Teaching Integrated Math & Science (TIMS) Project
Learning Sciences Research Institute

University of Illinois at Chicago

http://www.lsri.uic.edu/
Research to Practice: You Know What They Know, So, Now What?

Jennifer Mundt Leimberer (jleimb1@uic.edu)
Teaching Integrated Mathematics and Science (TIMS) Project
Learning Sciences Research Institute, University of Illinois at Chicago

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Indianapolis, Indiana
April 12, 2011
Discussion Question:

What can you learn about what students know and can do from this problem?
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You Know What They Know, So, Now What?

Session Overview

Introduction: A look at multiplication

Part 1: Framework for defining “What they Know”

Part 2: Design rational for “So, Now What” solutions
Math Trailblazers
Research and Revision Study

2006–2009 Revision and field test of new materials in grades 1–5
2008–2010 Student Achievement Study
2009–2012 Final revision of materials for publication
Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Fractions and Ratios–UMN 2004–2006
- Video Study–UIUC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC Current
<table>
<thead>
<tr>
<th>Data Sources*</th>
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<tbody>
<tr>
<td>Teacher Surveys</td>
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<td>Classroom Observations</td>
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<td>Teacher Interviews</td>
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<td>Student Interviews*</td>
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<td>Consultant Reviews</td>
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<td>(math, geometry, assessment¹)</td>
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<td>Feedback Meetings</td>
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Plus student work samples of open-response questions, end-of-year tests, labs, and other activities

¹(Pellegrion and Goldman, 2007)
Assessment Strengths and Weaknesses from CPS Review and Analysis

Strengths

• Reflects content in NCTM’s *Principles and Standards*

• Provides a balance of assessment types (e.g. tests, quizzes, projects (labs), open-response problems, and observations)

• Assesses students on what they were taught and represents activities in the curriculum

• The Assessment Indicators provide a developmental framework that can be used to evaluate student progress

• Answer keys or sample student work often provide performance criteria
Assessment Strengths and Weaknesses from CPS Review and Analysis

Weaknesses

• Few assessments require student to make mathematical connections within the assessment
• Does not provide many opportunities for self- or peer-assessment
• Only open-response questions include scoring guides and procedures
• None of the assessments make specific recommendations to support special needs students
• More structure is needed to assist teachers in making decisions based on the results of assessments

(Pellegrino & Goldman, 2004a, 2004b, 2007)
Context of Response to Intervention (RtI)

• Tier 1 occurs at the whole-class level and relies on the differentiation of instruction in the general curriculum.

• Tier 2 occurs in smaller groups and tailors instruction based on targeted student needs.

• Tier 3 occurs with students requiring individualized intervention.
Appropriate Use of Assessment: What to Avoid

“Today was definitely a day of inappropriate assessment in the school where I teach. We are starting [a Response to Intervention] program this year, and we all had to give our first round of Curriculum-Based Measures, which was a timed test with 35 straight-up computation problems—addition, subtraction, multiplication, division. We’re supposed to use our students’ scores to place them in groups for doing problem solving with fractions. So my question is, what does zipping through 35 basic computations have to do with solving problems? That test did not tell me anything about what my kids can do with problem solving, or fractions, or what they need to do to get better at either of those things.”
What Is Needed to Inform Instruction

- What does it look like to “Get it”?
- What has happened to help students “Get it”?
- So, now what should happen?
What Is Needed to Inform Instruction

What does it look like to “Get it”? 

• Create explicit expectations;
• Assess those expectations;
  Provide tasks that “force the issue” and attends to interconnections
  Provide tools and routines for feedback

A more in-depth analysis asks teachers to be aware of the “big ideas” in mathematics and then to connect the identified standards to these ideas.

-- Dacey & Lynch(2007)
Grade 4 Unit 11
Multiplication with Larger Numbers

Expectations
Use this list of expectations to assess students on the key concepts and skills in this unit. Students are able to do the following:

E1.* Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digit by 2-digit numbers (e.g., use the distributive property to multiply). [Algebra 4]

E2.* Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangle model for multiplication).

E3.* Estimate products of multidigit numbers.

E4.* Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, all-partial).

E5.* Multiply 2-digit by 2-digit numbers using the compact method.

E6.* Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.

E7.* Demonstrate fluency with the division facts for the last six facts.

E8.* Write the number sentences in the fact families for the last six facts.

* Denotes Benchmark Expectation
# Unit 11 Key Assessment Opportunities Chart

## Content

### Key Ideas in Unit 11

<table>
<thead>
<tr>
<th>Number</th>
<th>Operations: Understand the meaning of numerical operations and their application for solving problems.</th>
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<tbody>
<tr>
<td>2</td>
<td>E1*: Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digits by 2-digits numbers (e.g., use the distributive property to multiply) [Algebra 4]</td>
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<td>E2*: Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangle model for multiplication)</td>
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<td>E3*: Estimate products of multidigit numbers.</td>
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<td>E4*: Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, all-partial)</td>
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<td>E5*: Multiply 2-digit by 2-digit numbers using the compact method.</td>
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<td>E6*: Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.</td>
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### Unit 11 Expectations

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<td>111</td>
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<tr>
<td>112</td>
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<tr>
<td>120</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

* Denotes Benchmark Expectation
* Includes a Feedback Box
# Math Facts

<table>
<thead>
<tr>
<th>Number</th>
<th>Computation and Estimation: Use efficient and flexible procedures to compute accurately and make reasonable estimates.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E7</td>
<td>Demonstrate fluency with the division facts for the last six facts.</td>
</tr>
<tr>
<td>EB</td>
<td>Write the number sentences in the fact families for the last six facts.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E7</th>
<th>E8</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

# Problem Solving and Communication

<table>
<thead>
<tr>
<th>Problem Solving</th>
<th>SG All Porjects Revised Check In Q 18</th>
<th>DAE Drawing in 1994</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS81 Know the problem.</td>
<td>I read the problem carefully. I know the questions to answer and what information is important.</td>
<td>X</td>
</tr>
<tr>
<td>PS82 Find a strategy.</td>
<td>I choose a good and efficient strategy for solving the problem.</td>
<td>X</td>
</tr>
<tr>
<td>PS83 Check for reasonableness.</td>
<td>I look back at my solution to see if my answer makes sense. If it does not, I try again.</td>
<td>X</td>
</tr>
<tr>
<td>PS84 Check my calculations.</td>
<td>If I make mistakes, I correct them.</td>
<td>X</td>
</tr>
<tr>
<td>PS85 Show my work.</td>
<td>I show or tell how I arrived at my answer so that someone else can understand my thinking.</td>
<td>X</td>
</tr>
<tr>
<td>PS86 Use labels.</td>
<td>I use labels to show what numbers mean (such as 15 boys or 6 inches).</td>
<td>X</td>
</tr>
</tbody>
</table>

* Denotes Benchmark Expectation
** Includes a Feedback Box
What Is Needed to Inform Instruction

What does it look like to “Get it”?

- Create explicit expectations;
- Assess those expectations;

  Provide tasks that “force the issue” and attends to interconnections

  Provide tools and routines for feedback

A more in-depth analysis asks teachers to be aware of the “big ideas” in mathematics and then to connect the identified standards to these ideas.

-- Dacey & Lynch (2007)
# Unit 11 Key Assessment Opportunities Chart

## Content

<table>
<thead>
<tr>
<th>Key Ideas in Unit 11</th>
<th>11: DAB Solving Multiplication Problems Check-In Q#1-4</th>
<th>12: UBG Multiplication Quiz 1**</th>
<th>13: SG Multiplication with 2-digit Numbers Check-In Q#11</th>
<th>14: UBG Multiplication Quiz 2**</th>
<th>15: SG Compact Multiplication Revised Check-In Q#19-20</th>
<th>16: SG Checking Strategies to Multiply Check-In Q#23-27</th>
</tr>
</thead>
</table>

### Unit 11 Expectations

#### Number 2 Operations: Understand the meaning of numerical operations and their application for solving problems.

- **E1**: Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digit by 2-digit numbers (e.g., use the distributive property to multiply) (Algebra 4)
- **E2**: Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangular model for multiplication)  

#### Number 3 Computation and Estimation: Use efficient and flexible procedures to compute accurately and make reasonable estimates.

- **E3**: Estimate products of multidigit numbers.
- **E4**: Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, partials).
- **E5**: Multiply 2-digit by 2-digit numbers using the compact method.
- **E6**: Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.

---

* Denotes Benchmark Expectation  
** Includes a Feedback Box
Discussion Question:

What can you learn about what students know and can do from this problem?
The Confused Contessa solved the problems in Questions 7, 8, and 9. Estimate to see if her answers are reasonable. Find her mistakes and draw a circle around the incorrect part. Then, solve the problems correctly using her method.

### Problem 7

**Contessa’s Work**

\[
\begin{align*}
63 &= 60 + 3 \\
\times 27 &\quad 20 + 7 \\
21 &\quad 7 \times 3 \\
420 &\quad 7 \times 60 \\
140 &\quad 20 \times 7 \\
\end{align*}
\]

\[
\underline{641}
\]

**Your Estimate**

**Your Solution**

### Problem 8

**Contessa’s Work**

\[
\begin{align*}
95 &\quad 90 \\
\times 31 &\quad 5 \\
\end{align*}
\]

\[
\begin{array}{ccc}
30 & 30 \times 90 = 270 & 30 \times 5 = 150 \\
1 & 1 \times 90 = 90 & 1 \times 5 = 5 \\
\end{array}
\]

\[
\begin{align*}
270 &\quad 150 \\
&\quad 90 \\
\end{align*}
\]

\[
\begin{align*}
\frac{515}{5} \\
\end{align*}
\]

**Your Estimate**

**Your Solution**
## Multiplication Quiz 1 Feedback Box

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Expectation</th>
<th>Check In</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show how to use place value in multiplication [Q#7,8].</td>
<td>E1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate products [Q#6A].</td>
<td>E3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply multidigit numbers [Q#1–12].</td>
<td>E4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Using mental math [Q#6B]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Using rectangles [Q#8]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Using expanded form [Q#7]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Using the all-partial method [Q#9]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Choose appropriately from among mental math strategies and paper-and-pencil methods to multiply multidigit numbers [Q#1–12].</td>
<td>E6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What Is Needed to Inform Instruction

What does it look like to “Get it”?  
• Create explicit expectations;  
• Assess those expectations;  
  Provide tasks that “force the issue” and attends to interconnections  
  Provide tools and routines for feedback  

A more in-depth analysis asks teachers to be aware of the “big ideas” in mathematics and then to connect the identified standards to these ideas.  
-- Dacey & Lynch (2007)
What Is Needed to Inform Instruction

- What does it look like to “Get it”?  

- What has happened to help students “Get it”?

- So, now what should happen?
What Is Needed to Inform Instruction

What has happened to help students “Get it”?

What representations have they been using?
What have they been asked to do with them?
How did students “enter” this next string of content?
The Iceberg Model

Adapted from: Webb, Boswinkle, & Dekker, 2008
“The iceberg model supports teacher thinking about learning processes and strategies used by students (Boswinkle & Morelands, 2001). This model has proven to be a powerful metaphor for illustrating how students need to experience a broad range of mathematical models to make sense of formal mathematical representations.”

- Webb, Boswinkle, & Dekker, 2008
“To find one’s way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students’ conceptual understanding is related to the richness and extent of the connections they have made.”

- National Research Council, 2001
**Multiplication Strategies Menu**

**Breaking into tens and ones**

**Using Expanded Form**

\[
\begin{array}{c}
23 \\
\times 6
\end{array} =
\begin{array}{c}
20 \\
\times 6
\end{array} +
\begin{array}{c}
3 \\
\times 6
\end{array}
\]

\[
= 120 + 18 = 138
\]

or

\[
\begin{array}{c}
20 \\
6 \times 20 = 120
\end{array} +
\begin{array}{c}
3 \\
6 \times 3 = 18
\end{array}
\]

\[
= 120 + 18 = 138
\]

**Other ways to use simpler problems**

**Thinking about money**

\[
27 \times 4 = 25 \times 4 + 2 \times 4 = 100 + 8 = 108
\]

**Using Simpler Numbers**

\[
48 \times 6
\]

I know \(48 + 2 = 50\).
So, \(50 \times 6 = 300\) and \(2 \times 6 = 12\).
Then I subtracted \(300 - 12 = 288\).

**Compact Method**

\[
\begin{array}{c}
23 \\
\times 6
\end{array}
\]

\[
= 138
\]
### Multiplication Strategies Menu for Larger Numbers

#### Using Rectangles

\[
\begin{array}{c|c|c|c}
6 & 620 & 20 & 3 \\
\hline
6 \times 600 & 3600 & 6 \times 20 & 120 \\
6 \times 6 & 36 & 6 \times 3 & 18 \\
\end{array}
\]

\[
\begin{array}{c}
6 \times 600 = 3600 \\
6 \times 20 = 120 \\
6 \times 3 = 18 \\
\hline
3600 + 120 + 18 = 3738
\end{array}
\]

#### Using Expanded Form

\[
623 = 600 + 20 + 3 \\
\times 6
\]

\[
\begin{array}{c|c|c|c|c|c}
623 & \times & 6 & 3600 & 120 & 18 \\
\hline
600 & \times & 6 & 3600 & 120 & 18 \\
20 & \times & 6 & & & \\
3 & \times & 6 & & & \\
\hline
3600 & 120 & 18 & 3738
\end{array}
\]

#### Using All-Partials

\[
\begin{array}{c|c|c}
623 & \times & 6 \\
\hline
18 & 120 & 3600 \\
3600 & 120 & 18 \\
\hline
3738 & 3738 & 3738
\end{array}
\]

or

\[
\begin{array}{c|c}
623 & \times \\
\hline
120 & 3600 \\
3600 & 120 \\
\hline
3738 & 3738
\end{array}
\]

#### Using the Compact Method

\[
\begin{array}{c}
623 \\
\times 6 \\
\hline
3738
\end{array}
\]

#### Thinking About Money

\[
127 \times 4
\]

I think: 4 dollars + 4 quarters + (2 \times 4) pennies

\[
\begin{array}{c}
100 + 25 + 2 \\
\hline
400 + 100 + 8
\end{array}
\]

= 508

#### Using Simpler Numbers

\[
298 \times 4
\]

I know 298 + 2 = 300.

So, 300 \times 4 = 1200 and 2 \times 4 = 8.

Then I subtracted 1200 – 8 = 1192.

#### Halving and Doubling

\[
264 \times 5
\]

I know multiplying by 10 is easier than multiplying by 5. I double 5 to 10 and I take \(\frac{1}{2}\) of 264, which is 132.

\[
132 \times 10 = 1320.
\]

Or I could multiply 264 \times 10 = 2640 and take half of that: 1320.
Multiplication Strategies Menu
Paper-and-Pencil Methods

Using Rectangles

<table>
<thead>
<tr>
<th>7 × 326</th>
<th>28 × 63</th>
<th>60</th>
<th>3</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>300</td>
<td>20</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>7 × 300 = 2100</td>
<td>20</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>140</td>
<td>7 × 6 = 42</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>140</td>
<td>20 × 60 = 1200</td>
<td>20</td>
<td>3</td>
<td>1200</td>
</tr>
<tr>
<td>42</td>
<td>20 × 3 = 60</td>
<td>3</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>2282</td>
<td>8 × 3 = 24</td>
<td>1200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8 × 60 = 480</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All Partials

<table>
<thead>
<tr>
<th>326</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td>× 7</td>
<td>× 28</td>
</tr>
<tr>
<td>2100</td>
<td>1200</td>
</tr>
<tr>
<td>140</td>
<td>60</td>
</tr>
<tr>
<td>+ 42</td>
<td>60</td>
</tr>
</tbody>
</table>

Expanded Form

63 = 60 + 3
63 × 28 = 20 + 8

<table>
<thead>
<tr>
<th>326</th>
<th>300 + 20 + 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>× 7</td>
<td>7</td>
</tr>
<tr>
<td>2100 + 140 + 42 = 2282</td>
<td></td>
</tr>
</tbody>
</table>

Compact

326
× 7
2282

Combination

63
× 28
504

63
× 20
1260

63
× 8
504

1260 + 504 = 1764
**Multiplication Strategies Menu**

**Mental Math and Estimation**

**Using Simpler Numbers**

\[ 205 \times 8 = (200 \times 8) + (5 \times 8) \]
\[ = 1600 + 40 \]
\[ = 1640 \]

\[ 72 \times 99 = (72 \times 100) - 72 \]
\[ = 7200 - 72 \]
\[ = 7128 \]

**Using Convenient Numbers to Estimate**

\[ 63 \times 28 \]
\[ = \frac{60 \times 30}{1800} \]

1800 is a reasonable estimate.

**Halving and Doubling**

\[ 264 \times 5 \]

I know multiplying by 10 is easier than multiplying by 5. I double 5 to 10 and take \( \frac{1}{2} \) of 264 which is 132. \( 132 \times 10 = 1320 \). Or I could multiply 264 \( \times 10 = 2640 \) and take half of that: 1320.

**Finding a Range**

\[ 45 \times 65 \]

\[ 40 \times 60 = 2400 \]
\[ 50 \times 70 = 3500 \]

The exact answer will be between 2400 and 3500, or about 3000.
What Is Needed to Inform Instruction

• What does it look like to “Get it”? 

• What has happened to help students “Get it”? 

• So, now what should happen?
What are typical “so, now what” strategies?

- Lots of practice of $dd \times dd$ problems
- “Going through the same door” again and again and still not getting it...
- Grouping students by general ability
- Nothing...teachers just keep going
- Messages to teachers at the end of the unit in little boxes
What does research say about “now what”? 

• Provide instruction that explicitly helps connect models, representations, and concepts. (Fuson, 1992; Ginsburg, 1997)

• Provide (but guide) student choices of strategies. (Gersten and Chard, 2001)
What does research say about “now what”? 

- Practice on computational procedures should be designed to build on and extend understanding. (National Research Council, 2001)
- Develop shorter, carefully constructed problem sets because they may be more effective in helping develop fluency in facts and procedures. (Diezman et al, 2003)
What does research say about “now what”?

- Emphasis on thinking strategies (National Research Council, 2001)
- Develop shorter, carefully constructed problem sets because they may be more effective in helping develop fluency in facts and procedures. (Diezmann et al, 2003)
What does research say about “now what”? 

- Facilitate opportunities that provoke a stumble due to superficial understanding 
- Ask students to analyze other’s solutions 
- Ask students to reason with tools 
- Scaffold opportunities that help students uncover mathematical ideas 
- Need to enter content through a different door
Our “so, now what” design goals

• Put the practice and intervention where it is needed
• Build time right in (rather than Part 3 or in the boxes)
• Create models/tasks that are transportable
• Include “bottomless pits” of practice
• Something for everyone
• Create Targeted Practice based on expectations
# Grade 4 Unit 11 Outline

## Multiplication with Larger Numbers

### Lesson 1

**From the Fish Hatchery**

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>ACTIVITY: Students extend their understanding of multiplication concepts to multiplication of 2-digit by 2-digit numbers. They use mental math, rectangles, and expanded form to find solutions. <strong>EXPECTEDS:</strong> E1, E2, E3, E4, E5, E6, E7</td>
</tr>
</tbody>
</table>

**Supplies**

- SG pages 404-407
- DAB pages 581-592
- HP Parts 1-2
- URG pages 33-35
- DPP A-D

### Lesson 2

**All-Partial Revisited**

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>ACTIVITY: Students explore another paper and pencil method for multiplication of 2-digit by 2-digit numbers; the all-partial method. They compare this method to using the rectangle model and the expanded form. <strong>EXPECTEDS:</strong> E1, E2, E3, E4, E5, E6, E7, PSE1, PSE3, PSE5</td>
</tr>
</tbody>
</table>

**Supplies**

- SG pages 408-413
- DAB page 593
- HP pages 56-76
- URG E-H

### Lesson 3

**Workshop: Multiplication with Two-Digit Numbers**

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>ACTIVITY: Students practice solving multiplication problems involving 2-digit numbers. This Workshop allows students to choose from several problem sets and a game to review multiplication concepts and practice multiplication methods. <strong>EXPECTEDS:</strong> E1, E2, E3, E4</td>
</tr>
</tbody>
</table>

**Supplies**

- Multiplication Strategies Menu created in Lesson 2

### Lesson 4

**Compact Multiplication Revisited**

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>ACTIVITY: Students review the compact method of multiplication and then use it to multiply by two-digit numbers. They compare all paper-and-pencil methods and practice multiplication. <strong>EXPECTEDS:</strong> E1, E2, E3, E4, E5, E6, E8</td>
</tr>
</tbody>
</table>

**Supplies**

- SG pages 417-423
- DAB pages 96-116
- DPP K-N

### Lesson 5

**Choosing Strategies to Multiply**

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>ACTIVITY: Students apply strategies involving estimation, mental math, and paper-and-pencil methods to solve problems involving two-digit multiplication. This lesson focuses on choosing and applying strategies flexibly to match the context of problems. Students also use these strategies to solve an extended, open-response problem involving two-digit multiplication. <strong>EXPECTEDS:</strong> E3, E4, E5, E6, E7, PSE2, PSE3, PSE5</td>
</tr>
</tbody>
</table>

**Supplies**

- Multiplication Strategies Menu created in Lesson 2
- Rulers
- Division Facts / K Now charts

### Connections

A current list of connections is available at www.mathtrailblazers.com.

### Literature

Targeted Practice

All students need…

• Attention to conceptual underpinnings of the content and skills, and

• Opportunities to articulate concepts and communicate solutions.
Targeted Practice

1) Students who are “working on it” and need some extra help should circle the problem set marked with a star (★). These problems provide scaffolded support for developing the essential underlying concepts as well as some opportunities for practice.

2) Students who are “getting it” and just need more practice should circle the problem set marked with a circle (●). These problems mainly provide opportunities to practice with some concept reinforcement and some opportunities for extension.

3) Students who have “got it” and are ready for a challenge or extension should circle problems marked with a square (■). These problems provide some practice and then move into opportunities for extension.
## Multiplication with 2-Digit Numbers Workshop Menu

- Look at each row in the Can I Do This? column.
- For each row, decide whether you are “Working On It”, whether you are “Getting It”, or whether you already know it (Got It).
- Remember, you may feel you are “Working On It” for one row, but for another row, you know you have already “Got It.”
- On this page, draw a circle around each set of problems you decide to do.
- If one set of problems seems too easy or too hard, choose a different set from the same row.

<table>
<thead>
<tr>
<th>Can I Do This?</th>
<th>Working On It!</th>
<th>Getting It!</th>
<th>Got It!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show that I understand multiplication methods</td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
</tr>
<tr>
<td>Make connections between methods</td>
<td>★ Q# 1–3</td>
<td>★ Q# 2–4</td>
<td>□ Q# 3–4</td>
</tr>
<tr>
<td>Estimate products</td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
</tr>
<tr>
<td></td>
<td>★ Q# 12, 13, 21–24</td>
<td>□ Q# 12, 13, 21–26</td>
<td>□ Q# 13, 21, 25–27</td>
</tr>
<tr>
<td>Use different methods to multiply 2-digit numbers</td>
<td>Student Guide</td>
<td>Student Guide</td>
<td>Student Guide</td>
</tr>
<tr>
<td></td>
<td>★ Q# 1–6, 8, 11</td>
<td>★ Q# 1–6, 8, 11</td>
<td>★ Q# 1–6, 8, 11</td>
</tr>
<tr>
<td></td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
<td>Discovery Assignment Book</td>
</tr>
<tr>
<td></td>
<td>★ Q# 5–8, 12–15, 20</td>
<td>□ Q# 7–9, 11–18, 20</td>
<td>□ Q# 7–11, 13, 16–20</td>
</tr>
</tbody>
</table>

- Play Multiplication Digits Game

Pretend you are a student learning this content, choose your practice.
1. Irma solved $12 \times 52$ using rectangles.

   \[
   \begin{array}{c|c|c}
   & 10 & 2 \\
   \hline
   50 & 50 \times 10 = 500 & 2 \\
   2 & 50 \times 2 = 100 & 4 \\
   \hline
   & 10 \times 2 = 20 & 500 + 100 + 20 + 4 \\
   & \hline
   \end{array}
   \]

   $624$

   A. Use Irma’s rectangle to fill in the blank boxes for the same problem using the all-partial method.

   B. What numbers did Irma multiply in both problems to get 500?
■ 4. Professor Peabody’s cat had muddy feet and walked across some problems. Fill in the missing spots to show each solution correctly.

A. $156 \times 42$

\[
\begin{array}{c|cc|c}
40 & 30 & 4000 \\
100 & 50 & 40 \times 50 = \\
2 & 2 \times 12 & \\
\hline
& & + 4000 \\
\hline
\end{array}
\]

B. $64 \times 18$

\[
\begin{array}{c}
64 \\
\times 18 \\
\hline
10 + 8 \\
\hline
600 \\
\hline
40 \\
\hline
32 \\
\hline
1152
\end{array}
\]

C. $89 \times 46$

\[
\begin{array}{c}
360 \\
480 \\
54 \\
\hline
1152
\end{array}
\]

47
Math Practices

Solving a problem:

1. Know the problem. I read the problem carefully. I know the questions to answer and what information is important.

   The question tells me... I need to find out...

   Grace

2. Find a strategy. I choose good tools and an efficient strategy for solving the problem.

   Is there a pattern? Manipulative
   What tools should I use?
   What operations should I use?
   Is there a more efficient way?

   John

3. Check for reasonableness. I look back at my solution to see if my answer makes sense. If it does not, I try again.

   I used convenient numbers to estimate like this...
   Then I compared my answer to my estimate.

   Max

4. Check my calculations. If I make mistakes, I correct them.

   To check this: 715 - 250 = 465
   I did this: 265 + 350 = 715

   I check my calculations with a calculator.

   Maria

Showing or telling how I solve a problem:

5. Show my work. I show or tell how I arrived at my answer so that someone else can understand my thinking.

   Step 1...
   Step 2...
   Step 3...

   I show each step of my work using:
   Numerical sentence: 56 + 13 = 77 chairs
   Then I explained the chain of numbers.

   Sam

6. Use labels. I use labels to show what numbers mean.

   $5
   2 cars
   36 inches
   155 miles
   4 apples
   9 days

   Sam
Hour Walk

If you walked steadily for an hour, about how many steps would you take?

A. 500     B. 1000     C. 5000     D. 10,000     E. 50,000     F. 100,000

Make an estimate without walking for one hour. Explain how you made your estimate. Show all your work.
We decided to add 5,000 more heartbeats to the 10,000 already in the analysis. This increased the number of heartbeats to 15,000. After analyzing the data, we noted that the heartbeats were consistent and there were no abnormalities.
Keenya’s Work

8,000 is closest to our answer

1,059 steps 105 steps

10:00 min 1:00 min.

My partner and I walked for 10 min. We both got exactly 1,059 steps since we both have the same answer we don’t have to average out our work. 10 x 6 = 60 and thus are 60 steps in an hour. We multiply 10,59 x 0 = 6354. Step that’s our answer.

If we walked 1 hour we also walked for 1 min. Again we got the same answer. It was 105 steps. 1 x 60 = 60 x 60 = 60 steps we multiplied 105 x 60 = 6350 steps that’s our other answer for how many steps it would take for 1 hour. We know we are close because the answers are only 4 steps away we can find a better answer if we average out our answers.

Answers

<table>
<thead>
<tr>
<th>Time</th>
<th>Steps</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>1059</td>
<td>6354</td>
</tr>
<tr>
<td>60 min</td>
<td>6350</td>
<td>6354</td>
</tr>
</tbody>
</table>

First we added them we divide 12,704 = 2 6,352. This is the best answer we can come up with.
Research to Practice: You Know What They Know, So, Now What?

Jennifer Mundt Leimberger (jleimb1@uic.edu)
Teaching Integrated Mathematics and Science (TIMS) Project
Learning Sciences Research Institute, University of Illinois at Chicago
www.lsri.uic.edu go to Projects then go to
Math Trailblazers Research and Revision Study

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