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USEFUL, EASY, AND CONSEQUENTIAL: A PRACTICAL MEASUREMENT REPOSITORY TO ENHANCE THE WORK OF MATH EDUCATION INSTRUCTIONAL LEADERS AND TEACHERS

TEACHERS' DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS IN AN ELEMENTARY MATHEMATICS SPECIALIST PROGRAM

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DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS

TEACHERS' DEVELOPMENT OF PROFESSIONAL VISION AND LEADERSHIP CONCEPTIONS IN AN ELEMENTARY MATHEMATICS SPECIALIST PROGRAM

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ABSTRACT

In this paper, we share some ways a cohort of 24 elementary teachers developed over the course of a 2-year elementary mathematics specialist (EMS) certification program. We analyzed pre-, mid-, and post-program interviews to document the development of teachers' visions for high-quality mathematics instruction and their views about themselves as mathematics leaders in their schools. We also conducted end-of-program focus groups with a subset of participants (n = 13) to ask about program elements that helped them develop as teachers and leaders of mathematics. Participants identified several common elements of the program as impacting their knowledge, practice, confidence, leadership, and vision, including specific course assignments and cultivation of a supportive community in the cohort.

Elementary mathematics specialist (EMS) programs are designed to (a) support teachers' capacity for high-quality mathematics instruction and (b) support teachers' leadership development so they can, in turn, support the improvement of mathematics instruction in their schools and districts (de Araujo et al., 2017). Although research has found EMS programs can produce improvements in knowledge, beliefs, and practices for teachers (e.g., Myers et al., 2020), less research has documented the elements of EMS programs that contribute to teachers' development (e.g., Reys et al., 2017). In this paper, we share findings on EMS development in relation to two program goals—instructional vision and leadership capacity. We also examine participants' perspectives about how they developed these attributes and what elements of the EMS program supported this development.

LITERATURE REVIEW AND BACKGROUND

In our context, EMSs were teachers who had completed a 2-year graduate program (e.g., Goodman et al., 2017; Harrington et al., 2017) based on the 2013 Association of Mathematics Teacher Educators' Standards for Elementary Mathematics (recently updated; see Association of Mathematics Teacher Educators, 2024). This document provided research-based guidelines for preparation of EMSs in terms of mathematics content, pedagogy, and leadership. Accordingly, the program that served as this study's focus included many learning experiences designed to develop expertise in all three areas (de Araujo et al., 2017). Furthermore, several research studies have shown how graduates of the program have employed their expertise as teachers and teacher leaders (Conner et al., 2022; Nguyen et al., 2022, 2024; Webel et al., 2017, 2018, 2021, 2023). This work adds to a robust body of literature on EMS programs and their impacts on both participants and those served by EMSs.

Impact of EMS Programs

EMS programs generally have shown significant impact on teachers' knowledge and beliefs (e.g., Campbell & Malkus, 2014; Swars et al., 2018) and their teaching practices (Myers et al., 2020, 2021; Nickerson, 2010). EMSs who complete these programs have shown positive impact on student achievement when employed as teachers (Kutaka at al., 2017) or coaches (Campbell & Malkus, 2011). Some scholars have documented how graduates of EMS programs can exercise agency to push the boundaries of their contextual constraints to enact more ambitious practice (Nguyen et al., 2022; Webel et al., 2017, 2021).

In one example, we described how, despite building-wide expectations of daily differentiated instruction in small groups, multiple EMSs considered and implemented alternatives to homogeneous ability grouping, including using mixed ability groups and random grouping (Webel et al., 2021). In another example, we documented multiple structures for departmentalization (i.e., EMSs taught mathematics to multiple groups of students each day; Webel et al., 2017). In these cases, EMSs had some latitude to negotiate details of their departmentalization structure and reported departmentalizing enabled them to focus more effectively on planning and refining their mathematics instruction. These examples show how EMS teachers not only used their expertise to enact high-quality instruction but also exercised agency to create additional opportunities to provide high-quality mathematics instruction for students in their schools. These findings raise the question of how

EMS programs develop teachers' expertise and equip them to influence mathematics teaching at their schools.

Development of EMSs' Capacity for Delivering and Promoting High-Quality Instruction

One way EMS programs support the development of capacity for high-quality mathematics instruction is by integrating learning opportunities with teachers' practices. For example, in one EMS program, participants highlighted the importance of having opportunities as part of their coursework to implement observed lessons and receive feedback from instructors and opportunities to analyze student thinking, including their own students, using frameworks and progressions introduced in their coursework (Myers et al., 2020, 2021). In their analysis, researchers described how participants moved through stages of development: (a) exhibiting skepticism about new ideas for teaching mathematics, (b) demonstrating willingness to try out new practices, (c) making substantial shifts in their practice, and (d) expressing a desire for additional support to sustain these shifts (Myers et al., 2020). In addition, EMSs noted nonevaluative feedback and a comfortable, collaborative space for discussion about their teaching were key components of their learning experiences.

Such learning opportunities can lead to substantial changes in participants' beliefs about mathematics and mathematics teaching and learning (Campbell & Malkus, 2014; Swars Auslander, 2023; Swars et al., 2018; Webel et al., 2023). Beliefs are related to a slightly more specific construct: teachers' visions for instruction (Arbaugh et al., 2021; Jansen et al., 2020; Munter, 2014). Whereas beliefs suggest "a relatively static set of decontextualized ontological commitments . . . vision is intended to communicate a more dynamic view of the future" (Munter, 2014, p. 587), providing a sense of what teachers imagine for their teaching, even if it is not currently reflected in their practice. Vision is also more grounded in practice than beliefs; it is "a set of images of ideal classroom practice for which teachers" (Hammerness, 2001, p. 143) can strive. Indeed, it is difficult to conceive of teachers implementing instruction they have not imagined for themselves.

Elements of vision include the role of the teacher, the structure and nature of classroom discourse, and the nature of mathematical tasks students are asked to complete. In general, EMS programs have been designed around a vision of instruction articulated in standards documents published by the National Council of Teachers of Mathematics (NCTM, 2000, 2014, 2020). According to these documents, high-quality mathematics instruction empowers students to make sense of mathematical concepts; engages them with tasks that are cognitively demanding, mathematically rich, and often set in real-world contexts; and provides them with opportunities to discuss, explain, connect, and justify their reasoning through conversations with each other and with their teacher.

Researchers have found teachers with instructional visions more aligned with these principles are more likely to see improvements in their instructional practice (e.g., Munter & Correnti, 2017). Because EMS programs typically have been designed around this cohesive vision for mathematics instruction, we sought to explore whether and how teachers' images of their future instruction were impacted by their time in EMS programs, which could provide insights into whether and how these shifts likely impacted their future instruction. We anticipated this study could complement research on EMSs' beliefs and provide new insights into how EMSs' instructional vision develops through certain kinds of experiences, along with how this development corresponds to changes in their instruction.

Development of EMSs' Capacity for Leadership

In addition to developing expertise and capacity for high-quality mathematics teaching, researchers have found EMSs work in a variety of ways to support elementary mathematics instruction as leaders or coaches (Baker et al., 2022; Campbell & Griffin, 2017). For example, coaches engage in co-planning with individual or teams of teachers, modeling lessons, identifying and filtering mathematics resources to meet teacher and district needs, conducting workshops, developing assessments and organizing student data, and providing personal professional development to support teachers' work (Baker et al., 2022; Campbell & Griffin, 2017).

EMSs also engaged in both formal and informal leadership without becoming a coach or leaving their classroom teaching positions (Conner et al., 2022). For example, EMSs reported serving on mathematics committees in their schools or districts, mentoring new teachers, assisting in analyzing achievement data, leading community outreach events, and planning meetings with grade-level teams (Conner et al., 2022). EMSs also participated in spontaneous and informal conversations with colleagues where they provided advice and information about mathematics instruction (Nguyen et al., 2024). The fact that much of this work can be accomplished without EMSs leaving the classroom means they can be a cost-effective option for instructional leadership in under-resourced schools. In sum, these existing findings suggest well-prepared EMSs often have the capacity to influence instruction in classrooms beyond their own and can influence policies that impact mathematics instruction.

Additionally, research suggested one important trait that enables EMSs to be effective as mathematics leaders is confidence, which was associated with improvements in the knowledge, instruction, and self-efficacy of the teachers with whom leaders work (Yopp et al., 2019). Conversely, teacher–leaders who lack confidence may be reluctant to assume leadership positions and struggle to advocate for their work or gain legitimacy with their colleagues (Hunzicker, 2017; Wenner & Campbell, 2017). However, limited research has described how EMS programs help develop capacity for leadership. In one study, Swars Auslander et al. (2023) found EMSs experienced positive, significant shifts in beliefs about their coaching effectiveness, and EMSs with stronger self-efficacy beliefs reported using more coaching practices, especially those related to supporting teachers' mathematics content and pedagogical knowledge. EMSs attributed their increased confidence regarding leadership to (a)

participating in a community of other EMSs; (b) opportunities to engage in teacher leadership through the program; and (c) greater knowledge about mathematics content, pedagogy, and coaching strategies (Swars Auslander et al., 2023).

These findings point to the importance of teachers' self-perceptions as leaders and the relationships between teacher–leaders' confidence and their developing knowledge, awareness of, and participation in leadership. Because the transition from teacher to teacher–leader or coach entails shifts from work in which they are a recognized expert (i.e., teaching) to work in which they are relatively inexperienced (i.e., supporting teachers), it can require even veteran teachers to develop new identities and ways of seeing themselves and their work (Chval et al., 2010; Hanuscin & Zangori, 2016; Zuspan, 2013). For example, as teacher–leaders build trust with colleagues, learn to negotiate with administrators, and choose when to take risks, they may become aware of aspects and elements of schooling they had not considered previously (Zuspan, 2013).

Knapp (2017) described this widening of attention as emerging leaders taking up a system view of school improvement, supporting their transition from teacher to teacher–leader. This attention to broader contexts was also important in Hunzicker's (2017) study of teacher–leaders' self-perceptions, where teachers least likely to view themselves as teacher–leaders preferred leading on a smaller scale, in limited situations, or in their classrooms. Similarly, Brooks et al.'s (2004) typology of teacher–leaders foregrounds the parameters of their leadership responsibilities, moving from the classroom to the department to the whole school. This typology suggests a trajectory: As teachers gradually develop the skill and confidence to begin leading, they start with small steps and eventually take on larger responsibilities. As their responsibility expands, their scope of awareness also increases. They become aware of larger elements of the system, leading to the development of new skills and, potentially, increased confidence.

These findings raise questions about how leadership capacity is developed in EMS programs, specifically how teachers' conceptions about what leadership entails and their views about their own capacity to enact leadership evolve during their time in an EMS program. Additionally, further exploration may elucidate what a trajectory of mathematics leadership development looks like and reveal the kinds of experiences and activities that support teachers in feeling prepared to address the challenges noted previously and to develop the capacity to enact leadership in their contexts.

CURRENT STUDY

In the current project, we used a case study approach (Yin, 2018) to analyze a variety of data sources to examine EMS development across a 2-year graduate program, seeking to understand some dimensions that had not yet been explored and to understand what learning experiences were most powerful from the perspectives of the EMSs. We investigated two research questions (RQs):

1. How do EMSs' (a) visions for mathematics instruction and (b) conceptions of themselves as leaders develop over the course of their EMS program?
2. How do EMSs describe their development and the elements of the EMS program that supported this development?

Context and Methods

The context for this study was an EMS program comprised of 24 graduate credits earned over 2 years comprised of five content courses and two leadership courses. The program was co-designed by faculty at multiple institutions across the state of Missouri in the United States (Goodman et al., 2017). Common syllabi, lesson plans, and assignments were shared across sites, and representatives from each institution gathered each year to debrief experiences from the previous year and revise courses.

Participants were 24 elementary teachers who were recruited to become fellows as part of a National Science Foundation (NSF) grant, which paid the tuition for the EMS coursework and provided yearly stipends for 4 additional years of teaching and leadership in a high-need school district. Of these fellows, 13 participants attended Institution 1 and taught in District 1, and 11 participants attended Institution 2 and taught in District 2. To provide fellows with greater opportunity for collaboration on assignments and to use their collective expertise in their schools, program leaders at Institution 1 decided to select fellows from schools in pairs or trios (with one exception). All fellows who participated in the grant program agreed to participate in the research project.

Fellows attended in-person, week-long summer institutes at the beginning and end of the program, which focused on (a) understanding core research findings about mathematics teaching, learning, curriculum, equity, and assessment and (b) developing a vision and skills for leadership in mathematics education. Fellows completed the other five courses, which included online and in-person components, at each institution. Across both institutions, the five content courses covered number and operations, rational number, algebraic reasoning, geometry and measurement, and data and probability. These courses focused on developing an understanding of both mathematical content and research on student learning; a core textbook used across the program was Elementary and Middle School Mathematics: Teaching Developmentally (Van de Walle et al., 2014). Fellows also were expected to read numerous other books and articles, often published by the NCTM.

Each course integrated both content and pedagogy. For example, knowledge of content often was introduced through interaction with artifacts of instruction (e.g., student work, videos of teaching, mathematical tasks, representations of mathematics from elementary classrooms). Assignments often required fellows to solve mathematical tasks, anticipate how students would solve tasks, engage with research on student development of specific mathematical concepts, analyze student work, examine episodes of instruction and coaching, give tasks to their students, and discuss their teaching challenges and dilemmas. An example of an activity

was when fellows were asked to make conjectures about a number string (Bray & Maldonado, 2018) and then prove their conjectures with multiple representations (e.g., in words, with symbols, with an area model). The following number string provides an example:

$$3 \times 18$$

$$6 \times 9$$

$$5 \times 28$$

$$10 \times 14$$

During this activity, participants discussed how such sequences of problems could be used to help children develop a flexible understanding of the group structure of multiplication to solve a range of problems quickly and creatively. In doing so, fellows developed their own mathematical knowledge for teaching—that is, a deep understanding of mathematics used specifically in teaching (Hill et al., 2008).

Each content course was paired with a corequisite internship, which included assignments that engaged fellows in using their teaching contexts to apply ideas from the course. For example, each of four content courses introduced an instructional routine (Lampert et al., 2010): number talks (Parrish, 2011), number strings (Bray & Maldonado, 2018), sorting tasks (Baldinger et al., 2016), and contrasting cases (Teacher Education by Design, 2014). Fellows planned, enacted, and debriefed each routine with colleagues with the goal of developing not just knowledge about mathematics teaching but also the pedagogical skills needed to teach mathematics well (Loewenberg Ball & Forzani, 2009; NCTM, 2014).

Fellows also were expected to engage in leadership activities in their schools as part of these internships, such as creating and carrying out an action plan for supporting improvements to mathematics instruction, planning and implementing an outreach event, and planning professional development sessions for their colleagues. Fellows who taught at the same school were encouraged to complete these leadership assignments as a team. In one example, after a series of sessions on equitable grouping structures at the first summer institute, one school team developed an action plan to begin implementing alternatives to grouping students by ability (Webel et al., 2021). Each month they reported on their plan, made revisions, adapted their approach, and shared their progress with their peers in the program.

Data Collection

Data were collected via interviews at the beginning (Year 0), middle (Year 1), and end (Year 2) of the EMS program. To assess the development of participants' vision of mathematics instruction (RQ1a), we administered the Visions of High Quality Mathematics Instruction (VHQMI) interview protocol (Munter, 2014) to each of the 24 participants. Using the semistructured protocol (see Appendix), we asked participants to describe what they would look for in a mathematics classroom to determine whether the instruction was high quality. The protocol included questions about what the teacher would be doing, what the students would be doing, and what kinds of tasks would be presented to students.

To assess participants' development of their conceptions of themselves as leaders (RQ1b), we asked them to describe their ideas about possible leadership activities in which they would like to engage, what they would want to keep in mind in a leadership role, and what they were excited and apprehensive about regarding such a role. In the Year 1 and Year 2 interviews, we also asked whether anything had changed in terms of how participants engaged in leadership or were seen as leaders in their schools or districts.

To investigate participants' views about the impact of the program on their own development (RQ2), we conducted six focus groups with the 13 fellows at Institution 1. We asked how the program had helped them address teaching challenges; how their teaching of mathematics had changed; what readings, ideas, or activities had made the biggest impact on them; and what challenges they still faced regarding teaching and leadership in mathematics.

Data Analysis

To analyze the development of participants' visions for mathematics instruction (RQ1a), we focused on four VHQMI rubrics (Munter, 2014): (a) role of the teacher, (b) mathematical tasks, (c) patterns of classroom talk, and (d) nature of classroom talk. For each dimension, we used the rubrics to assign a score (0 to 4) to represent the sophistication of discourses teachers employed to characterize ideal classroom practice at the beginning of the program and after 1 year. The role of the teacher rubric characterizes the teacher's role along the dimensions of influence on classroom discourse, attribution of mathematical authority, and conception of typical activity structure, culminating in labels that include "teacher as more knowledgeable other," "teacher as facilitator," and "teacher as monitor." The mathematical tasks rubric addresses how teachers describe task elements such as cognitive demand, real-world application, multiple solution paths, and opportunities to generalize or make connections across mathematical ideas. The patterns of classroom talk rubric considers the extent to which student-to-student discourse is promoted, whether whole-class conversation is prioritized, and whether the students or the teacher initiate and carry out the talk. Finally, the nature of classroom talk rubric identifies whether talk focuses on concepts or calculations and the extent to which the talk is focused on mathematics. Ratings for each of these dimensions were assigned to each interview independently by two researchers, who then met to resolve discrepancies.

To categorize participants' conceptions of themselves as leaders (RQ1b), we used an open-coding process initially to identify ways participants answered questions about leadership. We noted variance in two dimensions: (a) confidence in themselves as leaders and (b) awareness of the nuances and responsibilities of leadership work. Confidence was judged through explicit statements referencing discomfort/comfort, anxiety/eagerness, and intimidation/assertiveness when discussing the prospect of leading others. Often these characterizations surfaced when participants

were asked what they were excited and apprehensive about regarding leadership. In some cases, especially at the higher end of the spectrum, confidence was not stated explicitly so much as inferred from the ease with which participants described their leadership or talked about their perspectives on teacher learning.

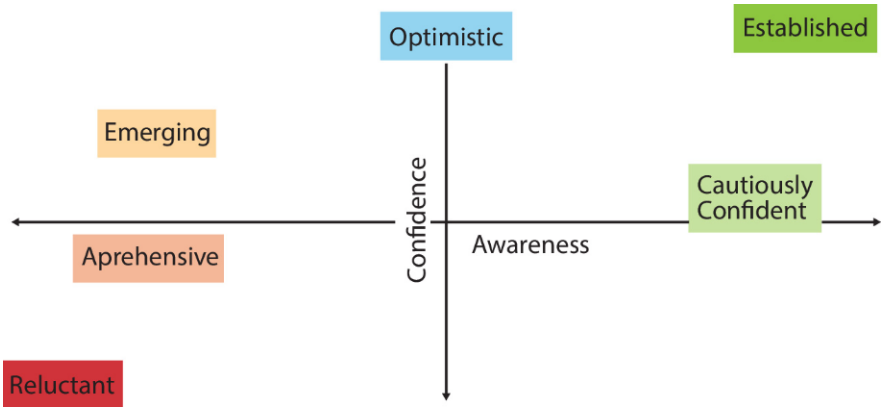
Awareness of the nuances and responsibilities of leadership work (hereafter referred to as "awareness") was determined through consideration of how detailed and specific participants were in their descriptions of leadership, including having realistic expectations for what kinds of changes can be accomplished in certain time frames, the roles of various policies and policymakers relevant to the changes they hoped to see, and their understandings of the limitations and challenges that would need to be confronted in their specific contexts. It was possible for these two dimensions (i.e., confidence and awareness) to fluctuate somewhat independently of each other, but our objective was to consider how a single trajectory of development could capture these changes and the relationships between them. For example, confidence might rise as participants developed deeper knowledge and teaching expertise, but opportunities to lead might cause greater awareness of systemic challenges, which could result in loss of confidence. We sought to develop a framework that would capture stages of development between varying levels of confidence and awareness and how they related to each other.

Looking across the cases, we created the following labels as a holistic characterization of how participants expressed their conceptions of themselves as leaders: established, cautiously confident, optimistic, emerging, apprehensive, and reluctant (see Table 1). We plotted a rough conception of how each characterization varied in terms of confidence and awareness in Figure 1. We revisited the data using these characterizations with each interview coded individually by two researchers. In cases of disagreement or uncertainty, we met as an author team to compare our evidence from the interviews, sometimes playing back portions of the interview until we agreed on the characterization for that participant. In this paper, we compared results from Year 0 and Year 2 interviews.

Table 1
Leadership Conceptions Framework

Leadership conception	Confidence	Awareness
Established	Identifies as a leader through experiences in which they have acted as a leader (formally or informally) in their school or district. Comfortably reflects on leadership experiences and challenges. Appreciates the complexity of the work. Actively seeks to address concerns or challenges in their context.	Specifically identifies leadership constraints that are relevant to building/district and considers ways to best navigate them. Shares general ideas (personal theories) about how teachers learn/develop.
Cautiously confident	Identifies as someone who is comfortable being seen as a leader in some aspects of their contexts but also expresses some reservations. Is somewhat daunted by the complexity of the work.	Specifically identifies constraints relevant to building or district and considers ways to best navigate them. Identifies specific goals for building or district.
Optimistic	Expresses excitement for developing leadership skills and potential status. Takes a learning orientation; excited to learn more. Conveys a sense of anticipation about what they will be able to do.	Identifies possible constraints that can exist in any space (not specific to context). Expresses some specific ideas regarding strategies for navigating constraints. Names broad goals that can be applied to any space.
Emerging	Expresses concern about the extent to which they are seen as knowledgeable. May say a goal is to get better at teaching first before engaging in leadership.	May be aware of possible constraints, but those are more likely to be based on assumptions than experiences. Not specific about how these constraints would be navigated. Expresses lack of prior consideration of personal leadership skills/goals (e.g., "I guess I'll just try it and see how it goes").
Apprehensive	Expresses doubt in leadership ability and/or is unsure about wanting to have a leader label. Expresses concern about their current level of expertise. Intimidated by more visible leadership opportunities that involve more responsibility.	Vaguely aware of constraints and worries about navigating them. Expresses a desire for relatively limited leadership opportunities with lower stakes (one on one, working with novices).
Reluctant	Does not see self as a leader and does not desire to become a person of leader status.	Aware of constraints but has few or no ideas for addressing them.

Figure 1
How Leadership Conceptions Fit Onto Dimensions of Confidence and Awareness



To understand how EMSs described their development and how the program supported this development (RQ2), we analyzed six focus groups and attended to development across five emergent dimensions: knowledge, practice, vision, leadership, and confidence (see Table 2).

Table 2
Categories for Self-Described Development

Category	Description	Examples
Knowledge	Descriptions of mathematical knowledge for teaching developed during the program	Progressions of children's thinking Common mathematical misconceptions Relationships between mathematical ideas, justifications for mathematical rules, meanings of mathematical symbols or procedures
Practice	Descriptions of how specific elements of instructional practice changed as a result of the EMS program	Using higher quality tasks Using routines explicitly emphasized in the program (e.g., number strings, which one doesn't belong) Using purposeful questions (e.g., "Is this always true?") More substantive math talk Reducing use of ability grouping, alternate approaches to differentiation Using different representations (e.g., number lines)
Vision	Descriptions of general shifts in perspectives about teaching mathematics as a result of the program	The importance of analyzing and understanding student work; focus on students' assets The importance of attending to students' experiences in math and their math identities Attention to the social dimensions of mathematics lessons Mathematics as a tool for change
Confidence	Descriptions of how the program impacted their confidence in teaching mathematics or in leading others in mathematics	Increased autonomy for adapting curricular materials Increased enjoyment of teaching mathematics, increased passion for mathematics Developing own mathematical identity More comfortable teaching mathematics More comfortable being positioned as a leader More willing to have others observe their instruction
Leadership	Descriptions of specific leadership activities attributed to the EMS program	Increased opportunities for formal professional development with teachers Informal conversations with colleagues, including novice and beginning teachers Providing resources for colleagues Leading by example Having practice identified as exemplary; being positioned as having expertise Resisting policies that conflict with vision for instruction Sharing mathematics teaching examples on social media

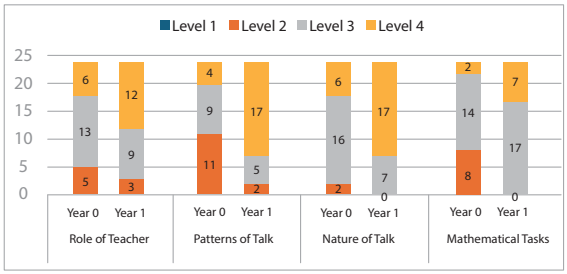
Segments where participants described development in these categories were identified by timestamps. We examined the content in each tagged segment and listed elements of the program referenced as supporting participants' development, taking note of specific assignments or activities. We then compared elements identified across the six focus groups to characterize ways participants perceived the program as supporting their development and together created a table to consolidate our findings.

FINDINGS

Visions of High-Quality Mathematics Instruction (RQ1a)

Results for our analysis of participants' visions of instruction are in Figure 2. Compared to the beginning of the program, more participants were rated at Levels 3 or 4 in all categories after the 1st year. In role of the teacher, patterns of talk, and nature of talk, at least half the participants were rated at the highest level, Level, 4, after their 1st year in the program, and only a few participants were at Level 2.

Figure 2
Development of Visions for High-Quality Mathematics Instruction



In the role of the teacher dimension, of the 18 participants who were not already rated at Level 4, 10 had higher ratings at the end of the 1st year than at the start of the program. For example, five participants initially were rated as Level 2, indicating a teacher as monitor conception, in which a high-quality mathematics lesson involves providing opportunities for children to work together on mathematical tasks, but the teacher is described as starting lessons by demonstrating or leading discussions on how problems should be solved and is treated as the primary source of knowledge. In one case, a participant named Candace talked about starting her lessons with "a little bit of that direct instruction" and then "a lot of group work." Candace emphasized the role of talk because, as she noted, "it's crazy how much they can learn from each other." However, Candace also talked about "pulling a group of students" to say, "hey, I just want to make sure you understand this." This description conveyed the idea that a teacher presents material, monitors students as they work, and intercedes as students experience struggle. Candace was mostly concerned students were "getting it" as they worked on problems.

After a year in the program, Candace's description of the teacher's role had shifted to a teacher as facilitator (i.e., Level 3) conception. She described a high-quality lesson involving the teacher:

> *[It's] not even leading the student into the answer maybe they're looking for, but just putting out the "tell me more*

[1] We used "knowledge" here to include knowledge of mathematical concepts and the validity of specific strategies and knowledge of how students' thinking develops, how mathematical ideas can be represented, etc. This approach is essentially the mathematical knowledge for teaching framework described by Hill et al. (2008), but, in this case, we analyzed participants' descriptions of their development of knowledge rather than as measured by the assessments developed by Hill and colleagues (e.g., Schilling & Hill, 2007).

about this" or "why did you do it this way?" and having the student explain their math, not what the answer is but the process of getting to that answer.

Candace also discussed the importance of the teacher "just sitting and listening to the kids . . . and if [they're] confused, just being like, 'tell me more, keep going,' instead of trying to push them to the answer." In this description, Candace was less concerned about monitoring the correctness of the children's work and instead encouraged students to work through their confusion. Candace conveyed more trust in students' abilities to resolve their own misconceptions through conversation.

Growth also was seen in the patterns of talk category, which addressed the structure of mathematical conversations (e.g., whether they occur as a whole class or small group), and the nature of talk category, which addressed more directly the content of the talk (e.g., whether it had a calculational or conceptual orientation). For example, in her initial interview, a participant named Erin was rated at Level 2 for patterns of talk and Level 3 for nature of talk. Erin talked about valuing student-to-student discourse but only in the context of small group work, not as a vital component of whole-class discussion. Erin described her use of small groups for math stations and explained, "Those are my times for more exploration time or practice time . . . I'm working with specific skills that I know kids are missing." Erin talked about how other students could play games or work on other tasks while she was "pulling a small group and teaching those missing skills." These descriptions suggested the focus of these small-group conversations was to enable students to solve problems correctly rather than to wrestle with ideas. Whole-group discussions of ideas were not an emphasis in Erin's description of high-quality instruction.

At the end of her 1st year in the EMS program, Erin was rated at Level 4 for both patterns of talk and nature of talk. She talked about "posing a problem or task" and "letting them [the students] explore," and then Erin would monitor children's thinking to plan for the whole-class discussion (e.g., "I want you guys to share your strategy when we go back to the whole group"). Erin discussed the importance of asking questions and noted she encouraged discussion by saying things like:

"How did you see that, how did you get that? Can you tell me more? Can you explain more?" And getting the kids to do most of the talking. Or, "I heard you say that you disagree. Can you tell me why you disagree?" And really encouraging students to do most of that.

Erin continued to talk about students working in small groups or with partners in this interview but also described these activities as designed to serve the whole-class conversation rather than being spaces for her to fill in missing gaps. She noted:

[I try] giving them different opportunities to partner talk. I think that's super powerful for them to feel confident and building that, "if I can tell a partner first, then I can share with the whole group." So partner talking and then engagement and just the idea of, like, using hand signals like "I agree" or "I have one strategy, two strategies, three strategies."

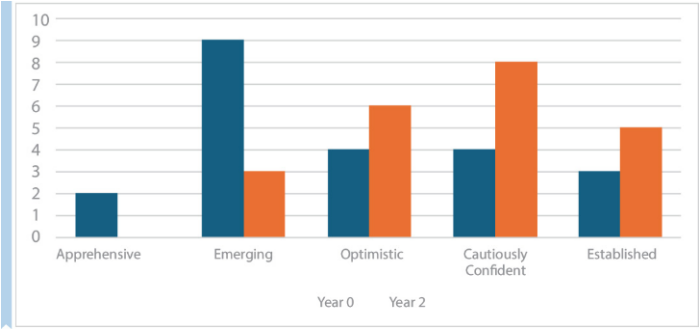
In general, Erin's description of the patterns of talk in a high-quality lesson changed markedly over the 1st year of the EMS program, moving from a structure emphasizing small-group work as the setting for resolving mathematical misconceptions to one in which whole-class conversation was designed to support mathematical argumentation about ideas students generated in small groups. These kinds of changes were evident for most participants across the sample (see Figure 2).

In their descriptions of mathematical tasks, there were fewer participants at Level 4 compared to the other categories, but nine of them received higher ratings in Year 1 than Year 0 (two already were at Level 4). Laura was a participant who moved from Level 3 to Level 4. In her initial interview, she described features of tasks in a general way; they would have multiple solution paths and would require students to do "something more than just memorized fact . . . but actually have to investigate something." She also talked about the importance of "having manipulatives out." After a year in the program, Laura was more specific about the function of multiple solution paths—that these would create opportunities for "analyzing each other's thinking" and to work through disagreements. Laura said, "I actually get really happy when a couple of my kids are disagreeing with each other because that's the one I'll pull back to [discuss with] the class." In this example, the task functioned to provide content for a whole-class discussion where students engaged in mathematical argumentation.

In another contrast with her Year 0 interview, Laura discussed tasks not in the abstract but instead with specific examples. She said, "There were five animals in a race, but 16 legs crossed the finish line; what animals could have been in the race?" Laura noted this task, a good example of doing mathematics (Stein et al., 1996), was valued because it "makes [students] think a lot more" about the mathematics. Overall, Laura's development represented a shift from broad descriptions of aspects of high-quality tasks to more specific descriptions and examples that included a rationale rooted in the potential of the task to generate discussions where strategies were compared and defended using mathematical reasoning. This case represents the kind of development we saw across multiple participants.

Fellows' Conceptions of Themselves as Leaders (RQ1b)
Over the 2 years of the program, our analysis revealed a decrease in the number of participants characterized as apprehensive and emerging and an increase in participants categorized as optimistic, cautiously confident, and established (see Figure 3).

Figure 3
Changes in Characterizations of Fellows' Conceptions of Themselves as Leaders



In one example of development, a participant named Maggie, initially characterized as apprehensive, began the program articulating her lack of confidence in her mathematics teaching and expressing hope her confidence would grow. She said, "Through these classes I'll develop even more confidence, like as a math teacher." Maggie expressed vague ideas about teacher development, stating, "Everybody is on a different journey and needs to grow in different areas." By the end of the program, Maggie expressed increased confidence, noting she felt "more comfortable in taking a leadership role" and was "excited to share what [she had] learned" with colleagues. Maggie talked about how one of the other participants was leaving her school, which was going to, in her words, "force me into more of leadership role . . . which will be good for me." However, at other times, Maggie continued to express doubts about her expertise, saying, "I try to be really honest about the fact that I'm not super knowledgeable." We characterized Maggie as emerging, showing some development from the initial interview but still demonstrating a substantial amount of hesitation about taking on leadership activities.

Janet, a participant who moved from emerging to cautiously confident, talked about her growth in how she saw herself in relation to leadership. Janet said:

> *When I started, I didn't really think of myself as a leader. I'm kind of quiet. I think about what I want to say. I always thought that was not a good characteristic of a leader. But really, it's good because maybe I'm more approachable. I'm not saying, "You have to do it this way." Let's talk about it.*

Janet originally described her quiet personality as a limitation, but she grew to recognize it as an asset, suggesting an expanded view of leadership that can include a quiet personality. Janet continued to express some apprehension in her Year 2 interview. She noted, "I've always been really careful . . . I don't like conflict all that much," and commented, "doing professional development has always been a little nerve-wracking for me." However, Janet also said, stepping out "would be good for me, too." At one point, she expressed concern about a new data teams policy she feared would increase emphasis on ranking and sorting students based on their mathematics achievement. Janet worried, "They want to group by ability for everything," and she expressed feeling an obligation to resist that trend. She said, "And I'm like 'No!' . . . I'm passionate about that, so it's like, I'm gonna have to really, you know, let everybody know, help share this information and how this [ability grouping] is hurting our children."

Janet ended the interview discussing her excitement for working with her colleagues from the EMS program in their building to create schoolwide interest in improving mathematics instruction. She said:

> *One of the things the three of us [fellows] have said is that we're going to try to do like a monthly PD session. . . . I'm hoping we're going to do a different routine each time. We're going to kind of see like what people want to talk about, like, if there's something they want to focus on. The three of us could maybe do something different to meet more teachers' needs . . . and I could do some number talk stuff with a small group of teachers or something, so I'm excited about that possibility.*

Overall, at the end of the program, Janet conveyed much more interest in stepping into leadership spaces, including sharing new strategies for more equitable approaches to grouping children for mathematics.

In another example, a participant named Gina began the program with vague ideas about how she might engage in leadership. She said, "I think just letting them know, 'Hey, I'm here. I'm doing this program. I'm more than willing to help you if you have questions.' Just being open and available for them." We characterized Gina's conceptions of leadership as emerging because she indicated a willingness to help colleagues but little evidence of awareness regarding the challenges and tensions involved in such work. By the end of the program, she was notably more specific about challenges and more intentional in her comments about leadership. In this excerpt, Gina described her desire to help children taught by an experienced teacher. She said:

> *She's been teaching for a long time, and it's hard sometimes when a younger teacher comes in and tries to offer suggestions or things that we could do differently. And I don't want to come off like, "I'm perfect, I know everything," like, "I have a degree and you don't," but I also know I have to help those kids. Because I, I grew up with not good math experience, and I honestly, I did not like math growing up because of the way it was presented to me.*

In this Year 2 excerpt, Gina, then characterized as cautiously confident, acknowledged tensions that can come with leading and was more intentional about leadership as she articulated an obligation to help children whom she feared were having negative experiences with mathematics ("I have to help those kids"). Like many participants in our sample, there was a significant shift across Gina's interviews between concern for "my kids" (i.e., the children in her classroom) and "our kids" (i.e., the children in her school), indicating an increased sense of responsibility for leadership beyond her classrooms.

Ella was a participant who initially described herself as having expertise in teaching language arts but who said she was excited "to have the opportunity to learn more about math and feel more confident about math." Characterized as cautiously confident, Ella was specific about some of the challenges of leading in school, noting the school had just completed a large reform effort with literacy instruction. She expressed caution about trying to change too much too fast. Ella said:

> *The first step might just be giving more information about providing student feedback. That's something that teachers, they are already doing that, just looking at how we are providing student feedback, doing some professional development on that and through our conversations in our grade levels . . . I thought that would be something really small. It's small to do as a teacher, but its huge in the direction of where students can grow as mathematicians.*

This excerpt shows Ella's awareness of the expectations in her building and sensitivity to what kinds of learning might be doable for teachers and impactful for students. In the Year 2 interview, Ella maintained a nuanced view of leadership. She stated, "I'm doing more listening and questioning, versus, 'this is what you need to do'" and described a number of leadership activities (e.g., "I have been on the ELA [English Language Arts] Committee and the SRG [Standards-Referenced Grading] Committee. I've been put in those positions."). However, Ella also noted an increase in her confidence, noting:

> *At the very beginning of the program, I would have never have ever said that I would be confident to share anything about math. Math was not my jam. This program has provided me with a lot of the tools that I would need . . . to really provide quality leadership.*

We saw the increase in confidence as development in our trajectory and characterized Ella's Year 2 interview as reflecting an established leader.

Overall, we saw substantial development over the course of the EMS program in how participants talked about leadership in mathematics and themselves as mathematical leaders in their contexts. In terms of confidence, several moved from expressing hesitation and apprehension to expressing excitement or even passion about promoting change outside their classrooms. They seemed to expand and refine some of their ideas about leadership, moving from vague or overly simplified notions of leadership to articulating specific tensions, challenges, or strategies for leading in their contexts.

Fellows' Impressions of How the EMS Program Impacted Their Development (RQ2)

In the following sections, we share some aspects of the EMS program that, according to participants, supported this and other types of development.

Knowledge

Several participants described developing new knowledge due to the program overall or because of particular assignments. Many participants talked generally about learning how students' mathematical conceptions develop over time in a particular domain (e.g., numbers and operations). One participant, who taught a lower grade, described how the program helped her "understand what kids are going to need when they leave [her classroom], and [knowing] what they're going to be doing in fourth grade and fifth grade, and middle school has been really powerful for me." Participants also described development of their own mathematical insights, such as a justification for the doubling/halving multiplication strategy (e.g., $12 \times 15 = 6 \times 30$) and an understanding of why and how number strings support students (Bray & Maldonado, 2018).

One assignment mentioned several times was related to the article "13 Rules That Expire" (Karp et al., 2014), which helped one participant realize "how many things I was saying to my kids [that] had expiration dates or how many things I was overgeneralizing." Another frequently mentioned activity was an assignment exploring how children often misinterpret the equals sign as a signal to compute. One participant reflected that, when she first started teaching, she wrote multiple equals signs, so the equation would not be accurate (e.g., $12 + 6 = 18 + 4 = 22$), and, by Year 2, she made sure to communicate to students "that that continuation is not going to work because you're [mis]representing what that equal sign means." Both "13 Rules That Expire" and the equals sign activity were mentioned by participants who taught across the lower and upper elementary grades.

Instructional Practice

Regarding impact on instructional practice, participants identified ways the program impacted their planning for and set-up of instruction and their facilitation of instruction. Several participants described being more intentional in their lesson planning, particularly in terms of selecting and adapting curricular resources. One participant talked about how she and other participants at her school worked hard to "analyze the lessons they were supposed to be teaching [from the assigned curriculum], find a problem . . . to tweak and modify in a certain way, and [then] spend most of their [instructional] time on that." Another participant gave an example of a "pretty good problem [from the curriculum] when you look at it on the surface" that she rewrote "as a compare and contrast routine" using the "Van de Walle text" (Van de Walle et al., 2014). Another participant talked about pushing back against homogeneous ability grouping—a common practice across her school—to instead do mixed-ability grouping (Webel et al., 2021). Several participants discussed creating space for children, rather than the teacher, to talk about mathematics.

Participants frequently mentioned instructional routines as a way the program impacted their facilitation of instruction. Each of the four routines introduced in the program (i.e., number talks, number strings, which one doesn't belong, contrasting cases) were named across the focus groups. One participant talked about her growth in understanding and using these routines. She said, "I will be honest that when I started this program, I had never ever done a number talk in my life. . . . [Initially] I was so confused and couldn't figure out what the difference was" between a number talk and

number string. However, by Year 2, she did them daily and noted she had "developed a rotation of a different routine every day." Several participants also discussed the program's impact on their questioning, including encouraging students to make and explore conjectures. Multiple participants described using the question "Is this always true?" to press students to generalize, which was a question emphasized in their algebraic reasoning course. One participant described an anchor chart in the focus group labeled "math rules we think are true" with examples of class conjectures such as, "When you add a double it will always equal an even number." All these examples show ways participants saw the program as directly impacting their instruction.

Vision

Complementing the findings from our individual VHQMI interviews, focus group participants described broad shifts in their visions for instruction, moving from thinking about teaching mathematics as helping children use specific strategies to helping them engage in sensemaking. One participant described this as a holistic transformation:

> *When I first started teaching, I did so much "I do, we do, you do," and I just drilled, and we practiced . . . how to apply an algorithm and how to do a procedure. . . . From the classes and the activities we did and the conversations with [other fellows], . . . my teaching has completely changed. I no longer try to get through every single problem in the book. We often start off with a task that students will complete on their own, and then we'll talk about it and share strategies.*

Other participants described their shifts in terms like mathematics no longer being quiet, allowing students to use strategies that make sense to them, and taking an asset-focused approach that emphasized what students can do rather than what they cannot do. Participants across focus groups described a desire to make mathematics enjoyable for students and cultivate positive mathematics identities.

In describing how the program impacted their visions for mathematics instruction, many participants referenced issues of equity. They frequently named The Impact of Identity in K–8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices (Aguirre et al., 2013), a book used in one of their leadership classes, as important to their considerations of how to engage typically underserved groups of students (e.g., underrepresented ethnic or racial groups, students learning English) and toward disrupting homogeneous ability grouping. Multiple participants also referenced an assignment that used the equity quantified in participation (EQUIP) tool (Reinholz & Shah, 2018) as integral to examining their own teaching practice, particularly as it related to expectations for and questions to different students. One participant, in connecting mathematics to literacy, mentioned the need for "windows and mirrors," where students "can all see themselves in the content" (i.e., a mirror for the student to see themself and a window to see stories of students who are different). These changes to instructional vision did not come up in the VHQMI interviews, but they were noticeably present in the focus groups at the end of the program.

Leadership

Leadership also surfaced in each focus group, though not as often as other themes, and participants were less likely to make connections to the EMS program relative to the other dimensions. Fellows described multiple ways they sought to share resources with others in their specific contexts, such as discussing cognitively demanding tasks with a small group of colleagues and posting images from their mathematics lessons on social media. One participant named the math talk posters she made "tons of teachers in [her] school" were beginning to use.

In an example of how the program supported their leadership, two participants who worked at the same school described their action plan assignment as an important tool to address "unsupportive leadership." They believed their administrators were overly focused on correct answers and standardized assessments, making it appear, as one participant noted, "like our students can't do math." In response, these participants' action plans focused on what one participant described as "really looking at what kids can do and not worrying about the cant's." When they shared their ideas for a more asset-based approach to assessment at a faculty meeting, both participants reported receiving significant positive feedback from their colleagues, which felt to them like a success.

Multiple participants also mentioned more direct ways of supporting colleagues. Several noted they had invited other teachers to observe their mathematics instruction, with one participant sharing she thought "welcoming teachers into your classroom" was the "way to get them on board" with new ways of instruction. Another participant, who worked as a mathematics interventionist, shared colleagues had approached her asking for support in analyzing student test results and planning subsequent instruction and coteaching lessons.

In addition to influencing their colleagues or other practicing teachers, multiple participants named working with and supporting teacher candidates as a next step in their leadership development. One participant, reflecting on the challenge of "finding my niche as a leader," said, "One-on-one with a student teacher is where I think I can make the most impact." Another participant described working with teacher candidates as "where [they] think change will happen over time." As with the previous participant regarding observations during math, this latter participant was considering a theory of change to teachers' instructional practice and how she could support growth.

Confidence

Increased confidence was mentioned in all focus groups, sometimes regarding teaching and sometimes regarding leadership, and often it was connected to development in a previously described area like knowledge. One participant stated:

> *[Prior to the program, I was] never a math person . . . math made me a little bit nervous. . . . I knew what I wanted the math classroom to look like but I didn't feel as confident about making it look like that.*

She recalled a time when school and district administrators visited her classroom during math after she had finished the program, and she "[was not] even bothered," which indicated to her the growth in her confidence in teaching math. On a smaller scale, a different participant stated the knowledge she gained from the program had supported her autonomy and decision making. She said, "[Given that we have to work] with a boxed curriculum, I feel more confident in myself to pull out what's important."

Regarding leadership, one participant mentioned, "[The program] gave me some practice in that leadership role, which I never would have probably taken on my own," and shared instead, "[I] would have kept all this knowledge to myself." Outside of confidence for mathematics instruction and leadership, one experienced participant described the program as a reinvigoration: It helped her rediscover her "zest" for teaching. She described, "In a sense [the program] . . . saved me. It kept me in teaching." These comments complemented our findings about the development of confidence in participants' capacity to enact leadership in mathematics and were all the more notable because they were raised in focus groups that did not ask about leadership. In general, these findings suggested participants had begun to see themselves, their roles, and their capacities differently, and they credited the EMS program with supporting these developments.

A Supportive Community

Across the dimensions described previously, one recurring theme went beyond a description of activities or content, instead focusing on the community established among the cohort members. One participant shared, "I love our cohort and the ability we have to work together and talk and do these things." Another talked about the importance of the synchronous meetings that occurred five times across each semester. She said, "Those times that we got to meet, I would learn so much from the other teachers in this program and what they were doing, what was working, and just engaging in all the different grade levels and experiences." One participant expressed appreciation for a community focused explicitly on mathematics; she said, "This has been a huge benefit to have the community and other people to bounce ideas off of. There's not a lot of elementary teachers, in my experience, that love teaching math." Finally, some participants noted how having another teacher in their building going through the program was something they valued about their experience, saying things like, "It's so nice to have someone else in the building where we've grown together in our teaching abilities and in our friendship." These comments echoed sentiments we noted across our data set, such as Janet's plans for working with their EMS colleagues at their school and the participants who reported working together to share an asset-based orientation of students with their colleagues as a response to school policies that functioned to sort and rank students (Webel et al., 2021).

Discussion

In relation to RQ1a, we found participants developed substantially over the course of the EMS program in their visions for high-quality mathematics instruction. We saw development across all areas of the VHQMI, with participants' descriptions of the teacher's role more likely to fall into the "teacher as facilitator" or "teacher as more knowledgeable other" categories, and their descriptions of student talk more likely to reflect a "mathematical discourse community" (Lampert, 1990) and tasks that constitute "doing mathematics" (Stein et al., 1996). Although these developments would be expected from an EMS program, they complemented other research showing impact on participants' beliefs (e.g., Swars et al., 2018) by providing insights into how participants imagined their future mathematics instruction. These visions are important because they are linked to instructional improvements (Munter & Correnti, 2017) and suggest fellows are relatively well prepared to enact the kind of mathematics instruction supported by research and professional organizations (NCTM, 2000, 2014, 2020).

In relation to RQ1b, we found participants also developed substantially over the course of the EMS program in their conceptions of themselves as leaders. We documented developments in terms of how teachers talked about leadership in mathematics and how they described their own comfort and ideas for enacting leadership in their schools. Our findings complemented other literature on teacher leadership, showing teacher–leaders gradually gain confidence as they develop (Hunzicker, 2017; Wenner & Campbell, 2017; Yopp et al., 2019) and begin to scale their leadership activities and perspectives to encompass larger systems (Brooks et al., 2004; Hunzicker, 2017; Knapp, 2017). In our analysis, we saw growth in confidence as participants moved from apprehensive to optimistic and excited about sharing their knowledge to improve students' mathematical experiences beyond their own classroom. Participants also increased their awareness of leadership practices and challenges, shifting from describing potential leadership activities in vague or hypothetical ways to giving concrete examples of leadership connected to a clearer theory of teacher learning and understanding of the systems that can impact opportunities for instructional change. Established participants, for instance, were not naïve about the resistance they might encounter in their leadership efforts, but they also were committed to making changes given those constraints (Nguyen et al., 2022).

As with vision for mathematics instruction, this development was not unexpected; fellows learned about and practiced leadership in several of their EMS courses, including working together to create and implement action plans in their contexts. These action plans created opportunities for fellows to deploy their increased confidence and awareness to challenge some of the barriers to instructional improvement they encountered in their school (Webel et al., 2021). The findings also showed how recruiting teachers to EMS programs in school-based teams might lead to increased impact; although some graduates of EMS programs struggled with feeling isolated and unseen in their schools (Webel et al., 2017), many of our participants had opportunities to coordinate their activities and present a united front when making requests of their administrators.

Although the development of EMSs' conceptions of leadership and themselves as leaders was not unexpected, the

conceptions of leadership framework extended previous literature by suggesting stages of growth that combine confidence and awareness. We saw that, as teachers grew in confidence, they also gained awareness about different ways to lead (e.g., Janet found it was possible to be a leader even with a "quiet personality") and different challenges that might arise when working to support their colleagues (e.g., the team that found opportunities to promote high-quality mathematics instruction despite "unsupportive leadership"). In general, we view the framework not only as useful for documenting leader development but also as a tool that could be used in other EMS or leadership programs.

Related to RQ2, focus groups conducted at the end of the EMS program allowed us to document other areas of development not captured by the VHQMI or our other interviews and to connect EMSs' development to specific elements of the program. Similar to Myers et al.'s (2020, 2021) findings, our participants appreciated the connection to practice, opportunities to apply learning progressions to their own students, opportunities for open discussions, and collaboration directly relevant to their teaching. Additionally, participants in this study discussed the importance of opportunities to plan, enact, and debrief instruction routines, explore mathematical content as learners, and consider issues of equity (for more on how fellows understood and worked toward equity, see Webel et al., 2021). The focus groups also elicited explicit statements of how the program helped fellows develop more confidence as teachers and leaders, which often was linked to their development of knowledge and skills. This growth was positive, and it also supported their ability to see themselves as leaders in mathematics. Overall, we saw the multiple areas of development (i.e., knowledge, skills, vision, leadership, confidence) as connected and interwoven throughout their experiences, with development in one area reinforcing development in other areas.

Future Work

One aspect of the program we want to understand more is the ways individual participant development interacted with the elements of their specific contexts. We know from previously conducted social network analyses that our fellows were sought after by their colleagues for advice and information, and we also know these patterns of advice seeking were different in different buildings, even in the same district, suggesting school structures and policies influence the impact leaders can have (Nguyen et al., 2024). Participants also emphasized the importance of the cohort community, sometimes in connection with their openness to start thinking of themselves as leaders. This emphasis was particularly the case for those fellows who were recruited as part of a school-based team to go through the program together. They shared stories of approaching their administrators together, conducting professional development as a team for their school, and advocating for resources and policies they believed would better support student learning. Moving forward, we hope to learn more about how going through an EMS program as a pair or team might better support mathematics leadership practices in schools.

Conclusion

In this paper, we shared how an EMS program supported teacher development in terms of vision and conceptions of leadership and used focus groups to identify elements of the program participants identified as important for their development. We found teachers incorporated new images of teaching into their visions emphasizing mathematical discourse, high-demand tasks, and student authority for mathematical reasoning. We saw participants expressing more confidence about serving as leaders in their contexts and more realistic expectations for the kinds of challenges they might encounter. We saw developing a supportive professional community was a key component of the learning experience for participants. We hope these findings can be useful for other EMS program personnel to inform their design and build a more robust knowledge base for what learning experiences are supportive for the varied work of EMSs.

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APPENDIX

Interview Questions (Munter, 2014)

I'd like to ask you a few questions about your view of high-quality mathematics instruction.

1. If you were asked to observe another teacher's math classroom for one or more lessons, what would you look for to decide whether the mathematics instruction is high quality?

Notes to interviewer:

- *Probe on **depth/specificity** of response until you understand what the participant describes (e.g., If a teacher says, "student engagement," ask, "Engaged in what?").*
- *Keep the **form/function** distinction in mind. Ask participants why they think ____ is important (e.g., Why do you think it's important for kids to work in groups? Why do you think it's important to hold a whole class discussion?).*
- *If the interviewee talks about the **structure of discourse** (who's talking to whom and when) probe on **content** (and vice versa). If the interviewee says, "Teachers (or students) should be asking questions," probe to find out the kinds of questions the teacher (or students) should ask and for what purpose, as well as whether they conceive of discussion as happening in whole class settings and/or in small groups alone.*

   a. Is there anything else you would look for? *(Ask BEFORE probing on the following issues.)*
   b. What are some of the things you would expect to find the teacher actually doing in the classroom for instruction to be of high quality?
   c. What kinds of problems or mathematical tasks would you expect to see the students working on for instruction to be of high quality?
      i. Can you please describe a _____[*use the word or phrase—e.g., "task" or "problem"— that the participant used for "task"*] that you would consider to be of high quality?
      ii. Can you please describe what classroom discussion would look and sound like if instruction were of high quality?
      iii. Would you expect to see the entire class participating in a single discussion, or would students be talking primarily in small groups?