questioning techniques, but I'm not really good with words. I'm not very good at asking questions on the fly, but when you actually get in to a lesson, you need to ask probing questions. So questioning is part of your planning."

"Just giving the right answer is not enough. You get them to explain how they got it and demonstrate different ways of doing problems. Just because a child gets the right answer that does not mean that they understand. And just because a child gets an answer wrong, that does not mean that they do not understand parts of the techniques. You really probe them. Why did you do it this way? Why does this work?"

"I'm the one that got hung up on get the right answer, not really explain it, just it's right or wrong. I was hung up in that kind of a rut, and this [the sessions] kind of got me out of that rut. I make my kids write more and I make them talk more in class. I make them explain what they did. I'll continue to give them problems where they have to really dig and make sure that they are looking for detail. They learn from each other. They learn from each other's mistakes."

Other Comments from Teachers

Other teachers in the field test responded positively to their interactions with the *DCA* materials.

I have told several people that this professional development program is the only "staff development" class I have actually put into action CONTINUOUSLY.

Effective mathematics instruction is more holistic than I previously thought. Processes and procedures and understanding of concepts aren't always equally developed in students.

I have found myself spending more time "assessing" my students' work. I evaluate more about their answers and give more feedback about what they are thinking.

I am listening to see if they understand the process or concept — not just whether they have the correct answer.

I think more about how students can grasp a concept rather than just pumping them with more information.

It is important to note that effects on teachers happened over time. No single activity produced dramatic effect. Rather, effects accumulated as teachers learned more, implemented ideas in their teaching, and reflected on changes in students' behavior and learning. This result speaks to the importance of treating the materials as a coherent entity. Extracting components of the program and using them in isolation should not be expected to have the desired effects.

In Closing

Any effective professional development program on classroom assessment needs to be long-term and classroom-focused, so that teachers can apply what they are learning fairly quickly. Participants need time to internalize the information they are learning and to become comfortable using that information in their own teaching. Any program will be deemed a success if teachers use classroom assessment to help students learn more. Teachers who understand what students know and can do are able to plan instruction so that it is better aligned with the needs of students.

One of the most important pieces of advice about implementation of classroom assessment strategies is that teachers should *talk less and listen more*. It is only by listening to the ways that students reason that we can expect to adapt instruction to fit students' needs. If we want students to learn more, we have to meet them on their ground and talk with them using language that will make sense to them. In a real sense, students are the clients of our instruction, and instruction must satisfy the needs of those clients.

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Mathematics Preparation for College:
Some Things We Learned the Hard Way, and What We Do About Them

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…it was the colleges and collegiate aspects of higher education that were visible and attractive…many colleges survived only by offering secondary education…frequently [secondary students] outnumbered collegians. (W. Bruce Weslie, 1997, p. 333, characterizing college student populations in the 1870s)

Our story evolved from a case study born out of frustration with students' poor mathematics performance at our university. Nearly half of our students, just out of high school, took remedial mathematics or no mathematics at all during their freshman year. The situation was not unique to our institution; voices from the other universities in our state and elsewhere echoed a similar concern: too many freshmen come to college deficient in mathematics. Our state universities required three years of math to enter, but students could graduate with only two. Compounding the problem was an open door policy of our well-developed community college network. That policy guarantees transfer to a four-year institution after completing a two-year degree. Nearly 30% of our state's students enter community colleges right out of high school, with another 20% following within 3 years. Such a policy seemed to permit an "end run" around college entrance requirements, particularly in mathematics. Furthermore, our statewide university mathematics placement test and our state standards (Essential Academic Learning Requirements) were not in alignment, partially because state standards for high school juniors and seniors did not exist.

In response to mounting concerns over these issues, higher education policy makers favored requiring a fourth year of secondary mathematics to enter state universities. Policy makers were not, however, adequately considering the rigor of the newly proposed senior course, nor were they considering raising high school graduation requirements in mathematics from the current two Carnegie credits. As researchers, we saw multiple disconnections in expectations, requirements, alignment, and articulation. Our case study addressed many of these issues, and we include, here, findings, interventions, and suggestions under two main headings: (1) We Don't Agree on What is Important, and (2) Students Get Mixed Messages.

PROBLEM: We Don't Agree On What Is Important

While raising critical issues, the National Council of Teachers of Mathematics [NCTM] standards (NCTM, 1989; 2000) movement and the calculus reform movement are not without a cadre of traditionalist detractors. "Math Wars" (Schoenfeld, 2003; Lundin, 2001, p. 197) between traditionalists and reformers continue to emphasize conflicting belief/value systems with respect to content, pedagogy, and assessment. Both sides have valid points, but a lack of coherence has led to confusion about what is important at many levels. The consequences for incoming freshmen college students, we think, are dire; they no longer know what to expect!

SOLUTION: Agree on Curriculum Intensity and Rigor Readiness

In an effort to cope with the polemic of the Math Wars and in the interest of conciliation, we have become believers in Clifford Adelman's (1999) notion of curriculum intensity and our own definition of rigor readiness.

Adelman, after examining high school curricula for content, scope, and sequence, graded the curricula on a scale of 1-40, from least to most academically intense. He then analyzed 13 years of data from the NCES High School and Beyond Study (U. S. Department of Education. National Center for Educational Statistics [NCES], 2004). In this comprehensive study, his academic intensity scale better predicted graduation from college by age 30 than did other more traditional variables, including college entrance exam scores or high school GPA/rank (Adelman, 1999, Executive summary, p. 2).

Of all pre-college curricula, the highest level of mathematics one studies in secondary school has the strongest continuing influence on bachelor's degree completion. Finishing a course beyond the level of Algebra 2 (for example, trigonometry or pre-calculus) more than doubles the odds that a student who enters post-secondary education will complete a bachelor's degree. (Executive Summary, p. 2)

Rigor Readiness is the level of preparedness to solve complex problems and logically communicate solutions or arguments. Isn't this what we mean when we plead, "I just wish my students would think?" We believe Rigor Readiness has been well conceived in the NCTM Principles and Standards for School Mathematics (NCTM, 2000). The concept of rigor is embodied in the Problem Solving and Connections standards, as well as in the Reasoning and Proof standard. Historically, Schoenfeld (1994, p. 55) elevated problem solving, or "doing mathematics," above the level of importance of curricular content. He gave (and we still give) thanks to George Polya for pioneering How to Solve It (Polya, 1957). More recently, Stigler and Hiebert (2004, p. 15) exposed the absence of making connections as detrimental to the performance of U.S. students in the Third International Mathematics and Science Study (TIMMS). Those authors concluded that our teachers tended to undermine students' learning of problem solving by reducing the process to procedures, rather than allowing students to construct connections. In any case, a rigorous argument must include constructing connections; the ability to communicate that argument is also critical.

If we want incoming students to do mathematics, to construct connections, to solve problems and to communicate solutions, there is not better framework is to guide them (and us) than the NCTM Standards. We readily acknowledge that symbolic manipulation and computation, often the mainstays of traditionalists, are tremendously important to mathematics and science. They are of particular importance in passing gateway tests in college. Incoming college students need both rigor readiness and computational and algebraic skills; but what messages do they get?

PROBLEM: Students Get Mixed Messages

High school graduation requirements are not equivalent to college entrance requirements. In 17 states, including ours, two credits of mathematics suffice to graduate from high school, even though three credits suffice in 28 states, and 4 credits, in 4 states. The remaining states had local laws governing requirements. (US Department of Education, National Center for Educational Statistics, 2001a, Table 153). Community colleges generally leave doors open, so, despite their own requirements, transferees often have deficiencies.

It is both permissible and popular for high school students to avoid rigorous senior courses. While 90 percent of high school freshmen expect to complete college, only about 44 percent take the college preparatory curriculum that equips them for high achievement (National Commission on the High School Senior Year, 2001a, p.1). While about two-thirds of all high school students complete a half-year of Algebra II, less than half take a fourth year of rigorous mathematics (The U.S. Department of Education, National Center for Education Statistics, 2002, Chapter 2). Overall, 27% of American high school students complete Math Analysis or Pre-Calculus, 12% complete calculus, and about 6% take statistics. We remark here that 22% of entering college freshmen, nationally, take remedial mathematics (Parsad, Lewis, & Greene, 2003, p. 18). Lest the reader attribute all of this remediation to non-traditional students, we remark that 18% of the 17-19 year olds entering our institution place into developmental mathematics courses. In the three-year sample of our case study, 44% of our students took no senior mathematics class, while about 30% took a full rigorous class (pre-calculus, calculus, or statistics).

Surprisingly, math avoidance begins even earlier in middle school. Results from the National Longitudinal Study of 1988 (U.S. Department of Education, 1997, p. 31) indicated that 51% of the students surveyed (n = 28,000), grades five through eleven, planned to quit taking mathematics as soon as possible. However, 89% of those students reported having college ambitions, and 91% of their parents harbored that dream for them (U.S. Department of Education, 1997, p. 18). More positively, eighth graders taking algebra tend to take advanced mathematics courses in high school, and taking advanced courses in high school can mitigate culturally linked deleterious effects in college performance. (Horn, Carol, & Kojaku, 2001, p. 38; U. S. Department of Education, 1997, p. 11).

In our case study, we sampled GPAs of traditional-aged freshman in the years 2001-2002 (n = 856). We disaggregated the data into two factor variables. The first factor, 1st High School Math Course, had three levels, (1) Pre-Algebra, (2) Algebra 1 or Integrated Math 1, and (3) Algebra 2, Geometry, or Integrated Math 2. The second factor, High School Senior Math Course Rigor, had five levels, (1) No Course, (2) Partial Course, (3) Non-Rigorous Course, (4) Rigorous Course, and (5) Advanced Course. Note that "Rigorous Course" here meant Math Analysis, Pre-Calculus, or Statistics, while "Advanced Course" meant Calculus or above. *See Table 1.*

TABLE 1:
Cross-Tabulation of First HS Math Course Rigor Level and Senior High School Math Course Rigor Level

			HIGH SCHOOL SENIOR MATH COURSE RIGOR LEVEL				
			No Course	Partial Course	Non-Rigorous Course	Rigorous Course	Advanced Course	Total
FIRST HIGH SCHOOL MATH COURSE RIGOR LEVEL	Pre-Algebra	Count	**23**	**4**	**69**	**10**	**0**	**106**
		% within First HS Course Vigor Level	21.7%	3.8%	65.1%	9.4%	.0%	100%
		% within Senior HS Course Rigor Level	6.1%	6.6%	42.3%	5.8%	.0%	12.4%
		% of Total	2.7%	.5%	8.1%	1.2%	.0%	12.4%
	Algebra or Int. Math 1	Count	**229**	**43**	**85**	**124**	**12**	**493**
		% within First HS Course Vigor Level	46.5%	8.7%	17.2%	25.2%	2.4%	100%
		% within Senior HS Course Rigor Level	60.4%	70.5%	52.1%	71.7%	15.8%	57.9%
		% of Total	26.9%	5.0%	10.0%	14.6%	1.4%	57.9%
	Algebra 2, Geometry, or Int. Math 2	Count	**127**	**14**	**9**	**39**	**64**	**253**
		% within First HS Course Vigor Level	50.2%	5.5%	3.6%	15.4%	25.3%	100%
		% within Senior HS Course Rigor Level	33.5%	23%	5.5%	22.5%	84.2%	29.7%
		% of Total	14.9%	1.6%	1.1%	4.6%	7.5%	29.7%
Total		Count	**379**	**61**	**163**	**173**	**76**	**852**
		% within First HS Course Vigor Level	44.5%	7.2%	19.1%	20.3%	8.9%	100%
		% within Senior HS Course Rigor Level	100%	100%	100%	100%	100%	100%
		% of Total	44.5%	7.2%	19.1%	20.3%	8.9%	100%

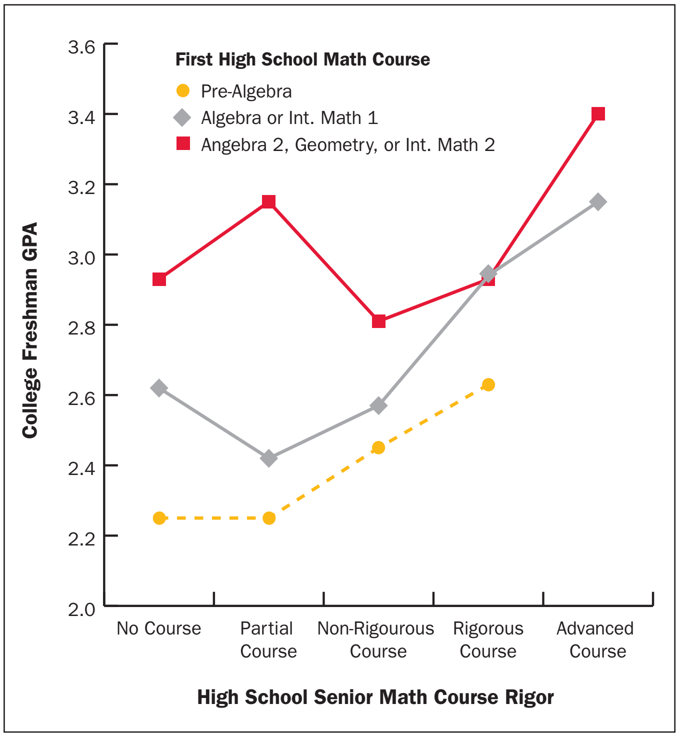

Figure 1

Students who took Algebra I or Integrated Mathematics as high school freshmen had a significantly higher mean college freshman GPA than those who began high school with a lower math course. Also, those taking Algebra II, Geometry, or Integrated Math II had significantly higher mean GPA than those taking Algebra I or IM 1. Furthermore, students who, as high school seniors, took No Course, a Partial Course, or a Non-rigorous Course had a mean freshman GPA significantly lower than those taking a Rigorous Course, and those taking an Advanced Course, had a mean GPA significantly higher than those taking a Rigorous Course. *See Figure 1.* Our research design did not control for cause; association, rather than cause-effect, is evident between the two factor variables and the independent variable.

SOLUTION: Intervene in Multiple Ways

1. Our university participates in Gear Up, an acronym for Gaining Early Awareness and Readiness. The Gear Up Program is funded by the Department of Education with the goal of enabling middle school students, especially those from low-income families, to choose a college path. The program focuses on sustaining achievement and interest in math, technology, science, and reading. Early reports of achievement gains are encouraging, and on-campus programs for middle school students and their teachers seem to produce the desired results. See more, including brief progress reports by state at www.ed.gov/gearup.

2. Algebra is important because of its connections to so many other areas. Research clearly shows that those who successfully experience it, perform better scholastically. We concede, however, that too much emphasis has been, and still is, placed on certain elements of symbol manipulation, even as the mathematics community continues to argue on import. In the interest of preparing students for college, where tradition reigns, we support the early introduction of algebraic concepts in middle school. Although we favor an integrated mathematics approach—see *Navigating Through Algebra* (Burke, Erickson, Lott, & Obert, 2001) for a compelling argument—we caution that manipulating symbols in a traditional sense is still important. It is a high stakes skill required for college admissions, placement, and in college mathematics and science courses.

3. To keep high school seniors interested in academics, the National Commission on the High School Senior Year recommended a "Triple A" solution: Align senior courses with college, raise the standard of Achievement, and provide course Alternatives, including those that are more rigorous (National Commission on the High School Year, 2001b, p. 19). We support dual enrollment programs that provide qualified high school students opportunities to take college courses either on campus or in their schools. Our Cornerstone Program (http://www.cwuce.org/cornerstone/) is becoming especially popular with high school students, teachers, and administrators, since students remain at school, but still earn college credit for taking pre-calculus or calculus. This program has strengthened the bond between our university, our mathematics department, and schools hosting Cornerstone courses. As an example, Mathematics teachers, wishing to qualify as Cornerstone Adjunct Instructors, have sought out our masters degree program.

Other options include our statewide Running Start Program, sponsoring high school students to attend regular college mathematics courses. Readers can compare the two programs at http://www.cwuce.org/cornerstone/cornerstone_vs_rs.asp. Finally, some schools in our area have had remarkable success preparing a majority of their students for college with AP mathematics. See for example, Bellevue High School's success story at http://www.bsd405.org/ap.html.

4. Our state does not have learning outcomes in mathematics for high school juniors and seniors. With a Bill and Melinda Gates Foundation grant and funding from the state legislature, members of the Transition Math Project [TMP] are now writing them. The grassroots committee, well-represented by key players in secondary and higher education policy, will take their completed recommendations to state agencies soon. Drafts of the new standards show innovative ideas, including a "student attribute" standard. The implementation of these state standards will, no doubt, smooth the transition to college. Readers may visit the TMP web site for more information at http://www.transitionmathproject.org/.

5. Finally James Rosenbaum (2004) recommended something we originally thought was a hard line approach to keeping standards high. He listed "New Rules of the Game" for college preparation: passing extra costs for college remediation down to students; increasing awareness in the high schools of the rigor of college coursework; retaining the burden of remediation at the high school level; and informing unprepared students of options other than immediate entry to higher education. We are no longer shocked by these suggestions. This year, our institution's doors closed early to new admissions; furthermore, some of our state's universities are already passing remediation service charges back to students, and the registrar just raised the admissions bar.

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Collaborative Partnership Helps Teachers Learn to Use Standards-Based Lessons and Analyze Data

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ABSTRACT:
This qualitative case study documented the presentation of a standards-based workshop to three groups of teachers. Collaboration of business, government and education groups created these professional development workshops for teachers. The sites for the workshops included one urban setting and one suburban setting. Three workshops composed of three once a month sessions were presented (two in the urban district, one in the suburban district) and analyzed to reveal the beliefs teachers held regarding: standards-based lessons, the use of technology for lesson plans and data analysis. The most glaring needs of these teachers were the abilities to collect and record data from student work, analyze the data, and reporting the conclusions reached from those analyses. Recommendations were made for incorporating more opportunities for teachers to engage in collaborative planning and examinations of teaching methods. The information gained from this study should be useful to any districts that are trying to answer the question "What is quality professional development and what are some creative ways to fund it?"

No Child Left Behind (NCLB) requires statistical evidence that students are improving their content mastery. Teachers are becoming anxious that more classroom instructional time will be sacrificed to standardized testing in order to provide data for school reports. In late September 2004, Secretary of Education Rod Paige stated "I think (NCLB) irreversibly changed the culture of K-12 education, I don't think it will ever … go back to the time when we did not focus on results"(Robelen, 2004). Teachers are spending hours of their personal time and their school planning time developing strategies to teach students how to be successful test takers. Issues surrounding "teaching to the test" are major concerns for teachers and administrators that affect instructional time, content depth, curriculum planning, and the scope and sequence of content as teachers prepare their yearly courses of instruction. How can teachers do what they are trained to do – teach – and respond to increasing demands to prepare students for multiple high stakes tests? Creative professional development is needed to help teachers make the transition into this new age of testing. The National Staff Development Council (NSDC) defines the ingredients necessary for quality professional development as results-driven, standards-based and job-embedded (Guskey & Sparks, 1991, Meyer, 2004). The Engineers Club of Dayton, Ohio, the Ohio Resource Center (ORC) and a mathematics educator joined forces to create a workshop focused on helping teachers learn how to use a new internet resource and to analyze their practice as well as student achievement using data from their students' work. All the work of the workshop is framed within the context of the teachers' regular instructional planning.

The intent of the federal legislation, NCLB, requires yearly standardized testing to measure student achievement. Teachers know that the results of high stake testing have dramatic impact upon the students, teachers, school buildings and school districts. Thus, it is imperative that teachers understand: 1) how to measure student success using data analysis; 2) how to collect data about their teaching; 3) how to analyze that data to improve their practice. To focus teachers on data throughout the workshop, the workshop introduces teachers to the ORC as a source for standards-based lessons. Teachers can collect and analyze data regarding their teaching strengths and areas that need improvement as well as to help them analyze student achievement.

Background

The collaboration of educators, business professionals, and a university mathematics educator made it possible to offer this series of professional development workshops. Each group brought an essential element to the project that was valuable as a single entity, but potent when put in combination. The history of the creation of this project involved the collaboration of the ORC, the Ohio Mathematics and Science Coalition, the Ohio Board of Regents, the Engineers Club of Dayton, and the University of Dayton. The collective wisdom of these groups produced a workshop that enhanced student achievement through teacher professional development.

The Ohio Board of Regents, at the suggestion of the State University Education Deans, established a unique system--the ORC-- for teachers to access standards-based lesson plans in mathematics, science, and reading that reflect best practice. Many of the web sites have video clips of lessons that allow teachers to see the selected lesson presented to a class. The ORC is a web site that anyone with access to the internet can reach at www.ohiorc.org.

The Engineers Club of Dayton is a professional organization that promotes mathematics and science by funding educational projects in the community. Through the intervention of the Ohio Mathematics and Science Coalition, an independent advocacy group from business, education, and government that works to improve PK -16 mathematics and science education for Ohio, the ORC, and the Engineers Club of Dayton were linked. The collaboration between these groups provided the funding needed to conduct the workshops. The mathematics educator created the delivery method of three once-a-month workshops. Each day of the workshop focused on one major component of quality professional development. The workshop trained teachers to recognize the components of best practice, to use the ORC web site, to employ methods for collecting and analyzing data, and to develop skills in analyzing classroom practice, in basing pedagogical decisions on data, and in reporting the results to multiple groups.

The workshop used a modified lesson study design to create a collaborative learning experience for the teachers. Lesson study in its classic form is the development of a lesson over an extended period of time with input from multiple teachers. Lewis (2002) summarized the lesson study format as a spiral in which teachers present and perfect a specific lesson. Lewis's cycle of development starts with teachers' recognition of the learning styles of their students. It moves to the development of a content specific lesson that addresses the identified student needs. Peer reviewers observe the lesson and discuss the lesson elements for effectiveness and those parts of the lesson that need to be changed. A number of teachers in the group present the same lesson and with each presentation, a review occurs and adaptations are made to improve the lesson. Due to the diversity of grades and buildings of the teachers attending the workshops, the lesson study format described by Pong, Chik and Tang (n.d.) was used as a framework for this workshop. This format includes the elements of Lewis's (2001) cycle of lesson study but has more focus on data analysis. The Pong, Chik and Tang (n.d.) method allows for data analysis of a lesson by a single teacher. Thus, the workshops employed the lesson study format of Pong, Chik and Tang to help the teachers examine their pedagogy through data-based decisions.

Methodology

This study examined three workshop series regarding how the teachers from two sites varied in their responses to using a web-based resource for standards-based lesson plans and how they generated and analyzed data. A qualitative design was the methodology chosen for this research, including individual case studies and a cross-case analysis. Data were collected as the result of teacher pre-workshop and post workshop surveys, projects and teacher reflection papers submitted at the conclusion of the workshop.

Participants

The workshop was presented twice in an urban school district and once in a suburban district in 2003. Approximately 65 teachers attended these programs. On the first day of the workshop, a questionnaire was administered to gather demographic information and data regarding the prior knowledge the teachers had about: the ORC; using computer programs, application of academic content standards to lessons. (See Appendix A.) The mathematics educator used this information to tailor the workshop to the needs of the attending teachers. She focused the grade level web sites, academic content standards, and grouping of the teachers by the majority grade level present at the workshop.

The school districts offered the workshop to middle school teachers. Several secondary level teachers and special needs teachers attended. While the teachers' primary content area was mathematics, other content areas were present such as science, reading, and health. All participants were volunteers who received course credit or funds for classroom materials for their participation. The teachers ranged in years of experience from first year teachers to those who were in their 32nd year of the profession. The mean number of years of experience was 14.2 years of classroom teaching. Thirty of the teachers were at the bachelor's degree level and 35 held master's degrees. There were very low numbers of males in each group. Ten percent of the first urban group were males, the second group was held at the suburban site where 8% were males, and the last group at the urban site had 20% male participants. Of the 65 participants, 24 teachers reported that their best computer access was their home computer with 41 teachers preferring to use their school computers. Slightly more than half of the teachers had some knowledge of the ORC web site prior to the workshop. Forty-seven of the teachers were interested in using the whole lesson plan found on the web site; whereas only 18 viewed the web site as a source of lesson parts.

Procedure

The workshop consisted of three once a month meetings. The objectives of the workshop focused on:

1) Learning to use the Ohio Resource Center (www.ohiorc.org) web site as a source for peer reviewed, best practice lesson plans
2) Collecting and analyzing data from student work using spreadsheets and graphs
3) Reporting the pretest/post test results of the lessons to administrators and parents.

The design of the workshop encouraged teachers to work as teams. The teams were taught how to find lessons that use best practice pedagogical methods at the ORC web page. The teams of teachers collected and analyzed their students' data that was used to produce reflections on their students' achievement and their own teaching practice.

Day One — Learning How to Use the Ohiorc.org Web Site

Workshop Elements:

1) Discussion and definition of best practices for teaching mathematics
2) Using the Ohio Resource Center web site
3) Selecting a lesson plan to be taught between meetings that fit into their curriculum pacing charts
4) Writing a pretest for the selected lesson plan

The first day of the series began with a discussion focused on identifying the constituent parts of best practice lessons for teachers of mathematics, science, and reading. The groups reviewed the fifteen criteria rubric definition of best practice used by the ORC lesson reviewers to classify lessons. The teachers worked with partners or in groups to facilitate conversations focused on their practice and how their lessons affected student learning. To control for variations in the rigor of lesson planning, the teachers were trained in the use of the ORC and were limited to selecting lessons from only this web site. The lesson selection requirement stated that the chosen lesson content had to map into the curriculum sequence of each teacher's school. The teachers modified their selected lesson to address the needs and backgrounds of their students. From the content of the selected ORC lesson, they constructed a pretest of five to ten questions that were not overwhelming to students, but challenging enough to be used as the lesson content post test. At the end of the first day, the teachers left with their pretest/post test and an ORC lesson that was modified to meet the needs of their students. Between the first and second sessions, the teachers were required to pretest their students, teach the lesson and post test their students.

Day Two — Learning to Use EXCEL Spreadsheets and Graphs

Workshop Elements:

1) Review discussion of the selected lesson plans and how these plans met the needs of the teachers
2) Workshop facilitator models how to use EXCEL spreadsheets to compare pretest and post test data.
3) Teachers create spreadsheets of their students' data
4) Teachers analyze their students' data for students' strengths and weaknesses
5) Teachers analyze their students' data as a reflection on what areas their teaching needs to improve and/or change

The teachers returned with their pretest and post test data. The primary objective for the second session was to review and use EXCEL spreadsheets. Instruction included how to enter student scores and how to express those results by using graphs was explained and practiced. Our discussions focused on multiple methods of statistical representations. The teachers experimented with data entry and modes of presenting the material graphically. The suggested form for data collection used in the Pong, Chik and Tang (n.d.) method examined each question and identified the response as right or wrong, allowing for no partial credit. The graphed data for each question was cumulative for the whole class. The data identified how many students answered the problems correctly. This was a modification from the Pong, Chik, and Tang method, which graphed the number of incorrect responses. Culturally, this was not what the teachers in these workshops preferred. These teachers wanted a positive graph that recorded the success levels of the students. The scores for the pretest and the post test were displayed on one graph to illustrate how each tested question changed by improving or regressing in student understanding. The workshops provided time for the groups to discuss the effectiveness of the lesson, how the students responded, what issues remained, and what could be changed in the presentation of the lesson to increase student achievement.

The element of lesson study that required teachers to conduct data analysis in order to examine which lesson elements needed improvement opened the groups to make insightful observations about the unintended objectives in their lesson. By examining the questions asked in the pretest and post test with the students' scores, the teachers were able to inspect the possible contributing factors to those scores. The mathematics educator asked the teachers to consider what could be changed in their pedagogical content knowledge to increase student achievement. Was the math presented in a way that built off experiences of their students? Was the math presented in an age and grade appropriate manner? Was the math in this lesson scaffolded appropriately for their students? What additional content should be taught next? What content or presentation method would be needed to increase your student achievement the next time they taught this lesson? These topics motivated the teachers to examine their content knowledge and how they taught a lesson.

Day Three — Learning to Compose Reports for Different Audiences: Administrators and Parents

Workshop Elements:

1) Review analysis of data using EXCEL spreadsheets and graphs
2) Learning how to explain the data to administrators, local professional development committees, and to parents
3) Preparation of the reports
4) Compiling two lessons, the data, the analyses, and the reports and presented as workshop evidence.
5) Writing reflections on the workshop effectiveness

The teachers returned to the third workshop session with the data analysis of their first ORC lesson and the data from their second ORC lesson presentation. The workshop content focused on having the teachers provide evidence that they could interpret the graphical information that they created. The teachers composed three separate reports about the lesson data: one for administrators, another report for local professional development committees to provide evidence of professional development, and a third report for parents. Each report briefly described the goals of the lesson, the graphical data of the pretest and post test and explained what the included graph represented. The teachers shared their second lessons that they taught, their data analysis, conclusions that they reached about student achievement and their teaching methods. The lessons learned during teacher collaboration motivated teachers to examine their individual analysis of where they used best practices and helped them to address the learning needs of more of their students. The teachers shared several noteworthy web sites. The third session concluded with the teachers' reflections regarding the impact and usefulness of the workshops, and completed the post workshop exit surveys. The funders received copies of the reflections for their review. Modifications were made to subsequent workshop presentations based on the teachers' survey comments.

The purpose of this project was to share with teachers an effective means of using technology to increase student achievement, collect and analyze data, and peer review their pedagogical methods of teaching content. The data analysis of student scores caused the teachers to make changes to their pedagogy, which met the NSDC demand for research-based professional development. The standards-based criteria were met by using the ORC as the sole source for lesson content. Teachers were required to review their curriculum sequence and their daily content pacing charts to select two timely lesson plans from the ORC web site, and to adapt the lessons for their students. This element satisfied the job-embedded requirement of staff development.

Procedure for Data Collection

A qualitative methodology was best suited for this study. The use of written documents (Patton, 1990) served as reliable sources of data collection. For the data collection, pre-workshop and post workshop surveys were developed. The design of the pre-workshop questions was to illuminate the experiences the teachers had using technology for lesson planning and to collect demographic data. (See Appendix A). The post workshop survey focused on teacher attitudes after working with presented materials. (See Appendix B). The documentation and information from the surveys supplied data for this study.

The researcher used sensitizing concepts to focus this study. Sensitizing concepts are starting points that guide data collections and direct a study where to examine data, what to examine, and they provide expectations about what will be produced (Denzin, 1989; Patton, 1990). Having taught at the high school level for 25 years, the researcher experienced many professional development in-service days and these experiences sensitized the focus of this study. The researcher's knowledge of relevant research and experiences with professional development served as an additional sensitizing concept and influenced the data analysis.

Results

In this section, the school districts are described, then a cross-case analysis follows that used the pre and post workshop surveys as data. A brief description of the demographics of the two school districts that offered the workshops provides the background for each site. The districts are identified with pseudonyms in order to maintain their anonymity. Two workshops were held in the Diversity district and one workshop occurred in the Target district.

Table 1

TWO SCHOOL DISTRICTS		
School Districts Characteristics	**Diversity**	**Target**
Student Population	26,000	8,000
Number of Teachers	1,700	458
Teacher – Years of Experience	15.3	15.0

Cross-Case Analysis

Based on a cross-case analysis of these three cases (workshops), three fields of information emerged: 1) themes found in the participants' reflections; 2) issues with the common requirements of the workshop where the teachers learned new methods; 3) post survey responses. Discussion of these three fields gave the researcher insights into how the teachers viewed professional development and the information presented in this workshop as well as what content areas needed additional information to be presented.

1) Participants Reflections

Four themes emerged from the participants reflections: a) ORC comments (ORC Lesson Plans), b) use of lesson study format (Lesson Study); c) how teachers plan to utilize the information from the workshop (Utilization); d) how the workshop encouraged teacher interactions and camaraderie (Camaraderie). Teacher responses to these themes are listed in Table 2 Teacher Reflection Themes.

2) Teachers Learning New Methods

The researcher observed four common areas within the structure of the workshop where the participants grappled with new pedagogical methods. The teachers focused their learning in the areas of: a) lesson adaptations, b) use of EXCEL spreadsheet program, c) reporting formats, d) lesson study where the teachers interacted when examining the lessons and student data.

a) Lesson Adaptations. The lesson adaptations made by the urban teachers for their students were cultural in nature and responded to urban student strengths. These adaptations included choral reading, additional group work, and oral reporting formats. The suburban teachers extended the lessons with additional assignments for those in their classes that needed greater challenges. Both groups of teachers added written assignments during the lessons in response to the Ohio Academic Content Standards requirements. All the teachers identified the importance of the ORC identifying specific academic content standards met in each lesson as helpful and time saving to their lesson planning.

b) Using EXCEL Spreadsheet Program. Several teachers were hesitant using the EXCEL program. They never used a spreadsheet program or forgot the procedural sequence for using the program. The workshop provided step-by-step instructions, which the teachers followed using their own student data to create a single graph of the pretest

TABLE 2:
Teacher Reflection Themes

ORC LESSON PLANS	LESSON STUDY	UTILIZATION	CAMARADERIE
1. I thought best practice lessons would be more complete. 2. The workshop provided a wealth of resources for teachers. 3. I have seen a lot of lesson plans, but the ORC are the best. 4. My students were impressed with the information I found in the lessons. 5. The ORC offers lessons rich in content and links to others. 6. The ORC is useful in giving teachers ideas to write their own standards-based lessons. 7. It was beneficial to see what resources are available to them to reduce their time creating lessons. 8. I thought the ORC lessons would have pretests and post tests in each lesson.	1. The lesson study format taught to us will be a boost to my professional development. 2. The workshop provided excellent opportunities for teachers to brainstorm lesson plans and their components. 3. I can actually go into my computer and do graphs on each student. 4. The ability to analyze the results will be of great value. 5. I feel this will take time to get use to. 6. It was helpful in representing student data graphically to see gains in learning. 7. By doing a lesson study, teachers can look at areas for remediation and plan activities. 8. The refresher on plotting in Excel was most informative.	1. I will definitely continue to use the ORC. 2. I will share the ORC with my department. 3. I hold myself accountable to present this material to my department. 4. Using best practice lessons will better prepare students for their futures. 5. Directly utilizing so much information from the web was awesome. 6. Basically, I will use this workshop as a catalyst to use the internet. 7. Teachers need to share this information with their colleagues and students in order to improve the whole educational process. 8. I plan to continue to communicate my findings to administrators and parents. 9. As department chair, I plan to share the ORC with my teachers and encourage their use of it.	1. Teachers need to know that there are people that care and support them. 2. The workshop provided collaboration among teachers to share and learn across the curriculum. 3. The workshop brought together teachers socially and in sharing academic ideas. 4. It is enjoyable to view the lessons of others and to adapt them to the needs of my students. 5. Workshops are sometimes the only time that I can associate with other math teachers in the district.

and post test results for each lesson. After practicing with EXCEL, the teachers became comfortable with the program, if not at ease with how to record data and present it in graphical form.

c) **Reporting Formats.** Reporting formats required detailed information for administrators and parents. The reporting forms were designed to provide data-based evidence that the teachers clearly understood their collected and recorded data. The reporting format for parents required the teachers to do a great deal of work translating pedagogical information into lay terminology. Continuous communication with parents has been a key to academic success for students (NBPTS, n.d.). Providing reports to parents enabled on going communication between parents and teachers about what was happening academically in the classroom and what the students understood about a specific lesson. These three reports were clear demonstrations of what the teachers understood of the workshop process and data analysis.

d) **Lesson Study Teacher Interactions.** Once the teachers mastered how to use spreadsheets and graph their student scores rich discussions took place within the teacher groups as they examined the graphs. One set of teachers found that their students could easily identify a quadrilateral, but had no success identifying a quadrilateral with no 90 degree angles. Their discussion examined their presentation of the material, what methods they used and how they planned the presentation to develop cognitive understanding of the concept of the quadrilateral. The teachers examined why their students were not able to identify a quadrilateral with no 90 degree angles as a parallelogram. The teachers revisited their curriculum map and their methods of teaching this unit, They concluded that they needed to scaffold the attributes of shapes to help their students learn shapes rather than just memorize the words used to identify shapes.

Another group of 6 inner city teachers introduced fractions, decimals and percents with a lesson that used the alphabet shapes from a cereal box. They found that when the students did not eat their data pieces, the comparisons written as fractions, decimals and percents came quickly and easily to their students. They brought a second box of cereal for the students to eat while doing the calculations. Several of these teachers shared that their students extended the lesson because the students wanted to know the ratio of the letter R to the letter O in one cereal box. After analyzing the fractions, decimals, percents and ratios the students drew consumer conclusions as to whether the cereal content was worth the price. The teachers were excited that the students wanted to do more mathematics than was in the original lesson plan. The students' post test scores were all in the 90% range. All the teachers in this group were very encouraged by what they learned about student centered learning and seeing data that verified the student learning.

3) Post Survey Responses

Responses to the post workshop survey were uniform in the opinions of the participants. Almost all 65 teachers responded to each post survey question with a Yes response except question #2 about how well the ORC lessons matched the students' learning levels (See Appendix B). The participants selected *b. Close Match* as the most common response. Selection *c. Need some additional work by me* was the second preferred response.

Discussion

Darling-Hammond, et al (1995) noted that professional development can not be done in a one shot experience if new applications to pedagogical practice were to become embedded in daily practice. The collaborators in this project recognized the importance of teachers working over a period of time to develop habits of mind that would lead to enhanced professionalism and higher student achievement. The Engineers Club of Dayton and the ORC donated funds with the proviso that this workshop had multiple sessions over a period of time to train teachers in the use of a new technology resource. The mathematics educator added the ORC web site pieces with the data analysis and reporting elements. Specifically, the purpose of this project was to instruct teachers in an effective means of using technology to increase student achievement, collect data, analyze data and conduct a peer review of their pedagogical methods of teaching. Standards-based best practices helped the teachers to transition into using and creating a standards–based curriculum as a basis for planning the scope and sequence of a content area. Practice with those new techniques to perfect their use, and the applications of the new materials to their teaching styles required several workshop sessions. All of the workshops were conducted on Saturdays, which demonstrated the professional commitment of the attending teachers.

The reflection sheets provided insights to what the teachers thought of the workshop. The teachers participated with great energy. They enjoyed learning new approaches to lesson presentation, data collection and earning their professional development credits while doing work that would enhance their classes. The teachers stated in their exit reports that they were excited to find a web site that would save them time searching for teaching materials that aligned with the Ohio Academic Content Standards and were engaging for students. Some teachers had difficulty using the computers, but with peer assistance and the workshop presenter motivating the teachers, they pushed themselves to learn how to access the ORC, search the site, find and extract lesson plans, and how to use spreadsheets to graph the results.

The question for future examination: Did the teachers who attended these sessions continue to use the format presented to examine their teaching and their student learning? The teachers who took part in these workshops could be surveyed next year. They could be asked if they employed the methodology presented in this workshop and, if so, how did they use the process, for what purpose, how often, and what modifications did they make to the process to best serve their schools. The collaboration of the business, government, and educational communities resulted in a workshop that enhanced teacher professional development and demonstrated to teachers that the community they serve values them.

Implementing long-term professional development for teachers is a time management issue. According to the National Staff Development Council (Meyer, 2004), 25% of a teacher's day should be focused on professional development. While this number is the ideal, implementing this much time for teachers away from students is not practical in today's schools. Conducting workshops on Saturdays was the alternative to finding substitute teachers and having teachers out of their classrooms. The number of teachers who attended the workshops was testament to how seriously teachers take professional development when they believe it will be a benefit to their teaching.

What can other groups of mathematics supervisors take away from this study? Educators need to be aware that the business community is vitally committed to helping improve the education of all children. Government agencies want schools to graduate productive citizens who can contribute to the economic development of their state. These groups are willing to help educators fund professional development when that development produces evidence of improved teaching. While a mathematics educator designed the content and delivery method of this workshop, professional organizations were the concept formulators. A key to keeping teachers returning to complete the workshop was that the requirements of the workshop respected the teachers' classroom requirements and curriculum. Addressing the needs of today's classroom teachers for models of standards-based lessons and data analysis skills rather than adding extraneous work on to their plates helped to make this workshop successful.

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APPENDIX A

Ohio Resource Center and the Dayton Engineers' Club Workshop

Please complete the following questions.

1) Did you know about the web site Ohiorc.org prior to the workshop on March 8, 2003?

a) Yes　　b) No

If yes, how did you learn about the ORC web site?____________________

If yes, had you used it as a source for lesson plans?

a. Yes　　b. No

2) Identify the degree of help this site will be to your lesson planning.

a. Great help　　b. Good help

c. Find ideas for my own lessons　　d. Not much help

3) Is the linking of lesson plans to the Ohio Academic Content Standards help you?

a. Yes　　b. No

4) Do you prefer lessons that add parts of a lesson, or the whole lesson plan from goals to evaluation?

a. Parts　　b. Whole

5) What will be the reason you reference the Ohio Resource center web site?

a. Whole lesson plans

b. Ideas to build upon for lessons

c. Resources to extend or add to my lesson

d. Identifying the standards met in a specific content lesson

e. I will not reference this web site

Demographics

1) How many years have you been teaching?____________________

2) What is the average age of your students?____________________

3) What is the grade level that you teach?____________________

4) What is your highest degree? Circle one

a. Associates　　b. Bachelor's

c. Master's　　d. Doctorate　　e. None of these

5) How many more years do you plan to teach?____________________

6) Where is your best computer access?

a. School　　b. Home　　c. Other____________

APPENDIX B

Ohio Resource Center and the Dayton Engineers' Club Post Workshop Survey

Please respond by circling Yes or NO (with the exception of statement #2):

1) The Ohio Resource Center web site helps me develop lesson plans focused on standards.

2) The lessons found on this web site match my students' learning levels
   a. Perfect match
   b. Close match
   c. Need some additional work by me
   d. Need a lot of work by me
   e. No help at all

3) The lessons on the ORC can be used for measuring student learning.

4) Lesson study is a tool that I plan to use in the future.

5) I would recommend the ORC to other teachers as a resource for lesson plans.

6) I can calculate my pretests and post tests on a graph for comparison purposes.

7) I can interpret my student data in order to report increases/decreases in student learning.

8) Working with another teacher made this experience better.

9) I plan to use the ORC lessons in my lesson planning.

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Address line 2 ____ Work Phone ____

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☐ State/Province
☐ District/County/City
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☐ University/College
☐ Senior High School
☐ Junior High/Middle School
☐ Elementary School
☐ Publisher
☐ Author
☐ Consultant/Independent
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☐ Retired
☐ Other________________

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☐ Asian American
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