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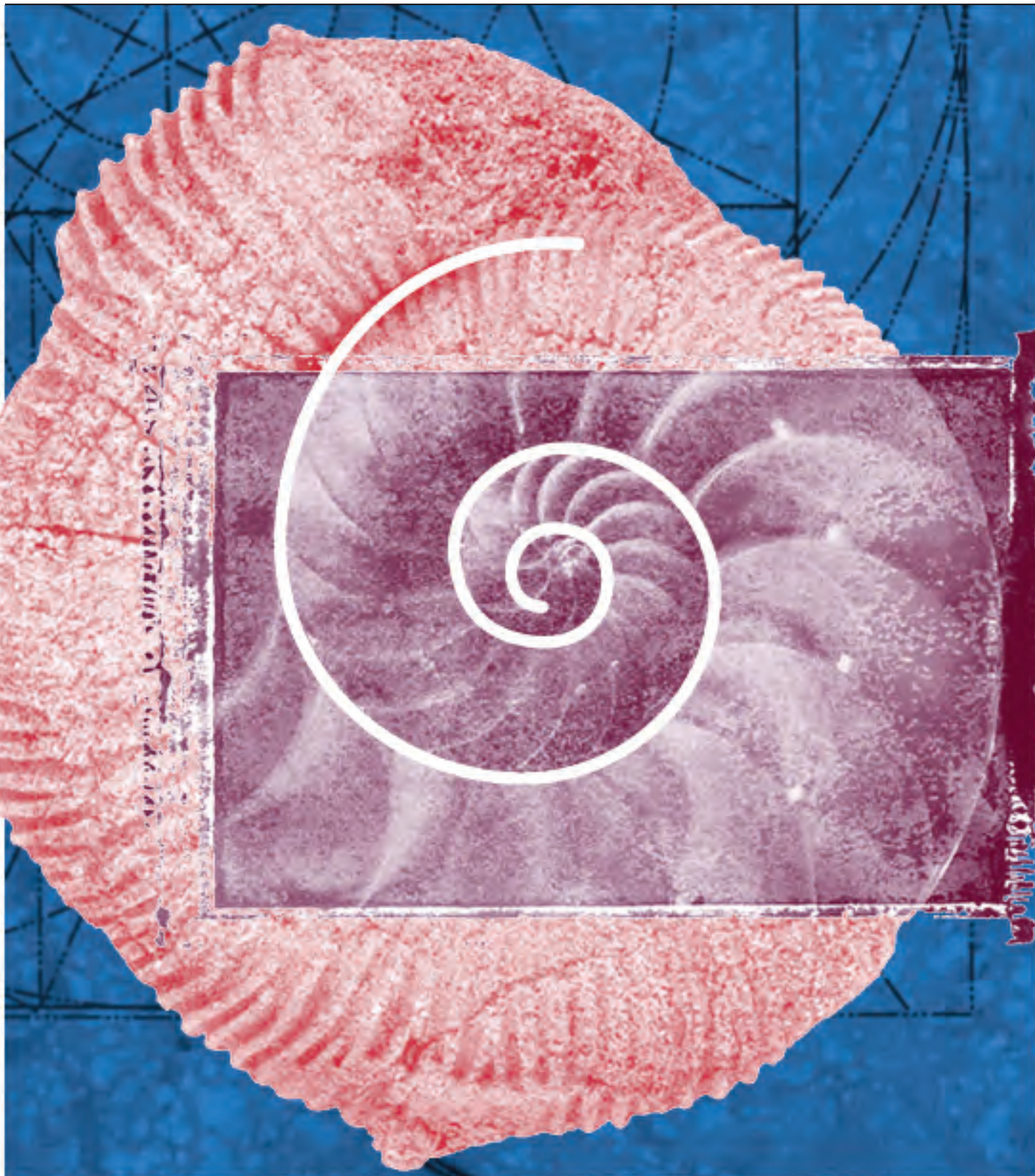
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*Note: The last two categories are intended for short pieces of 2 to 3 pages in length*

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### **On the cover:**

This issue's cover, created by Bonnie Katz, gets its inspiration from Randall Charles' article reflecting on Big Ideas in Mathematics, particularly ideas relating to two and three-dimensional shapes.

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## *Purpose Statement*

**T**he purpose of the NCSM Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership.

## Comments from the Editor

Mark Driscoll

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I have been privileged this past year to spend time, on a regular basis, in a South Bronx middle school, working alongside a mathematics coach as she engages teachers in reflecting on and improving their practice. The experience has enlivened my thinking about leadership and about NCSM, in particular. For one thing, it has reinforced my belief that educational leadership closely tied to mathematics content is extremely important in efforts to help teachers and students, and so what we stand for in NCSM has gained renewed luster in my eyes. Second, the experience has brought home to me in concrete ways how much the texture of mathematics leadership is changing in districts and schools — often in exciting ways. In this particular case, I have observed the coach doing the expected — directly impacting teachers' practice — but also the unexpected — for example, making herself an agent for distributing leadership in the school.

As a broadly applied change strategy in districts, mathematics coaching is relatively young, and sufficient data have not yet appeared revealing whether coaching is effective in improving teaching practice and, most importantly, in improving student outcomes. However, my instincts tell me that mathematics coaching has tremendous potential for improving teaching practice and student outcomes, in good part because coaches can work on several levels — they can directly impact practice and they can work indirectly, as well. In the third article in this issue, Middleton and Coleman write about the importance in school mathematics reform of creating "local experts," a variation on the theme of distributed leadership. To a large extent, that is what I have observed the New York coach do in her work — often in subtle ways. I have found it surprising, as well as inspiring, that much of her attention focuses on gathering data about where important knowledge is situated in

the school, then seeking ways to diffuse it throughout: *Which sixth grade teachers have a handle on planning, in order to keep pace with the pacing chart? Is there a sixth grade teacher who has focused successfully on diagnosing students' difficulties with fractions and using the information? Which seventh grade teachers have made strides in helping special needs students reach beyond the basics to learn challenging content? Who has strategies and techniques for managing rambunctious classes in that challenging period right after lunch?* In each case, she seeks opportunities for the knowledgeable person to take the lead in sharing his/her ideas with the others. In light of the first article in this issue, by Kitchen and DePree, one could argue that the coach is an agent for creating one of the characteristics the authors identify from highly effective schools: *Mathematics faculty collaborate and support each other.*

Also in that first article, the authors inform us that, in highly effective schools, a "tangible sense of hope" exists. My time with the coach has made me aware that a very important line of action for a mathematics coach — especially in areas where hope is a fragile commodity, as in this part of the South Bronx — is to persist in supporting teachers — individually and collectively — in ways that make hope tangible for them and, through them, for their students and the students' parents. Here the marriage of leadership and mathematics content is essential, in order for hope to be not only tangible but substantive, as well. Toward this end, the coach and I have created a set of professional-development experiences for the teachers around analyzing student work. Analysis is guided by a framework based on a few mathematical "big ideas," with particular resonance in the middle grades, such as operation sense, fraction units, and reasoning about size and shape. In his article in this issue, Charles argues for a wide role in teachers' lives for mathematical big ideas. His careful definition and detailed taxonomy will be a welcome guide to me in

future efforts to design programs around mathematical big ideas. I trust you will find them equally valuable.

Let me close with my own expressions of hope. I hope that teacher leaders, such as those described by Middleton and Coleman in their article, and mathematics coaches such as

the one I described above, continue to thrive and continue to reshape and enrich mathematics education leadership. I further hope that they have an increased presence in NCSM, in particular, sharing their work at our annual meeting and in this journal.

## Closing the Gap through an Explicit Focus on Learning and Teaching

Richard S. Kitchen, University of New Mexico  
Julie DePree, University of New Mexico, Valencia

*... When they teach you, they teach you with so much enthusiasm it makes you want to learn more than they're teaching you. You might get upset that you're leaving [the school] and it might feel early because the teachers are getting so enthusiastic with you and they're getting to a point that they're feeling like family. They're really like going for the gold and it's like their sole purpose is to teach you and to make you have fun.*

**T**his student at The Young Women's Leadership School (TYWLS) in East Harlem was describing the dedication and enthusiasm of her teachers. In 2001-02, TYWLS was one of only a handful of public, single-sex schools in the country. Like other highly effective, public schools that took part in a research study described here, the teachers at TYWLS made learning and teaching among their top priorities.

The Young Women's Leadership School was one of nine public, secondary-level schools selected in the spring of 2002 to participate in the High Achieving Schools (HAS) Initiative<sup>1</sup> for demonstrating exemplary achievement, while serving low-income communities. The nine schools from across the United States were chosen from more than 230 applicants specifically because they demonstrated: (1) free or reduced lunch rate of 50% or higher, and (2) sustained exemplary achievement or a significant increase in academic achievement, particularly in mathematics, over a minimum of 3-5 consecutive years. In addition to The Young Women's Leadership School, the following public, secondary-level

schools (middle schools and high schools) were selected to participate in the project: Emerald Middle School, J.D. O'Bryant School of Mathematics and Science, KIPP Academy Houston, KIPP Academy New York, Latta High School, Rockcastle County Middle School, YES College Preparatory School, and Ysleta Middle School.

As part of the HAS Initiative, the nine schools participated in a research study conducted by the University of New Mexico (UNM) during the 2002-2003 academic year. One goal of the research undertaken was to learn what characteristics distinguished TYWLS and the eight other highly effective schools, particularly in mathematics. In this article, a summary of the principal findings will be shared and, on occasion, illuminated with examples.

### Schooling in Low-Income Communities in the United States

Schools that serve low-income communities (defined here as schools in which 50% or more of the student population qualifies for free or reduced price lunch) have unique sets of problems that distinguish them from their more affluent, suburban counterparts. For example, at schools that serve low-income communities, students often attend classes in dilapidated facilities, have higher percentages of novice teachers, teachers without a teaching credential, and teachers who are teaching subjects in which they have neither a major nor a minor (Ingersoll, 1999; NRC, 2001). Schools that serve low-income communities are also characterized for their highly bureaucratic organizational structures (Kaestle, 1973); lack of support for change, particularly to

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<sup>1</sup> Funding from the Hewlett-Packard Company supported the High Achieving Schools Initiative.

personalize and individualize education (Louis & Miles, 1990); and standardized and uncoordinated instructional programs that encourage a custodial attitude towards children (Winfield & Manning, 1992).

Research has also shown that schools that serve low-income communities struggle to implement and benefit from school reform efforts (Jackson and Davis, 2000; Olsen, 1998). Teachers at schools that serve low-income communities face challenges specific to implementing mathematics education reforms such as those promoted by the National Council of Teachers of Mathematics (1989; 2000) and the National Science Foundation (1996). For instance, they often have minimal access to professional development opportunities to learn about standards-based curriculum and instruction, and may face resistance to the implementation of mathematics education reforms by administrators, colleagues, parents, students, and others (Kitchen, 2003). The research on effective teaching and school restructuring provides insight into classroom-level strategies that can be implemented to overcome these challenges.

### **Effective Teaching and School Restructuring**

The effective teaching literature (Brophy & Good, 1986; for mathematics see Good, Grouws, & Ebmeir, 1983) has consistently found that students taught by mathematics teachers who structure the lesson, maintain a decent pacing, and focus on the development of its main points outperform students whose teachers do not engage in a similar set of practices. Martin et al's (2000) analyses of TIMSS data found that opportunities provided at home (such as access to reading materials) to students were the most common school characteristics that discriminated schools whose students achieved high from those scoring low on the TIMSS mathematics and science assessment. Though not as important, the nature of mathematics and science instruction that was provided to students also made a difference.

In work on school restructuring and its relationship to student performance on high-level tasks, Newmann and Associates (1996) and Newmann & Wehlage (1995) reported that students enrolled in classes where the curriculum content and the instruction focused on depth (over mere coverage), analytic reasoning (over mere memorization), and the construction of value (over doing tasks as ends in themselves) outperformed their colleagues whose classrooms lacked these instructional features. Lee and Smith (2001) obtained similar results in their study of secondary schools. Secondary schools where mathematics and science

course offerings were predominantly academic, where teachers as a whole tended to report instruction that focused on depth, analytic (or higher order) thinking, and value were schools whose students began to close the social-class-based achievement gap.

In line with these findings, an hypothesis of the HAS study was that at the nine highly effective schools that served low-income communities, the majority of mathematics teachers had developed strategies to overcome challenges alluded to previously to support instruction that matched these characteristics (depth, analytic reasoning, and value). The full report can be found at [www.unm.edu/~jbrink/HASchools](http://www.unm.edu/~jbrink/HASchools). Here we summarize our methodology and findings.

### **Research Methodology**

**Classroom Observations.** During the 2002-2003 academic year, a team of UNM researchers visited all nine participating schools twice — once in the fall and once in the spring. During the fall of 2002 and the spring of 2003, we observed four teachers at each participating school. The participating teachers were selected by a school administrator to participate in the study. We requested that the four teachers be representative of the teachers at the school who taught the “regular” mathematics classes across multiple grade-levels. Overall, a total of eight observations were made at each participating school in the fall of 2002 and in the spring of 2003. School level and classroom level data were collected at the participating schools through classroom observations, interviews with teachers, administrators, and students, and through survey instruments.

**Data collection and analysis.** Qualitative methods were used to identify major patterns and themes related to the salient features that distinguished the participating schools as highly effective in mathematics, and to the teachers' conceptions and practices about mathematics curriculum, instruction, and assessment (Miles & Huberman, 1984; Strauss & Corbin, 1990). All qualitative data were analyzed by an iterative coding process (Emerson, Fretz, & Shaw, 1995). Codes were generated during the initial review of the interview texts. Relationships among the codes were explored in subsequent readings of responses and broad themes emerged. This process continued until consistent themes were achieved. The themes reported had to be confirmed by two or more teachers at more than 50% of the participating schools (i.e., two or more teachers from five or more schools).



**Relevant Findings**  
**Distinguishing Characteristics of the Highly Effective Schools**

The analysis of teacher and student narratives revealed seven primary findings: (1) Learning and teaching are prioritized to support high academic expectations for student learning; (2) Supplemental support is provided for student learning; (3) Mathematics faculty has a strong and well-defined sense of purpose; (4) Mathematics faculty collaborate and support each other; (5) Teachers prepare their students to be successful on standardized tests, but teach “beyond the test;” (6) Teaching resources are available; and (7) Teachers have regular access to professional development opportunities.

Table 1

THE SEVEN DISTINGUISHING CHARACTERISTICS OF THE NINE HIGHLY EFFECTIVE SCHOOLS
1. Learning and teaching are prioritized to support high academic expectations for student learning.
2. Supplemental support is provided for student learning.
3. Mathematics faculty has a strong and well-defined sense of purpose.
4. Mathematics faculty collaborate and support each other.
5. Teachers prepare their students to be successful on standardized tests, but teach “beyond the test.”
6. Teaching resources are available.
7. Teachers have regular access to professional development opportunities.

At all the participating middle schools, the discipline policy, class schedule, student support services, and professional development goals for teachers were established with one goal in mind: to positively impact student learning and achievement. Teachers valued problem solving and challenged students to think and reason. There was also an emphasis on instruction of mathematical facts and drill-and-practice approaches to teaching were used. However, because the primary goal was to challenge students with cognitively demanding mathematical content (e.g., students were expected to successfully complete a full year of Algebra I by the end of eighth grade), remediation was viewed as a means to that end. Teachers prioritized developing students’ problem solving skills and taught challenging mathematical content with the goal of impacting their students’ abilities to think critically. These findings lend strong support to the notion that highly effective schools implement

curriculum and instruction aligned with standards-based recommendations (see NCTM 1989, 2000; NSF 1996).

All the schools had discipline policies that reinforced the notion that learning was the top priority and obstructing the learning of others was a serious offense. For instance, at Ysleta Middle School in El Paso, the behavioral expectations were identical in all classrooms and teachers collaborated to uniformly uphold and enforce these expectations. At YES College Preparatory School in Houston and the two KIPP Academies, students attended summer school where they were introduced to the schools’ high behavioral expectations. A teacher at Emerald Middle School in El Cajon, California discussed how interdisciplinary teams at Emerald supported a focus on teaching rather than a focus on students with behavioral issues:

*... I don't care how great of a teacher you are, if you don't have good management skills the kids aren't going to get it. You can have the best person, the person who knows everything about mathematics come into the classroom. Most likely they won't succeed because they don't know how to relate to the kids. So, the fact that I have a team, that I work with people... it allows me more freedom to teach the math, to work in the math area, so I'm not always dealing with behaviors. Behaviors, people help me with that so I'm able to focus on my actual subject area.*

Slogans at the participating schools such as “Failure is Not an Option” and “Whatever it Takes” that communicated high academic expectations were not merely hollow rhetoric. Extensive academic support services for students were widely available to sustain these high academic expectations. All the participating schools had after-school tutorial programs (teachers were paid a stipend to tutor at some schools), Saturday study sessions, tutorial programs provided through university partnerships, and procedures to regularly assess student progress. At a few of the schools, students could even call their teachers at home for assistance. At KIPP Academy New York, students could be pulled out of the one elective that was available, school orchestra, if they needed tutoring in any of the core subjects.

In addition to supplemental academic assistance, teachers had extended class periods to teach mathematics at participating middle schools. Teachers took advantage of this extra time to meet students’ remediation needs and challenge them with cognitively demanding mathematical content. This two-pronged approach, instruction focused

on both remediation and challenging students with cognitively demanding mathematics curriculum, was possible because teachers had the time to do both. A teacher at YES College Preparatory School explained the benefits of having additional instructional time with students:

*I think one of the reasons that happened, if you think of the kids that we're serving, a lot of them, they come in 6th grade and they don't come in with the skills in order to take pre-algebra. They get double the time in math... Getting the kids in middle school with an hour and a half, it allows you to go over homework; it allows you to do a mini-lesson in between maybe even your lesson. It allows you the opportunity to give the class a chance to understand, give them class work. So having a double period is really awesome. I don't know if we could potentially teach as much as we do in a 45-minute block. I think that would be a disservice to them.*

Teachers across the highly effective schools spoke about how they worked with their colleagues to horizontally and vertically align curriculum, shared teaching ideas, discussed their students' mathematical strengths and weaknesses, and wrote and/or modified curriculum together. At the charter schools, time was built into the daily schedule for the mathematics teachers to meet. At Rockcastle County Middle School in Kentucky, teachers credited their extensive and long-term collaborations as key to the school's dramatic academic turnaround over the course of the past decade. The strong collaborations that existed among teachers clearly supported the implementation of challenging mathematics curriculum and instruction.

There is little doubt that the extraordinary collaborations that existed among faculty were among the primary reasons why the participating schools were highly effective. Teacher meetings often revolved around standardized testing. For instance, teachers engaged in test item analysis to identify students' weaknesses and wrote instructional units to prepare students for the test. Nonetheless, though teachers worked to help their students be successful on standardized tests, the test did not necessarily dictate mathematics curricula and instruction. Teachers spoke about teaching beyond the test. A teacher at KIPP Academy New York said: "... if you're teaching correctly, everything applies to the test. A test is just a basic problem solving situation, so if you're teaching them problem solving you won't have to worry so much about teaching [to] the test." The focus on high expectations for student learning at participating

schools coupled with the support mechanisms for students to thrive academically led to high achievement. This finding is an important one for schools given the high-stakes testing climate that currently exists in the United States.

Teachers talked about how fortunate they were to have so many teaching resources. They also spoke about how when they needed something, they could simply open their closets and pull out the desired materials. The resources were available to support the primary goals at the schools, learning and teaching. In general, teachers did not feel they had to beg for materials to be effective at their jobs. A teacher at Rockcastle County Middle School discussed how there were so many materials available at Rockcastle that it was actually a bit overwhelming: "... I think I have so many materials that it's hard to find what's what; it's almost too much. I guess it's a great thing, because we have so many materials to pull from that it's almost overwhelming." In addition, Rockcastle County Middle School and Ysleta Middle School employed a full-time mathematics consultant whose job was to support mathematics instruction at the school.

### **Final Remarks**

The focus on learning and teaching, support provided for student learning, and the availability of both professional development opportunities and teaching resources for the teachers promoted rigorous, enduring, and genuine learning environments at the nine highly effective schools that served low-income communities. Teachers came to school to teach and students came to learn. The culture at these schools was the exact opposite of what one may find at less effective schools: students who interrupted the learning of others were reprimanded not only by teachers, but by their peers as well. A teacher at The Young Women's Leadership School described the value students placed on academic success at the school: "It's cool to be good at math. The coolest girls, the most popular girls are also the ones who work the hardest and achieve the most."

Behavioral problems were minimal because of the steadfast focus on student learning at the schools. These highly effective schools were places where a tangible sense of hope existed. Teachers liked coming to work and students knew they were expected to take school seriously. Students also knew that they would be held accountable by multiple adults at the school for their actions. A teacher at The Young Women's Leadership School summarized how teachers and students approached teaching and learning at the

school: "In a lot of schools, there are a lot of teachers out there who are judged by the (amount of) time they are at the school because that's where they are. But I think that everybody here wants to be here..." Furthermore, "I think when they [the students] come here, they're going to learn and they want to be here to learn."

In addition to hope, a strong sense of caring was evident at the participating schools. For instance, at The Young Women's Leadership School, every student was in an "Advisory" group with a teacher who kept track of the student's academic performance. The Advisory groups also

promoted the development of strong personal relationships among teachers and students at the school. At Ysleta Middle School, there existed a very strong community outreach program that actively engaged parents in their students' educations. A student at KIPP Academy New York summarized the feeling of being cared for by teachers, a sentiment shared by many students at the participating schools: "They'll really do a lot of things for you, like they'll leave their cell phone on all night even if you have to call them just to say hello, or just to see how you're doing. Or they might call you to say hello and it's like, it's a real close relationship. It's like what you'd have with your parents."

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## Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics

Randall I. Charles, Carmel, CA

**E**ducation has always been grounded on the principle that high quality teaching is directly linked to high achievement and that high quality teaching begins with the teacher's deep subject matter knowledge. Mathematics education in the United States has been grounded on this principle, and most educators and other citizens have always believed that our teachers have adequate content knowledge given the high mathematics achievement of our students. Unfortunately, research conducted in the past ten years has shown that the United States is not among the highest achieving countries in the world, and that our teacher's subject matter knowledge and teaching practices are fundamentally different than those of teachers in higher achieving countries.

Research is beginning to identify important characteristics of highly effective teachers (Ma 1999, Stigler 2004; Weiss, Heck, and Shimkus, 2004). For example, effective teachers ask appropriate and timely questions, they are able to facilitate high-level classroom conversations focused on important content, and they are able to assess students' thinking and understanding during instruction. Another, and the focus of this paper, is the grounding of a teacher's mathematics content knowledge and their teaching practices around a set of Big Mathematical Ideas (*Big Ideas*).

The purpose of this paper is to initiate a conversation about the notion of Big Ideas in mathematics. Although Big Ideas have been talked about for some time, they have not become part of mainstream conversations about mathematics standards, curriculum, teaching, learning, and assessment. Given the growing evidence as to their importance, it is timely to start these conversations. A definition of a Big Idea is presented here along with a discus-

sion of their importance. Then a set of Big Ideas and Understandings for elementary and middle school mathematics is proposed. The paper closes with some suggestions for ways Big Ideas can be used.

In working with colleagues on the development of this paper I am rather certain that it is not possible to get one set of Big Ideas and Understandings that all mathematicians and mathematics educators can agree on. Fortunately, I do not think it's necessary to reach a consensus in this regard. Use the Big Mathematical Ideas and Understandings presented here as a starting point for the conversations they are intended to initiate.

### What is a *Big Idea* in mathematics?

*Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise.* (NCTM, 2000, p. 17)

Teachers are being encouraged more and more through statements such as the one above to teach to the big ideas of mathematics. Yet if you ask a group of teachers or any group of mathematics educators for examples of big ideas, you'll get quite a variety of answers. Some will suggest a topic, like equations, others will suggest a strand, like geometry, others will suggest an expectation, such as those found in *Principles and Standards for School Mathematics* (NCTM, 2000), and some will even suggest an objective, such as those found in many district and state curriculum standards. Although all of these are important, none seems sufficiently robust to qualify as a big idea in mathematics. Below is a proposed definition of a big idea, and it is the one that was used for the work shared in this paper.

**DEFINITION: A *Big Idea* is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole.**

There are several important components of this definition. First, a Big Idea is a statement; here's an example.

**Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.**

For ease of discussion each Big Idea below is given a word or phrase before the statement of the Big Idea (e.g., Equivalence). It is important to remember that this word or phrase is a *name* for the Big Idea; it is not the idea itself. Rather the Big Ideas are the statements that follow the name. Articulating a Big Idea as a statement forces one to come to grips with the essential mathematical meaning of that idea.

The second important component of the definition of a Big Idea given above is that it is an *idea central to the learning of mathematics*. For example, there are many mathematical concepts (e.g., number, equality, numeration) and there are many mathematical processes (e.g., solving linear equations using inverse operation and properties of equality) where understanding is grounded on knowing that mathematical objects like numbers, expressions, and equations can be represented in different ways without changing the value or solution, that is, equivalence. Also, knowing the kinds of changes in representations that maintain the same value or the same solution is a powerful problem-solving tool.

Ideas central to the learning of mathematics can be identified in different ways. One way is through the careful analysis of mathematics concepts and skills; a content analysis that looks for connections and commonalities that run across grades and topics. This approach was used to develop the Big Ideas presented here drawing on the work of others who have articulated ideas central to learning mathematics (see e.g., NCTM 1989, 1992, 2000; O'Daffer and others, 2005; Van de Walle 2001). Some additional thoughts are given later about identifying Big Ideas.

The third important component of the definition of a Big Idea is that it *links numerous mathematics understandings into a coherent whole*. Big Ideas make connections.<sup>1</sup> As an example, the early grades curriculum introduces several "strategies" for figuring out basic number combinations such as  $5 + 6$  and  $6 \times 7$ . The strategy of use a double involves thinking that  $5 + 6$  is the same as  $5 + 5$  and 1 more. The strategy of use a five fact involves thinking that  $6 \times 7$  is the same as  $5 \times 7$  and 7 more. Both of these strategies, and others, are connected through the idea of equivalence; both involve breaking the calculation apart into an equivalent representation that uses known facts to figure out the unknown fact. Good teaching should make these connections explicit.

A set of Big Ideas for elementary and middle school are given later in this paper. For each Big Idea examples of *mathematical understandings* are given. A *mathematical understanding* is an important idea students need to learn because it contributes to understanding the Big Idea. Some mathematical understandings for Big Ideas can be identified through a careful content analysis, but many must be identified by "listening to students, recognizing common areas of confusion, and analyzing issues that underlie that confusion" (Schifter, Russell, and Bastable 1999, p. 25). Research and classroom experience are important vehicles for the continuing search for mathematical understandings.

### **Why are Big Ideas Important?**

Big Ideas should be the foundation for one's mathematics content knowledge, for one's teaching practices, and for the mathematics curriculum. Grounding one's mathematics content knowledge on a relatively few Big Ideas establishes a robust understanding of mathematics. Hiebert and his colleagues say, "We understand something if we see how it is related or connected to other things we know" (1997, p. 4), and "The degree of understanding is determined by the number and strength of the connections" (Hiebert & Carpenter, 1992, p. 67). Because Big Ideas have connections to many other ideas, understanding Big Ideas develops a deep understanding of mathematics. When one understands Big Ideas, mathematics is no longer seen as a set of disconnected concepts, skills, and facts. Rather, mathematics becomes a coherent set of ideas. Also, understanding Big Ideas has other benefits.

<sup>1</sup> This attribute of a big idea is consistent with definitions others have provided- see Clements & DiBiase 2001; Ritchhart 1999; Southwest Consortium for the Improvement of Mathematics and Science Teaching 2002; Trafton & Reys, 2004.

Understanding:

- is motivating.
- promotes more understanding.
- promotes memory.
- influences beliefs.
- promotes the development of autonomous learners.
- enhances transfer.
- reduces the amount that must be remembered.

(Lambdin 2003).

Teachers who understand the Big Ideas of mathematics translate that to their teaching practices by consistently connecting new ideas to Big Ideas and by reinforcing Big Ideas throughout teaching (Ma 1999). Also, effective teachers know how Big Ideas connect topics across grades; they know the concepts and skills developed at each grade and how those connect to previous and subsequent grades.

And finally, Big Ideas are important in building and using curricula. *The Curriculum Principle from the Principles and Standards for School Mathematics* (NCTM, 2000) gives three attributes of a powerful curriculum.

- 1) A mathematics curriculum should be coherent.
- 2) A mathematics curriculum should focus on important mathematics.
- 3) A mathematics curriculum should be well articulated across the grades.

The National Research Council reinforced these ideas about curriculum: "...it is important that states and districts avoid long lists [of standards] that are not feasible and that would contribute to an unfocused and shallow mathematics curriculum" (2001, p. 35). By the definition given above, Big Ideas provide curriculum coherence and articulate the important mathematical ideas that should be the focus of curriculum.

### **What are Big Ideas for elementary and middle school mathematics?**

Twenty-one (21) Big Mathematical Ideas for elementary and middle school mathematics are given at the end of this paper. Knowing the process I used to develop this list and some issues I confronted in developing it might be helpful if you decide to modify it or build your own.

As part of a Kindergarten through Grade 8 curriculum development project, several colleagues and I articulated "math understandings" for every lesson we wrote in the program. Using the long list of math understandings we created, I organized these across content strands rather than grade levels. When I did that, it became apparent that there were clusters of math understandings, ideas that seem to be connected to something bigger. I then started the process of trying to articulate what it was that connected these ideas; I developed my definition of a Big Idea and used that as a guide. I next confronted a fundamental issue in doing this kind of work — how big (or small) is a Big Idea? Although I am not presumptuous enough to suggest an answer to this question, I can share some thinking that guided me. My sense is that Big Ideas need to be big enough that it is relatively easy to articulate several related ideas, what I called mathematical understandings. I also believe that Big Ideas need to be useful to teachers, curriculum developers, test developers, and to those responsible for developing state and district standards. If a Big Idea is too big, my sense is that its usefulness for these audiences diminishes. This thinking led to an initial list of 31 Big Ideas grouped into the traditional content strands. Reviews by colleagues suggested that articulating Big Ideas by content strands was not necessary; Big Ideas are BIG because many run across strands. This led to a reduction in the number of Big Ideas on my list. Further analyses of my list with regard to their usefulness for the audiences mentioned above led to the list offered in this paper.

Finally, it is important to note that there are relatively few Big Ideas in this list – this is what makes the notion of Big Ideas so powerful. One's content knowledge, teaching practices, and curriculum can all be grounded on a small number of ideas. This not only brings everything together for the teacher but most importantly it enables students to develop a deep understanding of mathematics.

### **What are some ways Big Ideas can be used?**

Here are a few ways that Big Mathematical Ideas and Understandings can be used.

#### **Curriculum Standards and Assessment**

- Revise/create district and state curriculum standards to incorporate Big Mathematical Ideas and Understandings. Many state standards emphasize mathematical skills. Curriculum coherence and effective mathematics instruction starts with standards that embrace not just skills but also big mathematical ideas and understandings.

- Develop individual teacher, district, state, or national assessments around Big Mathematical Ideas and Understandings. Alignment of standards and assessment is important for many reasons and both need to address big ideas, understandings, and skills.

### Professional Development

- Build *professional development courses* focused on mathematics content and anchored on Big Ideas and Understandings. Engage teachers with tasks that enable them to grapple with Big Ideas and Understandings.
- Do a *lesson study* where Big Ideas are used to connect content and teaching practices (See Takahashi & Yoshida, 2004).
- Develop *chapter/unit and individual lesson plans* by starting with Big Ideas. Generate mathematical understandings specific to the content and grade level(s) of interest.

Appendix A shows an example of one way Big Ideas might be infused into an existing curriculum. In this example, the teachers did an analysis of all of the lessons in a fourth grade chapter on multiplication. Based on that analysis, they created a chapter overview that started with their state content and reasoning standards but then connected them to Big Ideas. Individual lessons were then connected to the Standards and Big Ideas and to the specific mathematics understandings to be developed in that lesson.

### Conclusion

The purpose of this paper is to start a conversation about Big Ideas. Use the Big Ideas and Math Understandings presented here as a starting point; edit, add, and delete as you feel best. But, as you develop your own set keep these points in mind. First, do not lose the essence of a Big Idea as defined here, and second, do not allow your list of Big Ideas and Understandings to balloon to a point where content and curriculum coherence are lost. Big Ideas need to remain BIG and they need to be the anchors for most everything we do.

## Big Mathematical Ideas and Understandings for Elementary and Middle School Mathematics

A *Big Idea* is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole.

### BIG IDEA #1

**NUMBERS — The set of real numbers is infinite, and each real number can be associated with a unique point on the number line.**

#### Examples of Mathematical Understandings:

##### *Counting Numbers*

- Counting tells how many items there are altogether. When counting, the last number tells the total number of items; it is a cumulative count.
- Counting a set in a different order does not change the total.
- There is a number word and a matching symbol that tell exactly how many items are in a group.
- Each counting number can be associated with a unique point on the number line, but there are many points on the number line that cannot be named by the counting numbers.
- The distance between any two consecutive counting numbers on a given number line is the same.
- One is the least counting number and there is no greatest counting number on the number line.
- Numbers can also be used to tell the position of objects in a sequence (e.g., 3rd), and numbers can be used to name something (e.g., social security numbers).



### ***Whole Numbers***

- Zero is a number used to describe how many are in a group with no objects in it.
- Zero can be associated with a unique point on the number line.
- Each whole number can be associated with a unique point on the number line, but there are many points on the number line that cannot be named by the whole numbers.
- Zero is the least whole number and there is no greatest whole number on the number line.

### ***Integers***

- Integers are the whole numbers and their opposites on the number line, where zero is its own opposite.
- Each integer can be associated with a unique point on the number line, but there are many points on the number line that cannot be named by integers.
- An integer and its opposite are the same distance from zero on the number line.
- There is no greatest or least integer on the number line.

### ***Fractions/Rational Numbers***

- A fraction describes the division of a whole (region, set, segment) into equal parts.
- The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.
- A fraction is relative to the size of the whole or unit.
- A fraction describes division. ( $a/b = a \div b$ ,  $a$  &  $b$  are integers &  $b \neq 0$ ), and it can be interpreted on the number line in two ways. For example,  $2/3 = 2 \div 3$ . On the number line,  $2 \div 3$  can be interpreted as 2 segments where each is  $1/3$  of a unit ( $2 \times 1/3$ ) or  $1/3$  of 2 whole units ( $1/3 \times 2$ ); each is associated with the same point on the number line. (Rational number)
- Each fraction can be associated with a unique point on the number line, but not all of the points between integers can be named by fractions.
- There is no least or greatest fraction on the number line.
- There are an infinite number of fractions between any two fractions on the number line.
- A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
- Whole numbers and integers can be written as fractions (e.g.,  $4 = 4/1$ ,  $-2 = -2/1$ ).
- A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.
- Percent is relative to the size of the whole.

## **BIG IDEA #2**

### **THE BASE TEN NUMERATION SYSTEM — The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.**

#### **Examples of Mathematical Understandings:**

#### ***Whole Numbers***

- Numbers can be represented using objects, words, and symbols.
- For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
- Each place value to the left of another is ten times greater than the one to the right (e.g.,  $100 = 10 \times 10$ ).
- You can add the value of the digits together to get the value of the number.
- Sets of ten, one hundred and so forth must be perceived as single entities when interpreting numbers using place value (e.g., 1 hundred is one group, it is 10 tens or 100 ones).

**Decimals**

- Decimal place value is an extension of whole number place value.
- The base-ten numeration system extends infinitely to very large and very small numbers (e.g., millions & millionths).

**BIG IDEA #3**

**EQUIVALENCE: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.**

**Examples of Mathematical Understandings:**

***Numbers and Numeration***

- Numbers can be decomposed into parts in an infinite number of ways
- Numbers can be named in equivalent ways using place value (e.g., 2 hundreds 4 tens is equivalent to 24 tens).
- Numerical expressions can be named in an infinite number of different but equivalent ways (e.g.,  $4/6 \div 2/8 = 2/3 \div 1/4 = 2/3 \times 4/1$ ; also  $26 \times 4 = (20 + 6) \times 4$ ).
- Decimal numbers can be named in an infinite number of different but equivalent forms (e.g.,  $0.3 = 0.30 = 0.10 + 0.20$ ).

***Number Theory and Fractions***

- Every composite number can be expressed as the product of prime numbers in exactly one way, disregarding the order of the factors (Fundamental Theorem of Arithmetic).
- Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

***Algebraic Expressions and Equations***

- Algebraic expressions can be named in an infinite number of different but equivalent ways (e.g.,  $2(x - 12) = 2x - 24 = 2x - (28 - 4)$ ).
- A given equation can be represented in an infinite number of different ways that have the same solution (e.g.,  $3x - 5 = 16$  and  $3x = 21$  are equivalent equations; they have the same solution, 7).

***Measurement***

- Measurements can be represented in equivalent ways using different units (e.g., 2 ft 3 in = 27 in.).
- A given time of day can be represented in more than one way.
- For most money amounts, there are different, but finite combinations of currency that show the same amount; the number of coins in two sets does not necessarily indicate which of two sets has the greater value.

**BIG IDEA #4**

**COMPARISON: Numbers, expressions, and measures can be compared by their relative values.**

**Examples of Mathematical Understandings:**

***Numbers & Expressions***

- One-to-one correspondence can be used to compare sets.
- A number to the right of another on the number line is the greater number.
- Numbers can be compared using greater than, less than, or equal.
- Three or more numbers can be ordered by repeatedly doing pair-wise comparisons.
- Whole numbers and decimals can be compared by analyzing corresponding place values.
- Numerical and algebraic expressions can be compared using greater than, less than, or equal.

***Fractions, Ratios, & Percent***

- A comparison of a part to the whole can be represented using a fraction.
- A ratio is a multiplicative comparison of quantities; there are different types of comparisons that can be represented as ratios.
- Ratios give the relative sizes of the quantities being compared, not necessarily the actual sizes.
- Rates are special types of ratios where unlike quantities are being compared.
- A percent is a special type of ratio where a part is compared to a whole and the whole is 100.
- The probability of an event is a special type of ratio.

***Geometry and Measurement***

- Lengths can be compared using ideas such as longer, shorter, and equal.
- Mass/weights can be compared using ideas such as heavier, lighter, and equal.
- Measures of area, volume, capacity and temperature can each be compared using ideas such as greater than, less than, and equal.
- Time duration for events can be compared using ideas such as longer, shorter, and equal.
- Angles can be compared using ideas such as greater than, less than, and equal.

**BIG IDEA #5**

**OPERATION MEANINGS & RELATIONSHIPS: The same number sentence (e.g.  $12-4 = 8$ ) can be associated with different concrete or real-world situations, AND different number sentences can be associated with the same concrete or real-world situation.**

**Examples of Mathematical Understandings:**

***Whole Numbers***

- Some real-world problems involving joining, separating, part-part-whole, or comparison can be solved using addition; others can be solved using subtraction.
- Adding  $x$  is the inverse of subtracting  $x$ .
- Any subtraction calculation can be solved by adding up from the subtrahend.
- Adding quantities greater than zero gives a sum that's greater than any addend.
- Subtracting a whole number (except 0) from another whole number gives a difference that's less than the minuend.
- Some real-world problems involving joining equal groups, separating equal groups, comparison, or combinations can be solved using multiplication; others can be solved using division.
- Multiplying by  $x$  is the inverse of dividing by  $x$ .
- Any division calculation can be solved using multiplication.
- Multiplying two whole numbers greater than one gives a product greater than either factor.

***Rational Numbers (Fractions & Decimals)***

- The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.
- Different real-world interpretations can be associated with the product of a whole number and fraction (decimal), a fraction (decimal) and whole number, and a fraction and fraction (decimal and decimal).
- Different real-world interpretations can be associated with division calculations involving fractions (decimals).
- The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers.
- The product of two positive fractions each less than one is less than either factor.

*Integers*

- The real-world actions for operations with integers are the same for operations with whole numbers.

**BIG IDEA #6**

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**PROPERTIES: For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.**

**Examples of Mathematical Understandings:**

*Properties of Operations*

- Properties of whole numbers apply to certain operations but not others (e.g., The commutative property applies to addition and multiplication but not subtraction and division.).
- Two numbers can be added in any order; two numbers can be multiplied in any order.
- The sum of a number and zero is the number; the product of any non-zero number and one is the number.
- Three or more numbers can be grouped and added (or multiplied) in any order.

*Properties of Equality*

- If the same real number is added or subtracted to both sides of an equation, equality is maintained.
- If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained.
- Two quantities equal to the same third quantity are equal to each other.

**BIG IDEA #7**

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**BASIC FACTS & ALGORITHMS: Basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.**

**Examples of Mathematical Understandings:**

*Mental Calculations*

- Number relationships and sequences can be used for mental calculations (one more, one less; ten more, ten less; 30 is two more than 28; counting back by thousands from 50,000 is 49,000, 48,000, 47,000 etc.)
- Numbers can be broken apart and grouped in different ways to make calculations simpler.

*Whole Number Basic Facts & Algorithms*

- Some basic addition and multiplication facts can be found by breaking apart the unknown fact into known facts. Then the answers to the known facts are combined to give the final value.
- Subtraction facts can be found by thinking of the related addition fact.
- Division facts can be found by thinking about the related multiplication fact.
- When 0 is divided by any non-zero number, the quotient is zero, and 0 cannot be a divisor.
- Addition can be used to check subtraction, and multiplication can be used to check division.
- Powers of ten are important benchmarks in our numeration system, and thinking about numbers in relation to powers of ten can make addition and subtraction easier.
- When you divide whole numbers sometimes there is a remainder; the remainder must be less than the divisor.
- The real-world situation determines how a remainder needs to be interpreted when solving a problem.

*Rational Number Algorithms*

- Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.
- The product of two fractions can be found by multiplying numerators and multiplying denominators.

- A fraction division calculation can be changed to an equivalent multiplication calculation (i.e.,  $a/b \div c/d = a/b \times d/c$ , where  $b, c,$  and  $d \neq 0$ ).
- Division with a decimal divisor is changed to an equivalent calculation with a whole number divisor by multiplying the divisor and dividend by an appropriate power of ten.
- Money amounts represented as decimals can be added and subtracted using the same algorithms as with whole numbers.

#### *Measurement*

- Algorithms for operations with measures are modifications of algorithms for rational numbers.
- Length measurements in feet and inches can be added or subtracted where 1 foot is regrouped as 12 inches.
- Times in minutes and seconds can be added and subtracted where 1 minute is regrouped as 60 seconds.

### **BIG IDEA #8**

**ESTIMATION: Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.**

#### **Examples of Mathematical Understandings:**

##### *Numerical*

- The numbers used to make an estimate determine whether the estimate is over or under the exact answer.
- Division algorithms use numerical estimation and the relationship between division and multiplication to find quotients.
- Benchmark fractions like  $1/2$  (0.5) and  $1/4$  (0.25) can be used to estimate calculations involving fractions and decimals.
- Estimation can be used to check the reasonableness of exact answers found by paper/pencil or calculator methods.

##### *Measurement*

- Length, area, volume, and mass/weight measurements can be estimated using appropriate known referents.
- A large number of objects in a given area can be estimated by finding how many are in a sub-section and multiplying by the number of sub-sections.

### **BIG IDEA #9**

**PATTERNS: Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.**

#### **Examples of Mathematical Understandings:**

##### *Numbers*

- Skip counting on the number line generates number patterns.
- The structure of the base ten numeration system produces many numerical patterns.
- There are patterns in the products for multiplication facts with factors of 0, 1, 2, 5, and 9.
- There are patterns when multiplying or dividing whole numbers and decimals by powers of ten.
- The difference between successive terms in some sequences is constant.
- The ratio of successive terms in some sequences is a constant.
- Known elements in a pattern can be used to predict other elements.

##### *Geometry*

- Some sequences of geometric objects change in predictable ways.

## **BIG IDEA #10**

**VARIABLE: Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.**

### **Examples of Mathematical Understandings:**

- Letters are used in mathematics to represent generalized properties, unknowns in equations, and relationships between quantities.
- Some mathematical phrases can be represented as algebraic expressions (e.g. Five less than a number can be written as  $n - 5$ .)
- Some problem situations can be represented as algebraic expressions (e.g. Susan is twice as tall as Tom; If  $T =$  Tom's height, then  $2T =$  Susan's height.)
- Algebraic expressions can be used to generalize some transformations of objects in the plane.

## **BIG IDEA #11**

**PROPORTIONALITY: If two quantities vary proportionally, that relationship can be represented as a linear function.**

### **Examples of Mathematical Understandings:**

- A ratio is a multiplicative comparison of quantities.
- Ratios give the relative sizes of the quantities being compared, not necessarily the actual sizes.
- Ratios can be expressed as units by finding an equivalent ratio where the second term is one.
- A proportion is a relationship between relationships.
- If two quantities vary proportionally, the ratio of corresponding terms is constant.
- If two quantities vary proportionally, the constant ratio can be expressed in lowest terms (a composite unit) or as a unit amount; the constant ratio is the slope of the related linear function.
- There are several techniques for solving proportions (e.g., finding the unit amount, cross products).
- When you graph the terms of equal ratios as ordered pairs (first term, second term) and connect the points, the graph is a straight line.
- If two quantities vary proportionally, the quantities are either directly related (as one increases the other increases) or inversely related (as one increases the other decreases).
- Scale drawings involve similar figures, and corresponding parts of similar figures are proportional.
- In any circle, the ratio of the circumference to the diameter is always the same and is represented by the number  $\pi$ .
- Rates can be related using proportions as can percents and probabilities.

## **BIG IDEA #12**

**RELATIONS & FUNCTIONS: Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.**

### **Examples of Mathematical Understandings:**

- Mathematical relationships can be represented and analyzed using words, tables, graphs, and equations.
- In mathematical relationships, the value for one quantity depends on the value of the other quantity.
- The nature of the quantities in a relationship determines what values of the input and output quantities are reasonable.
- The graph of a relationship can be analyzed with regard to the change in one quantity relative to the change in the other quantity.

- The graph of a relation can be analyzed to determine if the relation is a function.
- In a linear function of the form  $y = ax$ ,  $a$  is the constant of variation and it represents the rate of change of  $y$  with respect to  $x$ .
- The solutions to a linear function form a straight line when graphed.
- A horizontal line has a slope of 0, and a vertical line does not have a slope.
- The parameters in an equation representing a function affect the graph of the function in predictable ways.

### **BIG IDEA #13**

**EQUATIONS & INEQUALITIES: Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.**

#### **Examples of Mathematical Understandings:**

- A solution to an equation is a value of the unknown or unknowns that makes the equation true.
- Properties of equality and reversible operations can be used to generate equivalent equations and find solutions.
- Techniques for solving equations start by transforming the equation into an equivalent one.
- A solution or solutions to a linear or quadratic equation can be found in the table of ordered pairs or from the graph of the related function.
- Techniques for solving equations can be applied to solving inequalities, but the direction of the inequality sign needs to be considered when negative numbers are involved.

### **BIG IDEA #14**

**SHAPES & SOLIDS: Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes.**

#### **Examples of Mathematical Understandings:**

- Point, line, line segment, and plane are the core attributes of space objects, and real-world situations can be used to think about these attributes.
- Polygons can be described uniquely by their sides and angles.
- Polygons can be constructed from or decomposed into other polygons.
- Triangles and quadrilaterals can be described, categorized, and named based on the relative lengths of their sides and the sizes of their angles.
- All polyhedra can be described completely by their faces, edges, and vertices.
- Some shapes or combinations of shapes can be put together without overlapping to completely cover the plane.
- There is more than one way to classify most shapes and solids.

### **BIG IDEA #15**

**ORIENTATION & LOCATION: Objects in space can be oriented in an infinite number of ways, and an object's location in space can be described quantitatively.**

#### **Examples of Mathematical Understandings:**

##### *Lines and Line Segments*

- Two distinct lines in the plane are either parallel or intersecting; two distinct lines in space are parallel, intersecting or skew.
- The angles formed by two intersecting lines in the plane are related in special ways (e.g., vertical angles).
- A number of degrees can be used to describe the size of an angle's opening.

- Some angles have special relationships based on their position or measures (e.g., complementary angles).
- In the plane, when a line intersects two parallel lines the angles formed are related in special ways.

### *Objects*

- The orientation of an object does not change the other attributes of the object.
- The Cartesian Coordinate System is a scheme that uses two perpendicular number lines intersecting at 0 on each to name the location of points in the plane; the system can be extended to name points in space.
- Every point in the plane can be described uniquely by an ordered pair of numbers; the first number tells the distance to the left or right of zero on the horizontal number line; the second tells the distance above or below zero on the vertical number line.

## **BIG IDEA #16**

**TRANSFORMATIONS: Objects in space can be transformed in an infinite number of ways, and those transformations can be described and analyzed mathematically.**

### **Examples of Mathematical Understandings:**

- Congruent figures remain congruent through translations, rotations, and reflections.
- Shapes can be transformed to similar shapes (but larger or smaller) with proportional corresponding sides and congruent corresponding angles
- Algebraic expressions can be used to generalize transformations for objects in the plane.
- Some shapes can be divided in half where one half folds exactly on top of the other (line symmetry).
- Some shapes can be rotated around a point in less than one complete turn and land exactly on top of themselves (rotational symmetry).

## **BIG IDEA #17**

**MEASUREMENT: Some attributes of objects are measurable and can be quantified using unit amounts.**

### **Examples of Mathematical Understandings:**

- Measurement involves a selected attribute of an object (length, area, mass, volume, capacity) and a comparison of the object being measured against a unit of the same attribute.
- The longer the unit of measure, the fewer units it takes to measure the object.
- The magnitude of the attribute to be measured and the accuracy needed determines the appropriate measurement unit.
- For a given perimeter there can be a shape with area close to zero. The maximum area for a given perimeter and a given number of sides is the regular polygon with that number of sides.

## **BIG IDEA #18**

**DATA COLLECTION: Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it.**

### **Examples of Mathematical Understandings:**

- An appropriately selected sample can be used to describe and make predictions about a population.
- The size of a sample determines how close data from the sample mirrors the population.



### **BIG IDEA #19**

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**DATA REPRESENTATION: Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.**

**Examples of Mathematical Understandings:**

- Each type of graph is most appropriate for certain types of data.
- Scale influences the patterns that can be observed in data.

### **BIG IDEA #20**

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**DATA DISTRIBUTION: There are special numerical measures that describe the center and spread of numerical data sets.**

**Examples of Mathematical Understandings:**

- The best descriptor of the center of a numerical data set (i.e., mean, median, mode) is determined by the nature of the data and the question to be answered.
- Outliers affect the mean, median, and mode in different ways.
- Data interpretation is enhanced by numerical measures telling how data are distributed.

### **BIG IDEA #21**

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**CHANCE: The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.**

**Examples of Mathematical Understandings:**

- Probability can provide a basis for making predictions.
- Some probabilities can only be determined through experimental trials.
- An event that is certain to happen will always happen (The probability is 1.) and an event that is impossible will never happen (The probability is 0.).

APPENDIX A:

*A Sample Chapter Analysis Using Standards and Big Ideas*

GRADE 3 CHAPTER 9: MULTIPLYING GREATER NUMBERS	
<b>CONTENT STANDARDS</b>	
<p><b>Focus</b> *NS 2.4: Solve simple problems involving multiplication of multi-digit numbers by one-digit numbers</p>	<p><b>Discussion</b> NS 2.4, Multiplying multi-digit numbers by 1-digit numbers, is the main focus of this chapter. Multiplicands up to four digits are used including money amounts such as \$43.98. The standard algorithm is developed with whole numbers and extended to money. All decimal quantities are related to money amounts.</p>
<p><b>Other</b> *NS 3.3: Solve problems involving operations with money amounts in decimal notation and multiply and divide money amounts in decimal notation using whole number multipliers and divisors</p>	
<b>BIG IDEAS</b>	
<p><b>Focus</b> + Algorithms: Algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.  + Operation Meanings &amp; Relationships: The same number sentence can be associated with different concrete or real-world situations, AND different number sentences can be associated with the same concrete or real-world situation.  + Properties: For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.</p>	<p><b>Discussion</b> The Big Idea of focus is Algorithms and it should be emphasized in every skill lesson. All calculations in this chapter involve changing the numerical expression to an equivalent one and breaking the calculation into simpler ones involving basic facts or 1-digit numbers times a multiple of 10, or 100. Lesson 9-1 develops the simpler calculations one needs to know for the other skill lessons in the chapter.  The array interpretation of multiplication is used to show how the standard algorithm involves breaking the calculation into simpler ones. The distributive property justifies the breaking apart process. For example, <math>3 \times 15 = 3 \times (10 + 5) = (3 \times 10) + (3 \times 5) = 30 + 5 = 35</math>. Notice that 15 is named in an equivalent way, <math>10 + 5</math>.</p>
<p><b>Other</b> + Equivalence: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.</p>	
<b>MATHEMATICAL REASONING</b>	
<p><b>Focus</b> MR 1.1 Analyze problems by identifying relationships, and so forth  MR 2.1 Use estimation to verify the reasonableness of results  MR 2.2 Apply strategies and results from simpler problems to more complex problems.</p>	<p><b>Discussion</b> Estimating products, developed in 9-4, should be emphasized in every lesson thereafter. Rounding might be used most often but compatible numbers also can be used (e.g., <math>2 \times 36</math> is about <math>2 \times 35</math>). The need for exact or approximate answers should be discussed.  Operation meanings should be emphasized. The repeated addition and array interpretations are emphasized throughout the chapter.  The Big Idea of Algorithms connects to MR 2.2, breaking a problem into simpler ones. This applies to the multiplication algorithm and to solving some types of word problems.  Students should generalize the multiplication process from 2-digit through 4-digit multiplicands; the process is the same, it is just repeated.</p>
<p><b>Other</b> MR 2.3 – 2.6, 3.0</p>	

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## Learner-Centered Teacher Leadership in Mathematics Education

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### Introduction

While mathematics education has been a key priority for professional development as long as we can remember, recently the push for increasing teachers' content and pedagogical knowledge mathematics has received increased attention and scrutiny at the local, state, and national levels. The stakes are higher. The expectations are higher. The purpose of this article is to provide a set of heuristics — rules of thumb — by which districts under fire can develop from within, the capacity for long-term positive change in mathematics teaching and learning.

The ideas we provide in this article stem from our own long-term reform effort, begun in Fall, 1994, to improve mathematics teaching and learning in a large, urban school district in Phoenix Arizona. We are happy to say that longitudinally, our efforts have resulted in grade by grade improvement in student achievement, and in general longitudinal change across all grade bands on the mathematics portion of the Stanford Achievement Test. Moreover, results on this and our state assessment (the Arizona Instrument to Measure Standards) reveal that achievement is not only improving, but it is at the highest level in comparison to other districts in the state, despite continually rising levels of poverty in the community. Details of our case can be found in (Middleton & Coleman, 2003) including student achievement data.

### First Principles

Two points must be made up front. First, the central orienting principle upon which we built our platform for

reform is that of increasing teachers' *understanding of children's mathematical thinking*. The most successful programs to date in the mathematics education literature were built on this premise: That the more teachers understand about the ways in which children interpret mathematical tasks, build informal knowledge about number, patterns, and relationships, and formalize that knowledge into skills and procedures, the better they are at providing appropriate tasks, questions, and feedback at appropriate times (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997; Fennema & Romberg, 1999). Second, *we rejected the trainer of trainers* model of professional development. A different (and better) model is that of a job-embedded, democratic, learner-centered leadership (e.g., Barth, 2001; Loucks-Horsley, Hewson, Love, & Stiles, 1998). Under such models, leaders are developed when departments or grade-level teams grapple with issues of student learning and how their own practices can contribute to that learning (e.g., Kennedy, 1999). Leadership development under this model can be defined as *the stimulation of the intellectual capacity of a district, aligned towards programmatic change* (adapted from McNamara, 1999). To enact such a definition, one must assume that the capacity for leadership exists in each and every one of the personnel in the district organization.

So, to share our experiences with colleagues in mathematics education supervision, we put forward several essential elements of a quality long-term professional learning program that prepares teachers to improve the mathematics learning ability of all students. We focus here on 1) Professionalizing the Role of the Teacher; and 2) Providing

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an Alternative to “Pyramid Schemes.” We end our discussion by projecting the Positive Outcomes of Learner-Centered Teacher Leadership in Mathematics Education.

### Professionalizing the Role of the Teacher

In the elementary and middle grades, the push for the development of deep content expertise in mathematics in teachers creates tremendous fiscal and human resource pressures. It is a fact of the political life of most districts that *more* teachers are expected to learn *more* about content and pedagogy in that content, in increasingly *deeper* and *more* coherent ways (Ball, 2002). To this we add, with *fewer and fewer resources*. But to constitute more than pie-in-the-sky thinking, this goal must be enacted in practice.

One way of developing this deep focus is to shift thinking about the role of the elementary school teacher from that of the multi-subject generalist to that of subject matter specialist in a subset of important curricular areas. Particularly in elementary schools, the model of the generalist teacher has not allowed teachers to become distinguished professionally from each other, to develop deep expertise in any one area of interest, or to take on the role of a leader in driving change in curriculum. Instead, curricular reform has been driven by textbook adoption cycles, state and local legislation, or administrator fiat.

In 1994, we began an extensive experiment, attempting to professionalize teaching in mathematics content. Through a combination of Federal Title monies and external grants, we were able to release grade-level Mathematics Teacher Leaders at each school in our district — time to work with their peers to deepen their understanding of children’s mathematical thinking, and its relation specifically to the district-adopted curriculum and the NCTM *Standards* (1989; 1991). Half-time release was a critical innovation. Results from other Systemic Initiatives showed that when leaders are released from their classroom duties full-time, they begin to lose credibility with their peers (i.e., they were no longer *teachers*). With their feet in the classroom half time, teacher leaders kept their own skills sharp and developed good “war stories” they could share with those they were helping in the other 50% of their time. Our Mathematics Teacher Leaders offered classroom demonstrations, peer coaching and other mentoring opportunities on an informal basis. Summer and Academic Year institutes in Algebra, Geometry, and Statistics were coordinated with the local University to enhance teachers’ mathematics content knowledge. Two of the Mathematics Teacher Leaders

obtained their Masters’ degrees during this time, and took over the content instruction from the professor of record (the first author of this paper), further enhancing their professional development and leadership capacity within the district.

The difficulties associated with this shift in culture must not be underestimated given the general attitudes of prospective teachers towards mathematics as a field (not too positive), and the current requirements for certification in most states that minimize the number of hours a teacher spends in any content area. However, providing each teacher with the opportunity to develop a personal area of expertise, i.e., a niche within the school and district culture is a key step towards teacher empowerment.

Pragmatically, it also allows for the development of adequate numbers of experts in mathematics so that each school in a district has a sufficient proportion of leaders driving the reform. With this in mind, as in our case, initial leaders can be identified at the building and grade level. Early adopters, converts, and even the healthy skeptics, who can articulate reforms in the language and practices of professional teachers are all critical to the successful long term structural change that is necessary for sustained reform (Middleton & Coleman, 2003).

This approach takes advantage of the extensive literature that shows that teachers-as-leaders do not exist on their own, but in a community of learners, devoted to better understanding content, pedagogy, and the institutional context within which their practices must be imbedded (DuFour & Eaker, 1998)

Lastly, this approach is developed in reaction to and stands in contradistinction to what we see as a current and prevalent attitude that ignores the power of ideas underlying public education. We maintain that disciplined practical knowledge is coherent, deriving on the best empirical evidence available. It is also generative, continuously striving for improvement as teachers grapple with new content and new contexts, and new student characteristics. Lastly, it must also be theoretical, in the sense that it is tied to a body of knowledge that explains *how* and *why* actions lead to particular forms of behavior and knowledge. This emphasis does not lessen the need for the “wisdom of practice,” rather it provides a structure by which this wisdom can be recognized and stimulated.

### Pyramid Schemes Don't Work!

The most common strategy for scale-up of reform in districts relies on what we call a *pyramid model* of diffusion — a trainer of trainers model — to exponentially increase the number and quality of teachers with particular expertise throughout large, particularly urban, school systems. The residue of these models (e.g., learnings and practices gleaned from the staff development experiences) often doesn't last for a very long time when the source of funding for the reform ends. Little personnel money is slated to maintain the release time, professional development experiences, and support of teachers identified as instructional leaders. As a result, these teachers go back full-time to the classroom, doing great work personally, but the entire structure of the professional development program collapses with no personnel to perform the tasks of instruction, mentoring, and curriculum development. Moreover, even with some sustained moneys, the nature of expertise shows that after only about two levels of trainers-of-trainers, the coherence of the original message becomes diluted and ineffective. A better approach is to begin with building- or grade-level *teams* that are charged to develop *their own* practice, and provide the highest level of support as needed for just-in-time learning. This practice takes longer, but has the potential for deeper, longer lasting change than the more sporadic workshop model.

Our Mathematics Teacher Leaders did not live outside the regular work day. They remained in their classes, teaching mathematics, and applying their own professional learning to the improvement of instruction. They also led grade-level teams in planning for instruction, sharing student strategies, and developing assessments to gauge their (the grade-level team's) success. Thus, when a MTL retired in 2001, another member of his team who had been mentored and supported as a second-tier leader was able to assume the leadership role. Leadership capacity must be built into the everyday interactions of identified leaders *and* potential leaders. This is job-embedded professional development that runs deeper and has (we think) potential for longer-lasting impact than a trainer-of-trainers model (Loucks-Horsley, Hewson, Love, & Stiles, 1998).

### Positive Outcomes of Learner-Centered Teacher Leadership in Mathematics

There are a number of critical reasons why a local colleague, who is a peer at the building or grade level, is a more credible and more effective staff developer, in the long run, than either an outside expert, or an expert at the

administrative level of a district. These reasons center on the place of a teacher in the local community, and the place of administration (particularly in the current politics of urban school districts).

**Sustained Reform Over Time.** As alluded to earlier, one of the key failures of systemic reform efforts is the inability to institutionalize and sustain the initiated reforms beyond the typical 5-year lifespan of most local, state, or federal projects. If continued activity is in fact a goal, there must be some administrative structure that 1) embeds the key learnings incurred in the professional development project in the building and grade levels; and that 2) provides a feedback loop regarding the success of the reforms to the project as a whole. With the size of modern districts constituting multiple schools with potentially hundreds of teachers, some personnel that have *direct access to each classroom on a regular basis* are required to staff such an administrative structure. When faced with the further constraint on the limited number of people with *both* the subject-matter expertise, *and* legitimacy in the eyes of the in-the-trenches practitioners, the pool of potential people to make up this staff is limited to teachers and a few experts at the district level and perhaps at local higher education institutions. A final constraint, cost, predicates that the structure for sustained activity in systemic reform be made up of current district employees—teachers.

Releasing our MTLs half-time maintained continuity in mathematics instruction for their own students thus insuring higher test scores, kept MTL's teaching skills honed, and as we said earlier, kept them legitimate in the eyes of their peers. The half-time release was also relatively cost-effective. By blocking special subjects, utilizing teaming, and by augmenting district funds with Federal dollars, the district was able to sustain at least two MTLs in each school (serving 2 grade levels each) for eleven years. Our data also suggest that it also kept MTLs and other teachers in district despite intense competition from neighboring schools.

Moreover, the development of local experts who have an investment in the community and institution is more likely to afford continued activity than hiring a set of paid consultants from the outside that bop in very once in a while. Our teachers by and large live and work in, or at least have a professional investment in, the communities within which their schools are located. And, while teacher mobility across districts is becoming more and more of a

staffing problem, our experience suggests that there are still large numbers of teachers who remain in district for extended periods of time, sustaining the institutional knowledge of the reform beyond the life of external funding.

***Transcending the Revolving Door Administrator.*** While acknowledging teacher mobility to be a difficult problem, the bigger problem in school leadership today is the tendency for high-level administration to move or leave office in 3- to 5-year cycles (i.e., a rate of approximately 30% each year) (Gates, Ringel, & Santibanez, 2003). As a result of this turnover, when teachers are faced with new mandates, policies, procedures and personalities regularly, they tend to perform their duties *in spite of* administration, with an attitude of “this too shall pass.” (Middleton & Webb, 1994).

In contrast to administrators, teachers move or leave the profession at an annual rate of only 15% nationally (Tabs, 2004). Research suggests that teachers who are provided leadership opportunities, ongoing professional development and who receive some material reward for their role as leader are less likely to move than the general population of teachers (Institute for Educational Leadership, 2001). We suggest that because of overall stability within districts, teacher leaders, may, if identified and supported properly, provide a more stable infrastructure upon which to hang systemic reform than say, superintendents and principals.

In our own case, we have had two associate superintendents, two superintendents, and several principals leave office during the 10 years of our reform efforts. We still have MTLs who joined up from the very beginning, and more importantly, we have the institutional capacity now to build new leaders from our junior ranks.

***Embeddedness in the Community.*** High quality teachers have the legitimacy to enact reforms which may be at first controversial, by virtue of their connectedness with parents. Both the fact that teachers may encounter multiple children from the same family year after year, and their presence in community affairs, makes them key brokers of information about reform and key advocates for the district.

***Authority by Virtue of Experience.*** While there are numerous cases of young and inexperienced teachers becoming leaders, our work leads us to characterize this as the exception rather than the rule. The level of experience working in classrooms with the same characteristics as

others in the school or districts is taken seriously by teachers, and they hold a healthy skepticism of any new reform proposed by someone who hasn't actually tried to implement it under authentic conditions of teaching. Moreover, as the community becomes more attuned to the difficulties new teachers experience during their first few years in the classroom, the natural leadership (both good and bad) that an experienced teacher can exert over the inexperienced colleague is powerful. It seems profitable, then, to harness this natural apprenticeship, identify good role models and support them with high quality experiences, tools, and materials. This influence may also be important for experienced teachers who are new to a school and who could use information about curriculum, available technologies, district expectations and philosophy, and school culture.

Our model builds this mentoring and support into the MTL job-description in a sustainable manner. But it is not the *role* we find most important, but the nurturance of a culture of support across all grade levels and subject-matter. Grade-level teams have time to meet and plan. District-level leaders, including MTLs, principals, and central administration, meet regularly, attend professional development sessions, and plan for the future. This model also is in place for literacy and is beginning for science.

***Capability for Moving Administration.*** As we speak of influence, the potential impact of teacher leaders on the coherence and consistency of service in the district, given the increased mobility of administrators, cannot be understated. In numerous districts we have worked with, the core leadership among the teaching cadre remains stable across multiple administrations, which often bring competing agendas that may countermand any current direction of reform. Teacher leaders, as successful agents of reform have been approached by new administrators for guidance in the implementation of new policies, for identification of appropriate sites for action, and for communication of new directions to the general district faculty. In some instances, initiative by the teachers in the district may actually provide impetus for administration to change or enact new policies, curriculum cycles, or priorities for professional development.

## Conclusions

In an earlier publication the first author described the notion that a system as complex as a public school district should have as a goal, coherence at all levels of the system, from classroom teacher to Superintendent (Middleton,



Sawada, Judson, Bloom, & Turley, 2002). The goals of the reform, even if they are fluid and evolving, must be understood by all, and their place in the overall support structure must be embraced. The push for reform in school mathematics is no different. This has been a key national priority for nearly fifteen years. While the entire education system is expected to align with these (sometimes conflicting) national goals it is the classroom teacher who has the responsibility for articulating the diverse sources of information, designing effective instructional strategies to meet the standards, and providing diagnostic, remedial, and advanced mathematical experiences for an ever more diverse student body.

Given the economics of professional development, and given the pressures of the No Child Left Behind Act, new models for leadership development are critical for deep and lasting educational improvement. In our experience, key learnings underscore the need for teacher leaders to continue to practice their own craft in the classroom, to utilize resources from higher education, and to plan for the long haul. In particular, we challenge the notion that “trainer-of-trainer” models are superior to “learning community” models. Briefly, trainer-of-trainer models assume that leaders are knowledge disseminators as opposed to mentors, colleagues, or coaches, i.e., as separate from people who are struggling under the same conditions of

practice as the classroom teacher. Moreover, the impact of trainer-of-trainer models degrade quickly as successive generations of trainers diffuse the original message of the professional development. The model of Learner-Centered Teacher Leadership we have developed is one alternative model for achieving this national goal under the real conditions of local implementation.

Finally, none of our modest inroads could have come to pass without the influence of a visionary leader at the district-level. Kay (the second author) maintained high expectations, focused the use of district professional development resources to begin the work, and acquired external dollars to support large scale change across the district. The mandate to achieve more in an economy with fewer resources is a critical dilemma for all educational leaders.

The achievement outcomes of our work show an improvement of 11 percentile ranks over the years 2000 through 2003. Given that this improvement really started to appear six years into our efforts, appropriate time scales for improvement given current mandates and sanctions must be considered carefully. An important discussion for leaders is how to manage both the short term time needs *and* the long term coherence in the face of ever greater challenges, shorter deadlines and trimmed budgets.

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