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OF MATHEMATICS EDUCATION LEADERSHIP

USEFUL, EASY, AND CONSEQUENTIAL: A PRACTICAL MEASUREMENT REPOSITORY TO ENHANCE THE WORK OF MATH EDUCATION INSTRUCTIONAL LEADERS AND TEACHERS

TEACHERS' DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS IN AN ELEMENTARY MATHEMATICS SPECIALIST PROGRAM

LEARNING TO FACILITATE CONTENT-FOCUSED COACHING CYCLES: A COMPREHENSIVE FRAMEWORK TO SUPPORT COACHES' PROFESSIONAL GROWTH

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The editors of the ***NCSM Journal of Mathematics Education Leadership (JMEL)*** are interested in manuscripts addressing issues of leadership in mathematics education which are aligned with the NCSM Vision.

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Table of Contents

04 **ABOUT NCSM**

05 **COMMENTS FROM THE EDITORS**
Evthokia Stephanie Saclarides, University of Cincinnati
Chadd McGlone, Mathkind Global

06 **USEFUL, EASY, AND CONSEQUENTIAL:**
A Practical Measurement Repository to Enhance the Work of Math Education Instructional Leaders and Teachers
By Kirk Walters, Angela Knotts, Andrew Brannegan, and Sola Takahashi

To improve and inspire high-quality mathematics teaching and learning, teachers and instructional leaders need access to data that are meaningfully connected to practice. Although most schools and districts are inundated with data (e.g., annual state test scores, data from any number of interim assessment systems), these data are not always helpful in terms of making timely adjustments to instruction, teacher professional learning, and other crucial factors affecting student mathematics outcomes. In this paper, we discuss the potential of practical measurement to fill this gap and address tensions facing math leaders. Unlike most data-driven accountability measures, practical measures are easy for teachers and leaders to collect and interpret data, enabling teachers to adjust instruction in a timely manner. We provide a repository of practical measures leaders can add to their instructional tool belts, discuss how middle-grade mathematics instructional leaders have used the repository to promote continuous improvement, and outline considerations for leaders and coaches in using practical measures to support their ongoing work with math educators.

19 **DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS**
Teachers' Development of Professional Vision and Leadership Conceptions in an Elementary Mathematics Specialist Program
by Corey Webel, Eric Partridge, and Phi Nguyen

In this paper, we share some ways a cohort of 24 elementary teachers developed over the course of a 2-year elementary mathematics specialist (EMS) certification program. We analyzed pre-, mid-, and post-program interviews to document the development of teachers' visions for high-quality mathematics instruction and their views about themselves as mathematics leaders in their schools. We also conducted end-of-program focus groups with a subset of participants (n = 13) to ask about program elements that helped them develop as teachers and leaders of mathematics. Participants identified several common elements of the program as impacting their knowledge, practice, confidence, leadership, and vision, including specific course assignments and cultivation of a supportive community in the cohort.

34 **LEARNING TO FACILITATE CONTENT-FOCUSED COACHING CYCLES:**
A Comprehensive Framework to Support Coaches' Professional Growth
by Ryan Gillespie, Jennifer Kruger, Cynthia Callard, and Kenley Ritter

This paper serves two purposes. First, we present the content-focused coaching implementation framework, a comprehensive tool to support mathematics coaches in navigating the complexities of facilitating coaching cycles with teachers. Second, we describe a research study in which we partnered with nine mathematics coaches and examined how the framework influenced coaches' perceptions of their professional growth when facilitating coaching cycles with local teachers. Through analysis of postcoaching cycle interviews, study participants reported the framework supported them to prepare intentionally for the three distinct phases of the coaching cycle (i.e., planning conversation, lesson implementation, debriefing conversation) and make responsive, "in-the-moment" decisions. Coaches also shared ways in which the framework sparked new insights about coaching. We discuss how our findings connected to and extended prior research on coach learning and the use of coaching tools. We also present implications of our framework and findings for practicing mathematics coaches and future researchers.

56 **INFORMATION FOR REVIEWERS**

57 **NCSM APPLICATION**

About NCSM

NCSM Mission Statement

NCSM is a mathematics education leadership organization that equips and empowers a diverse education community to engage in leadership that supports, sustains, and inspires high quality mathematics teaching and learning every day for each and every learner.

NCSM Vision Statement

NCSM is the premier mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high-quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

High-quality leadership is vital to this vision. NCSM is committed to:

Developing and Informing Vision

- Provide leadership to influence issues and policies affecting mathematics education in ways consistent with the mission and vision of NCSM;
- Equip leaders to be critical consumers of educational information, research, and policy to become change agents in their communities;
- Support leaders to develop an actionable vision of mathematics instruction consistent with a view of mathematics as a sense-making endeavor.

Ensuring Support to All Stakeholders

- Develop networking and communication opportunities that connect the mathematics education community as well as the broader education community;
- Equip leaders with the tools to create and sustain systems that fully align with the vision of mathematics and mathematics instruction promoted by NCSM;
- Equip leaders with the understanding, knowledge, and skills to continue their own personal growth, support emerging leaders, and further develop excellence in mathematics teaching.

Guaranteeing All Students Engage in Equitable, High-Quality Mathematical Experiences

- Provide advocacy and support regarding issues and policies affecting mathematics education in ways consistent with the mission and vision of NCSM;
- Provide resources for the implementation of research-informed instruction to ensure students engage in relevant and meaningful learning experiences that promote mathematics as a sense-making endeavor;
- Advocate for each and every student to have access to rigorous mathematics that develops their understanding, skills, and knowledge, along with the confidence to leverage their learning, in order to improve their world.

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COMMENTS FROM THE EDITORS

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For educators, summer often serves as a natural pause for reflection, to consider our growth, accomplishments, and ongoing areas for development. As leaders in mathematics education, we continually strive for clarity keeping our focus squarely on student learning and the practices of those who support it. In this spirit, we offer the Summer 2025 issue of JMEL, featuring articles that speak directly to the heart of instructional leadership in mathematics education.

Our first article, *"Useful, Easy, and Consequential: A Practical Measurement Repository to Enhance the Work of Math Education Instructional Leaders and Teachers,"* by Kirk Walters, Angela Knotts, Andrew Brannegan, and Sola Takahashi, presents a timely resource addressing the persistent challenges of how to effectively measure instructional quality in practical, meaningful ways. The authors introduce a collection of practical measurement tools designed specifically to help instructional leaders capture and respond to data rapidly, informing immediate and long-term decision-making. This article offers tangible guidance on integrating these accessible measurement tools into everyday leadership practice, significantly enhancing our collective ability to enact meaningful instructional improvements.

Next, Corey Webel, Eric Partridge, and Phi Nguyen's study, "*Teachers' Development of Professional Vision and Leadership Conceptions in an Elementary Mathematics Specialist Program,"* explores how focused professional learning experiences shape teachers' ability to notice and interpret classroom practices, as well as their evolving identities as leaders. Through a thoughtful analysis, the authors illustrate how structured learning opportunities strengthen teachers' instructional practices while encouraging them to adopt critical leadership roles within their educational communities. This work underscores the interconnectedness of classroom excellence and distributed leadership, emphasizing the crucial role of vision in teaching and leading.

Lastly, Ryan Gillespie, Jennifer Kruger, Cynthia Callard, and Kenley Ritter's piece, "*Learning to Facilitate Content-Focused Coaching Cycles: A Comprehensive Framework to Support Coaches' Professional Growth,"* offers a robust framework designed to enhance the professional learning of mathematics coaches. With a clear focus on structured coaching cycles and targeted skill development, the authors provide essential insights into how coaches can effectively support teachers in adopting ambitious teaching practices. Their comprehensive approach serves as a powerful reminder of the central role that purposeful, reflective coaching plays in advancing high-quality mathematics instruction.

> *Together, these articles illuminate the ways practical tools, professional vision, and focused coaching can significantly amplify our collective efforts.*

As you read and reflect, we encourage you to consider how these insights resonate with your current context and leadership goals. How might you leverage these ideas and resources to refine and strengthen your work in supporting educators and students alike?

Thank you, as always, for your continued engagement with JMEL. Your commitment to thoughtful, impactful leadership in mathematics education remains the core of our shared mission.

USEFUL, EASY, AND CONSEQUENTIAL:

A PRACTICAL MEASUREMENT REPOSITORY TO ENHANCE THE WORK OF MATH EDUCATION INSTRUCTIONAL LEADERS AND TEACHERS

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ABSTRACT

To improve and inspire high-quality mathematics teaching and learning, teachers and instructional leaders need access to data that are meaningfully connected to practice. Although most schools and districts are inundated with data (e.g., annual state test scores, data from any number of interim assessment systems), these data are not always helpful in terms of making timely adjustments to instruction, teacher professional learning, and other crucial factors affecting student mathematics outcomes. In this paper, we discuss the potential of practical measurement to fill this gap and address tensions facing math leaders. Unlike most data-driven accountability measures, practical measures are easy for teachers and leaders to collect and interpret data, enabling teachers to adjust instruction in a timely manner. We provide a repository of practical measures leaders can add to their instructional tool belts, discuss how middle-grade mathematics instructional leaders have used the repository to promote continuous improvement, and outline considerations for leaders and coaches in using practical measures to support their ongoing work with math educators.

Introduction

Educators must leverage data and measurement to reflect critically on their work, inform next steps, and advance their practice. In a series of prior issues, editors of the *Journal for Research in Mathematics Education* argued the field of math education is unlikely to advance unless teachers and instructional leaders have access to data that are meaningfully connected to practice (Cai et al., 2018a, 2018b, 2018c; Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, & Hiebert, 2020; Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, Heibert, & Bakker, 2020a, 2020b). Without such data and a broader infrastructure to house and support the uptake of these types of measures, attempts to fuel instructional improvement at scale are likely to continue to miss the mark. Accountability-based data systems (e.g., benchmark or interim assessments) have not been timely or actionable enough for educators to make better instructional decisions. The editors also argued educators, and the field more broadly, deserve better. This argument aligned with NCSM's (2019) vision of the importance of providing structures and resources for instructional leaders and teachers to ensure students consistently have access to rigorous mathematics. The structures include mechanisms to (a) drive continual job-embedded professional learning (Essential Action 3 – EA3) and (b) collectively collect, analyze, and celebrate evidence of student learning (Essential Action 7 – EA7).

This paper describes a multiyear project that has begun to tackle this thorny measurement problem. The Math Practical Measurement Project, funded by the Bill and Melinda Gates Foundation, includes a measurement repository and associated use cases that are proximal to the classroom. The measures focus on the processes of teaching and learning and were designed to support continuous improvement. At a high level, practical measures are easy, useful, and consequential for educators. They were designed to enhance learning and continuous, practice-based improvement, as opposed to measures primarily used to enforce accountability (see Figure 1). The free online repository can be accessed at https://mpm.wested.org.

Figure 1
Practical Measurement

> **Practical Measurement** is "the deliberate and routine gathering, analysis, and interpretation of information with the distinct purpose of enhancing the learning of system actors as they test changes and improve processes that are at the heart of their work" (Takahashi et al., 2022, p. 423). Measures are "practical" in that they can be collected, analyzed, and used in the daily work lives of practitioners.

They are also practical in that they reflect practice—they act as sensing mechanisms at the level at which work is carried out.

In what follows, we describe how practical measurement, which has been used extensively in fields outside education (often referred to as "process measurement" in fields such as health care), has begun to be used in education and why it is promising. Next, we explain how we, the research team, built the repository of practical measures, outlining the measures included and the organization of the repository, and provide suggestions for effective use of the practical measures. We conclude the paper with suggestions of continued opportunities for the field.

BACKGROUND

A Brief History of Practical Measures

Practical measurement grew out of quality improvement approaches in industry and health care (Provost & Murray, 2011; Solberg et al., 1997). These measures were used to break down organizational silos, prioritizing the work of frontline workers, who identified problems on the ground and were central to driving more systemic improvement efforts (Takahashi et al., 2022). A prominent example is Toyota, whose approach to continuous improvement helped the company become a highly respected global manufacturer (Morgan & Liker, 2020; Rother, 2009). Toyota's management principles focused on building quality throughout workplace systems, tasking everyone in the organization with being a quality control inspector. Under this system, problems identified by assembly line workers are flagged and solved collaboratively rather than waiting for the problem to emerge as a faulty component once the car has already been built.

Since 2015, researchers at the Carnegie Foundation for the Advancement of Teaching have argued practical measures and other aspects of continuous improvement from industries should be applied to educational contexts. In the auto industry, it is inefficient, and perhaps too late, to fix an interior engine bolt problem after the car has been built. In education, using test scores from the past academic year or even benchmark assessment data from the prior semester is similarly too late to deal with emergent issues in the instructional environment that necessitated improvement. Through networked improvement communities (NICs), communities of educators and researchers work together to solve well-specified problems through rapid inquiry cycles. The NIC model requires a measurement system that is aligned to the problems of practice and can be leveraged practically by educators in the network (Bryk et al., 2015).

A core principle of NICs is the belief that "[one] cannot improve at scale what [one] cannot measure" (Bryk et al., 2015, p. 111), meaning organizations seeking to create improvement (e.g., schools) must think carefully about the properties of measures that will allow them to learn in and through practice (Yeager et al., 2013). The measures that most centrally benefit improvement efforts are practical measures, measures taken directly from practice that are easy to use in working to change key processes (Bryk et al., 2015). Such measures are "practical" both in the sense that they are relatively easy to use and in that they are proximal signals of practice (A. Bryk, personal communication, October 10, 2014). Practical measures have five key features:

- They connect to aspects of systemic structures, processes, or norms believed to be critical to achieve an aim.
- They are meaningful to system actors who use the data.
- They are actionable.
- They are minimally burdensome to the system actors who use the data.
- They provide timely and regular information. (Bryk et al., 2015)

In short, practical measures are easy, useful, and consequential.

One of Carnegie's first NICs focused on improving outcomes for community college students enrolled in developmental mathematics, a course the NIC eventually redesigned (Bryk et al., 2015). For many institutions, most students are tracked into these non-credit-bearing development courses, yet very few succeed. Not succeeding in these courses creates a huge roadblock for students because college-level mathematics credit is required to transfer to 4-year universities or to pursue many occupational programs. This NIC, which later became the Carnegie Math Pathways (CMP) program, identified several key root causes of the program and, from that, established a working theory of improvement and associated change ideas to test and refine. One set of change ideas focused on development of productive student mindsets during their 1st week in developmental mathematics. For example, the NIC redesigned lessons with activities to build students' mindsets and created professional development sessions for teachers on the importance of developing a productive mindset. To know whether these and other change ideas were working, the NIC needed a coherent yet nimble measurement system. One practical measure they developed was a brief survey on student mindsets that teachers could implement easily in their classrooms. The NIC also collected other practical measures from teachers related to their professional development experiences and instruction. Such a system of measures helped the NIC carry out quick-cycle continuous improvement cycles as it worked to meet its larger aim of significantly improving outcomes for developmental mathematics students, which it accomplished (Hoang et al., 2017; Yamada & Bryk, 2016; Yamada et al., 2018).

Measurement for Improvement, Accountability, and Research

The emergence of practical measurement in education reflects the contrast between these measurement tools and practices and the tools and practices that predominate the U.S. educational system and are designed for accountability or research purposes (Solberg et al., 1997; Takahashi et al.,

[1] To hear more about how Toyota's approach compares with other manufacturers, including a General Motors plan in California, see This American Life's NUMMI episode (Langfitt & Glass, 2015).

2022). Accountability measures often have been used to judge summative performance or inform broader policy decisions retrospectively, whereas practical measures have been designed to stimulate rapid improvement and are not punitive in nature. Practical measures prioritize the need for school- and district-based practitioners (and practitioners in other types of educational organizations) to have regular and timely data that serve as quick measures to access feedback on how continuous improvement efforts are progressing before the end of a course or school year (Jackson et al., 2016). Accountability and research measures commonly used in the education system frequently have been time and labor intensive to collect, typically have been collected only after the end of some cycle (e.g., a school year) when those impacted can no longer benefit from the data, and have been tied to global measures of outcomes resulting from such a complex system that causes of those outcomes cannot be tied meaningfully to specific practices delivered at a specific time. Data collection techniques that keep teachers and learners engaged in the work of teaching and learning, in contrast, can be an important resource for improving teaching and learning (Yeager et al., 2013).

This is not to say that the relationship between practical measures and accountability measures should not be examined if the improvement goal is aligned to one of these measures. In fact, this relationship is one of the key analytic means to evaluate whether practical measures capture information that is consequential (Takahashi et al., 2022). In the Carnegie NIC example, the student survey measure of productive persistence was sensitive to change ideas being tested, but student answer patterns also predicted end-of-course performance (Bryk et al., 2015). However, importantly, the practical measures used to drive continuous improvement were not used to evaluate student or teacher performance for accountability purposes.

Researchers sometimes develop measures to test theories; for example, a researcher might hypothesize elementary teachers' increased understanding and use of the double number line representation might improve their ability to teach problems involving fractions with unlike denominators. The researcher might design teacher- and student-facing materials with the double number line, an observation instrument, and an assessment of teacher and student knowledge. The primary purpose of these measures would be for theory development, not rapid cycle improvement. The researcher would assume most of the burden related to data collection and analysis. Eventually, the findings would be shared with teachers, but not before the research project had been completed, which could take months or even years.

Practical measures offer tangible benefits to both classroom teachers and educational leaders. These measures:

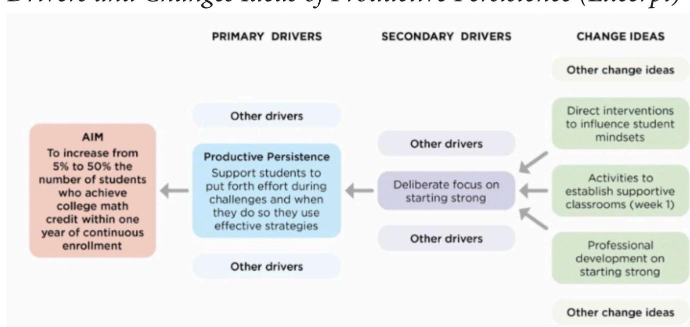
- shed light on how improvement efforts are going on a regular basis, not just infrequently or after the fact;
- bring discipline to the work of testing change ideas by providing data to confirm or refute teachers' and leaders' general sense of how things are going;
- focus attention on a particular challenge across a learning community (e.g., across math teachers at a particular grade level, across schools in a district);
- illuminate variation across a learning community and identify "bright spots" (e.g., schools or classrooms where a change idea seems to be effective) and instances where the same change seems to be less effective and additional support may be needed; and
- elevate the voices and experiences of people "closest to the problem," often students or teachers.

A Practical Example

Returning to the earlier example of developmental mathematics reform, the CMP program was designed to increase the proportion of students who achieve college math credit in 1 year of continuous enrollment. More specifically, the aim of this NIC was to increase the math credit achievement rate from 5% to 50%. As shown in Figure 2, the aim was part of a working theory of improvement called a driver diagram, which continuous improvement researchers use to organize their efforts to test and refine strategies to reach an aim. We present an excerpt of the driver diagram to illustrate connections between the aim, a working theory of improvement, change ideas, and practical measures. As shown, the NIC theorized productive persistence was a key driver in shifting student outcomes. They theorized that, by developing change ideas, including strategies to use as students began their math courses, students would develop healthy habits of mind and learn strategies to help them persevere when facing academic challenges.

Figure 2

Drivers and Changes Ideas of Productive Persistence (Excerpt)



Having a working theory of improvement is critical to solving complex problems; however, without practical measures, the theory might not inform concrete actions. For example, suppose a math department designed a 2-day professional development training focused on the importance of starting strong and included direct interventions to influence student mindsets and activities to create supportive classrooms. The department must determine if change ideas contribute to improvement. At a high level, department personnel would want to ensure the professional development was attended, received, and implemented well by faculty. This assessment could be accomplished through quick surveys and informal observations. Digging deeper, the department might develop surveys that measure students' attitudes toward learning and

[2] For a more detailed driver diagram and related discussion, see *Learning to Improve: How America's Schools Can Get Better at Getting Better* by A. S. Bryk, L. M. Gomez, A. Grunow, & P. G. LeMahieu (2015), pp. 75–79. Harvard Education Press.

their beliefs about persistence, which they could use to assess the effects of the change ideas. In fact, the CMP program found students' academic mindsets shifted in expected directions after 3 weeks of implementing associated change ideas (Bryk et al., 2015). The surveys proved to be timely, useful, and consequential for the NIC.

ABOUT THE MATH PRACTICAL MEASUREMENT PROJECT

The Middle Grades: A Crucial Time for Mathematics Success

In recent decades, math education researchers and policymakers have identified the middle grades (i.e., Grades 5–9) as critical years in the trajectory of math students, with Algebra I widely considered a gatekeeper to college- and career-ready math classes and beyond (Adelman, 1999; Finkelstein et al., 2012). Despite various types of reforms implemented over this timeframe, student achievement in mathematics has remained lackluster, with only about one third of U.S. eighth graders scoring at or above the proficient level on the most recent National Assessment of Education Progress report in mathematics (National Center for Education Statistics, 2023). Rates were even lower for Black, Latinx, and low-income students. U.S. 15-year-olds performed well below the international average on the most recent Program for International Student Assessment (PISA), a test of math problem-solving skills, but again, the results were even worse for underserved students (Organisation for Economic Co-operation and Development, 2018, 2022). The concerns were even greater in the COVID-19 global pandemic context, in which some researchers projected greater learning loss due to school closures compared to what had been seen due to "summer slide" in typical pre-COVID-19 conditions (Kufeld & Tarasawa, 2020). Given these disappointing outcomes and concerning trends, improving mathematics teaching and learning in middle grades has remained an important area of reform, including recent reforms that have drawn on continuous improvement approaches.

Practical Measures for Middle Grades Mathematics Improvement Networks

When teachers and instructional leaders have worked to improve math teaching and learning using a continuous improvement approach, they often have lacked access to a coherent set of high-quality measures to understand their progress and inform their next steps. Of course, teachers often use their own classroom data to inform instructional improvements in their day-to-day work (Rothkopf, 2009), but the means to gather information are typically informal, idiosyncratic, and not widely shared (Cai et al., 2020b). Math education researchers also have created measures that reach many classrooms, such as tools for elementary teachers to understand student approaches to solving arithmetic problems (e.g., Cognitively Guided Instruction; Carpenter et al., 1996, 1998), strategies to promote and assess student mathematical discourse (Smith & Stein, 2011), and various techniques and measures that fall under the broad category of formative assessment (Burton et al., 2018; Fennell et al., 2016). Yet, these measures are typically only accessible to practitioners who participate in the respective research projects or for purchase after the research concludes.

Reflecting this policy and reform context, the Bill and Melinda Gates Foundation launched the Networks for School Improvement (NSI) grant portfolio in 2018, which funded improvement networks focused on advancing middle and high school math and English language arts (ELA) outcomes for students in historically underserved communities. As the NSIs began to conduct their work, the foundation determined the overall project lacked a resource of existing practical measures (i.e., timely data about processes that are the focus of improvement and that can be collected in a minimally burdensome way with the least disruption to classroom learning) and a categorization of such measures in ways that are useful for educators and education leaders. The foundation partnered with WestEd to build the repository, strengthen the capacity of math-focused NSIs to incorporate measurement into their work, and develop two new practical measures aligned to focus areas of the NSIs (see Figure 3).

Figure 3
Goals of the Mathematics Practical Measurement Project (October 2020–June 2024)

1. to develop a repository of math practical measures;
2. to build the capacity of math-focused NSIs to effectively use measurement in their continuous improvement work; and
3. to develop two practical measures focused on improving middle grades mathematics for historically underserved populations.

Existing research supported grounding our approach to this problem in best practices of math teaching and learning, such as

- explicitly focusing on the development of conceptual understanding and providing students with problems that promote productive struggle (Hiebert & Grouws, 2007);
- giving students prompts to monitor problem-solving approaches and encouraging the use of multiple approaches (Woodward et al., 2012); and
- providing all students with regular opportunities to engage in mathematical discourse (National Governors Association for Best Practices, Council of Chief State School Officers, 2010).

In addition, the nature of this work lent itself particularly well to continuous improvement theories, such as the model for improvement (Langley et al., 2009); the Carnegie Foundation's model of NICs and their six principles of improvement (Bryk et al., 2015); the measurement work adapted, developed, and refined in the work of quality improvement in health care at the Institute for Healthcare Improvement; and others (e.g., Provost & Murray, 2022). We also sought to incorporate key lessons learned from a mature, math-focused NIC, The Better Math Teaching Network, which had been engaged in math instructional improvement work since 2015.

The vision for the project also was informed by interviews with six math-focused NSIs. Consistently, the NSI leaders and their data and analytic specialists discussed the difficulty of identifying and enacting timely and regular measurement for teachers and instructional leaders to inform continuous improvement in math instruction. They spoke of challenges in identifying measures aligned with the aims of the work and simultaneously provided a feedback loop for the changes being tested in the classroom. They discussed wanting to "make practice visible" to practitioners but not having the right data to do so. They also spoke about the challenges in getting teachers on board with measurement work, which often required additional time and effort (e.g., data collection) on top of teachers' existing responsibilities.

THE MATH PRACTICAL MEASUREMENT REPOSITORY[3]

To address the aforementioned challenges related to the field's lack of access to existing practical measures, we sought to develop an online repository to house measures that have been useful in practice; their associated instruments; tools of collection, analysis, and visualization; information about the validity and warrant of the measurement tools to inform continuous improvement; and, in many cases, written vignettes or illustrations of development and use. In particular, the project was designed to attend to measures and their uses particularly connected to increasing the success of Black, Latinx, and low-income students in middle-grade mathematics.

Developing the Math Practical Measurement Repository

To develop the repository, we performed a scan of the field adapting a 90-day cycle process to understand what measures researchers and educators were using for improvement in math. This process, adopted from its use in the health care quality improvement field, involved gathering information through a review of select literature and interviews with experts, broadly defined as experts of practice and scholars. Information was gathered and then synthesized and organized at 30- and 60-day points in time where external review and reflection enhanced the emerging product. We spoke with math education scholars and researchers, instructional leaders, and continuous improvement specialists with knowledge of data useful for instructional improvement. Interviewees included Karen Givvin (University of California, Los Angeles), Kara Jackson (University of Washington), Grace Kelemanik (Fostering Math Practices), Ann Edwards (WestEd), Skip Fennell (McDaniel College), and Christine Roberts (Tulare County Office of Education), among many others.

During our interviews, we asked the experts about the characteristics and use of their practical measures. We asked the following questions:

- How did the measure support an improvement effort?
- Who was using the measure, what did they learn, and how did they use what they learned?
- What contextual factors supported the effective use of the measure?

Practical measures included in the repository are useful in a continuous improvement effort, are relatively easy for practitioners to use and enact in their work, and are consequential in that they capture something that matters for the improvement of math teaching and learning. Although we encountered numerous measures that might be considered "practical," we did not include everything we encountered. Rather, we prioritized including measures that

- were clearly connected to math learning or best practices recognized by the field of math educators;
- had demonstrated evidence of support for instructional improvement, either at a single site or in a more comprehensive study;
- yielded actionable data;
- could be used across multiple contexts and settings;
- were easy to administer and produced data that were easy to analyze and interpret (e.g., could fit into educators' regular routines, required minimal training, were not overly time consuming); and
- were free or low cost.

Throughout the development and refinement of the repository, we received and incorporated feedback on the repository from our advisory committee and NSI leaders.

Measures in the Repository

The repository includes 18 measures, along with associated guidance. The modes of the measures include surveys (10), classroom observation tools (3), quick student reflections (2), artificial intelligence (AI)-powered apps (2), and a rubric (1). The content of the measures is organized by focus areas arrayed along the instructional triangle of teachers and students interacting with content, including socioemotional learning (student focused), teacher mindset (teacher focused), and processes of teaching and learning (the interaction of the three nodes) but also by guiding questions. These guiding questions provide measures for educators looking to understand certain topics, such as:

- What does mathematical discourse look like in our classrooms?
- How are students making sense of the rigorous math tasks they are given?
- What mindset and beliefs do students hold about themselves as learners in a math classroom?
- How are teachers experiencing feedback about their practice?

For each measure in the repository, there is guidance for users on how to use the measure in their improvement work; for most measures, there are vignettes that tell the story of educators using the measurement tool.

Three Snapshots of Measures in the Repository

Figure 3 provides descriptions of three questions faced by teachers and instructional leaders along with measurement tools in the repository that can be used to gain insights on

[3] The repository can be accessed at https://mpm.wested.org/

[4] The 90-day cycles were described in the handbook on the Carnegie Foundation for the Advancement of Teaching website: https://www.carnegiefoundation.org/improvement-products-and-services/articles/ninety-day-cycle-handbook/

these questions. Included in the measure descriptions is information about how they have been used in practice and the analytic infrastructure and social processes and routines that support measure use.

Figure 3
Repository Measures and Tools for Teachers and Instructional Leaders

Challenge faced by teachers and instructional leaders: How are students participating in classroom discourse? Where are the opportunities for teachers to initiate more academically rigorous, student-led discussion?
Measurement Tool
TeachFX: An AI-powered app focused on classroom discourse
Description
An app that measures the amount of teacher talk, student talk, student group talk, and silence occurring during a lesson
User
Classroom teachers and coaches
The measure in practice
As a 1st-year teacher and the only math instructor at a newly launched high school, Daniel knew they needed a way to focus their improvement efforts around classroom discourse and chose to use the TeachFX tool. For Daniel, the app's ease of use (just hit "record") and automatically generated data reports showing ratios of teacher-to-student talk time made it easy to integrate the tool into their routine. Importantly, Daniel emphasized that reports were a launching point to reflect more deeply on their practice: Visual displays of talk patterns allowed him to home in quickly on stretches of their class that were worth digging into. Daniel said: I really like how you can see where there are interesting blocks [of teacher, student, or group talk time] . . . from there, you can spend 10 minutes and get a lot of rich insights, without having to dive into the 60-minute-long recording and having to find those spots.
Analytic infrastructure that supports use of this measure
TeachFX is a full-service app that supports data collection, storage, and analysis. After downloading the TeachFX app to their device, the teacher opens the app before class begins and clicks "record." Afterward, the app emails the teacher a report of talk patterns for the class. The report shows the breakdown of talk and distribution of types of talk throughout the lesson. Teachers may work toward a goal of simply increasing the percentage of student talk over time, but teachers and coaches also can delve deeper into this breakdown to uncover talk patterns that suggest strong instructional practices.
Supporting the social routines of data sensemaking with this measure
• Reviewing talk pattern data can better support instructional improvements when paired with professional learning around high-level questioning; student discourse; and the relevant, authentic, and rigorous tasks that support high-quality student talk. • A school culture in which student voice is explicitly valued can set the groundwork for authentic and reflective engagement with TeachFX data. • Although individual teachers can look at data on their own, incorporating the tool into professional learning communities or using it as a tool to aid coaching conversations will support grade-level or schoolwide improvement. • TeachFX has seen success in schools that begin using the app with a small, enthusiastic cohort of teachers and then expand to the rest of the school as teachers become comfortable with the app. • School leaders should take care to communicate that teachers' individual data are private, and use of the app is voluntary. TeachFX data should never be used for evaluation purposes.

Figure 3 continued on next page…

Challenge faced by teachers and instructional leaders:

How are students experiencing learning through whole-class and small-group discussion? Where are there opportunities for teachers to enhance classroom discussion and foster access to rigorous mathematics?

Measurement Tool

Practical Measures, Routines and Representations (PMRR) whole-class discussion and small-group work in specific lesson student surveys

Description

Student survey about student experiences with the whole-class discussion in their classroom

User

Classroom teachers, instructional coaches, and district leaders

The measure in practice

To help understand if the instructional changes teachers were making around whole-class discussion were resulting in improvement, teacher–coach pairs embedded the whole class survey into their one-on-one coaching cycles. Teacher–coach pairs co-planned to set goals and select tasks around the whole-group discussion, administered the survey as part of classroom instruction, and used data from select items to inform their debrief discussions.

As an example, at the end of one teacher's coaching cycle, only about half of students responded "no" to the item, "Did you have trouble understanding other students' thinking in today's whole class discussion?" As the teacher and coach unpacked why this might be the case, the data point provided an opening for the coach–teacher pair to discuss one of the coach's observations: The teacher tended to rephrase students' thinking during whole-class discussion, and students tended to share without building on and making sense of their peers' thinking.

This measure also can be used by district leaders as a window into the quality of math instruction at scale. The results of these surveys across classrooms can provide leaders with insight into matching instructional coaches and teachers and targeting professional development experiences (Jackson et al., 2016). Critically, effective and meaningful use of these measures depends on their use for purposes of improvement, not for evaluation. The authors cautioned a collective inquiry stance is critical for use of these measures, which must be explicit and reinforced, especially when district leaders are involved.

Analytic infrastructure that supports use of this measure

Teachers administer the relevant survey either immediately following a whole-class discussion or small-group work or at the end of a lesson that incorporates a whole-class discussion or small-group work. Students can take the survey using paper and pencil or online via Google Forms (or another online survey platform) to allow for quicker representation of student responses.

To help a teacher assess whether a new instructional strategy has improved students' learning, the teacher–coach pair might compare responses from the end of the previous coaching cycle to responses at the end of the current coaching cycle. To best support teachers in making sense of why students responded the ways they did, survey data should be analyzed alongside student work, coach observations, and teacher reflections

Supporting social routines of data sensemaking with this measure

- Using surveys in the context of regular coaching or a professional learning community helps teachers make sense of their data and connect data to targeted instructional changes.
- To best make sense of student responses, survey data should be analyzed alongside other information, such as student work, coach observations, and teacher reflections.
- Positioning the surveys to elicit student feedback and voice can help users understand the survey as a tool for exploring practice rather than as an accountability or evaluation tool.
- When discussing survey results, users should bring an asset-based perspective and a willingness to reflect on their own practices to avoid data being used to reinforce existing perspectives. A context of ongoing professional learning can allow a coach or school leader to shape conversations about survey data and can prevent data from reinforcing problematic ways of characterizing students.
- If district leaders engage teachers around these data, this must be done in a context where it is abundantly clear that the goal is support and learning, not evaluation. Practical measures, including these, are not designed for evaluation purposes and should not be used as such.

Challenge faced by teachers and instructional leaders:

How do students feel about their identities as math learners? Where are there opportunities for teachers and school leaders to improve student attitudes toward rigorous mathematics content?

Measurement Tool

High-Tech High Mathematical Agency Improvement Community (MAIC): Student Agency Survey

Description

Student survey on perceptions of their agency, mathematical identity, and group work experiences in their math class

User

Classroom teachers, instructional coaches, and network leaders

The measure in practice

The MAIC, a network of K–12 schools in Southern California, wanted to understand how student-centered practices support mathematical agency and success. Students of MAIC teachers completed the student agency survey three times in the academic year. Teachers appreciated the way the survey data provided insights into student perceptions and needs. Furthermore, when data were disaggregated, teachers were able to focus on subgroups of students who were having adverse experiences or whose needs were not being met as well as others' needs. One teacher explained, "We really tried to visualize data according to very specific target groups of students, and to help us unearth our own biases and things we're bringing to teaching we didn't realize we had." This same teacher reviewed survey data alongside grades and test scores to see if "there were students who were not getting what they needed in [their] classroom." One teacher stressed that the culture of learning in the MAIC network was critical; they explained:

> Sometimes you see data, and it hurts . . . but because MAIC felt like a learning environment, I felt very comfortable sharing my data and wonderings about it with others. When you come at it with a lens of curiosity . . . that allows the conversation to move and be authentic.

MAIC network leaders also used these data to understand trends across teachers and schools involved in this work. For many of the concepts captured through this practical measure, network leaders saw improvements over time, but some concepts appeared more intractable. For example, math classroom status hierarchies among students have proven to be an ongoing area of challenge. Network leaders have used this information to focus on specific practices teachers can try in their classrooms.

Analytic infrastructure that supports use of this measure

The MAIC network developed a survey infrastructure that allowed teachers to administer surveys using a digital platform, connected individual student responses to the student information system, and generated reports and data visualizations for teachers. These data could be viewed by network leaders at an aggregate level to see overall trends. They also viewed data in a "small multiples" display that showed improvements at each site together, so they could know where there were "bright spots" and where teachers were struggling and needed more support.

Supporting social routines of data sensemaking with this measure

- Students should be informed as to why teachers are collecting the survey data. Without understanding the purpose of the surveys, especially for a survey meant to be given multiple times, students may resist or disengage from the survey leading to less accurate data and reducing students' sense of agency.
- The MAIC network provided important context and support for educators to analyze survey results and connect survey data to specific change ideas and instructional strategies.
- Teachers in the MAIC network used the network's data protocol to review their data reports and network-wide data together to identify trends, patterns, and longitudinal changes. Data review and discussion was coupled with a focus on practices and strategies around student math agency and growth mindset that were shared and modeled at MAIC convenings. Teachers could bring these strategies back to their classrooms and test them through a series of plan, do, study, act improvement cycles.

Reception of the Repository

Since the launch of the repository in 2021, it has been visited by over 5,000 unique users across 21 countries. Introduced to 16 leaders from the NSI in August 2021, all 16 said in a survey that it was very likely or likely the repository would be helpful to their network. Ten of the respondents said there were measures they may want to try using in their network, whereas the other six were unsure. At the following meeting with NSI leaders, 8 of 11 survey respondents reported they had looked at the repository since it launched, and 6 of 11 said they had shared it with someone else in their NSI.

USING PRACTICAL MEASURES EFFECTIVELY

Practical measurement is a tool that can be leveraged to powerful effect in a continuous improvement approach to systemic change. However, measures and data do not hold meaning in themselves; any measure only holds meaning in the social learning system in which it is used. Here, we highlight four key practices for math leaders looking to incorporate practical measures in their improvement efforts. These key practices derive from challenges faced by those who have used practical measurement in the field. We drew upon several sources to arrive at these four practices: the stories we heard through the 90-day cycle scanning process, including interviews with experts in the field; our communications with the NSI over the course of the Math Practical Measurement Project; and our own personal experiences attempting to leverage practical measurement in our support of K–12 school district improvement efforts.

Start With a Clear Theory of Improvement

Practical measures alone cannot improve math teaching and learning. To generate useful, actionable data, practical measures must be connected to a clear and specific theory that outlines how the team believes changes to the system will lead to improvements in service of an aim (Bennett & Provost, 2015; Bryk et al., 2015; Takahashi et al., 2022). A common pitfall in using practical measures is selecting or developing measures without first coming to consensus as a team regarding a clear and sufficiently detailed theory of improvement, including (a) what one is trying to achieve (i.e., aim statement); (b) what structures, processes, and norms of the organization must be improved to achieve that aim (i.e., drivers); and (c) what change ideas (e.g., classroom or professional learning activities and strategies) are planned to create the desired improvements via those drivers. Without going through the process of identifying drivers and making explicit how team members believe a change idea will lead to improvement, selecting and implementing practical measures is unlikely to produce interpretable, usable data regarding whether and how a change idea is helping an organization make progress toward achieving its aim.

The theory of improvement is the basis for identifying the right measures (Bryk et al., 2015; Takahashi et al., 2022). Measures can help to answer questions about whether changes have been enacted consistently across sites and over time, to what degree these changes are occurring with high quality, and whether the changes are leading to desired proximal and distal outcomes (i.e., goals). Educators and education leaders can use resulting data to gain insight into what is working well, for whom, and under what conditions. They can identify bright spots and areas of growth, focusing their attention where it is needed. A set of measures connected to a theory of improvement, also referred to as a "family of measures" (Provost & Murray, 2022), can illuminate the larger theory of improvement. In other words, educators and leaders can determine if the changes actually led to the hoped for outcomes and, if not, whether the theory needs to be revised.

Try to Avoid Developing a Measurement Tool From Scratch

Development of a practical measure can be a substantial undertaking (Jackson et al., 2016; Takahashi et al., 2022). These measures are meant to be practical to use, but that does not mean they are practical to develop. In fact, prioritizing usefulness of the measures and their ease of use requires work during the development phase that is typically not called for in development of research measures, such as involving potential users in the development process and studying how a measure is used in practice. Teams engaging in continuous improvement efforts should first consider measures already collected in their organization. School systems typically collect many kinds of data. Math leaders should consider if anything can be leveraged from extant data. These data may not be housed in core data storage systems in a school district or other education organization. In one project in which one of the authors was involved, data about students' attempts to revise their own work were gleaned from assignment-completion data teachers kept, which existed outside the larger learning management system where core district data were stored.

If relevant data do not exist in the organization, the next question the team might ask is if there are valid measures other practitioners in the field have used successfully (Walston & Conley, 2022), such as those that can be found in the math practical measurement repository. These types of measures might also be found in related improvement research focused on similar aims, drivers, or change ideas. Once potential measures have been identified, they (or elements of them) can be mapped to the team's theory of improvement. This mapping process allows the team to prioritize items for certain change ideas or drivers, identify which items may need to be modified to better match the team's work, and identify gaps where new measures are needed.

Minimize Measurement Burden

A crucial design feature of any practical measure is that it is minimally burdensome and that it ideally takes little to no time to collect the data, especially for staff engaged in the core work that is the focus of the improvement effort (e.g., teachers who are already stretched thin with responsibilities; Takahashi et al., 2022). This is particularly important considering practical measures should be collected on multiple occasions to see change over time. Burdensome measures will be abandoned over time if they are not sustainable. Measurement burden can be minimized in the following ways:

- Focus on "just enough" data. What are the minimal amount of data that could give educators a signal their work is heading in the right direction or might need to be modified (or abandoned)?
- Use already-collected data rather than engaging in new data collection (as discussed previously).
- Reduce the data collection task (e.g., using a 3-item survey to check in with students rather than a full survey).
- Leverage technology and automation to ease data collection and analysis. A tool like TeachFX requires a tap on a phone to collect data. A tool like the PERTS Elevate survey automatically processes data so graphs summarizing survey results can be accessed in days. More examples like this have continued to emerge.
- Embed measurement in existing workflows. For example, if teachers are already entering data about student learning into their gradebook, this process could be leveraged to capture information valuable to the improvement effort at hand. If instructional coaches are already taking observation notes from classroom visits, these kinds of data may be leveraged.
- Make measurement meaningful to people who collect, give, and use the data. There is burden associated with the amount of time or resources data collection and analysis may take, but there is also the perception of burden affected by how meaningful individuals find the measures. If key organizational members (and here we include students and families) understand why data are being collected, know how those data are being used, and see data used to benefit teachers and students, they are more likely to continue to participate in data giving and data collecting. Data can be shared with teachers and students so they can partake in sensemaking and collaborate in development of next steps and actions.

A clear, well-specified data collection and analysis plan focused on just enough data and involving core interest holders is key to creating routines that ensure measurement activities occur in a sustainable way for ongoing learning.

Design Data Discussions That Enable Critical Reflections and Thoughtful Next Steps

The "rubber meets the road" concept reflects the work of practical measurement when educators collectively engage with data to make sense of it and use learning to reflect on practice and generate next steps. The meaning that comes from practical measurement happens in dedicated routines of social sensemaking. We highlight five factors that contribute to these sensemaking spaces:

- Structures and routines for data sensemaking conversations (Coburn & Turner, 2011). These routines include identifying who is meeting with whom, how often, for how long, and with what purpose. These collaborative spaces are where data sensemaking can find a natural home, but they require aligning schedules, protecting time, and prioritizing these conversations. This approach may involve creation of new routines or embedding data sensemaking in existing structures of collaboration.
- Identifying participants. Walston and Conley (2022) recommended teams keep in mind that the primary determining factor for participating in data sensemaking meetings is whether the person is affected directly by the condition the group is attempting to change and has the power to implement or enable any decisions resulting from data inquiry (e.g., teachers, students, parents). We would add, when teams are prioritizing equity work, it is particularly important to be mindful of interest holders who are traditionally "not at the table" or underserved by the system. For example, educators and leaders may examine how students and families can be included in these spaces to make sense of and improve the system intended to benefit them (Takahashi & Norman, 2025).
- Norms of openness, transparency, and innovation (Takahashi & Norman, 2025). Fruitful data conversations require participants to talk with transparency about what is not going well, perhaps even more than discussing what is going well (although this also can be productive). This practice, in turn, requires a culture of safety and trust where individuals can discuss their own areas of growth and feel supported in taking risks by trying new practices.
- Data discussion protocols and meeting agendas (Little & Curry, 2009; Takahashi & Norman, 2025). The data conversation can be structured and designed to enable participation of all voices, to stay grounded in data before introducing hypotheses, and to lead toward collectively shared learning connected to next steps. A protocol used by a skilled facilitator can deepen the learning that can be had from practical measures.
- Visual representations of data. Most people tend to process and understand information better when it is presented visually, so it is critical to provide clear and organized representations of data designed to inform actionable discussion around identified research questions. Data visualizations are most effective when they reflect data accurately, use clear labels and uncluttered design, and take advantage of the graph type best suited to the data and research questions (Evergreen, 2017). Walston and Conley (2022) recommended organizing data by the questions they are intended to address. For example, if one wants to know how often a teacher implemented a particular change idea (e.g., conducting a number talk, orchestrating a whole-group summary discussion), one might look at data from a teacher's self-report log, a student survey, an observer's checklist, or some combination of all three.

OPPORTUNITIES FOR THE FIELD

This project illustrated the relevance and importance of practical measures to the work of instruction leaders and educators working in continuous improvement networks, yet there is much more to learn about practical measurement in this context and, more broadly, in the field of math education. The following subsections outline three concrete opportunities that would extend what is known about practical measurement and potentially provide greater support to the work of instructional leaders.

Employ Practical Measures in Common Professional Learning Structures

Most districts and schools are not part of large, externally funded improvement networks, yet districts and schools have resources for professional learning and, at a high level, work toward continuous improvement. Instructional leaders might integrate practical measures in regular professional learning community meetings or coaching. For example, if a fourth-grade math team identifies comparing fractions as an area for improvement after reviewing formative assessment data, they could spend time as a group identifying other sources of data that could be used to address the issue. Perhaps the team has not yet gotten information from students about why they struggle with this topic, and they could introduce a short survey. Perhaps the issue is related to instruction: Teachers may not be using representations or different types of representations with students, and they could collect data on how different types of instructional routines resonate with students. This type of inquiry could be applied to instructional coaching cycles and be the basis of empirical discussions about instructional improvement.

Apply Practical Measurement to Implementation of High-Quality Instructional Materials

Independent organizations, like EdReports, analyze the quality of curricular materials in terms of rigor, usability, and coherence, providing district leaders with additional data to inform their selection of instructional materials. Curricula are rated higher in quality when they include more mathematically rigorous content and integrate the mathematical practices into their instructional routines. Such materials provide opportunities for practical measures to support implementation and continuous improvement. For example, a program emphasizing student discourse through one or more instructional routines could benefit from a practical measure that captures the quantity of discourse or students' perceptions of the routine. The math practical measures repository has measures that could be used for this purpose. Instructional leaders could apply practical measures to understand better how teachers use their planning time and improve the support they provide. These sorts of efforts could provide more nuanced information about implementation of high-quality instructional materials rather than the all-too typical "the program was too difficult for teachers to implement" storyline.

Leverage Technology to Improve the Efficiency of Instructional Improvement Work

Technology in education has continued to advance at a rapid pace, and some technological tools have the potential to make the work of instructional leaders and educators more effective. Although this message is not new, and the mixed effects of technology as part of instruction are well documented (Campuzano et al., 2009; Higgins et al., 2019), some tools are promising. For example, TeachFX could make small group or 1:1 coaching sessions more productive because of the classroom data it automatically generates. This tool is also less invasive than video observations and might expand the number of teachers willing to share artifacts from their own classrooms. Survey data collection tools have become better and easier to use over time, including the types of displays that can be automatically generated to guide the work of instructional leaders and educators. Countless other examples exist of technological tools that could support the work of instructional leaders. The question is why certain tools are helpful and how those tools can be used in authentic educational settings.

Educators and leaders must attend to ongoing tension between authentic data interpretation and organizational pressures. Because the data cannot speak for itself (Coburn & Turner, 2011), a key role of the instructional leader is to facilitate active sensemaking among educators as they evolve into concrete, authentic actions for continuous improvement. The more opportunities teachers have to practice improvement with their coaches, the stronger the uptake (Biag & Sherer, 2021). Still, maintaining an inquiry stance with educators can be challenging in the face of intense accountability pressures from school and district leaders. It is important for leaders to acknowledge this tension and find opportunities for synergy. For example, as teachers use data from practical measures to shift instruction to deepen student engagement, leaders can underscore the connection between engagement and performance on the next formative or benchmark assessment. Such advocacy allows educators to thrive from ongoing, job-embedded professional development (EA3) and identify a broader evidence base that celebrates student learning (EA7).

We urge leaders to consider these field-building opportunities and document and share lessons learned with other instructional leaders and educators. It is only through creating a larger professional community, informed by evidence from practitioners, that the field will move forward. Indeed, such work is the engine for the useful, dynamic knowledge base for teaching that Hiebert et al. (2002) envisioned 2 decades ago, and development of that engine is long overdue.

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DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS

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TEACHERS' DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS IN AN ELEMENTARY MATHEMATICS SPECIALIST PROGRAM

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ABSTRACT

In this paper, we share some ways a cohort of 24 elementary teachers developed over the course of a 2-year elementary mathematics specialist (EMS) certification program. We analyzed pre-, mid-, and post-program interviews to document the development of teachers' visions for high-quality mathematics instruction and their views about themselves as mathematics leaders in their schools. We also conducted end-of-program focus groups with a subset of participants (n = 13) to ask about program elements that helped them develop as teachers and leaders of mathematics. Participants identified several common elements of the program as impacting their knowledge, practice, confidence, leadership, and vision, including specific course assignments and cultivation of a supportive community in the cohort.

Elementary mathematics specialist (EMS) programs are designed to (a) support teachers' capacity for high-quality mathematics instruction and (b) support teachers' leadership development so they can, in turn, support the improvement of mathematics instruction in their schools and districts (de Araujo et al., 2017). Although research has found EMS programs can produce improvements in knowledge, beliefs, and practices for teachers (e.g., Myers et al., 2020), less research has documented the elements of EMS programs that contribute to teachers' development (e.g., Reys et al., 2017). In this paper, we share findings on EMS development in relation to two program goals—instructional vision and leadership capacity. We also examine participants' perspectives about how they developed these attributes and what elements of the EMS program supported this development.

LITERATURE REVIEW AND BACKGROUND

In our context, EMSs were teachers who had completed a 2-year graduate program (e.g., Goodman et al., 2017; Harrington et al., 2017) based on the 2013 Association of Mathematics Teacher Educators' Standards for Elementary Mathematics (recently updated; see Association of Mathematics Teacher Educators, 2024). This document provided research-based guidelines for preparation of EMSs in terms of mathematics content, pedagogy, and leadership. Accordingly, the program that served as this study's focus included many learning experiences designed to develop expertise in all three areas (de Araujo et al., 2017). Furthermore, several research studies have shown how graduates of the program have employed their expertise as teachers and teacher leaders (Conner et al., 2022; Nguyen et al., 2022, 2024; Webel et al., 2017, 2018, 2021, 2023). This work adds to a robust body of literature on EMS programs and their impacts on both participants and those served by EMSs.

Impact of EMS Programs

EMS programs generally have shown significant impact on teachers' knowledge and beliefs (e.g., Campbell & Malkus, 2014; Swars et al., 2018) and their teaching practices (Myers et al., 2020, 2021; Nickerson, 2010). EMSs who complete these programs have shown positive impact on student achievement when employed as teachers (Kutaka at al., 2017) or coaches (Campbell & Malkus, 2011). Some scholars have documented how graduates of EMS programs can exercise agency to push the boundaries of their contextual constraints to enact more ambitious practice (Nguyen et al., 2022; Webel et al., 2017, 2021).

In one example, we described how, despite building-wide expectations of daily differentiated instruction in small groups, multiple EMSs considered and implemented alternatives to homogeneous ability grouping, including using mixed ability groups and random grouping (Webel et al., 2021). In another example, we documented multiple structures for departmentalization (i.e., EMSs taught mathematics to multiple groups of students each day; Webel et al., 2017). In these cases, EMSs had some latitude to negotiate details of their departmentalization structure and reported departmentalizing enabled them to focus more effectively on planning and refining their mathematics instruction. These examples show how EMS teachers not only used their expertise to enact high-quality instruction but also exercised agency to create additional opportunities to provide high-quality mathematics instruction for students in their schools. These findings raise the question of how

EMS programs develop teachers' expertise and equip them to influence mathematics teaching at their schools.

Development of EMSs' Capacity for Delivering and Promoting High-Quality Instruction

One way EMS programs support the development of capacity for high-quality mathematics instruction is by integrating learning opportunities with teachers' practices. For example, in one EMS program, participants highlighted the importance of having opportunities as part of their coursework to implement observed lessons and receive feedback from instructors and opportunities to analyze student thinking, including their own students, using frameworks and progressions introduced in their coursework (Myers et al., 2020, 2021). In their analysis, researchers described how participants moved through stages of development: (a) exhibiting skepticism about new ideas for teaching mathematics, (b) demonstrating willingness to try out new practices, (c) making substantial shifts in their practice, and (d) expressing a desire for additional support to sustain these shifts (Myers et al., 2020). In addition, EMSs noted nonevaluative feedback and a comfortable, collaborative space for discussion about their teaching were key components of their learning experiences.

Such learning opportunities can lead to substantial changes in participants' beliefs about mathematics and mathematics teaching and learning (Campbell & Malkus, 2014; Swars Auslander, 2023; Swars et al., 2018; Webel et al., 2023). Beliefs are related to a slightly more specific construct: teachers' visions for instruction (Arbaugh et al., 2021; Jansen et al., 2020; Munter, 2014). Whereas beliefs suggest "a relatively static set of decontextualized ontological commitments . . . vision is intended to communicate a more dynamic view of the future" (Munter, 2014, p. 587), providing a sense of what teachers imagine for their teaching, even if it is not currently reflected in their practice. Vision is also more grounded in practice than beliefs; it is "a set of images of ideal classroom practice for which teachers" (Hammerness, 2001, p. 143) can strive. Indeed, it is difficult to conceive of teachers implementing instruction they have not imagined for themselves.

Elements of vision include the role of the teacher, the structure and nature of classroom discourse, and the nature of mathematical tasks students are asked to complete. In general, EMS programs have been designed around a vision of instruction articulated in standards documents published by the National Council of Teachers of Mathematics (NCTM, 2000, 2014, 2020). According to these documents, high-quality mathematics instruction empowers students to make sense of mathematical concepts; engages them with tasks that are cognitively demanding, mathematically rich, and often set in real-world contexts; and provides them with opportunities to discuss, explain, connect, and justify their reasoning through conversations with each other and with their teacher.

Researchers have found teachers with instructional visions more aligned with these principles are more likely to see improvements in their instructional practice (e.g., Munter & Correnti, 2017). Because EMS programs typically have been designed around this cohesive vision for mathematics instruction, we sought to explore whether and how teachers' images of their future instruction were impacted by their time in EMS programs, which could provide insights into whether and how these shifts likely impacted their future instruction. We anticipated this study could complement research on EMSs' beliefs and provide new insights into how EMSs' instructional vision develops through certain kinds of experiences, along with how this development corresponds to changes in their instruction.

Development of EMSs' Capacity for Leadership

In addition to developing expertise and capacity for high-quality mathematics teaching, researchers have found EMSs work in a variety of ways to support elementary mathematics instruction as leaders or coaches (Baker et al., 2022; Campbell & Griffin, 2017). For example, coaches engage in co-planning with individual or teams of teachers, modeling lessons, identifying and filtering mathematics resources to meet teacher and district needs, conducting workshops, developing assessments and organizing student data, and providing personal professional development to support teachers' work (Baker et al., 2022; Campbell & Griffin, 2017).

EMSs also engaged in both formal and informal leadership without becoming a coach or leaving their classroom teaching positions (Conner et al., 2022). For example, EMSs reported serving on mathematics committees in their schools or districts, mentoring new teachers, assisting in analyzing achievement data, leading community outreach events, and planning meetings with grade-level teams (Conner et al., 2022). EMSs also participated in spontaneous and informal conversations with colleagues where they provided advice and information about mathematics instruction (Nguyen et al., 2024). The fact that much of this work can be accomplished without EMSs leaving the classroom means they can be a cost-effective option for instructional leadership in under-resourced schools. In sum, these existing findings suggest well-prepared EMSs often have the capacity to influence instruction in classrooms beyond their own and can influence policies that impact mathematics instruction.

Additionally, research suggested one important trait that enables EMSs to be effective as mathematics leaders is confidence, which was associated with improvements in the knowledge, instruction, and self-efficacy of the teachers with whom leaders work (Yopp et al., 2019). Conversely, teacher–leaders who lack confidence may be reluctant to assume leadership positions and struggle to advocate for their work or gain legitimacy with their colleagues (Hunzicker, 2017; Wenner & Campbell, 2017). However, limited research has described how EMS programs help develop capacity for leadership. In one study, Swars Auslander et al. (2023) found EMSs experienced positive, significant shifts in beliefs about their coaching effectiveness, and EMSs with stronger self-efficacy beliefs reported using more coaching practices, especially those related to supporting teachers' mathematics content and pedagogical knowledge. EMSs attributed their increased confidence regarding leadership to (a) participating in a community of other EMSs; (b) opportunities to engage in teacher leadership through the program; and (c) greater knowledge about mathematics content, pedagogy, and coaching strategies (Swars Auslander et al., 2023).

These findings point to the importance of teachers' self-perceptions as leaders and the relationships between teacher–leaders' confidence and their developing knowledge, awareness of, and participation in leadership. Because the transition from teacher to teacher–leader or coach entails shifts from work in which they are a recognized expert (i.e., teaching) to work in which they are relatively inexperienced (i.e., supporting teachers), it can require even veteran teachers to develop new identities and ways of seeing themselves and their work (Chval et al., 2010; Hanuscin & Zangori, 2016; Zuspan, 2013). For example, as teacher–leaders build trust with colleagues, learn to negotiate with administrators, and choose when to take risks, they may become aware of aspects and elements of schooling they had not considered previously (Zuspan, 2013).

Knapp (2017) described this widening of attention as emerging leaders taking up a system view of school improvement, supporting their transition from teacher to teacher–leader. This attention to broader contexts was also important in Hunzicker's (2017) study of teacher–leaders' self-perceptions, where teachers least likely to view themselves as teacher–leaders preferred leading on a smaller scale, in limited situations, or in their classrooms. Similarly, Brooks et al.'s (2004) typology of teacher–leaders foregrounds the parameters of their leadership responsibilities, moving from the classroom to the department to the whole school. This typology suggests a trajectory: As teachers gradually develop the skill and confidence to begin leading, they start with small steps and eventually take on larger responsibilities. As their responsibility expands, their scope of awareness also increases. They become aware of larger elements of the system, leading to the development of new skills and, potentially, increased confidence.

These findings raise questions about how leadership capacity is developed in EMS programs, specifically how teachers' conceptions about what leadership entails and their views about their own capacity to enact leadership evolve during their time in an EMS program. Additionally, further exploration may elucidate what a trajectory of mathematics leadership development looks like and reveal the kinds of experiences and activities that support teachers in feeling prepared to address the challenges noted previously and to develop the capacity to enact leadership in their contexts.

CURRENT STUDY

In the current project, we used a case study approach (Yin, 2018) to analyze a variety of data sources to examine EMS development across a 2-year graduate program, seeking to understand some dimensions that had not yet been explored and to understand what learning experiences were most powerful from the perspectives of the EMSs. We investigated two research questions (RQs):

1. How do EMSs' (a) visions for mathematics instruction and (b) conceptions of themselves as leaders develop over the course of their EMS program?
2. How do EMSs describe their development and the elements of the EMS program that supported this development?

Context and Methods

The context for this study was an EMS program comprised of 24 graduate credits earned over 2 years comprised of five content courses and two leadership courses. The program was co-designed by faculty at multiple institutions across the state of Missouri in the United States (Goodman et al., 2017). Common syllabi, lesson plans, and assignments were shared across sites, and representatives from each institution gathered each year to debrief experiences from the previous year and revise courses.

Participants were 24 elementary teachers who were recruited to become fellows as part of a National Science Foundation (NSF) grant, which paid the tuition for the EMS coursework and provided yearly stipends for 4 additional years of teaching and leadership in a high-need school district. Of these fellows, 13 participants attended Institution 1 and taught in District 1, and 11 participants attended Institution 2 and taught in District 2. To provide fellows with greater opportunity for collaboration on assignments and to use their collective expertise in their schools, program leaders at Institution 1 decided to select fellows from schools in pairs or trios (with one exception). All fellows who participated in the grant program agreed to participate in the research project.

Fellows attended in-person, week-long summer institutes at the beginning and end of the program, which focused on (a) understanding core research findings about mathematics teaching, learning, curriculum, equity, and assessment and (b) developing a vision and skills for leadership in mathematics education. Fellows completed the other five courses, which included online and in-person components, at each institution. Across both institutions, the five content courses covered number and operations, rational number, algebraic reasoning, geometry and measurement, and data and probability. These courses focused on developing an understanding of both mathematical content and research on student learning; a core textbook used across the program was Elementary and Middle School Mathematics: Teaching Developmentally (Van de Walle et al., 2014). Fellows also were expected to read numerous other books and articles, often published by the NCTM.

Each course integrated both content and pedagogy. For example, knowledge of content often was introduced through interaction with artifacts of instruction (e.g., student work, videos of teaching, mathematical tasks, representations of mathematics from elementary classrooms). Assignments often required fellows to solve mathematical tasks, anticipate how students would solve tasks, engage with research on student development of specific mathematical concepts, analyze student work, examine episodes of instruction and coaching, give tasks to their students, and discuss their teaching challenges and dilemmas. An example of an activity was when fellows were asked to make conjectures about a number string (Bray & Maldonado, 2018) and then prove their conjectures with multiple representations (e.g., in words, with symbols, with an area model). The following number string provides an example:

$$3 \times 18$$
$$6 \times 9$$
$$5 \times 28$$
$$10 \times 14$$

During this activity, participants discussed how such sequences of problems could be used to help children develop a flexible understanding of the group structure of multiplication to solve a range of problems quickly and creatively. In doing so, fellows developed their own mathematical knowledge for teaching—that is, a deep understanding of mathematics used specifically in teaching (Hill et al., 2008).

Each content course was paired with a corequisite internship, which included assignments that engaged fellows in using their teaching contexts to apply ideas from the course. For example, each of four content courses introduced an instructional routine (Lampert et al., 2010): number talks (Parrish, 2011), number strings (Bray & Maldonado, 2018), sorting tasks (Baldinger et al., 2016), and contrasting cases (Teacher Education by Design, 2014). Fellows planned, enacted, and debriefed each routine with colleagues with the goal of developing not just knowledge about mathematics teaching but also the pedagogical skills needed to teach mathematics well (Loewenberg Ball & Forzani, 2009; NCTM, 2014).

Fellows also were expected to engage in leadership activities in their schools as part of these internships, such as creating and carrying out an action plan for supporting improvements to mathematics instruction, planning and implementing an outreach event, and planning professional development sessions for their colleagues. Fellows who taught at the same school were encouraged to complete these leadership assignments as a team. In one example, after a series of sessions on equitable grouping structures at the first summer institute, one school team developed an action plan to begin implementing alternatives to grouping students by ability (Webel et al., 2021). Each month they reported on their plan, made revisions, adapted their approach, and shared their progress with their peers in the program.

Data Collection

Data were collected via interviews at the beginning (Year 0), middle (Year 1), and end (Year 2) of the EMS program. To assess the development of participants' vision of mathematics instruction (RQ1a), we administered the Visions of High Quality Mathematics Instruction (VHQMI) interview protocol (Munter, 2014) to each of the 24 participants. Using the semistructured protocol (see Appendix), we asked participants to describe what they would look for in a mathematics classroom to determine whether the instruction was high quality. The protocol included questions about what the teacher would be doing, what the students would be doing, and what kinds of tasks would be presented to students.

To assess participants' development of their conceptions of themselves as leaders (RQ1b), we asked them to describe their ideas about possible leadership activities in which they would like to engage, what they would want to keep in mind in a leadership role, and what they were excited and apprehensive about regarding such a role. In the Year 1 and Year 2 interviews, we also asked whether anything had changed in terms of how participants engaged in leadership or were seen as leaders in their schools or districts.

To investigate participants' views about the impact of the program on their own development (RQ2), we conducted six focus groups with the 13 fellows at Institution 1. We asked how the program had helped them address teaching challenges; how their teaching of mathematics had changed; what readings, ideas, or activities had made the biggest impact on them; and what challenges they still faced regarding teaching and leadership in mathematics.

Data Analysis

To analyze the development of participants' visions for mathematics instruction (RQ1a), we focused on four VHQMI rubrics (Munter, 2014): (a) role of the teacher, (b) mathematical tasks, (c) patterns of classroom talk, and (d) nature of classroom talk. For each dimension, we used the rubrics to assign a score (0 to 4) to represent the sophistication of discourses teachers employed to characterize ideal classroom practice at the beginning of the program and after 1 year. The role of the teacher rubric characterizes the teacher's role along the dimensions of influence on classroom discourse, attribution of mathematical authority, and conception of typical activity structure, culminating in labels that include "teacher as more knowledgeable other," "teacher as facilitator," and "teacher as monitor." The mathematical tasks rubric addresses how teachers describe task elements such as cognitive demand, real-world application, multiple solution paths, and opportunities to generalize or make connections across mathematical ideas. The patterns of classroom talk rubric considers the extent to which student-to-student discourse is promoted, whether whole-class conversation is prioritized, and whether the students or the teacher initiate and carry out the talk. Finally, the nature of classroom talk rubric identifies whether talk focuses on concepts or calculations and the extent to which the talk is focused on mathematics. Ratings for each of these dimensions were assigned to each interview independently by two researchers, who then met to resolve discrepancies.

To categorize participants' conceptions of themselves as leaders (RQ1b), we used an open-coding process initially to identify ways participants answered questions about leadership. We noted variance in two dimensions: (a) confidence in themselves as leaders and (b) awareness of the nuances and responsibilities of leadership work. Confidence was judged through explicit statements referencing discomfort/comfort, anxiety/eagerness, and intimidation/assertiveness when discussing the prospect of leading others. Often these characterizations surfaced when participants were asked what they were excited and apprehensive about regarding leadership. In some cases, especially at the higher end of the spectrum, confidence was not stated explicitly so much as inferred from the ease with which participants described their leadership or talked about their perspectives on teacher learning.

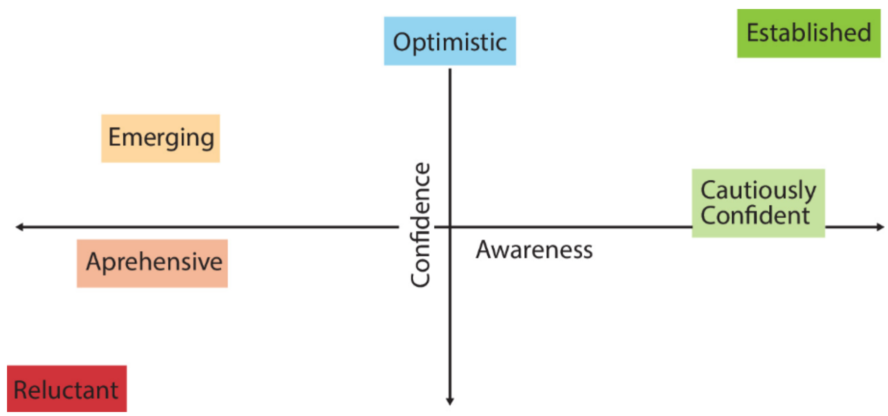
Awareness of the nuances and responsibilities of leadership work (hereafter referred to as "awareness") was determined through consideration of how detailed and specific participants were in their descriptions of leadership, including having realistic expectations for what kinds of changes can be accomplished in certain time frames, the roles of various policies and policymakers relevant to the changes they hoped to see, and their understandings of the limitations and challenges that would need to be confronted in their specific contexts. It was possible for these two dimensions (i.e., confidence and awareness) to fluctuate somewhat independently of each other, but our objective was to consider how a single trajectory of development could capture these changes and the relationships between them. For example, confidence might rise as participants developed deeper knowledge and teaching expertise, but opportunities to lead might cause greater awareness of systemic challenges, which could result in loss of confidence. We sought to develop a framework that would capture stages of development between varying levels of confidence and awareness and how they related to each other.

Looking across the cases, we created the following labels as a holistic characterization of how participants expressed their conceptions of themselves as leaders: established, cautiously confident, optimistic, emerging, apprehensive, and reluctant (see Table 1). We plotted a rough conception of how each characterization varied in terms of confidence and awareness in Figure 1. We revisited the data using these characterizations with each interview coded individually by two researchers. In cases of disagreement or uncertainty, we met as an author team to compare our evidence from the interviews, sometimes playing back portions of the interview until we agreed on the characterization for that participant. In this paper, we compared results from Year 0 and Year 2 interviews.

Table 1
Leadership Conceptions Framework

Leadership conception	Confidence	Awareness
Established	Identifies as a leader through experiences in which they have acted as a leader (formally or informally) in their school or district. Comfortably reflects on leadership experiences and challenges. Appreciates the complexity of the work. Actively seeks to address concerns or challenges in their context.	Specifically identifies leadership constraints that are relevant to building/district and considers ways to best navigate them. Shares general ideas (personal theories) about how teachers learn/develop.
Cautiously confident	Identifies as someone who is comfortable being seen as a leader in some aspects of their contexts but also expresses some reservations. Is somewhat daunted by the complexity of the work.	Specifically identifies constraints relevant to building or district and considers ways to best navigate them. Identifies specific goals for building or district.
Optimistic	Expresses excitement for developing leadership skills and potential status. Takes a learning orientation; excited to learn more. Conveys a sense of anticipation about what they will be able to do.	Identifies possible constraints that can exist in any space (not specific to context). Expresses some specific ideas regarding strategies for navigating constraints. Names broad goals that can be applied to any space.
Emerging	Expresses concern about the extent to which they are seen as knowledgeable. May say a goal is to get better at teaching first before engaging in leadership.	May be aware of possible constraints, but those are more likely to be based on assumptions than experiences. Not specific about how these constraints would be navigated. Expresses lack of prior consideration of personal leadership skills/goals (e.g., "I guess I'll just try it and see how it goes").
Apprehensive	Expresses doubt in leadership ability and/or is unsure about wanting to have a leader label. Expresses concern about their current level of expertise. Intimidated by more visible leadership opportunities that involve more responsibility.	Vaguely aware of constraints and worries about navigating them. Expresses a desire for relatively limited leadership opportunities with lower stakes (one on one, working with novices).
Reluctant	Does not see self as a leader and does not desire to become a person of leader status.	Aware of constraints but has few or no ideas for addressing them.

Figure 1
How Leadership Conceptions Fit Onto Dimensions of Confidence and Awareness



To understand how EMSs described their development and how the program supported this development (RQ2), we analyzed six focus groups and attended to development across five emergent dimensions: knowledge, practice, vision, leadership, and confidence (see Table 2).

Table 2
Categories for Self-Described Development

Category	Description	Examples
Knowledge	Descriptions of mathematical knowledge for teaching developed during the program	Progressions of children's thinking Common mathematical misconceptions Relationships between mathematical ideas, justifications for mathematical rules, meanings of mathematical symbols or procedures
Practice	Descriptions of how specific elements of instructional practice changed as a result of the EMS program	Using higher quality tasks Using routines explicitly emphasized in the program (e.g., number strings, which one doesn't belong) Using purposeful questions (e.g., "Is this always true?") More substantive math talk Reducing use of ability grouping, alternate approaches to differentiation Using different representations (e.g., number lines)
Vision	Descriptions of general shifts in perspectives about teaching mathematics as a result of the program	The importance of analyzing and understanding student work; focus on students' assets The importance of attending to students' experiences in math and their math identities Attention to the social dimensions of mathematics lessons Mathematics as a tool for change
Confidence	Descriptions of how the program impacted their confidence in teaching mathematics or in leading others in mathematics	Increased autonomy for adapting curricular materials Increased enjoyment of teaching mathematics, increased passion for mathematics Developing own mathematical identity More comfortable teaching mathematics More comfortable being positioned as a leader More willing to have others observe their instruction
Leadership	Descriptions of specific leadership activities attributed to the EMS program	Increased opportunities for formal professional development with teachers Informal conversations with colleagues, including novice and beginning teachers Providing resources for colleagues Leading by example Having practice identified as exemplary; being positioned as having expertise Resisting policies that conflict with vision for instruction Sharing mathematics teaching examples on social media

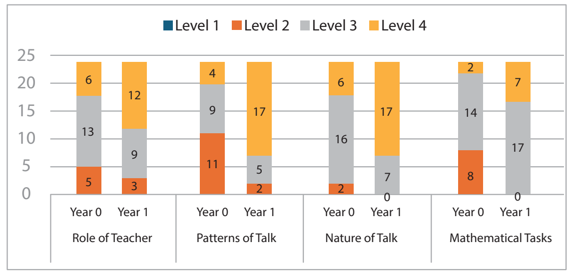
Segments where participants described development in these categories were identified by timestamps. We examined the content in each tagged segment and listed elements of the program referenced as supporting participants' development, taking note of specific assignments or activities. We then compared elements identified across the six focus groups to characterize ways participants perceived the program as supporting their development and together created a table to consolidate our findings.

FINDINGS

Visions of High-Quality Mathematics Instruction (RQ1a)

Results for our analysis of participants' visions of instruction are in Figure 2. Compared to the beginning of the program, more participants were rated at Levels 3 or 4 in all categories after the 1st year. In role of the teacher, patterns of talk, and nature of talk, at least half the participants were rated at the highest level, Level, 4, after their 1st year in the program, and only a few participants were at Level 2.

Figure 2
Development of Visions for High-Quality Mathematics Instruction



In the role of the teacher dimension, of the 18 participants who were not already rated at Level 4, 10 had higher ratings at the end of the 1st year than at the start of the program. For example, five participants initially were rated as Level 2, indicating a teacher as monitor conception, in which a high-quality mathematics lesson involves providing opportunities for children to work together on mathematical tasks, but the teacher is described as starting lessons by demonstrating or leading discussions on how problems should be solved and is treated as the primary source of knowledge. In one case, a participant named Candace talked about starting her lessons with "a little bit of that direct instruction" and then "a lot of group work." Candace emphasized the role of talk because, as she noted, "it's crazy how much they can learn from each other." However, Candace also talked about "pulling a group of students" to say, "hey, I just want to make sure you understand this." This description conveyed the idea that a teacher presents material, monitors students as they work, and intercedes as students experience struggle. Candace was mostly concerned students were "getting it" as they worked on problems.

After a year in the program, Candace's description of the teacher's role had shifted to a teacher as facilitator (i.e., Level 3) conception. She described a high-quality lesson involving the teacher:

> *[It's] not even leading the student into the answer maybe they're looking for, but just putting out the "tell me more*

[1] We used "knowledge" here to include knowledge of mathematical concepts and the validity of specific strategies and knowledge of how students' thinking develops, how mathematical ideas can be represented, etc. This approach is essentially the mathematical knowledge for teaching framework described by Hill et al. (2008), but, in this case, we analyzed participants' descriptions of their development of knowledge rather than as measured by the assessments developed by Hill and colleagues (e.g., Schilling & Hill, 2007).

> *about this" or "why did you do it this way?" and having the student explain their math, not what the answer is but the process of getting to that answer.*

Candace also discussed the importance of the teacher "just sitting and listening to the kids . . . and if [they're] confused, just being like, 'tell me more, keep going,' instead of trying to push them to the answer." In this description, Candace was less concerned about monitoring the correctness of the children's work and instead encouraged students to work through their confusion. Candace conveyed more trust in students' abilities to resolve their own misconceptions through conversation.

Growth also was seen in the patterns of talk category, which addressed the structure of mathematical conversations (e.g., whether they occur as a whole class or small group), and the nature of talk category, which addressed more directly the content of the talk (e.g., whether it had a calculational or conceptual orientation). For example, in her initial interview, a participant named Erin was rated at Level 2 for patterns of talk and Level 3 for nature of talk. Erin talked about valuing student-to-student discourse but only in the context of small group work, not as a vital component of whole-class discussion. Erin described her use of small groups for math stations and explained, "Those are my times for more exploration time or practice time . . . I'm working with specific skills that I know kids are missing." Erin talked about how other students could play games or work on other tasks while she was "pulling a small group and teaching those missing skills." These descriptions suggested the focus of these small-group conversations was to enable students to solve problems correctly rather than to wrestle with ideas. Whole-group discussions of ideas were not an emphasis in Erin's description of high-quality instruction.

At the end of her 1st year in the EMS program, Erin was rated at Level 4 for both patterns of talk and nature of talk. She talked about "posing a problem or task" and "letting them [the students] explore," and then Erin would monitor children's thinking to plan for the whole-class discussion (e.g., "I want you guys to share your strategy when we go back to the whole group"). Erin discussed the importance of asking questions and noted she encouraged discussion by saying things like:

> *"How did you see that, how did you get that? Can you tell me more? Can you explain more?" And getting the kids to do most of the talking. Or, "I heard you say that you disagree. Can you tell me why you disagree?" And really encouraging students to do most of that.*

Erin continued to talk about students working in small groups or with partners in this interview but also described these activities as designed to serve the whole-class conversation rather than being spaces for her to fill in missing gaps. She noted:

> *[I try] giving them different opportunities to partner talk. I think that's super powerful for them to feel confident and building that, "if I can tell a partner first, then I can share with the whole group." So partner talking and then engagement and just the idea of, like, using hand signals like "I agree" or "I have one strategy, two strategies, three strategies."*

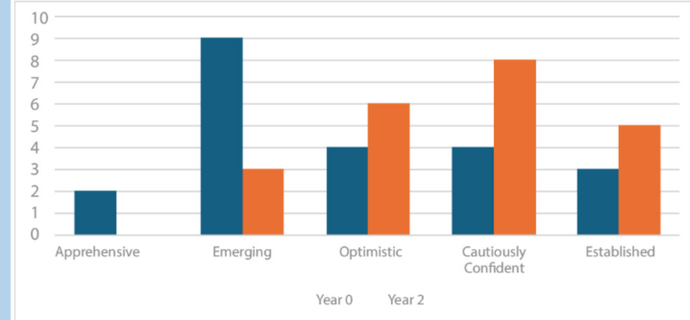
In general, Erin's description of the patterns of talk in a high-quality lesson changed markedly over the 1st year of the EMS program, moving from a structure emphasizing small-group work as the setting for resolving mathematical misconceptions to one in which whole-class conversation was designed to support mathematical argumentation about ideas students generated in small groups. These kinds of changes were evident for most participants across the sample (see Figure 2).

In their descriptions of mathematical tasks, there were fewer participants at Level 4 compared to the other categories, but nine of them received higher ratings in Year 1 than Year 0 (two already were at Level 4). Laura was a participant who moved from Level 3 to Level 4. In her initial interview, she described features of tasks in a general way; they would have multiple solution paths and would require students to do "something more than just memorized fact . . . but actually have to investigate something." She also talked about the importance of "having manipulatives out." After a year in the program, Laura was more specific about the function of multiple solution paths—that these would create opportunities for "analyzing each other's thinking" and to work through disagreements. Laura said, "I actually get really happy when a couple of my kids are disagreeing with each other because that's the one I'll pull back to [discuss with] the class." In this example, the task functioned to provide content for a whole-class discussion where students engaged in mathematical argumentation.

In another contrast with her Year 0 interview, Laura discussed tasks not in the abstract but instead with specific examples. She said, "There were five animals in a race, but 16 legs crossed the finish line; what animals could have been in the race?" Laura noted this task, a good example of doing mathematics (Stein et al., 1996), was valued because it "makes [students] think a lot more" about the mathematics. Overall, Laura's development represented a shift from broad descriptions of aspects of high-quality tasks to more specific descriptions and examples that included a rationale rooted in the potential of the task to generate discussions where strategies were compared and defended using mathematical reasoning. This case represents the kind of development we saw across multiple participants.

Fellows' Conceptions of Themselves as Leaders (RQ1b)
Over the 2 years of the program, our analysis revealed a decrease in the number of participants characterized as apprehensive and emerging and an increase in participants categorized as optimistic, cautiously confident, and established (see Figure 3).

Figure 3
Changes in Characterizations of Fellows' Conceptions of Themselves as Leaders



In one example of development, a participant named Maggie, initially characterized as apprehensive, began the program articulating her lack of confidence in her mathematics teaching and expressing hope her confidence would grow. She said, "Through these classes I'll develop even more confidence, like as a math teacher." Maggie expressed vague ideas about teacher development, stating, "Everybody is on a different journey and needs to grow in different areas." By the end of the program, Maggie expressed increased confidence, noting she felt "more comfortable in taking a leadership role" and was "excited to share what [she had] learned" with colleagues. Maggie talked about how one of the other participants was leaving her school, which was going to, in her words, "force me into more of leadership role . . . which will be good for me." However, at other times, Maggie continued to express doubts about her expertise, saying, "I try to be really honest about the fact that I'm not super knowledgeable." We characterized Maggie as emerging, showing some development from the initial interview but still demonstrating a substantial amount of hesitation about taking on leadership activities.

Janet, a participant who moved from emerging to cautiously confident, talked about her growth in how she saw herself in relation to leadership. Janet said:

> *When I started, I didn't really think of myself as a leader. I'm kind of quiet. I think about what I want to say. I always thought that was not a good characteristic of a leader. But really, it's good because maybe I'm more approachable. I'm not saying, "You have to do it this way." Let's talk about it.*

Janet originally described her quiet personality as a limitation, but she grew to recognize it as an asset, suggesting an expanded view of leadership that can include a quiet personality. Janet continued to express some apprehension in her Year 2 interview. She noted, "I've always been really careful . . . I don't like conflict all that much," and commented, "doing professional development has always been a little nerve-wracking for me." However, Janet also said, stepping out "would be good for me, too." At one point, she expressed concern about a new data teams policy she feared would increase emphasis on ranking and sorting students based on their mathematics achievement. Janet worried, "They want to group by ability for everything," and she expressed feeling an obligation to resist that trend. She said, "And I'm like 'No!' . . . I'm passionate about that, so it's like, I'm gonna have to really, you know, let everybody know, help share this information and how this [ability grouping] is hurting our children."

Janet ended the interview discussing her excitement for working with her colleagues from the EMS program in their building to create schoolwide interest in improving mathematics instruction. She said:

> *One of the things the three of us [fellows] have said is that we're going to try to do like a monthly PD session. . . . I'm hoping we're going to do a different routine each time. We're going to kind of see like what people want to talk about, like, if there's something they want to focus on. The three of us could maybe do something different to meet more teachers' needs . . . and I could do some number talk stuff with a small group of teachers or something, so I'm excited about that possibility.*

Overall, at the end of the program, Janet conveyed much more interest in stepping into leadership spaces, including sharing new strategies for more equitable approaches to grouping children for mathematics.

In another example, a participant named Gina began the program with vague ideas about how she might engage in leadership. She said, "I think just letting them know, 'Hey, I'm here. I'm doing this program. I'm more than willing to help you if you have questions.' Just being open and available for them." We characterized Gina's conceptions of leadership as emerging because she indicated a willingness to help colleagues but little evidence of awareness regarding the challenges and tensions involved in such work. By the end of the program, she was notably more specific about challenges and more intentional in her comments about leadership. In this excerpt, Gina described her desire to help children taught by an experienced teacher. She said:

> *She's been teaching for a long time, and it's hard sometimes when a younger teacher comes in and tries to offer suggestions or things that we could do differently. And I don't want to come off like, "I'm perfect, I know everything," like, "I have a degree and you don't," but I also know I have to help those kids. Because I, I grew up with not good math experience, and I honestly, I did not like math growing up because of the way it was presented to me.*

In this Year 2 excerpt, Gina, then characterized as cautiously confident, acknowledged tensions that can come with leading and was more intentional about leadership as she articulated an obligation to help children whom she feared were having negative experiences with mathematics ("I have to help those kids"). Like many participants in our sample, there was a significant shift across Gina's interviews between concern for "my kids" (i.e., the children in her classroom) and "our kids" (i.e., the children in her school), indicating an increased sense of responsibility for leadership beyond her classrooms.

Ella was a participant who initially described herself as having expertise in teaching language arts but who said she was excited "to have the opportunity to learn more about math and feel more confident about math." Characterized as cautiously confident, Ella was specific about some of the challenges of leading in school, noting the school had just completed a large reform effort with literacy instruction. She expressed caution about trying to change too much too fast. Ella said:

> *The first step might just be giving more information about providing student feedback. That's something that teachers, they are already doing that, just looking at how we are providing student feedback, doing some professional development on that and through our conversations in our grade levels . . . I thought that would be something really small. It's small to do as a teacher, but its huge in the direction of where students can grow as mathematicians.*

This excerpt shows Ella's awareness of the expectations in her building and sensitivity to what kinds of learning might be doable for teachers and impactful for students. In the Year 2 interview, Ella maintained a nuanced view of leadership. She stated, "I'm doing more listening and questioning, versus, 'this is what you need to do'" and described a number of leadership activities (e.g., "I have been on the ELA [English Language Arts] Committee and the SRG [Standards-Referenced Grading] Committee. I've been put in those positions."). However, Ella also noted an increase in her confidence, noting:

> *At the very beginning of the program, I would have never have ever said that I would be confident to share anything about math. Math was not my jam. This program has provided me with a lot of the tools that I would need . . . to really provide quality leadership.*

We saw the increase in confidence as development in our trajectory and characterized Ella's Year 2 interview as reflecting an established leader.

Overall, we saw substantial development over the course of the EMS program in how participants talked about leadership in mathematics and themselves as mathematical leaders in their contexts. In terms of confidence, several moved from expressing hesitation and apprehension to expressing excitement or even passion about promoting change outside their classrooms. They seemed to expand and refine some of their ideas about leadership, moving from vague or overly simplified notions of leadership to articulating specific tensions, challenges, or strategies for leading in their contexts.

Fellows' Impressions of How the EMS Program Impacted Their Development (RQ2)

In the following sections, we share some aspects of the EMS program that, according to participants, supported this and other types of development.

Knowledge

Several participants described developing new knowledge due to the program overall or because of particular assignments. Many participants talked generally about learning how students' mathematical conceptions develop over time in a particular domain (e.g., numbers and operations). One participant, who taught a lower grade, described how the program helped her "understand what kids are going to need when they leave [her classroom], and [knowing] what they're going to be doing in fourth grade and fifth grade, and middle school has been really powerful for me." Participants also described development of their own mathematical insights, such as a justification for the doubling/halving multiplication strategy (e.g., $12 \times 15 = 6 \times 30$) and an understanding of why and how number strings support students (Bray & Maldonado, 2018).

One assignment mentioned several times was related to the article "13 Rules That Expire" (Karp et al., 2014), which helped one participant realize "how many things I was saying to my kids [that] had expiration dates or how many things I was overgeneralizing." Another frequently mentioned activity was an assignment exploring how children often misinterpret the equals sign as a signal to compute. One participant reflected that, when she first started teaching, she wrote multiple equals signs, so the equation would not be accurate (e.g., $12 + 6 = 18 + 4 = 22$), and, by Year 2, she made sure to communicate to students "that that continuation is not going to work because you're [mis]representing what that equal sign means." Both "13 Rules That Expire" and the equals sign activity were mentioned by participants who taught across the lower and upper elementary grades.

Instructional Practice

Regarding impact on instructional practice, participants identified ways the program impacted their planning for and set-up of instruction and their facilitation of instruction. Several participants described being more intentional in their lesson planning, particularly in terms of selecting and adapting curricular resources. One participant talked about how she and other participants at her school worked hard to "analyze the lessons they were supposed to be teaching [from the assigned curriculum], find a problem . . . to tweak and modify in a certain way, and [then] spend most of their [instructional] time on that." Another participant gave an example of a "pretty good problem [from the curriculum] when you look at it on the surface" that she rewrote "as a compare and contrast routine" using the "Van de Walle text" (Van de Walle et al., 2014). Another participant talked about pushing back against homogeneous ability grouping—a common practice across her school—to instead do mixed-ability grouping (Webel et al., 2021). Several participants discussed creating space for children, rather than the teacher, to talk about mathematics.

Participants frequently mentioned instructional routines as a way the program impacted their facilitation of instruction. Each of the four routines introduced in the program (i.e., number talks, number strings, which one doesn't belong, contrasting cases) were named across the focus groups. One participant talked about her growth in understanding and using these routines. She said, "I will be honest that when I started this program, I had never ever done a number talk in my life. . . . [Initially] I was so confused and couldn't figure out what the difference was" between a number talk and number string. However, by Year 2, she did them daily and noted she had "developed a rotation of a different routine every day." Several participants also discussed the program's impact on their questioning, including encouraging students to make and explore conjectures. Multiple participants described using the question "Is this always true?" to press students to generalize, which was a question emphasized in their algebraic reasoning course. One participant described an anchor chart in the focus group labeled "math rules we think are true" with examples of class conjectures such as, "When you add a double it will always equal an even number." All these examples show ways participants saw the program as directly impacting their instruction.

Vision

Complementing the findings from our individual VHQMI interviews, focus group participants described broad shifts in their visions for instruction, moving from thinking about teaching mathematics as helping children use specific strategies to helping them engage in sensemaking. One participant described this as a holistic transformation:

> *When I first started teaching, I did so much "I do, we do, you do," and I just drilled, and we practiced . . . how to apply an algorithm and how to do a procedure. . . . From the classes and the activities we did and the conversations with [other fellows], . . . my teaching has completely changed. I no longer try to get through every single problem in the book. We often start off with a task that students will complete on their own, and then we'll talk about it and share strategies.*

Other participants described their shifts in terms like mathematics no longer being quiet, allowing students to use strategies that make sense to them, and taking an asset-focused approach that emphasized what students can do rather than what they cannot do. Participants across focus groups described a desire to make mathematics enjoyable for students and cultivate positive mathematics identities.

In describing how the program impacted their visions for mathematics instruction, many participants referenced issues of equity. They frequently named The Impact of Identity in K–8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices (Aguirre et al., 2013), a book used in one of their leadership classes, as important to their considerations of how to engage typically underserved groups of students (e.g., underrepresented ethnic or racial groups, students learning English) and toward disrupting homogeneous ability grouping. Multiple participants also referenced an assignment that used the equity quantified in participation (EQUIP) tool (Reinholz & Shah, 2018) as integral to examining their own teaching practice, particularly as it related to expectations for and questions to different students. One participant, in connecting mathematics to literacy, mentioned the need for "windows and mirrors," where students "can all see themselves in the content" (i.e., a mirror for the student to see themself and a window to see stories of students who are different). These changes to instructional vision did not come up in the VHQMI interviews, but they were noticeably present in the focus groups at the end of the program.

Leadership

Leadership also surfaced in each focus group, though not as often as other themes, and participants were less likely to make connections to the EMS program relative to the other dimensions. Fellows described multiple ways they sought to share resources with others in their specific contexts, such as discussing cognitively demanding tasks with a small group of colleagues and posting images from their mathematics lessons on social media. One participant named the math talk posters she made "tons of teachers in [her] school" were beginning to use.

In an example of how the program supported their leadership, two participants who worked at the same school described their action plan assignment as an important tool to address "unsupportive leadership." They believed their administrators were overly focused on correct answers and standardized assessments, making it appear, as one participant noted, "like our students can't do math." In response, these participants' action plans focused on what one participant described as "really looking at what kids can do and not worrying about the cant's." When they shared their ideas for a more asset-based approach to assessment at a faculty meeting, both participants reported receiving significant positive feedback from their colleagues, which felt to them like a success.

Multiple participants also mentioned more direct ways of supporting colleagues. Several noted they had invited other teachers to observe their mathematics instruction, with one participant sharing she thought "welcoming teachers into your classroom" was the "way to get them on board" with new ways of instruction. Another participant, who worked as a mathematics interventionist, shared colleagues had approached her asking for support in analyzing student test results and planning subsequent instruction and coteaching lessons.

In addition to influencing their colleagues or other practicing teachers, multiple participants named working with and supporting teacher candidates as a next step in their leadership development. One participant, reflecting on the challenge of "finding my niche as a leader," said, "One-on-one with a student teacher is where I think I can make the most impact." Another participant described working with teacher candidates as "where [they] think change will happen over time." As with the previous participant regarding observations during math, this latter participant was considering a theory of change to teachers' instructional practice and how she could support growth.

Confidence

Increased confidence was mentioned in all focus groups, sometimes regarding teaching and sometimes regarding leadership, and often it was connected to development in a previously described area like knowledge. One participant stated:

> *[Prior to the program, I was] never a math person . . . math made me a little bit nervous. . . . I knew what I wanted the math classroom to look like but I didn't feel as confident about making it look like that.*

She recalled a time when school and district administrators visited her classroom during math after she had finished the program, and she "[was not] even bothered," which indicated to her the growth in her confidence in teaching math. On a smaller scale, a different participant stated the knowledge she gained from the program had supported her autonomy and decision making. She said, "[Given that we have to work] with a boxed curriculum, I feel more confident in myself to pull out what's important."

Regarding leadership, one participant mentioned, "[The program] gave me some practice in that leadership role, which I never would have probably taken on my own," and shared instead, "[I] would have kept all this knowledge to myself." Outside of confidence for mathematics instruction and leadership, one experienced participant described the program as a reinvigoration: It helped her rediscover her "zest" for teaching. She described, "In a sense [the program] . . . saved me. It kept me in teaching." These comments complemented our findings about the development of confidence in participants' capacity to enact leadership in mathematics and were all the more notable because they were raised in focus groups that did not ask about leadership. In general, these findings suggested participants had begun to see themselves, their roles, and their capacities differently, and they credited the EMS program with supporting these developments.

A Supportive Community

Across the dimensions described previously, one recurring theme went beyond a description of activities or content, instead focusing on the community established among the cohort members. One participant shared, "I love our cohort and the ability we have to work together and talk and do these things." Another talked about the importance of the synchronous meetings that occurred five times across each semester. She said, "Those times that we got to meet, I would learn so much from the other teachers in this program and what they were doing, what was working, and just engaging in all the different grade levels and experiences." One participant expressed appreciation for a community focused explicitly on mathematics; she said, "This has been a huge benefit to have the community and other people to bounce ideas off of. There's not a lot of elementary teachers, in my experience, that love teaching math." Finally, some participants noted how having another teacher in their building going through the program was something they valued about their experience, saying things like, "It's so nice to have someone else in the building where we've grown together in our teaching abilities and in our friendship." These comments echoed sentiments we noted across our data set, such as Janet's plans for working with their EMS colleagues at their school and the participants who reported working together to share an asset-based orientation of students with their colleagues as a response to school policies that functioned to sort and rank students (Webel et al., 2021).

Discussion

In relation to RQ1a, we found participants developed substantially over the course of the EMS program in their visions for high-quality mathematics instruction. We saw development across all areas of the VHQMI, with participants' descriptions of the teacher's role more likely to fall into the "teacher as facilitator" or "teacher as more knowledgeable other" categories, and their descriptions of student talk more likely to reflect a "mathematical discourse community" (Lampert, 1990) and tasks that constitute "doing mathematics" (Stein et al., 1996). Although these developments would be expected from an EMS program, they complemented other research showing impact on participants' beliefs (e.g., Swars et al., 2018) by providing insights into how participants imagined their future mathematics instruction. These visions are important because they are linked to instructional improvements (Munter & Correnti, 2017) and suggest fellows are relatively well prepared to enact the kind of mathematics instruction supported by research and professional organizations (NCTM, 2000, 2014, 2020).

In relation to RQ1b, we found participants also developed substantially over the course of the EMS program in their conceptions of themselves as leaders. We documented developments in terms of how teachers talked about leadership in mathematics and how they described their own comfort and ideas for enacting leadership in their schools. Our findings complemented other literature on teacher leadership, showing teacher–leaders gradually gain confidence as they develop (Hunzicker, 2017; Wenner & Campbell, 2017; Yopp et al., 2019) and begin to scale their leadership activities and perspectives to encompass larger systems (Brooks et al., 2004; Hunzicker, 2017; Knapp, 2017). In our analysis, we saw growth in confidence as participants moved from apprehensive to optimistic and excited about sharing their knowledge to improve students' mathematical experiences beyond their own classroom. Participants also increased their awareness of leadership practices and challenges, shifting from describing potential leadership activities in vague or hypothetical ways to giving concrete examples of leadership connected to a clearer theory of teacher learning and understanding of the systems that can impact opportunities for instructional change. Established participants, for instance, were not naïve about the resistance they might encounter in their leadership efforts, but they also were committed to making changes given those constraints (Nguyen et al., 2022).

As with vision for mathematics instruction, this development was not unexpected; fellows learned about and practiced leadership in several of their EMS courses, including working together to create and implement action plans in their contexts. These action plans created opportunities for fellows to deploy their increased confidence and awareness to challenge some of the barriers to instructional improvement they encountered in their school (Webel et al., 2021). The findings also showed how recruiting teachers to EMS programs in school-based teams might lead to increased impact; although some graduates of EMS programs struggled with feeling isolated and unseen in their schools (Webel et al., 2017), many of our participants had opportunities to coordinate their activities and present a united front when making requests of their administrators.

Although the development of EMSs' conceptions of leadership and themselves as leaders was not unexpected, the conceptions of leadership framework extended previous literature by suggesting stages of growth that combine confidence and awareness. We saw that, as teachers grew in confidence, they also gained awareness about different ways to lead (e.g., Janet found it was possible to be a leader even with a "quiet personality") and different challenges that might arise when working to support their colleagues (e.g., the team that found opportunities to promote high-quality mathematics instruction despite "unsupportive leadership"). In general, we view the framework not only as useful for documenting leader development but also as a tool that could be used in other EMS or leadership programs.

Related to RQ2, focus groups conducted at the end of the EMS program allowed us to document other areas of development not captured by the VHQMI or our other interviews and to connect EMSs' development to specific elements of the program. Similar to Myers et al.'s (2020, 2021) findings, our participants appreciated the connection to practice, opportunities to apply learning progressions to their own students, opportunities for open discussions, and collaboration directly relevant to their teaching. Additionally, participants in this study discussed the importance of opportunities to plan, enact, and debrief instruction routines, explore mathematical content as learners, and consider issues of equity (for more on how fellows understood and worked toward equity, see Webel et al., 2021). The focus groups also elicited explicit statements of how the program helped fellows develop more confidence as teachers and leaders, which often was linked to their development of knowledge and skills. This growth was positive, and it also supported their ability to see themselves as leaders in mathematics. Overall, we saw the multiple areas of development (i.e., knowledge, skills, vision, leadership, confidence) as connected and interwoven throughout their experiences, with development in one area reinforcing development in other areas.

Future Work

One aspect of the program we want to understand more is the ways individual participant development interacted with the elements of their specific contexts. We know from previously conducted social network analyses that our fellows were sought after by their colleagues for advice and information, and we also know these patterns of advice seeking were different in different buildings, even in the same district, suggesting school structures and policies influence the impact leaders can have (Nguyen et al., 2024). Participants also emphasized the importance of the cohort community, sometimes in connection with their openness to start thinking of themselves as leaders. This emphasis was particularly the case for those fellows who were recruited as part of a school-based team to go through the program together. They shared stories of approaching their administrators together, conducting professional development as a team for their school, and advocating for resources and policies they believed would better support student learning. Moving forward, we hope to learn more about how going through an EMS program as a pair or team might better support mathematics leadership practices in schools.

Conclusion

In this paper, we shared how an EMS program supported teacher development in terms of vision and conceptions of leadership and used focus groups to identify elements of the program participants identified as important for their development. We found teachers incorporated new images of teaching into their visions emphasizing mathematical discourse, high-demand tasks, and student authority for mathematical reasoning. We saw participants expressing more confidence about serving as leaders in their contexts and more realistic expectations for the kinds of challenges they might encounter. We saw developing a supportive professional community was a key component of the learning experience for participants. We hope these findings can be useful for other EMS program personnel to inform their design and build a more robust knowledge base for what learning experiences are supportive for the varied work of EMSs.

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APPENDIX

Interview Questions (Munter, 2014)

I'd like to ask you a few questions about your view of high-quality mathematics instruction.

1. If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?

Notes to interviewer:

- *Probe on **depth/specificity** of response until you understand what the participant describes (e.g., If a teacher says, "student engagement," ask, "Engaged in what?").*
- *Keep the **form/function** distinction in mind. Ask participants why they think ____ is important (e.g., Why do you think it's important for kids to work in groups? Why do you think it's important to hold a whole class discussion?).*
- *If the interviewee talks about the **structure of discourse** (who's talking to whom and when) probe on **content** (and vice versa). If the interviewee says, "Teachers (or students) should be asking questions," probe to find out the kinds of questions the teacher (or students) should ask and for what purpose, as well as whether they conceive of discussion as happening in whole class settings and/or in small groups alone.*

   a. Is there anything else you would look for? *(Ask BEFORE probing on the following issues.)*
   b. What are some of the things you would expect to find the teacher actually doing in the classroom for instruction to be of high quality?
   c. What kinds of problems or mathematical tasks would you expect to see the students working on for instruction to be of high quality?
      i. Can you please describe a _____[*use the word or phrase—e.g., "task" or "problem"— that the participant used for "task"*] that you would consider to be of high quality?
      ii. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
      iii. Would you expect to see the entire class participating in a single discussion, or would students be talking primarily in small groups?

LEARNING TO FACILITATE CONTENT-FOCUSED COACHING CYCLES:

A COMPREHENSIVE FRAMEWORK TO SUPPORT COACHES' PROFESSIONAL GROWTH

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ABSTRACT

This paper serves two purposes. First, we present the content-focused coaching implementation framework, a comprehensive tool to support mathematics coaches in navigating the complexities of facilitating coaching cycles with teachers. Second, we describe a research study in which we partnered with nine mathematics coaches and examined how the framework influenced coaches' perceptions of their professional growth when facilitating coaching cycles with local teachers. Through analysis of postcoaching cycle interviews, study participants reported the framework supported them to prepare intentionally for the three distinct phases of the coaching cycle (i.e., planning conversation, lesson implementation, debriefing conversation) and make responsive, "in-the-moment" decisions. Coaches also shared ways in which the framework sparked new insights about coaching. We discuss how our findings connected to and extended prior research on coach learning and the use of coaching tools. We also present implications of our framework and findings for practicing mathematics coaches and future researchers.

Coaching teachers has been shown to be an effective method for improving teachers' pedagogical practices and content knowledge (Desimone & Pak, 2017; McGatha et al., 2015; West & Cameron, 2013). In this paper, we considered a mathematics coach to be a mathematics specialist who has full- or part-time release from teaching responsibilities to collaborate with teachers (Baker et al., 2022; Mudzimiri et al., 2014; NCSM, 2019; Sutton et al., 2011). Mathematics coaches work to address teachers' individual needs through ongoing professional learning focused on content development, pedagogy, assessment, and curriculum (Campbell & Griffin, 2017; Neufeld & Roper, 2003; Obara & Sloan, 2009; Thomas et al., 2015).

In the wide array of professional learning activities available to coaches and teachers (Gibbons & Cobb, 2017), content-focused coaching (CFC) cycles have become a prominent activity in mathematics education (Bengo, 2016). A CFC cycle is a 3-phase activity in which a coach and teacher collaboratively plan, implement, and reflect upon a lesson. Aligned with principles of a CFC model (Callard et al., 2022; West & Cameron, 2013), the coach and teacher maintain focus on mathematical content and how students learn this content throughout a CFC cycle to provide individualized and job-embedded support that can deepen a teacher's knowledge of content and pedagogy. This continual focus on mathematics content distinguishes CFC from other forms of coaching, such as (a) instructional coaching, which focuses on general instructional and assessment strategies (Knight, 2007) and (b) cognitive coaching, which focuses on mediating teachers' thinking (Costa & Garmston, 2016).

The emergence of CFC cycles has been connected to educational policy shifts in the United States (Common Core Standards Initiative, 2010; Every Student Succeeds Act, 2015) which significantly raised expectations for improved student outcomes in mathematics. These shifts increased demands on teachers, requiring them to adapt their instructional strategies rapidly to align with ambitious and equitable mathematics teaching (Horn & Garner, 2022; Lampert & Graziani, 2009) to meet new or revised mathematics standards. To support teachers in shifting their instructional practices and increasing student achievement, school district personnel have turned increasingly to mathematics coaches and CFC cycles as a professional learning option (Desimone & Pak, 2017; McGatha et al., 2015; West & Cameron, 2013). Facilitating coaching cycles is complex, and the effectiveness of a CFC cycle relies on the coach's ability to act intentionally yet responsively (Stein et al., 2022). In planning conversations, coaches must learn to guide interactions in ways that maintain focus on essential ideas related to teacher development (West & Cameron, 2013) while also being responsive to each teacher's individual needs in producing a high-quality lesson (Carlson et al., 2017). When collaboratively teaching, coaches must select forms of coteaching strategically based on a teacher's current practice and student needs (Saclarides, 2023). While reflecting on the implementation of lessons, coaches must have honest conversations about teachers' content knowledge and pedagogical practices that are grounded in evidence of student thinking (Varghese et al., 2023; West & Cameron, 2013). In sum, facilitating CFC cycles requires coaches to intentionally plan for and responsively enact three distinct yet interrelated interactions with a teacher, which formed the basis of our research.

The purpose of this paper was twofold. First, we presented the CFC implementation framework, a comprehensive tool to support mathematics coaches in navigating the complexities of facilitating one-on-one coaching cycles with teachers (see Appendix), and we described the processes used to create the framework. Second, we described a research study in which we analyzed nine mathematics coaches' perceptions of their professional growth when using this framework during coaching cycles with teachers in their local contexts. Our study answered the research question, "How did the CFC implementation framework influence coaches' perceptions of their professional growth when facilitating coaching cycles?" Exploring mathematics coaches' perceptions provided insights into how the framework influenced coaches' work with teachers, supported the growth of coaches, and generated authentic feedback to further refine the framework.

As a final opening note, our framework is directed at the use of CFC cycles because our work was grounded in this particular coaching model (Callard et al., 2022; West & Cameron, 2013; West & Staub, 2003). However, we believe the framework and results of this study apply to any coach, specialist, or teacher educator using coaching cycles to help mathematics teachers deepen their content and pedagogical content knowledge.

FRAMING LITERATURE

We framed our study using three bodies of existing literature. First, we discuss ambitious and equitable mathematics teaching because our coaching framework is designed to support coaches in working with teachers to engage in practices in line with this pedagogical model. Second, we present literature on CFC, the coaching model underpinning our framework. Finally, we review existing literature on coach learning and how coaching tools and structures support coaches' professional learning.

Ambitious and Equitable Mathematics Teaching

For over 3 decades, educational organizations and researchers have advocated for ambitious and equitable mathematics teaching, an instructional approach that strives to provide all students with access to rigorous mathematical learning opportunities (Horn & Garner, 2022; National Council of Teachers of Mathematics [NCTM], 1991, 2014; National Research Council, 2001). According to Smith et al. (2017):

> *In ambitious teaching, the teacher engages students in challenging tasks and then observes and listens while they work so that he or she can offer an appropriate level of support to diverse learners. The goal is to ensure that each and every student succeeds in doing high-quality academic work rather than merely executing procedures with speed and accuracy. (p. 4)*

Building from this broad vision, mathematics education organizations (NCSM, 2019; NCTM, 2014) and various researchers have highlighted the following high-leverage practices as critical components of ambitious and equitable mathematics teaching:

- establishing mathematics learning goals focused on understanding big ideas in mathematics (Hiebert & Grouws, 2007),
- implementing high-cognitive demand tasks that promote inquiry and provide access to all students (Smith & Stein, 2018),
- eliciting and responding to students' thinking (Leahy et al., 2005; Smith & Stein, 2018),
- facilitating meaningful and productive mathematical discourse that builds from students' reasoning and connects mathematical strategies and representations (Chapin et al., 2009; Smith & Stein, 2018; Staples, 2007), and
- supporting and promoting productive struggle for all students (Kapur, 2010; Warshauer, 2015).

Ambitious and equitable mathematics teaching often is found to be in sharp contrast to prevailing instructional methods and may be significantly different from teachers' own mathematics learning experiences (Horn & Garner, 2022). Thus, implementing such practices can present formidable challenges because implementation often requires mathematics teachers to make significant changes in, or even to completely overhaul, multiple facets of their professional practice (Star, 2016; Valoyes-Chávez, 2019). These changes often include adopting new beliefs about students' capacities for thinking and reasoning (NCTM, 2014), learning to use high-leverage instructional practices in responsive ways (Witherspoon et al., 2021), and acquiring new knowledge about mathematical content and how students learn this content (West & Cameron, 2013). Hence, mathematics teachers deserve and require high-quality professional learning experiences, including coaching (Smith et al., 2025), to support their implementation and refinement of ambitious and equitable mathematics teaching as they strive to help all students reach high levels of mathematical proficiency.

CFC

Schools and school districts across the United States have invested in mathematics coaches to provide professional learning opportunities that support mathematics teachers' development of ambitious and equitable mathematics teaching (Desimone & Pak, 2017; McGatha et al., 2015; West & Cameron, 2013). The term coach covers a range of definitions and descriptions and is often linked to a specific coaching model (Bengo, 2016). CFC, one such coaching model, emphasizes the subject matter taught and the use of students' thinking to design and reflect upon instruction (West & Cameron, 2013; West & Staub, 2003). In line with principles of ambitious and equitable mathematics teaching, a content-focused coach's primary goals are to (a) increase a teacher's content knowledge and (b) increase a teacher's pedagogical content knowledge, which pertains to effective instructional practices related to the subject matter (Ball et al., 2008; Shulman, 1987).

The core activity of CFC is a coaching cycle, which consists of three distinct yet interconnected phases: the planning conversation, the lesson implementation (often referred to as coteaching), and the debriefing conversation (Callard et al., 2022; West & Cameron, 2013; West & Staub, 2003). During the planning conversation, the teacher and coach coconstruct a lesson that includes clear learning goals and cognitively demanding tasks (Callard et al., 2022). The lesson implementation phase involves the teacher and coach coteaching the coconstructed lesson with the coach assuming an active role in implementation based on the teacher's learning needs (Gillespie & Kruger, 2022). The debriefing conversation elicits reflections on the lesson's effectiveness by examining evidence of student thinking, considering contributing factors that may have impacted the lesson's effectiveness, and establishing implications for future practice (Gillespie et al., 2023). This approach to coaching promotes an authentic partnership as both the teacher and coach share responsibility for the success of the lesson across all three phases (Bickel et al., 2017; West & Cameron, 2013; West & Staub, 2003).

CFC cycles hold the potential to support teachers in implementing ambitious and equitable mathematics teaching effectively (Bickel et al., 2017; Witherspoon et al., 2021). However, existing literature on mathematics coaching continuously has highlighted the complexity of coaching, emphasizing that it takes time, often years, to develop the expertise necessary to facilitate one-on-one professional learning activities with teachers successfully (Carlson et al., 2017; Saclarides & Kane, 2021; Stoetzel & Shedrow, 2020). In the context of CFC cycles, this expertise begins with coaches possessing deep knowledge of mathematical content and instructional practices (Coburn & Russell, 2008; Gibbons & Cobb, 2016; Yopp et al., 2019). However, being knowledgeable of mathematical content and instructional practices associated with ambitious and equitable mathematics teaching is not enough. Coaches also must know how to facilitate coaching cycles with clear goals and intentionality while also being responsive to teachers' individual needs (Costa & Garmston, 2016; West & Cameron, 2013). The ability to act intentionally and responsively relies on a coach's understanding of how a coaching cycle can support a teacher's professional growth and how individual phases support the larger purpose. Building from this understanding, coaches must plan for interactions in each phase in ways that maintain focus on important pedagogical and mathematical ideas while being flexible to incorporate unique needs of individual teachers (Baker & Knapp, 2019; Russell et al., 2020). In addition, coaches need specific techniques to elicit and deepen a teacher's thinking about both content and pedagogy (Russell et al., 2020). During such interactions, coaches must be mindful of power dynamics because coaches often are positioned and perceived as more knowledgeable experts (Chval et al., 2010; MacPhee & Jewett, 2017; West & Cameron, 2013). Being mindful of power dynamics includes balancing the use of (a) directive coaching moves (e.g., suggestions and explanations) where the coach positions themself momentarily as a knowledgeable expert and (b) invitational questions where the coach positions the teacher to be the intellectual authority (Gillespie et al., 2025; Witherspoon et al., 2021). In short, the promise of CFC cycles is contingent upon a coach's knowledge and ability to navigate the inherent complexity of three separate but interconnected one-on-one interactions.

Tools That Support the Enactment of Specific Coaching Practices

In response to the complexity of coaching, numerous authors have created tools for coaches that recommend specific practices for use in coaching cycles. In some cases, a coaching tool has applications in a particular part of a coaching cycle. For example, Wills and Rawding (2019) created a protocol that describes a set of six practices a coach can use in a preliminary conversation with a teacher to set the foundation for upcoming coaching cycles. Kochmanski and Cobb (2023) designed a decision-making tool coaches can use to identify productive instructional goals for teachers when preparing for debriefing conversations. Smith et al. (2025) developed a four-move routine (i.e., invite, rehearse, suggest, and generalize) for coaches to use during planning conversations. Other authors have generated larger texts that provide coaches with comprehensive guidance for facilitating coaching cycles aligned with particular coaching models. For example, Costa and Garmston (2016), in their work involving cognitive coaching, presented "maps" highlighting broad structures and questions coaches can use when facilitating planning and debriefing conversations. Similarly, Knight (2007) and Sweeney (2011) have provided coaches with detailed instructions and recommendations for all three phases of coaching cycles that align with instructional and student-centered coaching models, respectively.

Complementing practitioner-facing literature, empirical evidence has suggested tools that articulate specific coaching practices (e.g., protocols, routines, models, frameworks) can support coaches' professional learning. We highlighted three examples of such tools in this paper. First, Baker and Knapp (2019, 2023) examined how their decision-making protocol for mathematics coaching (DMPMC) supports content-focused mathematics coaches to facilitate productive and targeted coaching interactions including, but not limited to, coaching conversations. Baker and Knapp found their protocol guided coaches to plan for coaching interactions and anticipate teacher responses, which supported coaches to act responsively during coaching conversations with teachers. Furthermore, findings suggested the DMPMC promoted self-reflection, enabling coaches to critically examine and refine their practices. Second, Russell et al. (2020) examined how coaches enacted specific coaching practices outlined in an inquiry-based mathematics coaching model. Russell et al. found coaches implemented and adapted presented practices during coaching cycles with teachers, leading to conversations with greater depth and specificity. Third, Gillespie et al. (2023) examined how a conversational structure for debriefing a lesson with a teacher supported content-focused coaches in facilitating effective reflective conversations. Gillespie et al. found their debriefing conversational structure enabled coaches to balance the amount of teacher and coach dialogue and increased coaches' use of discourse that contained fundamental aspects of reflective thinking.

These examples illustrated that tools appear to be a promising way to support coaches to navigate the complexity of coaching and adopt new practices when striving to support professional growth of mathematics teachers. However, mathematics coaches lack a comprehensive and cohesive framework that outlines specific practices for all three phases of a coaching cycle aligned with CFC. Moreover, little is known about how such a framework might support coaches to adopt new practices when engaging in one-on-one interactions with teachers aiming to implement practices associated with ambitious and equitable mathematics teaching.

OUR CFC FRAMEWORK

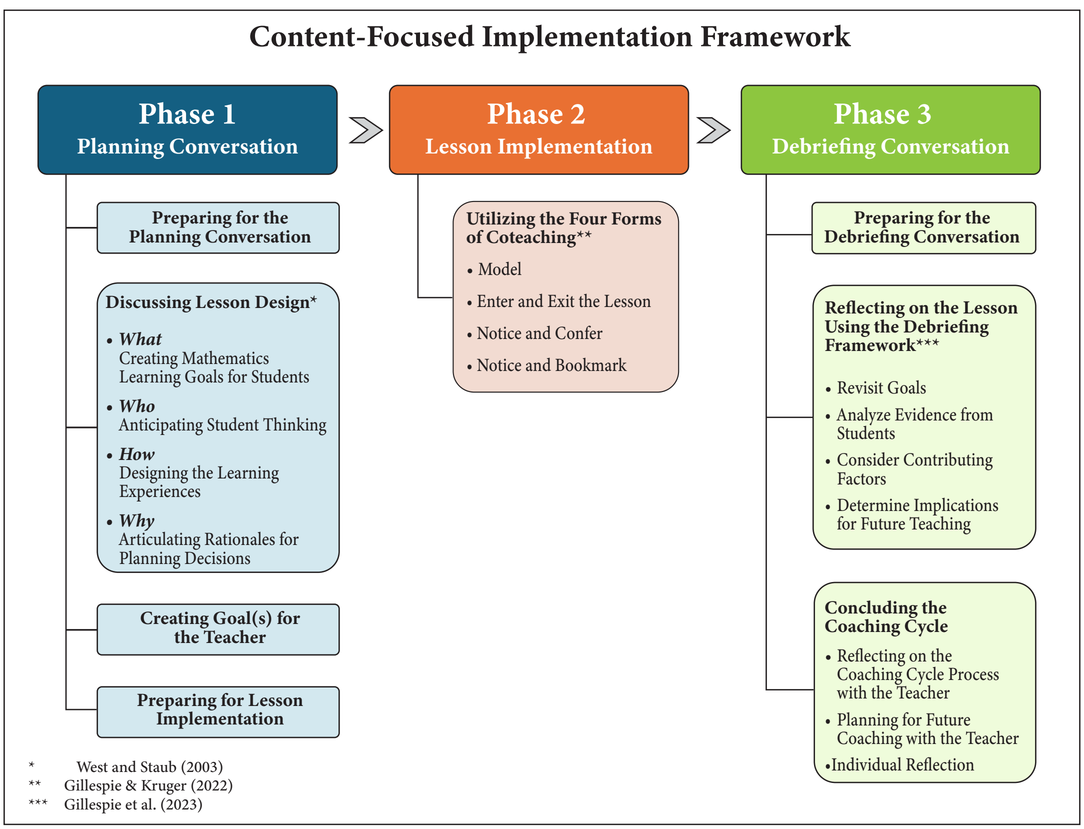
In a recent project funded by the National Science Foundation (Grant #2006353), our author team, as part of a larger team of researchers and professional learning providers, designed, implemented, and researched a professional learning model to support mathematics coaches to facilitate CFC cycles. Through our work, we discovered existing CFC literature provided fragmented tips and guidance for facilitating coaching cycles. A detailed, actionable, and comprehensive guide did not exist to help coaches facilitate each phase of a CFC cycle with intentionality and coherence. This absence stood in noticeable contrast to the presence of actionable guidance from other coaching models. For example, Costa and Garmston (2016) provided conversational maps to guide coaches in facilitating both planning and debriefing conversations in ways that aligned with the guiding principles of the cognitive coaching model. Similarly, Knight (2017) detailed specific processes and coaching behaviors to implement all phases of a coaching cycle that aligned with the guiding principles of the instructional coaching model.

To address this gap and support coaches in our project in learning to facilitate CFC cycles, we created the CFC implementation framework (see Appendix). Figure 1 provides an overview of the content of this framework, which we describe in greater detail in the following section.

Creation Process

To create the CFC implementation framework, we engaged in biweekly conversations for approximately 2 years. We used a backward design process that leveraged our extensive experience as content-focused coaches and professional learning providers who support content-focused coaches. First, we articulated the outcomes and guiding principle of a CFC cycle (found on the first page of the framework in the Appendix). Next, we crafted ideal outcomes of each phase of the cycle that connected to overarching outcomes and the guiding principle (found in the top row of each part of the framework in the Appendix). With these outcomes as a foundation, we systematically deconstructed each phase of a CFC cycle into discrete elements and identified specific coaching practices associated with each element. Throughout the 2-year development of the framework, project researchers outside our author team provided ongoing feedback. Upon completing the first draft of the framework, we received extensive feedback from (a) project advisory board members, who were researchers with expertise on using coaching to support ambitious and equitable mathematics teaching and (b) the coaches who participated in this study. We compiled this feedback, which resulted in the CFC implementation framework.

Figure 1
The Elements of Each Phase of the CFC Implementation Framework



As illustrated in Figure 1, our framework has three parts: Phase 1: Planning Conversation, Phase 2: Lesson Implementation, and Phase 3: Debriefing Conversation. Each part is designed to guide practicing coaches to grow in their ability to facilitate a particular phase of a CFC cycle while also helping coaches recognize and appreciate coherence across the three phases. Each part of the framework contains multiple elements, and each element is decomposed into CFC practices distributed across three descriptive columns. The first column, "First Steps," outlines basic yet essential actions coaches can implement immediately in a CFC cycle with a teacher. The second column, "Next Steps," articulates coaching actions that build on the first steps to support coaches in fostering deeper thinking from a teacher. The final column, "Peak Performance," describes coach's actions that reflect the desired outcomes and guiding principle of a CFC cycle. The framework provides intentional support for coaches to manage potential power dynamics through their discursive choices (Witherspoon et al., 2021). For example, actions in the first steps tend to be invitational in nature to encourage coaches to position teachers as the knowledge authority at the start of discussion about a particular topic (Smith et al., 2025). Furthermore, outcomes in the peak performance column reinforce the big idea that the goal of coaching is to help teachers develop new habits and knowledge that will support effective teaching without the presence of a coach. Coaches may shift to use directive coaching moves when enacting descriptors in the next steps phase, but the descriptors of first steps and peak performance illustrate the importance of coaches incorporating ample opportunities to position teachers as the intellectual authority.

Foundational Models for Each Part of the Framework

Each part of the framework is a synthesis of our experiences and our research team's thorough review of coaching support structures available in the existing literature. Elements of Phase 1: Planning Conversation are grounded in the work of West and Staub (2003) who articulated a 4-part structure for a CFC planning conversation. In this structure, the coach engages collaboratively with a teacher to consider the (a) what (i.e., mathematical content goals), (b) who (i.e., anticipated student thinking), (c) how (i.e., lesson design), and (d) why (i.e., decision rationales) when preparing a lesson. During typical lesson planning, teachers often focus their attention on how curricular materials will be implemented without addressing what, why, and who associated with the lesson (West & Cameron, 2013). West and Cameron (2013) argued coaching questions beyond the how of lesson design "essentially change teacher thinking and trigger more reflective lesson planning habits of mind" (p. 98).

Phase 1: Planning Conversation includes these four elements and further articulates actions a coach can use when coconstructing a lesson with a teacher. As an example, Figure 2 illustrates how we incorporated the "what" component of West and Staub's (2003) work as a single element in Phase 1 of the framework. Phase 1 contains three additional elements beyond the Guide to Core Issues that focus on (a) preparing for the planning conversation (Smith et al., 2025), (b) creating instructional practice goals with teachers (Kochmanski & Cobb, 2023), and (c) preparing for different forms of coteaching (Saclarides, 2023) during the lesson.

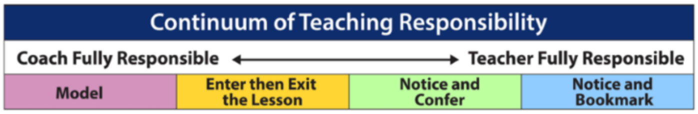
Elements of Phase 2: Lesson Implementation incorporate our previous work in which we developed a coteaching structure, the continuum of teaching responsibility (Gillespie & Kruger, 2022), to describe various coaching roles for collaborative lesson implementation (see Figure 3). A core feature of CFC is that the teacher and coach coteach the lesson, sharing accountability for both the lesson design and implementation, which, in turn, supports authentic collaboration (Gillespie & Kruger, 2022). Hence, when coteaching, the teacher and coach actively collaborate during lesson implementation, as opposed to other models of coaching where the teacher and coach might work with separate groups of students or where the coach solely observes the teacher's instruction (Saclarides, 2023; West & Cameron, 2013). The continuum of teaching responsibility presents four options for a coach's coteaching roles differentiated by the amount of responsibility the coach assumes for teaching.

Figure 2

Example of the Integration of West and Staub (2003) Into the Framework

Elements	First Steps	Next Steps	Peak Performance
***What*: Creating Mathematics Learning Goal(s) for Students**	Teacher or coach states the mathematics learning goal(s) for students.	Teacher and coach discuss: • ways to make the goal(s) more focused on learning and understanding. • how the lesson/task aligns with the mathematical goal(s). • evidence of student thinking that would indicate the goal(s) was met. • how the goal(s) relates to important mathematical ideas. • how the goal(s) connects to students' prior learning. • how the goal(s) of this lesson are part of a coherent set of learning experiences.	Teacher and coach discuss how *creating mathematics learning goal(s) for students* supports ambitious mathematics teaching and improves student learning. Teacher and coach discuss the teacher's next steps in making *creating mathematics learning goal(s) for students* a planning habit.

Figure 3
Continuum of Teaching Responsibility

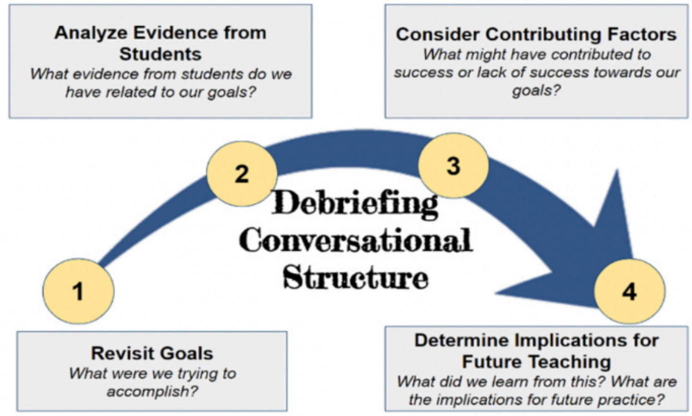


Note. From "With the Right Strategies, Coaches Can Leverage Co-Teaching," by R. Gillespie and J. S. Kruger, 2022, *The Learning Professional,* 43(2), p. 45 (https://learningforward.org/journal/coaching-for-change/with-the-right-strategies-coaches-can-leverage-co-teaching/). Used with permission of Learning Forward, www.learningforward.org. All rights reserved.

These forms of coteaching can shift throughout a lesson and support a coach to work toward the goals of the teacher and students. Figure 4 illustrates how the model component from the continuum of teaching responsibility is integrated into Phase 2 of the framework.

Elements of Phase 3: Debriefing Conversation also expanded on our earlier work in which we created a structure to guide content-focused coaches in facilitating debriefing conversations in the final phase of a CFC cycle (Gillespie et al., 2023). The debriefing conversational structure phase (see Figure 5) merges structures from other coaching models (Costa & Garmston, 2016; Knight, 2007) with the primary goal of CFC, which is increasing the teacher's content and pedagogical content knowledge (West & Cameron, 2013). The debriefing conversational structure comprises four phases to guide coaches in facilitating debriefing conversations: (a) revisit goals, (b) analyze evidence from students, (c) consider contributing factors, and (d) determine implications for future teaching. Phase 3 uses these four elements to articulate actions a coach can use to engage in a productive debriefing conversation.

Figure 5
Debriefing Conversational Structure



Note. From "Learning to Facilitate Reflective Conversations: Exploring Changes in the Practices of Mathematics Coaches" by R. Gillespie, J. Kruger, A. Hanan, and J. Amador, in T. Lamberg and D. Moss (Eds.), *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 686–695). International Group for the Psychology of Mathematics. Copyright by the International Group for the Psychology of Mathematics, 2023.

Figure 6 depicts how we integrated the component of "analyze evidence from students" from the debriefing conversational structure into our framework. Phase 3 contains four elements beyond debriefing conversational structure that focus on (a) preparing for the debriefing conversation, (b) reflecting on the coaching cycle process with the teacher, (c) planning for future coaching, and (d) engaging in individual reflection. The Peak Performance section of Phase 3 emphasizes the interconnectedness of each element of the debriefing conversational structure and the integration of teacher learning into future practice. The concept of peak performance remains consistent for all components of the debriefing conversational structure phase.

Figure 4
Example of the Integration of the Continuum of Teaching Responsibility Into the Framework

Elements	First Steps	Next Steps	Peak Performance
Model	Coach models a portion of the lesson while the teacher observes.	Coach finds moments while modeling to make their decision- making process explicit with the teacher Coach and teacher find opportunities to discuss what they are noticing about student thinking. Coach provides the teacher with opportunities to retake responsibility of the teaching.	Coach models the lesson in ways that draw attention to aspects of practices related to the teacher's learning needs and instructional practice goals. Coach and teacher find opportunities to listen to each other's noticings about student thinking and the coach makes instructional decisions based on these shared noticings.

Figure 6
Example of the Integration of the Debriefing Conversational Structure Into the Framework

Elements	First Steps	Next Steps	Peak Performance
Analyze Evidence from Students	Coach and teacher discuss specific instances of student thinking using artifacts such as student work or observation notes.	Coach and teacher: • discuss how the evidence of student thinking connects to the mathematical learning goals. • use evidence of student thinking to collaboratively make and support claims about student understanding.	Coach and teacher make connections between the mathematical learning goals, evidence of student thinking, and contributing factors to identify important and specific implications for the teacher's future practice. Coach and teacher co-construct specific next steps that support the teacher to embed new learning in their regularly occurring practice.

METHODS

Project Context

This study was an extension of a 2-year professional learning project for mathematics coaches learning to facilitate CFC cycles with teachers (see Amador et al., 2021). The larger, 2-year project supported 30 mathematics coaches working in rural areas across the United States to improve their abilities to (a) facilitate productive planning and debriefing conversations with teachers, (b) notice salient CFC practices and their impact on teachers' thinking, and (c) use evidence of teacher learning to make decisions about their own coaching practices. All activities were fully online and included three parts: (a) five 2-hour synchronous course sessions, (b) eight video coaching clubs, and (c) two mentor-supported CFC cycles. The course sessions focused on concepts and practices for facilitating the three phases of a CFC cycle (Callard et al., 2022; West & Cameron, 2013). The video coaching clubs aimed to grow participants' abilities to notice and interpret coaching moves and teacher thinking in video clips of CFC conversations. The mentor-supported CFC cycles involved each participant working individually with a mentor–coach to prepare for and reflect on a CFC cycle with a mathematics teacher.

The study described in this paper extended research and professional learning activities beyond the initial project. These extended professional learning activities provided previous project participants with the CFC implementation framework as they engaged in a CFC cycle with a teacher in their local area. The extended research activities examined how the CFC framework supported these coaches' professional growth when facilitating CFC cycles.

First, all participants took part in a preparticipation interview, which included questions about participants' background experiences with coaching and a series of questions exploring their experiences and beliefs about each part of a CFC cycle. Next, participants engaged in a 90-minute online professional learning session in which project personnel introduced participants to the CFC implementation framework. During this session, participants were invited to discuss their initial reactions to the framework, describe how the framework related to their prior learning and current coaching, and consider implications of use of the framework on their future practice. Additionally, project personnel shared parameters and directions for a coaching cycle, which stipulated the coaching cycle needed to include a planning and debriefing conversation lasting at least 30 minutes and a lesson implementation in which the coach and teacher could coteach the lesson. Study participants were then asked to facilitate a CFC cycle with a teacher in their context using the CFC implementation framework as the primary tool to guide their preparation and facilitation. Participants also were asked to share any artifacts that captured their planning process for planning and debriefing conversations. However, there were no expectations or instructions for how participants should plan or what they should create in preparation for the CFC cycle. Finally, we encouraged study participants to ask teachers to bring a rough draft of a lesson plan or math task, along with mathematical learning goals for students and an instructional goal for themselves, to their planning conversations.

Participants

Nine coaches from the initial 2-year professional learning project participated in this study. Study participants had varied levels of coaching experience (2–15 years) and held various full-time positions in K–12 schools but were classified in two ways: (a) practicing mathematics coaches employed by a single school, school district, or regional organization with full-time release to work one on one with teachers or (b) classroom teachers who engaged in coaching work with colleagues as part of additional teacher leadership roles. We noted participants without full-time release from teaching likely had less time for planning and preparing for coaching interactions than participants with full-time release from teaching, a point to which we return when discussing implications of the findings. Table 1 displays participants' years of coaching experience, teaching experience, and role at the onset of the project; all names are pseudonyms.

Table 1
Participant Demographics

Participant	Coaching experience (years)	Teaching experience (years)	Role
Howard	15	5	Full-time coach in a single school
Logan	12	3	Full-time coach in a region
Arnold	7	13	Full-time coach in a school district
Clark*	6	17	Classroom teacher engaging in coaching
Bell	4	17	Full-time coach in a single school
Haynes*	3	33	Classroom teacher engaging in coaching
Peters	3	31	Full-time coach in a region
Vargas*	3	20	Classroom teacher engaging in coaching
Briggs	2	4	Full-time coach in a region

Note. *All years of coaching experience were not mutually exclusive from the years of teaching experience.

Data Collected

Primary data collected for the study were responses from semistructured interviews conducted after participants facilitated their full coaching cycle. Interviews consisted of questions asking participants to reflect on each of the three phases of their coaching cycle and share perceptions about how the CFC implementation framework may have influenced their coaching in each phase. For example, when discussing the planning conversation, participants first provided a summary of their overall experience during the planning conversation. Next, participants described how they used the CFC implementation framework to prepare for and facilitate their planning conversation. Finally, participants identified two moments in their planning conversation they found particularly productive. For each productive moment they shared, they included what made the moment productive and how the framework may have contributed to the moment. Participants then shared related comments for the lesson implementation and debriefing conversations from their coaching cycles. Participants also completed a postcoaching cycle survey in which they were asked to provide feedback on the framework. Survey data were not used to respond to the research question and instead were used to refine the CFC implementation framework further.

Data Analysis

Our team of four researchers qualitatively examined coaches' perceptions of how the framework supported them throughout their CFC cycle. To begin the analysis, two of our team members individually wrote low-inference paraphrases of all participants' responses and then met to reach consensus about an accurate paraphrase (Corbin & Strauss, 2014). Next, all four researchers met to distill paraphrases into prominent themes that emerged across participants' responses. Because our goal was to understand how the framework supported coaches during their CFC cycle, we only included paraphrases in which coaches communicated a connection between their coaching and their use of the framework. Three themes emerged during this second layer of analysis, which are presented in the following section. For the postcoaching cycle surveys, two members of our team read all participant responses and considered the feedback to make adjustments and final revisions to the CFC implementation framework. Survey responses were used to refine the framework and were not analyzed to answer the research question.

FINDINGS

Our analysis of participants' postcoaching cycle interviews and surveys revealed three interconnected themes involving coaches' perceptions of professional growth because of engaging with the framework during their CFC cycles. First, coaches detailed ways the framework supported them to intentionally plan and prepare for the three distinct phases of the CFC cycle. In a second theme, coaches described how intentional preparation using the framework facilitated improvements in responsive, "in-the-moment" decision making during planning, lesson implementation, and debriefing interactions. Coaches connected these improvements in their responsive decision making to using the framework to prepare for coaching interactions. In a third theme, coaches shared how using the framework led to powerful interactions with teachers, which in turn catalyzed new insights about coaching. We considered such insights as comments that transcended the single coaching cycle in this study as participants made connections between their use of the framework and their prior or future coaching practices.

Theme 1: Planning and Preparing for Phases of Coaching Cycles

All nine coaches reported various ways in which the framework supported them effectively in creating an intentional plan for the planning, lesson implementation, and debriefing phases of their coaching cycle. Amid this variety of responses, one central pattern was that all nine coaches used the different elements of the framework to create personalized and practical tools to guide their planning and debriefing conversations. When introducing the framework to coaches initially, we instructed them to use it to plan for their cycle interactions and to share any artifacts they created when preparing, which could include something as simple as handwritten notes. However, all nine coaches produced unique and comprehensive tools to guide their conversations, weaving together elements of the framework with their individual coaching practices skillfully. These actions exceeded our expectations, given that we did not recommend the level of specificity the coaches provided, nor did we ask coaches to generate their own unique coaching tools for planning and debriefing conversations. To create these tools and plans for their interactions, most coaches described first reviewing the framework broadly and then creating an outline for an upcoming conversation that mirrored the structure of the framework in terms of major focal topics. Then, coaches lifted specific questions from the framework to their personal tools based on the anticipated needs of the teacher. For example, Clark shared the following about how they created their conversation tool:

> *I typed up an outline of what to do, but I had the framework right next to me as I was doing that. So, I went through it all and made sure I would ask about the student learning goal, like you suggested, the instructional goals, what our model for coteaching was, how we want the students to be engaged. I also made sure we looked at the math problems . . . Then I added specific questions to ask, which I took right from your framework.*

Haynes similarly shared that they created an overarching plan for both their planning and debriefing conversations, using each row in the framework as a section on their personal conversation tool. Additionally, Haynes shared that they used the Peak Performance section to consider their overarching goals for themself and the teacher and the Next Steps section to select questions to ask based on their prior knowledge of the teacher. Haynes shared:

> *I looked at the peak performance pieces to think about, "What do we want as an outcome beyond just this one experience?" This included thinking about myself and what I needed to do well as a coach. I then looked specifically at that middle piece, next steps, and I highlighted questions that I wanted [the teacher] and I to work off of. That was a wonderful help and supported us to be very focused in our discussion.*

Each of the nine participants also shared that the framework supported dialogue in the planning conversation about intentionally selecting coteaching roles with the teacher for lesson implementation. To illustrate this trend, we highlight the actions of Logan, who shared a visual of the continuum of teaching responsibility (Gillespie & Kruger, 2022) with the teacher during the planning conversation and used the framework to facilitate discussion about which forms of coteaching would be most beneficial. Logan shared:

> *[The coteaching portion of the framework] offered a lot of clarity about the continuum of teaching responsibility . . . I said, "Here's this continuum," and I asked, "Where would you like my responsibility to be during the lesson?" [The teacher] could point and describe what she wanted from me. Then we were able to dive into the actual language in the framework about the "notice and confer" and "notice and bookmark" and kind of say, "Okay, so if I'm noticing and conferring, this is what would happen." And then I was able to ask her, "What would you like me to bookmark? Your goal is to ask better thinking questions, so do you want me to bookmark the questions you ask?" It felt really productive in the planning because the framework and the visual helped me to know my role and responsibility. And I think it helped her be really clear about what she wanted from me.*

Beyond the trends for all nine coaches of creating conversation tools and preparing for coteaching roles, coaches also shared additional perceptions, unique to individual or small groups of coaches, about ways the framework supported their preparation efforts. For example, Briggs and Vargas both shared that they spent time anticipating teacher thinking and planning possible responses to their anticipated thinking. Arnold, Briggs, and Vargas reported using the framework to engage in preplanning interactions with teachers. In short, all coaches shared how the framework supported them to create personal conversation tools and collaboratively prepare for coteaching the lesson.

Theme 2: Responsive Decision Making

A second theme, reported by all nine coaches, was that the framework effectively supported them in making productive, in-the-moment decisions that were responsive to teachers' learning needs. Examples of these decisions varied for each coach and traversed the planning, lesson implementation, and debriefing phases of the coaching cycles uniquely. For example, Bell talked about how the framework supported him in pacing both planning and debriefing conversations in a fixed amount of time. Briggs and Peters recounted making intentional decisions about coteaching, informed by the framework, during lesson implementation. Clark described using various moves from the framework to sustain an in-depth planning conversation about specific strategies to support students with productive struggle.

Across these differences, all nine coaches described instances in which their preparation using the framework helped them deepen planning discussions about the purpose of teaching mathematical content, how to respond to anticipated student strategies, or the rationale for instructional decisions. For example, Howard shared that they prepared a list of possible questions they might ask the teacher about anticipated student strategies despite "not being a huge fan of having a list of questions to ask somebody." During the planning conversation, Howard described how they pressed the teacher to think more deeply about possible student strategies using these prepared questions. Howard shared:

> *We were anticipating some student work . . . and she was describing what she thinks kids will do. I was able to ask, "What does it look like when you say this? What do you mean? What does that mean when this happens?" I think that was productive for us. It really helped us kind of decompose the task into where would that cognitive effort be needed for kids.*

Haynes shared a similar story. Prior to her planning conversation, she prepared a series of "why" questions to help the teacher think more deeply about various lesson design decisions. Haynes shared:

> *We were very happy with [the lesson plan] and how we would assess what students learned. We got into the nitty gritty of how we were going to teach the lesson. But it came back to the "whys" of the lesson. That was a big piece of our planning. Being able to ask questions about our decisions like, "Why is this content important?" This got us into mathematical discussion that the why of this lesson was really to be able to discover a property and then be able to use it to solve problems. "Why are we doing groups in this way? Why are we choosing this lesson format?"*

In both cases, Howard and Haynes shared stories representative of those found in the perceptions of all nine coaches. The framework first supported coaches to create a broad structure for their conversations along with possible questions to ask in different phases in the conversations. Such support, in turn, supported coaches to act responsively during various moments, using questions to facilitate deeper discussion about planning decisions.

We also identified this theme in coaches' descriptions of their lesson implementation, as five coaches reported improved preparation translated into improved decision making when collaboratively teaching the lesson. Briggs shared:

> *There was a time in the lesson where kids were not progressing from one model representation to the next in the way that we hoped. We both kind of recognized this was happening. And so, with maybe 13 minutes to spare, we decided together to pivot the lesson. And that completely changed the students' feeling from "I'm starting to be frustrated" to "I have tools." As a result of that pivot, I think students really had the opportunity to dig into the math and really be engaged with it. If we had not had that content focus in the planning session, not done the math or anticipated strategies, we would not have been prepared to recognize that that pivot needed to happen. I also think that I was ready to do a notice and confer. The framework definitely helped make that pivot happen.*

In this instance, Briggs attributed a successful, in-the-moment decision to collaboratively adjust the lesson to two preparation activities involving the framework. First, Briggs and the teacher engaged in a planning conversation about the mathematical content and possible student strategies, which supported them in effectively noticing student thinking during lesson implementation. Second, Briggs shared the framework prepared him to notice and confer (Gillespie & Kruger, 2022), 1 of 4 options for coteaching, which Briggs selected in that moment, allowing the coach and teacher to share observations and make a quick decision to adjust the lesson.

Theme 3: New Insights About Coaching

The third theme highlighted that 5 of 9 coaches reported gaining new insights, or "ah-ha moments," about coaching. We considered insights about coaching to be statements that made connections between the ideas in the framework and either the coaches' prior experiences or the coaches' practice in future coaching cycles. In each instance of insight, coaches reported that the framework supported them to experiment with a new coaching action, and the resulting interactions with teachers triggered new ideas about their coaching practice. To illustrate how the framework supported new coaching behaviors, which translated into new insights, Briggs, Haynes, Howard, and Vargas reported the framework sparked a realization regarding the potential of reflective discussion after a lesson to support teacher growth. For example, Howard stated:

> *I have not been intentional about asking teachers, "How are you planning to incorporate this into your typical practices now?" Because I had the framework, that became very intentional on my part, and it hasn't always been in the past. And I can see now why that's so important. . . . I think a lot of times teachers reflect on what they could have done differently or what kids might have learned or didn't learn, and how they might go about addressing it. But I think a big part of it for [the teacher] was really thinking about what she learned. This got cemented in my head as far as what's important.*

Similar, Vargas shared:

> *The implications section in the debrief part of the framework, I feel like that's important. It made me realize that a major goal of ours as coaches is to make sure that the teacher carries this practice forward. That it doesn't just stop because you're not there in the classroom.*

In both examples, Vargas and Howard shared how the implications section of the debriefing conversational structure component of the framework prompted them to implement new coaching behaviors; the resulting experiences illuminated how prompting teachers to consider implications for their future teaching is a critical coaching move.

Briggs shared a different realization regarding how the examination of specific student thinking during a debriefing conversation, as prompted by the framework, can have a significant impact on teacher learning. Briggs shared:

> *We talked about the student who was visibly changing her math mindset during class. I had bookmarked a moment involving a student that is classified as special education. We talk about how this student was not only keeping up with the lesson but also making conclusions that other students were not. We spent a lot of time talking about that. "Why did this happen? What were the contributing factors? What made this happen? How can we do this again in the future?" He got kind of emotional, in a good way. And he started to tear up and said, "This is the first time I've seen my kids wanting to do math and being that engaged and being that open and willing to share what they were thinking. This was totally transformative; it was the best lesson I've ever been a part of." I just can't even tell you how awesome it was. Here I am thinking, "We screwed up," and he's saying, "Wow, this was transformative for my kids." So, I can see from the framework that, honestly, this came from looking carefully at student evidence and then considering contributing factors.*

Although most new coaching insights centered on the debriefing conversations, Logan reflected about the power of asking teachers "why" questions from the framework during planning conversations. Logan shared:

> *Talking more about "the why" was powerful. Why are we going to teach the lesson the way we were going to do it? It felt productive because it helped her refer back to the goals for the students. . . . We were able to talk about, "Does this actually meet our learning goals and the instructional practice goals?" We could really be intentional about the time we were going to spend in the lesson. It felt productive to me because now we weren't just wasting time and, you know, that's a big thing teachers say: "I don't have enough time." So, we could really be intentional about every decision we are making for the lesson. . . . Without the framework, I don't think I would have spent that time in the planning session doing that.*

In this example, Logan shared how the "why" of the planning conversation component of the framework supported them to consider planning decisions carefully in relation to the learning goals for students and instructional practice goals for the teacher. Logan also shared how the "why" questions on the framework helped them and the teacher to coconstruct a lesson that used limited instructional time, which was a challenge she had often encountered when working with teachers.

Taken together, these examples illustrate how the framework supported coaches to purposefully plan and prepare for their coaching cycle; improve their responsive decision making in their planning conversations, lesson implementation, and debriefing conversations; and gain new insights into their work as coaches. Furthermore, these insights primarily involved debriefing conversations but also included examples from the planning conversation and lesson implementation phases of the coaching cycle.

DISCUSSION

This study aimed to understand how a CFC framework influenced coaches' professional growth with respect to facilitating one-on-one coaching cycles. Through our analysis of coaches' perceptions when using the framework with teachers, we found the framework supported coaches to prepare for and act responsively during CFC cycle interactions. Furthermore, findings showed over half of participants shared new insights about coaching that transcended the coaching cycle they facilitated for this study. In the following section, we describe how our framework and study provided two key contributions to the field of mathematics education.

CFC Implementation Framework

The first contribution is the CFC implementation framework, a practical tool for any mathematics coach, specialist, or teacher educator engaging teachers in collaborative planning, coteaching, and debriefing interactions. The framework was constructed through a robust process that merged our extensive experience coaching teachers and coaches with theoretical structures underpinning the three phases of a CFC cycle. This process included iterative cycles of drafting and revision based on feedback among our team of project researchers, who were actively analyzing mathematics coaching, and external reviewers whom we sought out because of their expertise in the field. The final round of feedback and revision occurred with practicing coaches (i.e., our study participants) as they used the framework in coaching cycles with teachers in their local settings. This feedback and our findings, which were grounded in participants' stories from using the framework in the field, provided evidence that the framework supported professional improvements in coaches' abilities to facilitate CFC cycles.

Beyond the robust construction process, our framework has at least three characteristics that make it a unique extension of existing protocols, structures, and frameworks that have similarly intended to support coaches in acting intentionally, yet responsively, when supporting teachers. Our framework (a) provides specific and actionable behaviors that align with the three distinct phases of a coaching cycle; (b) articulates connections between these three phases, so coaches recognize the components of a coaching cycle are a coherent set of learning experiences; and (c) applies directly to coaching cycles in which the primary goal is improving teachers' content and pedagogical content knowledge. Our CFC implementation framework builds on prior tools that contain subsets of these three characteristics. For example, Wills and Rawding (2019) created protocols to help coaches set goals with teachers for future interactions. Baker and Knapp (2019, 2023) developed a protocol that is inclusive of coaching cycles while extending beyond this single coaching activity. The protocol is a tool that supports coaches in planning and reflecting on coaching interactions aimed at supporting teachers in the implementation of ambitious and equitable teaching practices (NCTM, 2014). These tools articulated specific and actionable behaviors, yet they were not connected to CFC cycles. Knight (2007) and Costa and Garmston (2016) have provided structures to guide coaches to facilitate all three phases of a coaching cycle that align to the principles of instructional and cognitive coaching, respectively. However, the conversational structures in these models did not target improving teachers' content and pedagogical content knowledge and instead aimed to cultivate general instructional habits and cognition. West and Cameron (2013) described CFC cycles as a tool to improve teachers' content and pedagogical content knowledge; however, they did not provide a comprehensive set of actionable practices to guide coaches through all three phases of the coaching cycle.

Coach Learning

As a second contribution to the field of mathematics education, our study provides new insights into how coaches learn to coach, with a focus on how a coaching tool (i.e., our framework) supported professional growth. Research on professional development for mathematics specialists (Jarry-Shore et al., 2023; Swars Auslander et al., 2023) and coaches working one on one with teachers (Gibbons & Cobb, 2016; Saclarides & Kane, 2021; Saclarides & Kane, 2024) remains an emerging focus. Coaches often have been "anointed and/or appointed" (Fennell, 2017, p. 9) without ongoing professional learning. Findings from our study build on prior studies of coach learning, such as those from Stein et al. (2022) and Kane and Saclarides (2022), who have investigated how specific professional learning activities influenced coaches and their practices and called upon future researchers to investigate coach learning. In response, our study generated new knowledge about how a coaching tool, as opposed to a set of professional learning activities, supported coaches' professional growth in facilitating coaching cycles. Our findings connected closely to those of Baker and Knapp (2019), who showed a coaching protocol improved content-focused coaches' abilities to plan for and reflect upon coaching interactions. As in Baker and Knapp's study, our participants also reported improvements in their abilities to plan and prepare for all three phases of the coaching cycle. We found the framework supported coaches' abilities to make responsive decisions during coaching interactions and triggered new insights about the larger goals of coaching.

To illustrate this claim, we synthesize experiences shared by Briggs across the full coaching cycle. Recall that Briggs described how the planning portion of the framework prepared the coach and teacher to adjust the lesson responsively based on observations of student thinking using a notice and confer coteaching move. Then, Briggs shared how the debriefing portion of the framework helped the coach prepare for and facilitate a debriefing conversation centered around analyzing evidence of student thinking. This conversation supported the teacher to have transformative realizations about students' capacities to think and reason and the use of more equitable mathematics teaching practices (NCSM, 2019; NCTM, 2014). In response, Briggs described his own realization about coaching, catalyzed from use of the framework, regarding the power of examining interplay between student thinking and instructional decisions during debriefing conversations.

Implications and Future Directions

This study holds implications for both practitioners and future research. For practitioners, our framework is a practical tool that articulates specific and actionable behaviors for any mathematics coach, specialist, or teacher educator wanting to develop teachers' mathematical content and pedagogical content knowledge through collaborative planning, lesson implementation, and debriefing interactions. Based on findings and professional learning experiences, we found the framework supported professional growth for a group of coaches with varied levels of coaching experience and diverse roles, including coaches with limited release time from teaching for coaching colleagues. We encourage any practitioner who supports mathematics teachers through one-on-one interactions to consider adopting our framework into practice.

However, we also found all participating coaches, regardless of their prior coaching experience, discussed the connected nature of their own professional learning experiences and framework use. Coaches' perceptions suggested understanding the guiding principles of CFC and theoretical structures underpinning the framework phases appeared to be prerequisite knowledge for effectively engaging with the framework. Using the framework also supported participating coaches in deepening their understandings of these principles and structures. Thus, for leaders interested in using the framework to support the learning of mathematics coaches and specialists, we recommend designing experiences to help coaches develop at least a basic understanding of principles and theoretical structures in the framework. Then, coaches would benefit from opportunities to use the framework with teachers to continue to deepen this understanding of CFC concepts. For example, district leaders responsible for coaching teams might structure a series of collaborative learning sessions to unpack the structure and individual elements of a particular phase of the framework. Then, they might encourage coaches to plan for and use that phase of the framework when working with teachers and follow up with collaborative discussions to reflect on their coaching experiences. It is plausible that, based on our findings, such actions from district leaders could improve coaches' abilities to facilitate CFC cycles and deepen their understandings of coaching. Additionally, such learning opportunities might also naturally catalyze discussion about inherent challenges in coaching not explicitly addressed in the framework (e.g., ways to manage power dynamics in coaching relationships through balancing directive and invitational discourse moves; Gillespie et al., 2025; Smith et al., 2025; Witherspoon et al., 2021).

Our findings also suggest new research questions about how coaching tools (e.g., frameworks, protocols) influence coach development. For example, future researchers should examine features of coaching tools that make them productive and practical resources. Although it was not the primary focus of our analysis, multiple coaches mentioned framework features that increased the usability of the tool. These features may provide a starting point for other researchers and professional learning specialists interested in coaching tools. For example, participants highlighted the power of pairing visual, theoretical structures with specific and actionable coaching behaviors. Across the three themes of our findings, coaches continually mentioned connections between structures, such as the Guide to Core Issues (West & Cameron, 2013), the continuum of teaching responsibility (Gillespie & Kruger, 2022), and the debriefing conversational structure (Gillespie et al., 2023), along with specific behaviors to enact the structures played a key role in supporting growth.

As a second example, multiple coaches referenced how the framework's inclusion of first steps, next steps, and peak performance played distinct yet interrelated roles when they interacted with the framework. Coaches shared that the first steps and next steps provided more immediate and actionable guidance for each phase of the coaching cycle, whereas the peak performance descriptors helped coaches consider the overarching purpose of their interactions with teachers and set personal coaching goals. Thus, future researchers who establish new tools might consider ways to couple short-term behaviors with long-term outcomes to support coaches with limited time to prepare for coaching interactions.

Third, coaches consistently referenced how the framework helped them view a coaching cycle as a coherent whole, comprised of three interconnected phases. Furthermore, coaches mentioned having separate guidance for each of the three phases was productive because descriptions made explicit connections between the phases—another important consideration for future coaching tools. In sum, these three features appear to unveil practical actions and larger coaching concepts for participating coaches simultaneously, and we encourage future researchers to follow this initial path, examining what features make coaching tools useful.

Future research is also needed to understand better the interplay between professional learning experiences and coaching tools. In this study, participants had 2 years of learning prior to receiving the framework. Without this prior learning, we speculate our participants may not have been prepared to use the framework adequately and grow in the ways they reported. On the other hand, the same participants requested a comprehensive tool that summarized and operationalized their learning experiences (from the prior 2-year study) in ways that prepared them for real interactions with teachers. Thus, we conjecture high-quality professional learning for coaches involves collaborative experiences connected to practical tools. We encourage future research to investigate this relationship between learning experiences and tools with coach learning. Finally, the field would benefit from further research on ways this coaching tool may be applicable beyond mathematics education or one-on-one coaching. It is possible the framework could be used in diverse contexts in which one educator strives to help fellow educators build their content and pedagogical content knowledge. For example, Kraft and Blazar (2018) raised the possibility of coaches working with small groups of teachers to address scalability issues inherent to one-on-one coaching. We can envision how a coach might use the framework to facilitate content-focused planning and debriefing conversations with small groups of teachers around a shared lesson in addition to engaging in one-on-one coaching cycles. Although such use would likely require modifications and is beyond the findings of our analysis, we encourage both practitioners and researchers to explore use of our CFC implementation framework in a wider variety of contexts.

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Appendix: Content-Focused Coaching Implementation Framework

Content-Focused Coaching Implementation Framework

The material is based upon work supported by the National Science Foundation Grant #2006353 & #2006263. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Guiding Principle

Building from prior research and literature on coaching, the guiding principle for the design and implementation of our content-focused coaching model is as follows: *If coaches maintain an intentional focus on specific, high-leverage teaching practices during all coaching conversations and activities, they can reliably achieve positive outcomes related to teacher growth and student learning.*

These high-leverage teaching practices, components of ambitious and equitable mathematics teaching (Horn & Garner, 2022), include

- establishing mathematics learning goals focused on understanding big ideas in mathematics (Hiebert & Grouws, 2007),
- implementing high cognitive demand tasks that promote inquiry and provide access to all students (Smith & Stein, 2018),
- eliciting and responding to students' thinking (Leahy et al., 2005; Smith & Stein, 2018),
- facilitating meaningful and productive mathematical discourse that builds from students' reasoning and connects mathematical strategies and representations (Chapin et al., 2009; Smith & Stein, 2018; Staples, 2007), and
- supporting and promoting productive struggle (Kapur, 2010; Warshauer, 2015).

As a result, outcomes of content-focused coaching in mathematics are the development of a teacher's

- mathematical content knowledge (West & Cameron, 2013; West & Staub, 2003),
- pedagogical content knowledge (Ball et al., 2008; Shulman, 1987), and
- ability to use ambitious and equitable mathematics teaching practices (Lampert & Graziani, 2009).

Phase 1: Planning Conversation

Outcomes of Planning Conversations Related to the Guiding Principle

- Teacher can explain how their understanding of (a) mathematical content knowledge, (b) how students learn mathematical content, and (c) ambitious and equitable mathematics teaching grew as a result of the planning conversation.
- Teacher develops planning habits focused on (a) mathematical content, (b) how students learn mathematical content, and (c) use of ambitious and equitable mathematics teaching practices without the support of a coach.

Elements	First Steps	Next Steps	Peak Performance
Preparing for the Planning Conversation			
Preparing for Planning Conversation	Coach asks teacher to provide student mathematics learning goal(s) and lesson activities/tasks. Coach asks teacher to provide instructional goal for themself.	Coach: • asks teacher to anticipate student thinking and related responses by completing lesson activities/tasks themself as a learner. • anticipates student thinking and related responses by completing lesson activities/tasks themself as a learner. • designs plan for conversation including questions to ask the teacher.	Coach uses knowledge of the teacher, lesson activities, and goals to design plan for conversation. Coach anticipates teacher's responses to planned questions and prepares potential coaching moves.

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Elements	First Steps	Next Steps	Peak Performance
Discussing the *What, Who, How, and Why* of Lesson Design (West & Staub, 2003)			
***What:* Creating Mathematics Learning Goal(s) for Students**	Teacher or coach states the mathematics learning goal(s) for students.	Teacher and coach discuss: • ways to make goal(s) more focused on learning and understanding. • how lesson/task aligns with mathematical goal(s). • evidence of student thinking that would indicate goal(s) was/were met. • how goal(s) relate(s) to important mathematical ideas. • how goal(s) connect(s) to students' prior learning. how goal(s) of this lesson are part of a coherent set of learning experiences.	Teacher and coach discuss how *creating mathematics learning goal(s) for students* supports ambitious and equitable mathematics teaching and improves student learning. Teacher and coach discuss teacher's next steps in making *creating mathematics learning goal(s) for students* a planning habit.
***Who:* Anticipating Student Thinking**	Coach and teacher discuss anticipated student thinking, including possible strategies or related responses.	Teacher and coach: discuss how to productively build from and/or respond to anticipated student thinking. • use anticipated thinking to design or refine lesson activities. • discuss how anticipated thinking is evidence of progress toward mathematics learning goals.	Teacher and coach discuss how *anticipating student thinking* supports ambitious and equitable mathematics teaching and improves student learning. Teacher and coach discuss teacher's next steps in making *anticipating student thinking* a planning habit
***How:* Designing the Learning Experiences**	Coach and teacher engage in discussion about designing the learning experiences.	Coach and teacher discuss: how learning experiences connect to each other and to goal(s) for students, which may result in refining experiences, tasks, or goals. • how to begin each learning experience in ways that provide all students with access without declining the cognitive demand. • how students will engage in learning experiences and teacher's role in facilitating the experiences (e.g., providing independent thinking time, using discussion protocols in small groups). • how to summarize each learning experience by making student thinking visible to support all students in advancing toward mathematical goal(s). • how to assess student learning including artifacts to collect and examine in the debrief.	Teacher and coach discuss how designing the learning experiences supports ambitious and equitable mathematics teaching and improves student learning. Teacher and coach discuss teacher's next steps in making designing the learning experiences a planning habit.

Elements	First Steps	Next Steps	Peak Performance
Discussing the *What, Who, How, and Why* of Lesson Design (West & Staub, 2003)			
***Why:* Articulating Rationales for Planning Decisions**	Coach invites teacher to share rationales for certain planning decisions.	Coach embeds frequent questions to: support teacher to consider affordances and drawbacks of their planning decisions. • support teacher to make connections between the *what, who, and how* components of lesson design.	Teacher and coach discuss how *articulating rationales for planning* decisions supports ambitious and equitable mathematics teaching and improves student learning. Teacher and coach discuss teacher's next steps in making *articulating rationales for planning* decisions a planning habit.

Creating Goal(s) for the Teacher			
Creating Instructional Practice Goal(s) for the Teacher	Coach invites teacher to share rationales for certain planning decisions.	Coach and teacher discuss: how to refine instructional goal(s) so it/they is/are sufficiently specific to reflect upon in the debrief. • evidence of both student and teacher actions that would indicate progress toward teacher's instructional practice goal(s). • how the lesson/task affords opportunities to use practices related to goal(s) for the teacher. • why the goal(s) is/are high leverage and how this/these goal(s) will support student learning of important mathematical content.	Teacher and coach discuss how *creating instructional practice goal(s) for the teacher* supports ambitious and equitable mathematics teaching and improves student learning. Teacher and coach discuss teacher's next steps in making *creating instructional practice goal(s) for the teacher* a planning habit.
Preparing for Lesson Implementation			
Preparing for Coteaching	Coach asks teacher how they would like to coteach the lesson during planning conversation.	Coach and teacher consider four forms of coteaching (see Phase 2: Lesson implementation) to design plan for lesson implementation that assigns teaching responsibility for each lesson activity.	Coach and teacher consider four forms of coteaching (see Phase 2: Lesson implementation), teacher's instructional goals, and teacher's learning needs to design plan for lesson implementation that provides coach and teacher with a clear understanding of their roles for each lesson activity. Coach and teacher are prepared for "in-the-moment" decisions and adjustments based on teacher's and students' learning needs.

Phase 2: Lesson Implementation			
Outcomes of Lesson Implementation Related to the Guiding Principle Teacher is intentionally supported by coach to experiment with new instructional practices related to (a) implementing lessons focused on important mathematical concepts, (b) noticing and responding to how students are learning mathematical content, and (c) ambitious and equitable mathematics teaching. • Coach and teacher execute a coteaching plan that allows for collaborative decisions to fluidly shift between forms of coteaching in response to in-the-moment opportunities and the teacher's instructional goal and emotional needs.			
Elements	**First Steps**	**Next Steps**	**Peak Performance**
Using the Four Forms of Coteaching (Gillespie & Kruger, 2022)			
Model	Coach models portion of lesson while teacher observes.	Coach finds moments while modeling to make their decision-making process explicit with teacher. • Coach and teacher find opportunities to discuss what they notice about student thinking. • Coach provides teacher with opportunities to retake responsibility of the teaching.	Coach models lesson in ways that draw attention to aspects of practices related to teacher's learning needs and instructional practice goals. Coach and teacher find opportunities to listen to each other's noticings about student thinking, and coach makes instructional decisions based on these shared noticings.
Enter and Exit the Lesson	Coach enters and exits moments in the lesson.	Coach intentionally enters and exits moments in the lesson to draw attention to aspects of practices • Coach is mindful about how they enter the lesson and for how long they maintain teaching responsibility.	Coach intentionally enters and exits critical moments in the lesson to draw attention to aspects of practices related to teacher's learning needs and instructional practice goals. Coach enters the lesson for smallest amount of time needed to draw attention to important aspects of practice. Coach uses knowledge of teacher to enter the lesson safely (without causing interruption) and fluidly returns teaching responsibility to teacher when exiting.
Notice and Confer	Coach records noticings and confers with teacher at various moments in the lesson.	Coach (a) notices particular students' thinking and aspects of teacher's instructional practice and (b) confers with teacher at various moments in the lesson. • Coach and teacher find opportunities to listen to each other's noticings about student thinking.	Coach (a) intentionally notices students' thinking and attends to relationship between students' thinking and instructional practices and (b) confers with teacher at critical moments in the lesson that address teacher's learning needs and instructional practice goals. Coach and teacher find opportunities to listen to each other's noticings about student thinking, and teacher makes instructional decisions based on these shared noticings.

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Elements	First Steps	Next Steps	Peak Performance
Using the Four Forms of Coteaching (Gillespie & Kruger, 2022)			
Notice and Bookmark	Coach notices and bookmarks lesson events at various moments in the lesson.	Coach (a) notices particular students' thinking and aspects of teacher's instructional practice and (b) bookmarks moments in the lesson. Coach bookmarks lesson events using details that allow the events to be accurately recalled during the debrief.	Coach (a) intentionally notices students' thinking and attends to relationship between students' thinking and instructional practices and (b) bookmarks critical moments in the lesson that address teacher's learning needs and instructional practice goals to discuss in the debrief conversation. Coach bookmarks lesson events using details that allow the events to be accurately recalled during the debrief as evidence of progress toward goals for both students and teacher.

Phase 3: Debriefing Conversation			
Outcomes of Debriefing Conversations Related to the Guiding Principle Teacher can explain how their understanding of (a) mathematical content knowledge, (b) how students learn mathematical content, and (c) ambitious and equitable mathematics teaching grew as a result of the entire coaching cycle. Teacher can transfer new understanding of (a) mathematical content knowledge, (b) how students learn mathematical content, and (c) ambitious and equitable mathematics teaching to support growth in future practice. • Teacher develops reflective habits focused on (a) mathematical content, (b) how students learn mathematical content, and (c) use of ambitious and equitable mathematics teaching practices without the support of a coach.			
Elements	**First Steps**	**Next Steps**	**Peak Performance**
Preparing for the Debriefing Conversation			
Preparing for Debriefing Conversation	Coach reviews goals established during planning conversations. Coach reviews notes about observed events during coteaching.	Coach designs a plan for the debriefing conversation that accounts for goals for students and goal for teacher established during planning conversation and observed events during coteaching. • prioritizes moments from coteaching to highlight (a) evidence of students' mathematical thinking or (b) factors that supported or limited progress toward the goals. • includes questions to ask teacher that elicit teacher reflection.	Coach designs plan for debriefing conversation that integrates mathematical goal(s) established during planning conversation and observed events during coteaching to support teacher to transfer new learning to future practice. When designing plan for the conversation, coach prioritizes moments from coteaching that (a) highlight evidence of student thinking, (b) showcase factors that supported or limited progress toward goals, and (c) inform future practice. Coach anticipates teacher's responses to planned questions and prepares potential coaching moves.

Continued on next page…

<table>
<tr><th>Elements</th><th>First Steps</th><th>Next Steps</th><th>Peak Performance</th></tr>
<tr><td colspan="4">Reflecting on the Lesson Using the Debriefing Framework (Gillespie et al., 2023)</td></tr>
<tr><td>Revisit Goals</td><td>Coach reminds teacher about goal(s) for students and goal(s) for teacher established during planning conversation.</td><td>Coach and teacher discuss mathematics learning goal(s) for students and instructional practice goal(s) for teacher to ensure shared understanding.</td><td rowspan="4">Coach and teacher make connections between mathematical learning goals, evidence of student thinking, and contributing factors to identify important and specific implications for teacher’s future practice.

Coach and teacher coconstruct next steps that support the teacher to embed new learning in their regularly occurring practice.</td></tr>
<tr><td>Analyze Evidence From Students</td><td>Coach and teacher discuss instances of student thinking using artifacts such as student work or observation notes.</td><td>Coach and teacher:
• discuss how the evidence of student thinking connects to the mathematical learning goals.
• use evidence of student thinking to collaboratively make and support claims about student understanding.</td></tr>
<tr><td>Consider Contributing Factors</td><td>Coach and teacher discuss features of lesson design and implementation that may have contributed to student understanding of mathematical learning goals.</td><td>Coach and teacher:
• discuss connections between evidence of student thinking and features of lesson design and implementation.
• collaboratively make and support claims about how features of lesson design and implementation supported or limited success toward goals.</td></tr>
<tr><td>Determine Implications for Future Teaching</td><td>Coach asks teacher what they learned from preceding discussion that will inform teacher’s future practice.</td><td>At multiple points throughout the conversation, coach and teacher consider how ideas being discussed will inform future practice.
• Coach invites teacher to share actions they will take to enact their new learning.</td></tr>
</table>

Elements	First Steps	Next Steps	Peak Performance
Concluding the Coaching Cycle			
Reflecting on Coaching Cycle Process With Teacher	Coach asks teacher to share their thoughts about coaching cycle process.	Coach and teacher consider ways to improve future coaching interactions.	Coach and teacher collaboratively name aspects of coaching cycle process that were beneficial, or in need of improvement, to guide future coaching interactions. Coach reflects on and records ideas from process reflection conversation and uses these ideas during future coaching work.
Planning for Future Coaching With Teacher	Coach and teacher discuss and/or schedule future coaching work.	Coach and teacher anticipate teacher's needs for continued growth as they experiment with new practices.	Coach and teacher collaboratively create plan for future coaching based on teacher's new learning and anticipated needs for continued professional growth. Coach records and uses ideas from this discussion to create connections between current and future coaching work.
Individual Reflection	Coach takes time to individually reflect on effectiveness of their coaching during the cycle.	Coach reflects on: • evidence of teacher learning. • how their coaching supported or limited teacher learning during the cycle.	Coach uses their individual reflection to determine next steps to continue growing their coaching practice.

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NCSM sometimes provides its mailing list to outside companies. These companies have been approved by NCSM to send catalogs, publications, announcements, ads, gifts, etc. Please check the box below to remove your name from mailing lists. In addition, by checking this box, only your name without contact information will be included in the NCSM Directory. ☐

Employer____

Title ____

Telephone (_____________)______________________________

Fax (_____________) ___________________________________

Email ____

Please check all that apply. I currently work as:

☐ State/Provincial Department of Education Employee
☐ Superintendent
☐ Graduate Student
☐ Department Chair
☐ District Mathematics Supervisor/ Curriculum Director
☐ Building Administrator
☐ Teacher Leader
☐ Coach/Mentor
☐ Consultant
☐ Higher Education
☐ Other______________________________

Since designations vary over time, check the one you feel best describes you:

☐ African American/Black ☐ Asian American ☐ European Descent ☐ Hispanic Descent
☐ Native American ☐ Pacific Islander ☐ Bi-Racial/Multi-Racial ☐ Other_____________________

Check the area you serve:
☐ Rural ☐ Suburban ☐ Urban

Do you influence purchasing decisions?
☐ Yes ☐ No

Age: ☐ under 25 ☐ 25 - 34 ☐ 35 - 44 ☐ 45 - 54 ☐ 55-64 and over

_____NCSM Member Pin $2 $ __________

Merchandise Total: $ __________

Individual Dues $85 $ __________

Emeritus Membership $55 $** __________

Student Membership* $55 $ __________

Institutional Membership $100 $ __________

Institution Name: _______________________________

____

TOTAL ORDER: $ __________

Payment Method: ☐ Visa ☐ MasterCard ☐ Discover Card
☐ Check/Money Order *(U.S. Funds only)*
☐ Purchase order***

**Purchase Order # ____________________

Credit Card #______________________________Exp _____/ _____

Cardholder Name________________________Billing Zip__________

Cardholder Signature ______________________________________

Please Note: An invoice will NOT be sent. Please use this form as your invoice.

***Student Membership:** Requirements for student membership are 1) This is your first membership in NCSM (student members are eligible for one renewal at the student member rate for a maximum of two years and; 2) You are currently enrolled in a graduate program to become a mathematics leader (proof of enrollment must be provided). This membership must be approved by the Membership & Marketing Committee.

****Emeritus Membership:** Requirements for emeritus membership are being age 65 and a member in good standing of NCSM for a mini-mum of 15 years or more. Visit mathedleadership.org for additional information.

*****Purchase Orders:** POs must be paid within 60 days or the membership will be suspended until payment is received.

Institutional Membership: With an Institutional Subscription membership, your organization will receive all of NCSM's print material during the time of your membership. Institutional Subscriptions can only be renewed by mail or fax.

NCSM Tax ID: 39-1556438

Please return this form to:
NCSM Member and Conference Services
6000 E. Evans Avenue 3-205, Denver, CO 80222
Phone: 303-758-9611; Fax: 303-758-9616
Email: office@mathleadership.org **Web: mathedleadership.org**